SLAC-PUB-7949
CERN-TH/98-297
SU-ITP-98/54

Stabilization of Sub-Millimeter Dimensions: The New Guise of the Hierarchy Problem

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Abstract

A new framework for solving the hierarchy problem was recently proposed which does not rely on low energy supersymmetry or technicolor. The fundamental Planck mass is at a TeV and the observed weakness of gravity at long distances is due the existence of new sub-millimeter spatial dimensions. In this picture the standard model fields are localized to a $(3+1)$-dimensional wall or "3-brane". The hierarchy problem becomes isomorphic to the problem of the largeness of the extra dimensions. This is in turn inextricably linked to the cosmological constant problem, suggesting the possibility of a common solution. The radii of the extra dimensions must be prevented from both expanding to too great a size, and collapsing to the fundamental Planck length TeV^{-1}. In this paper we propose a number of mechanisms addressing this question. We argue that a positive bulk cosmological constant $\bar{\Lambda}$ can stabilize the internal manifold against expansion, and that the value of $\bar{\Lambda}$ is not unstable to radiative corrections provided that the supersymmetries of string theory are broken by dynamics on our 3-brane. We further argue that the extra dimensions can be stabilized against collapse in a phenomenologically successful way by either of two methods: 1) Large, topologically conserved quantum numbers associated with higher-form bulk U(1) gauge fields, such as the naturally occurring Ramond-Ramond gauge fields, or the winding number of bulk scalar fields. 2) The brane-lattice-crystallization of a large number of 3-branes in the bulk. These mechanisms are consistent with theoretical, laboratory, and cosmological considerations such as the absence of large time variations in Newton's constant during and after primordial nucleosynthesis, and millimeter-scale tests of gravity.

1 New Guise of the Hierarchy Problem

A new proposal for solving the hierarchy problem was recently introduced [1, 2, 3] which circumvents the need for supersymmetry or technicolor. Instead the hierarchy problem for the standard model (SM) is solved by bringing the fundamental Planck scale down to the TeV scale. Gravity becomes comparable in strength to the other interactions at this scale, and the observed weakness of gravity at long distances is then explained by the presence of n new "large" spatial dimensions.

Gauss' Law relates the Planck scales of the $(4+n)$ dimensional theory, M_*, and the long-distance 4-dimensional theory, $M_{\rm pl}$,

$$M_{\rm pl}^2 \sim r_n^n M_*^{n+2} \tag{1}$$

where r_n is the size of the extra dimensions. Putting $M_* \sim 1\,{\rm TeV}$ then yields

$$r_n \sim 10^{-17+\frac{30}{n}}{\rm cm} \tag{2}$$

For $n = 1$, $r_1 \sim 10^{13}$ cm, so this case is excluded since it would modify Newtonian gravitation at solar-system distances. Already for $n = 2$, however, $r_2 \sim 1$ mm, which happens to be the distance where our present experimental knowledge of gravitational strength forces ends. For larger n, $1/r_n$ slowly approaches the fundamental Planck scale M_*.

While the gravitational force has not been measured beneath a millimeter, the success of the SM up to $\sim 100\,{\rm GeV}$ implies that the SM fields can not feel these extra large dimensions; that is, they must be stuck on a 3-dimensional wall, or "3-brane", in the higher dimensional space. Thus, in this framework the universe is $(4+n)$-dimensional with fundamental Planck scale near the weak scale, with $n \geq 2$ new sub-mm sized dimensions where gravity, and perhaps other fields, can freely propagate, but where the SM particles are localised on a 3-brane in the higher-dimensional space. The most attractive possibility for localizing the SM fields to the brane is to employ the D-branes that naturally occur in type I or type II string theory [4, 2]. Gauge and other degrees of freedom are naturally confined to such D-branes [4], and furthermore this approach has the obvious advantage of being formulated within a consistent theory of gravity. However, from a practical point of view, the most important question is whether this framework is experimentally excluded. This was the subject of [3] where laboratory, astrophysical, and cosmological constraints were studied and found not to exclude these ideas.

There are also a number of other papers discussing related suggestions. Refs. [5] examine the idea of lowering the GUT scale by utilizing higher dimensions. Further papers concern themselves with the construction of string models with extra dimensions larger than the string scale [6, 7, 8], and gauge coupling unification in higher dimensions without lowering the unification scale [9]. There are also important papers by Sundrum on the effective theory of the low energy degrees of freedom in realizations of our world as a brane [10].

In our framework the hierarchy problem becomes the problem of explaining the size and stability of the large extra dimensions. The main purpose of this paper is to exhibit mechanisms which accomplish these objectives, and examine some aspects of their phenomenology. Since a rather wide collection of possible stabilization mechanisms are discussed in this paper, only some of which we believe to be successful, we think it necessary to provide the reader with a guide to our main results: In Section 1.1 we discuss a very general consistency constraint on the bulk cosmological constant; and in Section 2 we describe some basic kinematics pertaining to the radial oscillation field, whose mass will turn out to provide significant constraints on stabilization scenarios. In particular this is the constraint that will force us to have a large conserved integer parameter in our models. We also briefly describe the reasons for the cosmological safety of this scenario. Further details of the early universe cosmology will be presented in [16]. The most important results of this paper are contained in Section 3 where we discuss long-distance (IR) and, particularly, short-distance (UV) stabilization mechanisms, and put these together to obtain a variety of complete stabilization models. We find that two methods of UV stabilization are particularly attractive: "brane-lattice-crystallization" discussed in Section 3.1, with the analysis of a complete model presented in Section 3.1.I; and "topological stabilization" discussed in Section 3.2, with the analysis of another complete model presented in Section 3.2.IV. Finally in Section 4 we present a summary of our results.

1.1 The Hierarchy and the Bulk Cosmological Constant.

Let us begin with some necessary conditions that must be satisfied to ensure the existence of large radii. As we know from experience with our 4-dimensional world, to ensure that our three ordinary spatial dimensions are very large the radius of curvature of the universe must be no less than the present horizon size. This leads to the requirement that the cosmological constant of the universe is less than the critical density. An identical line of reasoning for the case of n-extra dimensions also leads to an upper limit on the bulk cosmological constant as we now explain.

The curvature radius $L_{\rm curv}$ of the bulk space in the presence of energy density or an effective cosmological constant, $\bar{\Lambda}$, in the bulk, is

$$L_{\rm curv} \sim \left(\frac{M_*^{n+2}}{\bar{\Lambda}}\right)^{1/2} . \tag{3}$$

This curvature radius must be larger than the physical size of the transverse dimensions r_n in order to insure that the bulk space does not "split off" into separate inflating universes separated by horizons of size $L_{\rm curv}$, or collapse into black holes. This gives an upper bound on $\bar{\Lambda}$ [11]:

$$\bar{\Lambda} \lesssim M_*^{(4+n)} \left(\frac{M_*}{M_{\rm pl}}\right)^{4/n} \tag{4}$$

This constraint will play an important role in what follows. It already implies that the magnitude $\bar{\Lambda}$ must be smaller than the fundamental scale of M_*. This was to be expected since in this case there is one scale in the problem and the bulk would split into a collection of non-communicating 1/ TeV size regions, outside of each others' particle horizons. An important corollary of this is that one cannot use the Scherk-Schwarz mechanism to break supersymmetry at M_* since this would induce a bulk cosmological constant of the order of M_*^4, which exceeds the limit Eq. (4).

Of course the effective 4-dimensional cosmological constant measured at long distances (greater than the size of the extra dimensions) must to a very high degree of accuracy vanish. This can be achieved by cancelling the wall and bulk contributions:

$$0 = f^4 + (r_n)^n \bar{\Lambda} \tag{5}$$

We see that if the bulk energy is negative, a postive f^4 will cancel the 4-d cosmological constant, while if the bulk energy is positive, we need a negative f^4. Clearly a positve f^4 is reasonable, if the wall can fluctuate in the extra dimensions, f^4 is just the tension of the wall, and provides the correct sign kinetic term for the goldstones of spontaneously broken $(4+n)$-d Poincare invariance which live on the wall. This reasoning seems to exclude the possibility of a negative f^4, since this gives the wrong sign kinetic term to the goldstones. This is correct if the goldstone fields are indeed present, that is, if the $(4+n)-$d Poincare invariance is spontaneously broken. On the other hand, suppose that the wall is "stuck" and cannot fluctuate in the extra dimensions, due to explicit breaking of $(4+n)$-d Poincare invariance. As an example, we can consider SM fields to be twisted sector fields living at an orbifold fixed point. In this case, f^4 is just the wall energy density acting as a source for gravity, but there are no goldstones on the wall to receive a wrong-sign kinetic term. Another way of saying this is as follows. The wall can have an energy density as a source for gravity f^4_{grav}, and a tension under "bending " f^4_{bend}. It is f^4_{grav} which should appear in 5. If the $4+n$-d Poincare invariance is only spontaneously broken, its non-linear realisation forces $f^4_{grav} = f^4_{bend}$ (they both come from expanding $-\int d^4\sqrt{-g_{ind}})f^4$ where g_{ind} is the induced metric on the wall). Since $f^4_{bend} > 0$, $f^4_{grav} > 0$. On the other hand, if the $(4+n)$-d Poincare invariance is explicitly broken, there need not be any relationship between the two. Indeed, if the wall can not fluctuate, effectively $f^4_{bend} = \infty$, while f^4_{grav} can be finite and of any sign. Therefore, we will allow the possiblity that the bulk energy can be either positve or negative [1]

Given Eq. (4) we learn that the effective wall-localized cosmological constant, f^4, is bounded above by

$$f \lesssim M_* \left(\frac{M_{\rm pl}}{M_*}\right)^{(n-2)/2n} \tag{6}$$

This is not too severe a constraint though, varying between 10 TeV for $n = 2$, to $\sim 10^8$ GeV for $n = 6$. Of course, the relation, Eq. (5), can be turned around to determine the effective bulk cosmological constant, $\bar{\Lambda}$, given f. A natural assumption for the wall-localized cosmological constant, given our state of knowledge of the standard model interactions on the wall, is $f^4 = (1\,\text{TeV})^4 \sim M_*^4$. Thus

$$\bar{\Lambda} = M_*^{4+n}\left(\frac{M_*}{M_{\rm pl}}\right)^2 \tag{7}$$

is the value of the bulk cosmological constant necessary to cancel the total long-distance cosmological constant in our world.

The cosmological constant is bounded from below from another consideration. As we will see later in Section 2, there are light gravitationally coupled particles in the spectrum whose (mass)^2 is proportional to $\bar{\Lambda}$ (see Eq. (26)). The requirement that these particles do not conflict with measurements of gravity imply that they weigh more than a meV and consequently put a lower limit on $\bar{\Lambda}$. This in turn implies that the large size of the new dimensions, in all cases studied here, cannot be solely due to the smallness of $\bar{\Lambda}$. We need additional dynamics to boost the size of the extra dimensions. This can easily come about if there is a conserved charge, analogous to baryon number. Just as humans are large because they carry large baryon number, the extra dimensions can be large because they carry some large topological or other charge Q.

[1]We thank Eva Silverstein for discussions about this point.

1.2 Stable and Calculable Hierarchy

In this paper we will not search for dynamical mechanisms where the hierarchy between the size of the extra dimensions and the fundamental scale is *calculable*. We will instead be content to enforce this hierarchy by choosing the bulk cosmological constant to be small and/or the above-mentioned topological or other charge to be large. This is analogous to the early days of the supersymmetric standard model (MSSM) [12] where the soft supersymmetry breaking terms were postulated without any reference to a dynamical mechanism which generates them. The idea there was that since the problem of supersymmetry breaking is connected with the cosmological constant problem it seemed premature to adopt a specific SUSY-breaking mechanism and it seemed more prudent to study consequences that were independent of the details of the SUSY-breaking mechanics. Similarly, in our new framework the hierarchy and cosmological constant problems are even more closely intertwined so we will adopt a similar philosophy of not insisting on a detailed dynamical mechanism for a *calculable* hierarchy and will be content to instead parametrize our ignorance by a choice of $\bar{\Lambda}$ and an integer Q.[2]

The second aspect of the hierarchy problem is its stability against radiative corrections. In the MSSM this is guaranteed by low energy supersymmetry, which protects the Higgs mass against large radiative corrections. Presumably, the analogous question in our framework is the behaviour of the pair of parameters $(\bar{\Lambda}, Q)$ under radiative corrections. The integer Q is automatically protected since it refers to charge of a configuration. Since $\bar{\Lambda}$ is a bulk cosmological constant one can imagine two possibilities. One is that whatever solves the cosmological constant problem will also prevent $\bar{\Lambda}$ from becoming as large as the cutoff M_*. The second more explicit and perhaps more satisfactory viewpoint is to invoke bulk-supersymmetry to protect $\bar{\Lambda}$ from large radiative corrections. Indeed, as pointed out in reference [2], if supersymmetry is broken solely on our 3-brane by an amount $\sim M_* \sim 1\,\text{TeV}$, the Fermi-Bose splittings that this induces in the bulk are miniscule $\sim \text{TeV}^2/M_{\text{pl}} \sim 10^{-3}\,\text{eV}$ and therefore the bulk cosmological constant $\bar{\Lambda}$ is protected by the approximate bulk-supersymmetry.

[2]For clarity we will use the notation k for the generalization of the integer monopole number which in the context of topological stabilization plays the same role as Q.

2 Kinematics of Radius Stabilization

Suppose that we have an N-brane embedded in a space with N large spatial dimensions and n small dimensions we wish to stabilize. The total action is comprised of a bulk part,

$$S_{\text{bulk}} = -\int d^{1+N+n}x\sqrt{-\det G_{(1+N+n)}}\left(M_*^{(n+N-1)}\mathcal{R} + \Lambda - \mathcal{L}_{\text{matter}} + \dots\right), \tag{8}$$

and a brane part,

$$S_{\text{brane}} = -\int d^{1+N}x\sqrt{-\det g_{(1+N)}^{\text{induced}}}\left(f^{N+1} + \dots\right), \tag{9}$$

where $\mathcal{L}_{\text{matter}}$ is the Lagrangian of bulk gauge or scalar fields, and the ellipses denote higher-derivative terms that can be ignored in the regime of interest as we will demonstrate below. Take the background metric for the $(1+N+n)$-dimensional spacetime to be of the form

$$g_{\mu\nu} = \begin{pmatrix} 1 & & \\ & -R(t)^2 g_{IJ} & \\ & & -r(t)^2 g_{ij} \end{pmatrix}, \tag{10}$$

where R is the scale factor of the N-dimensional space, and r is the scale factor of the internal n-dimensional space, with geometry set by g_{ij} where $\det(g_{ij}) = 1$.

With this metric the Ricci scalar is

$$-\mathcal{R} = 2N\frac{\ddot{R}}{R} + N(N-1)\left(\frac{\dot{R}}{R}\right)^2 + 2n\frac{\ddot{r}}{r} + n(n-1)\left(\frac{\dot{r}}{r}\right)^2 + 2Nn\left(\frac{\dot{r}\dot{R}}{rR}\right) + \frac{\kappa n(n-1)}{r^2}, \tag{11}$$

where the internal curvature term is present for n-spheres ($\kappa = 1$), but vanishes for tori ($\kappa = 0$), and we have ignored a similar curvature term for the large dimensions. After integrating over all spatial coordinates we obtain,

$$S = \int dt(\mathcal{L}_{\text{KE}}(\dot{R}, \dot{r}) - R^N V_{\text{tot}}(r)), \tag{12}$$

where the total potential is given by

$$V_{\text{tot}}(r) = \Lambda r^n - n(n-1)\kappa M_*^{n+N-1} r^{n-2} + V_{\text{matter}}(r) + f^{N+1}, \tag{13}$$

where,

$$V_{\text{matter}}(r) = -\int d^n x\,(r^n \mathcal{L}_{\text{matter}}). \tag{14}$$

After integrating the $\ddot{R}$ and $\ddot{r}$ terms by parts, the kinetic part of the action for the radii, R and r, becomes

$$S = -M_*^{N+n-1}\int dt\, R^N r^n \left(N(N-1)\left(\frac{\dot{R}}{R}\right)^2 + n(n-1)\left(\frac{\dot{r}}{r}\right)^2 + 2Nn\left(\frac{\dot{r}\dot{R}}{rR}\right)\right). \tag{15}$$

Note the overall negative sign of these kinetic terms. This is connected to the well-known phenomenon that the conformal mode of gravity has the opposite sign kinetic term to the transverse graviton kinetic term (and which bedevils attempts at defining gravity via the Euclidean functional integral).

In any case there is clearly an extremum of the action with $\dot{R} = \dot{r} = 0$, when the condition $\partial_R (R^N V_{\rm tot}(r))|_{R=R_0, r=r_0} = 0$, and similar with $\partial_R \to \partial_r$ are met. These imply (for $R_0 \neq 0$)

$$\begin{aligned} V_{\rm tot}(r_0) &= 0, \qquad \text{and} \\ V'_{\rm tot}(r_0) &= 0. \end{aligned} \tag{16}$$

This is as one would have naively expected. However, because of the negative sign for the kinetic term for the radial degrees of freedom, the stability analysis for such static solutions has to be treated with care. The analysis starts by expanding the action, Eq. (15), in small fluctuations around the extremum: $R(t) = R_0 + \delta R(t)$, and $r(t) = r_0 + \delta r(t)$. Then to quadratic order, and defining $\Delta \equiv \delta R / R_0$ and $\delta \equiv \delta r / r_0$, the expansion gives the coupled equations of motion

$$\begin{pmatrix} N(N-1) & Nn \\ Nn & n(n-1) \end{pmatrix} \begin{pmatrix} \ddot{\Delta} \\ \ddot{\delta} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & \omega^2 \end{pmatrix} \begin{pmatrix} \Delta \\ \delta \end{pmatrix}, \tag{17}$$

where

$$\omega^2 = \frac{1}{2} \frac{(r_0)^2 V''_{\rm tot}(r_0)}{M_*^{N+n-1} (r_0)^n} = \frac{1}{2} \frac{(r_0)^2 V''_{\rm tot}(r_0)}{M_{(N+1)}^{N-1}}. \tag{18}$$

Here $M_{(N+1)}$ is to be understood as the effective Planck mass in the large $(N+1)$-dimensional spacetime ($M_{(4)} \equiv M_{\rm pl}$). We now search for oscillating solutions, $(\Delta, \delta) = \exp(i\Omega t)(\Delta_0, \delta_0)$ of the stability equations. From Eq. (17), Ω^2 is thus given by the eigenvalues of the matrix

$$+\frac{\omega^2}{nN(N+n-1)} \begin{pmatrix} 0 & -Nn \\ 0 & N(N-1) \end{pmatrix}, \tag{19}$$

namely, $\Omega^2 = 0$, and

$$\Omega^2 = \frac{(N-1)}{n(N+n-1)} \omega^2. \tag{20}$$

The zero eigenvalue just corresponds to the fact that R_0 is a flat direction since, by assumption, there is no potential for R. The crucial expression is Eq. (20), which gives us the condition for stability of our static solution. Stability requires $\Omega^2 > 0$, which for $N > 1$ implies

$$\omega^2 > 0 \quad \Rightarrow V''_{\rm tot}(r_0) > 0. \tag{21}$$

This is the main result of this Section. Even though it seems trivial that stability is equivalent to requiring the second derivative of the potential around the extremum to be positive, this condition is *a priori* not at all obvious given the negative kinetic terms for the radii fields. As an example of this consider the case $N = 0$, which corresponds to r being thought of as the radius of a Friedman-Robertson-Walker universe. *In this case* stability requires $\omega^2 < 0$, or equivalently $V''_{\rm tot}(r_0) < 0$. This accords with our usual understanding: for example take the only term in V to be a positive cosmological term $V_{\rm tot}(r) = \Lambda r^n$. Then around the minimum at $r = 0$ the solution is unstable to inflationary growth as we expect.

The end result of this analysis is simply that we can think in terms of a total potential $V(r)$ that one can minimize to find the stable static solutions for the size of the internal dimensions. Also note that from Eqs. (20) and (18) we can extract the mass of the canonically normalized radial oscillation field ϕ (we will refer to ϕ as the "radion") in the case of interest, $N = 3$, n arbitrary:

$$m^2_{\rm radial} = \frac{1}{n(n+2)} \frac{(r_0)^2 V''_{\rm tot}(r_0)}{M^2_{\rm pl}} \tag{22}$$

Finally, consider a quite general form of the possible stabilizing potential

$$V(r) = \epsilon r^\alpha + \frac{N}{r^\beta} \tag{23}$$

Equating the minimum of this potential with the required radius of the extra dimensions we find the constraint

$$r_0 = \left(\frac{N\beta}{\alpha\epsilon}\right)^{1/(\alpha+\beta)} \tag{24}$$

Thus for reasonable exponents α and β, a large radius r_0 requires either large N, small ϵ or both.

It is interesting that independent of the details of the stabilizing potential there is an upper bound on the mass of the radial excitation field: By equipartition, the second derivative of the general potential $V(r)$ of Eq. (23) around the minimum is given by $V'' \sim N/(r_0)^{\beta+2}$. In addition the mean bulk value of the cosmological constant is defined by

$$\bar{\Lambda} \equiv \frac{V(r_0)}{(r_0)^n} \tag{25}$$

and since $V(r_0) \sim N/(r_0)^\beta$, we have that $\bar{\Lambda} = N/(r_0)^{(n+\beta)}$. Thus using the definition of the canonically normalized radial excitation, Eq. (22), it is easy to see that physical mass of the radial excitations is

$$m_{\rm radial}^2 \sim \frac{N/(r_0)^{2+\beta}}{M_*^{2+n}(r_0)^{n-2}} = \frac{\bar{\Lambda}}{M_*^{2+n}}. \tag{26}$$

But now we can apply the curvature radius bound on $\bar{\Lambda}$, Eq. (4), to find

$$m_{\rm radial}^2 \lesssim M_*^2 \left(\frac{M_*}{M_{\rm pl}}\right)^{4/n} \tag{27}$$

independent (up to the $\mathcal{O}(1)$ coefficients we have dropped) of any details of the stabilizing potential or mechanism. Evaluating this for the desired values of M_* leads to a mass for the radial field that varies between $10^{-2}\,\mathrm{eV}$ or less for $n = 2$, to $\sim 20\,\mathrm{MeV}$ or less for $n = 6$. Note that the reason why the radion mass is much smaller than M_* is that $\bar{\Lambda}$ must be relatively small to allow a large extra dimension. Naively one might think that low n cases with very light radial excitation fields would be excluded by considerations of early universe cosmology, or by astrophysical constraints. But this is not the case. First of all, these constraints can not be any worse than those for the couplings to the 4-d graviton and its KK excitations, which were analysed in [3] and found to be safe. But there is a more important point: there is no direct coupling of SM fields on the wall to the radions. The reason is simple: the couplings of gravity to the SM fields on the wall come entirly from the induced metric on the wall, which is independent of the radions g_{ij}, unless the fluctuations of the wall in the extra dimensions are also inculded. Therefore, the only coupling to g_{ij} invloves the goldstones on the wall, but there is no direct coupling to SM fields. Further details of the early universe cosmology in our scenario will be presented in [16].

3 Radius Stabilization Mechanisms

Two issues must be distinguished in discussing radius stabilization: the mechanism by which the internal dimensions are prevented from collapsing to $1/M_*$, and the mechanism by which they are prevented from expanding to a size much larger than a millimeter or fermi.

The most obvious idea for limiting the expansion of the internal dimensions is to employ a component of the potential energy that scales like the volume of the internal space: $V \sim r^n$. Such an effective potential energy density results from a *positive* bulk cosmological constant term, as shown in Section 2. Recall that Eq. (4) shows that there are significant constraints on the size of this bulk cosmological constant, independent of the cancellation of the effective 4-dimensional cosmological constant. Nevertheless, as we will shortly argue models of stabilization consistent with early universe and laboratory phenomenology do exist.

We now turn to the ways in which the radii of the extra dimensions can be stopped from collapsing to small values. We will see that a wide range of mechanisms are in principle possible, leading to a variety of power-law potentials of the form $1/r^\ell$ for various ℓ. (One possibility that we will not discuss in detail in this section is that stabilization in both the UV and IR domains is due to a non-trivial function of $\log(r)$. Such a possibility was first discussed in Ref. [3].)

3.1 Radius Stabilization from Brane Lattice Crystallization

The largeness of the internal dimensions compared to $(1\,\mathrm{TeV})^{-1}$ could also arise from the existence of a large (conserved) number of branes populating the bulk. There can exist inter-brane forces which act like the Van der Walls and hard-core forces between atoms in a crystal. The inter-brane distance is set by these forces, and might be quite small, but the size of the whole internal space is set by the total number of branes, just as the total extent of a crystal is set by the number of atoms, rather than just the inter-atom distance which is much smaller.

Before we discuss the inter-brane forces we note that there is a constraint on the total number of branes that can populate the internal dimensions. If the transverse inter-brane separation becomes comparable to $1/M_*$, then there will be new light open string modes that arise from strings starting on one brane and ending on a neighbor. These will lead to a large number of new gauge bosons with masses of order a TeV, which because of their large multiplicity would be excluded. Thus the maximum number of branes that can occupy the extra dimensions is

$$Q_{\max} \sim (r_0)^n M_*^n \sim \left(\frac{M_{\rm pl}}{M_*}\right)^2 \sim 10^{32}. \tag{28}$$

We now discuss some concrete possibilities for the inter-brane forces.

[I] Classical forces between branes.

It is a well-known fact that two infinite parallel domain walls in (3+1)-dimensional spacetime do not attract gravitationally, but rather repel [13]. This can be seen from a simple analysis in the post-Newtonian approximation to general relativity, which generalizes to the case of p-branes in a higher dimensional space, as we now present.

Consider the 0-0 component of the linearized $(4+n)$-dimensional Einstein's equation in the background of a p-dimensional domain wall localized at $x_{p+1} = x_{p+2} = \ldots = x_{3+n} = 0$. The Newtonian potential energy ϕ is related to the metric components by $g_{00} = 1 + 2\phi$, and substituting this into the linearized form of Einstein's equations gives

$$\nabla^2 \phi = \frac{4\pi}{M_*^{n+2}} (\rho + p_1 + \ldots + p_{3+n}) \quad (29)$$

The stress-energy tensor for the wall configuration has the form

$$T_{\mu\nu} = \text{diag}(1, -1, \ldots, -1, 0, 0, \ldots, 0) \rho_0 \delta(x_{p+1}) \ldots \delta(x_{3+n}), \quad (30)$$

where there are $(4 + n - p)$ zeroes. Thus the Einstein equation reduces to

$$\nabla^2 \phi = -\frac{4(p-1)\pi}{M_*^{n+2}} \rho_0 \delta(x_{p+1}) \ldots \delta(x_{3+n}). \quad (31)$$

Note the sign in this equation for ϕ. The form of the general solution to Eq. (31) with respect to the transverse radial direction r depends on the co-dimension of the p-brane; for D transverse dimensions the solution has the form $\phi(r) = a + br^{(2-D)}$, except for $D = 2$ in which there is a logarithmic r dependence, corresponding to a conical singularity at the position of the brane with associated deficit angle. The sign of the coefficient b is $b > 0$ for $D > 2$ and $b < 0$ for $D < 2$. In any case, the important point is that the sign of the potential is reversed with respect to the usual sign appropriate for a mass distribution, and therefore branes generically gravitationally repel.

With just two 3-branes populating the internal space the potential energy varies as

$$V(r) \sim \frac{f^8}{M_*^{n+2}} \frac{r^{2-n}}{(n-2)}. \quad (32)$$

The inter-brane distance can be estimated from balancing this repulsive force against a bulk cosmological constant term $V(r) \sim \bar{\Lambda} r^n$. However, in doing this it is very important to remember that the effective 4-dimensional cosmological constant must be tuned to zero; this is enforced by Eq. (5), which gives $\bar{\Lambda} r^n = -f^4$. Thus the predicted inter-brane separation r_I is

$$(r_I)^{n-2} \sim \frac{f^4}{M_*^{n+2}} \sim (1\,\text{TeV})^{(2-n)}. \quad (33)$$

What happens when Q branes occupy the internal space? One may think that the size of the internal volume will just be Q times larger than $(r_I)^n$ calculated above. However this is incorrect. The reasons for this are two-fold. The first is that, unlike in a normal crystal, there is no necessity that the inter-brane forces are screened. Thus the total potential energy density due to the gravitational inter-brane forces increases as Q^2, just as in a star, and the UV stabilizing part of the potential has the form

$$V \sim \frac{f^8 Q^2}{M_*^{n+2}} \frac{r^{2-n}}{(n-2)}, \tag{34}$$

where r is now roughly the total extent of the system. The second reason why the two brane calculation is inappropriate is that the equation for the cancellation of the effective 4-dimensional IR cosmological constant is modified. At the minimum of the potential, where the size of the extra dimensions is stabilized at a value r_0, we must have

$$0 = Qf^4 + V(r_0). \tag{35}$$

Putting all parts of the potential together, and for the moment making the simple assumption that the IR stabilizing potential is due to a bulk cosmological constant $V \sim \bar{\Lambda} r^n$, we have

$$V(r)_{\rm tot} \sim \frac{f^8 Q^2}{M_*^{n+2}} \frac{r^{2-n}}{(n-2)} + \bar{\Lambda} r^n + Qf^4. \tag{36}$$

Solving for the size of the system gives,

$$r_0 \sim \left(\frac{Q^2 f^8}{n \bar{\Lambda} M_*^{n+2}} \right)^{1/2(n-1)}. \tag{37}$$

However $\bar{\Lambda}$ can be eliminated by imposing Eq. (35), leading to

$$\bar{\Lambda} \sim \frac{f^{-8/(n-2)} M_*^{n(n+2)/(n-2)}}{Q^{2/(n-2)}}, \tag{38}$$

and thus the final expression for r_0

$$r_0 \sim \frac{f^{4/(n-2)} Q^{1/(n-2)}}{M_*^{(n+2)/(n-2)}}. \tag{39}$$

If the assumption is made that $f \sim M_*$ (as well, of course, as the implicit assumption made above that all Q of the 3-branes are broadly similar), and utilizing the formula for the required size of the extra dimensions, $(r_0)^n M_*^{n+2} = M_{\rm pl}^2$, then we can solve for the necessary brane-number

$$Q \sim \left(\frac{M_{\rm pl}}{M_*} \right)^{2(n-2)/n}. \tag{40}$$

Numerically this varies from $Q \sim 10^{10}$ for $n = 3$ (the smallest number of extra dimensions for which the above analysis applies), to $Q \sim 10^{20}$ for $n = 6$.

The first comment to make about this result is that the number of 3-branes satisfies the bound, Eq. (28). The second is that if one substitutes this value back into the equation for $\bar{\Lambda}$, Eq. (38), then one finds

$$\bar{\Lambda} \sim M_*^{n+4} \left(\frac{M_*}{M_{\rm pl}} \right)^{4/n} , \tag{41}$$

which is smaller than the naive value M_*^{n+4}, showing that indeed one component of the hierarchy problem in this framework is the (bulk) cosmological constant problem.

There is one other requirement that needs to be satisfied. The mean curvature radius on scales smaller than the inter-brane separation needs to be larger than the inter-brane separation itself. This is the generalization of the curvature radius condition used to derive Eq. (4). In any case, the average inter-brane transverse separation is now

$$r_I \equiv \left(\frac{(r_0)^n}{Q} \right)^{1/n} \sim \frac{1}{M_*} \left(\frac{M_{\rm pl}}{M_*} \right)^{4/n^2} , \tag{42}$$

whilst the curvature radius resulting from our potential is

$$L_{\rm curv} \sim \frac{1}{M_*} \left(\frac{M_{\rm pl}}{M_*} \right)^{2/n} . \tag{43}$$

For the case of $n > 2$ where the above analysis applies, one always has $L_{\rm curv} > r_I$ as required.

Some remarks are now in order. One may worry that not only is the required value of the bulk cosmological constant small, but more possibly seriously that it suffers from large radiative corrections, so that it's size is not even technically natural. One rather nice answer to this concern is provided by the following scenario. Suppose that the supersymmetries of string theory are broken only by on-the-wall dynamics at a scale $\sim M_* \sim 1\,{\rm TeV}$. Then the mass splittings so induced among the bulk supergravity multiplet are $\sim M_*^2/M_{\rm pl}$, and a bulk cosmological constant of order $\Lambda \sim M_*^{2(4+n)}/M_{\rm pl}^{(4+n)}$ arises. Then at the minimum where $r = r_0$ the ratio of this new term to the $\bar{\Lambda} r^n$ term is

$$\frac{V_{\rm correction}}{V_{\rm tree}} \sim \left(\frac{M_*}{M_{\rm pl}} \right)^{(n^2+4n-4)/n} \ll 1. \tag{44}$$

Therefore the value of the bulk cosmological constant can be technically natural.

Secondly, we have used in this subsection the classical forces between 3-branes embedded in a $(4+n)$-dimensional space. Polchinski's now classic calculation of the forces between Dp-branes demonstrated that the forces due to the RR gauge fields precisely cancelled the gravitational forces in the supersymmetric limit, as they must for a pair of BPS states [4]. One component of this cancellation is that the RR charge density $\rho^{(p)}$ of the p-branes is equal to their tension $T^{(p)}$ in the supersymmetric limit. When supersymmetry is broken on the wall at a scale $\sim M_*$ it is natural to expect a mismatch of order $(M_*)^4$ to arise between these quantities. It is this mismatch we have been calling f^4.

In summary, we have made a number of simplifying assumptions which can be questioned and modified. These include the simplification that all 3-branes are broadly similar and have similar brane-localized cosmological constants of order $f^4 \sim (1\,\mathrm{TeV})^4$ at short distances. Nevertheless, it is encouraging that the large-brane-number scenario for stabilizing the volume of the internal dimensions at large values passes the first tests.

[II] Non-extensive Bulk Cosmological Constant.

We now discuss an idea for *producing* a mean bulk cosmological constant $\bar{\Lambda}$ of the correct size. Suppose that the effective bulk cosmological constant is *not extensive* as a function of volume, but that apart from this non-extensivity, its' value is of the order of the fundamental scale. Concretely suppose the IR potential is λr^a, for $a < n$. The total potential reads

$$V(r)_{\rm tot} \sim \frac{f^8 Q^2}{M_*^{n+2}} \frac{r^{2-n}}{(n-2)} + \lambda r^a + Q f^4. \tag{45}$$

Solving for the size of the system once again gives,

$$r_0 \sim \left(\frac{Q^2 f^8}{a \lambda M_*^{n+2}} \right)^{1/(a+n-2)}. \tag{46}$$

Eliminating λ by imposing Eq. (35), now leads to the *requirement* that

$$\lambda \sim \frac{f^{-4(2+a-n)/(n-2)} M_*^{a(n+2)/(n-2)}}{Q^{(a+2-n)/(n-2)}}. \tag{47}$$

Note that, despite the different potential, the final expression Eq. (39) for r_0 remains unchanged, basically because of energy equipartition, and thus so does the required Q: It is still given by Eq. (40). In any case, substituting this value of Q back into the expression for λ, Eq. (47), and taking $f \sim M_*$ gives

$$\lambda|_{\text{required}} \sim M_*^{4+a} \left(\frac{M_*}{M_{\text{pl}}}\right)^{2(a+2-n)/n} . \qquad (48)$$

Thus for $a = n - 2$ the required value of λ agrees with the "natural" value $\lambda \sim M_*^{4+a}$. It is not clear to us whether there is a physical mechanism (such a wrapped p-branes, or holography) that leads to a non-extensive bulk "cosmological constant". In any case, in this situation hierarchy problem maps purely to the question of why Q has the large value, Eq. (40).

[III] Casimir forces between branes.

Another potentially attractive idea for UV stabilization at the quantum level is to use the Casimir force to maintain the size of the internal space [14]. The effective 4d potential energy density corresponding to the Casimir effect in a $(4+n)$-dimensional spacetime is

$$V(r) \sim \frac{C}{r^4}, \qquad (49)$$

where C is a calculable coefficient in any given model. Even with a general non-extensive stabilizing potential, $\Delta V \sim \lambda r^a$ this leads to a inter-brane distance of

$$r_I \sim \left(\frac{C}{\lambda}\right)^{1/(4+a)} . \qquad (50)$$

Given that the "natural" value of λ is expected to be $M_*^{(4+a)}$, this clearly doesn't allow us to stabilize at large radii. What about many branes? However, when we go to Q 3-branes in the bulk, the Casimir energy does not increase with Q for $n \geq 2$. But the total wall cosmological constant Qf^4 does, and thus the situation gets worse.

In summary, the Casimir force idea, even with a large brane number $Q \gg 1$, fails to stabilize the internal dimensions at large radii, at least under the simplifying assumptions we have made.

3.2 Topological Stabilization

One of the most attractive ways of preventing collapse is to imagine that there is a topologically conserved quantity which holds up the size of the extra dimensions. A prototypical example of this is provided by the monopole stabilization mechanisms discussed in Ref. [15] and in the context of our scheme by Sundrum [10]. Consider the simple case of two extra dimensions and where the internal manifold has the topology of a 2-sphere, S^2. Further suppose that in the bulk there exists not only the graviton, but also a U(1) gauge field, which might naturally be a Ramond-Ramond (RR) gauge field of the string theory in question. Then it is possible to take the gauge field configuration on S^2 to be topologically non-trivial with quantized "monopole number" k (the first Chern number of the U(1) bundle) given by[3]

$$\frac{1}{2\pi}\int_{S^2} H^{(2)} = k. \tag{51}$$

If the area of the S^2 is denoted $V_{(2)}$ then we have $H \sim k/V_{(2)}$ and since the kinetic term for the U(1) gauge field is (expressed in form notation, with M^4 denoting 4-dimensional Minkowski space)

$$S_{KE} \sim \frac{M_*^2}{g^2}\int_{M^4\times S^2} H \wedge^* H, \tag{52}$$

we have that the 4-dimensional potential energy density of the monopole field on the S^2 scales like

$$V \sim \frac{M_*^2}{g^2}\frac{k^2}{V_{(2)}}. \tag{53}$$

In other words we get an energy density that scales like $k^2 M_*^2/(g^2 r^2)$. For large enough monopole number, k, this will stabilize the internal S^2 at any desired size.

This basic mechanism has a wide variety of generalizations. One such is to use the topological invariants of the higher-form RR gauge fields that naturally arise in the type II and type I string theories with D-branes.

[I] Higher-form RR fields

Denote the manifold of the extra n dimensions by E^n, and suppose that the bulk theory contains an $(n-1)$-form U(1) gauge field, with n-form field strength $F^{(n)}$. Then once again there is the topological invariant

$$\frac{M_*^{n-2}}{2\pi}\int_{E^n} H^{(n)} = k. \tag{54}$$

The kinetic energy of $H^{(n)}$ is the generalization of the usual 1-form gauge kinetic term

$$S_{KE,n} \sim \frac{M_*^n}{g^2}\int_{M^4\times E^n} H^{(n)} \wedge^* H^{(n)}, \tag{55}$$

[3] We will always use H for field strengths of gauge fields that live in the bulk. Quite often we will think of these as being RR gauge fields.

and thus the potential energy density depends on the volume $V_{(n)}$ of E^n as

$$V \sim \frac{1}{g^2 M_*^{n-4}} \frac{k^2}{V_{(n)}}. \tag{56}$$

In the case of the chiral type IIB string theory there exists 1 and 3-form RR field strengths and a self dual 5-form RR field strength (together, of course, with their magnetic duals). There also exists the usual NS-NS 3-form field strength. The type I string theory has a 3-form RR field strength and it's 7-form magnetic dual. Thus using the invariants so far described, it is natural to stabilize 1, 3, and 5-manifolds.

However, invariants that lead to $1/r^n$ potentials for E^n are not the only possibility. Consider the situation in which our 3-brane world is the boundary of (a set of) higher-dimensional branes which are in turn embedded in the full $(4+n)$-dimensional space. We can then use topological invariants of the world-volume gauge fields of these higher-dimensional branes to stabilize the internal dimensions. To make this clear consider the following very simple example: In the $n = 4$ case take the internal manifold to be $E^4 = T_1^2 \times T_2^2$. Suppose further that there exist 2 5-branes that intersect at the position of our 3-brane but are perpendicular in the extra 4 dimensions, so that one 5-brane lives in $M^4 \times T_1^2$ and the second lives in $M^4 \times T_2^2$. Then we have the two topological invariants

$$\frac{1}{2\pi} \int_{T_i^2} F_i = k_i, \qquad i = 1, 2, \tag{57}$$

where F_i, $i = 1, 2$ are world-volume U(1) 2-form field strengths of the first and second 5-brane. The brane-localized kinetic terms for these gauge fields then leads to an effective 4-dimensional potential energy density of the form

$$\Delta V(r) \sim \frac{M_*^2 k_1^2}{r_1^2} + \frac{M_*^2 k_2^2}{r_2^2}, \tag{58}$$

where r_1 and r_2 are the radii of the two S^2's. Note that since we have used torii, there is no negative curvature term $\sim -M_*^2 r^2$ in the potential. This, then, is an UV stabilizing potential for $E^4 = T_1^2 \times T_2^2$ not of the form $1/r^4$. Clearly this type of mechanism admits many generalizations.

Finally, one can also consider higher "reducible" invariants such as the second Chern class of the usual 2-form U(1) field strength defined wrt a 4-manifold

$$\frac{1}{8\pi^2} \int_{E^4} H^{(2)} \wedge H^{(2)} = c_2, \tag{59}$$

but such invariants typically lead to a potential energy varying as r^α with $\alpha \geq 0$.

[II] Metric topological invariants.

Purely metric topological invariants are possible, for example the Euler number of a 2-manifold component E^2 of the internal space

$$\chi = \frac{1}{2\pi}\int_{E^2} R, \tag{60}$$

where R is the curvature 2-form. Other possibilities include the Pontrjagin classes of the tangent bundle of the internal manifold. However, because the leading term in the gravitational effective action is only linear in the curvature, this does not provide a UV stabilizing potential unless higher derivative terms, such as

$$\Delta S \sim M_*^n \int \mathrm{tr}(R^2) \tag{61}$$

are included in the effective action. For the simple case of $n = 2$ this leads to a potential $V \sim \chi^2 M_*^2/r^2$. For this to balance, at the appropriate r_0, even a best-case stabilizing potential of the form $M_*^5 r$, an Euler number of $\chi \sim (M_{\rm pl}/M_*)^{3/2}$ is required. So clearly the internal manifold is very highly curved. In particular, the leading gravitational action $M_*^4 \int \mathcal{R}$ dominates the other terms by an amount $(M_{\rm pl}/M_*)^{1/2}$, and leads to an unacceptably large bulk cosmological constant. This seems to be a generic problem with this type of topological stabilization, although we have not investigated the question in detail.

[III] Scalar-field and other non-gauge invariants.[4]

One can also imagine stabilizing the size of the internal space by the use of non-gauge or metric topological invariants. For example, consider a complex scalar field that lives on a 1-dimensional higher brane that has as boundary our 3-brane. Then the phase of this field can wind as an S^1 cycle of the internal space is transversed, with topologically conserved winding number

$$k = \int_{S^1} d\phi. \tag{62}$$

Once again the kinetic energy of this configuration increases as the size of the internal space is reduced, and thus a stabilizing potential results. More sophisticated scalar field invariants are also conceivable, the Hopf winding number of the map $\pi : S^3 \to S^2$ being one among many such examples. In general this leads to quite similar results to the gauge field topological stabilization mechanisms, but possibly without the natural advantage of gauge fields of their constrained

[4]Gia Dvali has independently considered this possibility. We thank him for discussions.

couplings. (For instance it is easy to arrange that the stabilizing gauge fields do not lead to dangerous flavor-changing neutral current processes on the wall, while this requires additional input in the scalar case.)

[IV] Phenomenologically successful topological stabilization.

In the previous subsections we have seen that a variety of UV stabilizing potential energy densities of the general form N/r^β are possible. We now wish to show that these lead to successful means by which the internal radii can be stabilized at the required distances.

First, though, we explain a simple argument which demonstrates that there is a β-dependent lower bound on the topological number k, *independent* of any other details of the potential. This bound comes from demanding that the radius oscillation field not be too light. Since the experiments on gravitational strength forces at a distance of 1mm and above do not observe any deviation from Newton's law, the mass of the radion field cannot be less than $\sim 10^{-3}\,\text{eV} \sim (1\,\text{TeV})^2/M_{\rm pl}$. This, together with Eqs. (22 – 24), leads to a lower bound on the dimensionless quantity $k^2 \equiv N/M_*^{4-\beta}$:

$$k^2 \gtrsim (M_* r_0)^\beta \left(\frac{1\,\text{TeV}}{M_*}\right)^4 = \left(\frac{M_{\rm pl}}{M_*}\right)^{2\beta/n} \left(\frac{1\,\text{TeV}}{M_*}\right)^4 . \qquad (63)$$

In the above examples of topological radius stabilization the quantity k is directly proportional to the "monopole" number. From this expression the smallest k clearly occurs when the ratio β/n is as small as possible. As an example, if $\beta = 1$ and there 6 extra dimensions, then $k \gtrsim 3 \times 10^2$ is required to stabilize at a sufficiently large radius. If $\beta = 2$ and there are 6 extra dimensions, then $k \gtrsim 10^5$ is necessary. The $\beta = 2$ case is particularly interesting since it is the first case we can realize with gauge-field topological invariants rather than scalar field invariants. In any case, note that the bound on k follows *independent* of the IR potential, once the bound on the radial excitation mass is employed. The worst case, requiring the largest k, occurs when β/n takes on its largest value. A typical "worst-case" is provided by the irreducible topological stabilization mechanism involving bulk RR fields (for example). This gives $\beta = n$, and leads to $k \gtrsim 10^{15}$.

Now let study what values of the "tension" ϵ in the IR restoring part of the potential are required to stabilize at the appropriate value of $r = r_0$. From Eq. (24), and the expression for N in terms of k and the formula for the required radius $(r_0)^n M_*^{n+2} = M_{\rm pl}^2$, we derive

$$\epsilon \sim k^2 M_*^{4-\beta} \left(\frac{M_*^{n+2}}{M_{\rm pl}^2}\right)^{(\alpha+\beta)/n} . \tag{64}$$

Given a choice of k we can compare this with the "expected" ϵ from various mechanisms. Let us substitute the value of k needed to satisfy the radion mass constraint Eq. (63): we find

$$\epsilon \gtrsim M_*^{4+\alpha} \left(\frac{M_*}{M_{\rm pl}}\right)^{2\alpha/n} . \tag{65}$$

The very important point to note about this value is that it is much *bigger* than the natural value we would expect if supersymmetry was broken on the wall by a large amount $\sim M_* \sim 1\,\mathrm{TeV}$, and the SUSY breaking was communicated to the bulk. Concretely the mass splitting between bulk superpartners would be $\sim (M_*)^2/M_{\rm pl}$, leading to a bulk cosmological constant of order $\Lambda \sim (M_*^2/M_{\rm pl})^{4+n}$, and a IR potential of the form Λr^n (so $\alpha = n$). The required cosmological constant given in Eq. (64) with $\alpha = n$ is much larger than Λ. Furthermore Eq. (64) with $\alpha = n$ is the value needed to cancel the wall-localized cosmological constant, $f^4 \sim M_*^4$, to give zero cosmological constant in the IR in our universe. See Eq. (7).

What this tells us is that the required bulk cosmological constant is *natural* in the sense that it is not disrupted by radiative corrections once supersymmetry is broken on the wall.

In summary we have shown that the topological stabilization mechanism successfully meets all our phenomenological requirements, with a price of a large, but in some cases not too large integer k.

[V] Corrections to leading-order potentials.

One may worry that in the regime of interest, when $r \sim r_0$, the semiclassical reasoning that we have applied to the leading-order kinetic and non-derivative terms in the effective action suffers from large corrections due to the presence of other terms. Such corrections are, in actual fact, entirely negligible. For example, if one included higher-order derivative terms, such as

$$\Delta S \sim M_*^{n-4} \int_{M^4 \times E^n} (H^{(p)} \wedge H^{(p)}) \wedge^* (H^{(p)} \wedge H^{(p)}), \tag{66}$$

in the effective action, then they would lead to corrections in the 4-dimensional effective potential energy density, V, of order

$$\Delta V \sim M_*^4 \frac{k^4}{(M_* r_0)^{3n}} \tag{67}$$

at the minimum r_0. Compared to the leading kinetic term this is a fractional change of order

$$\frac{\Delta V}{V} \sim k^2 \left(\frac{M_*}{M_{\rm pl}}\right)^4, \tag{68}$$

negligible unless $k \gtrsim 10^{30}$. Such statements generally apply for $r \sim r_0$, and are basically due the fact that $r_0 \gg (1\,\mathrm{TeV})^{-1}$. This is not quite trivial because of the potentially large dimensionless factor k which could have overcome this suppression. In any case we see that the leading-order analysis is entirely sufficient unless we are interested in physics at radii $r \ll r_0$.

4 Remarks and Summary

The hierarchy problem in our framework is replaced by the problem of obtaining large new dimensions, of a size which varies between a millimeter and a fermi depending of the number of new dimensions, in a theory with a much smaller fundamental length $\sim$ TeV^{-1}. In this paper we exhibited mechanisms which provide such large extra dimensions. These mechanisms relied on two ingredients:

- A large conserved integer Q or k, respectively the brane number or the topological charge of the vacuum configuration. This large integer should be regarded as analogous to the net conserved baryon number which accounts for the large size of macroscopic objects relative to that of atoms. The necessity for such a large number was *not* forced on us by the need for large internal dimensions, but rather by the requirement that the radial oscillation field (or "radion") be sufficiently heavy to have escaped tests of gravity at the millimeter-scale and above. The value of k or Q depends on the details of the stabilization scenario; it varies from $Q \sim 10^{10}$ to $Q \sim 10^{20}$ in the brane-lattice-crystallization scenario, while in the topological stabilization scenario it varies from $k \sim 3 \times 10^2$ to $k \sim 10^{15}$.
- A small bulk cosmological constant, analogous to the 4-dimensional cosmological constant whose smallness accounts for the size of our universe relative to the

Planck length. However, as we discuss in detail in, for example, Section 3.1.IV, the value of this bulk cosmological constant is *stable against radiative corrections* if supersymmetry-breaking of order the fundamental Planck mass $\sim M_* \sim 1\,\mathrm{TeV}$ takes place on our 3-brane. Of course we must still impose a fine tuning to get a vanishing effective 4-dimensional, brane-localized cosmological constant in the IR in our world. This is expressed in Eq. (5) or (35), depending on the stabilization scenario.

A valid criticism of our analysis is that we have not provided a *dynamical* framework in which, for instance, the largeness of Q or k is explained. As discussed in the introduction our viewpoint on this issue is that this is closely analogous to the situation in the MSSM where soft supersymmetry-breaking operators of order (1 TeV) are introduced [12].

With the advent of many quantum-field-theoretic (QFT) models of dynamical supersymmetry breaking it is commonly believed that the problem of the size of these soft operators has been solved, at least in principle. However, from a fundamental vantage-point this belief is not correct. Concretely, what is the situation in the standard model or MSSM, where the usual (reduced) Planck mass $M_{\rm pl} \sim 2 \times 10^{18}\,\mathrm{GeV}$ is taken as fundamental? We must now explain the ratio of this Planck scale to the weak scale $\sim 10^{15}$. There too the "dilaton runaway problem" prevents us from having a calculational framework for this number. This point is important to emphasize. Although in the context of QFT dynamical SUSY breaking solves the hierarchy problem, in that it generates the small scale by dimensional transmutation, in the context of string theory the couplings and thus the scale of SUSY breaking are dynamical, and there is a ground state at zero coupling with unbroken supersymmetry [19]. This means that there exists no known solution to the hierarchy problem in usual 4-dimensional QFT once it is embedded in string theory. Therefore both frameworks face similar challenges.

Acknowledgements

It is a pleasure to thank Philip Argyres, Gia Dvali, Vadim Kaplunovsky, Yaron Oz and Eva Silverstein for valuable discussions. SD thanks the CERN theory group for its hospitality. The work of NAH is supported by the Department of Energy under contract DE-AC03-76SF00515. The work of SD is supported in part by NSF grant PHY-9219345-004. The work of JMR is supported in part by an A.P. Sloan Foundation

Fellowship.

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