

Gauge-invariant strings in the 3d U(1)+Higgs theory

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We describe how the strings, which are classical solutions of the continuum three-dimensional U(1)+Higgs theory, can be studied on the lattice. The effect of an external magnetic field is also discussed and the first results on the string free energy are presented. It is shown that the string free energy can be used as an order parameter when the scalar self-coupling is large and the transition is continuous.

1. Three-dimensional U(1)+Higgs theory

The three-dimensional U(1)+Higgs theory contains a complex scalar field $\phi(x) \equiv R(x)e^{i\varphi(x)}$ and an abelian gauge field $\alpha_i(x) \equiv eaA_i(x)$ and is defined by the action

$$S = \int d^3x \left[\frac{1}{4} F_{ij} F_{ij} + |D_i \phi|^2 + m^2 \phi^* \phi + \lambda (\phi^* \phi)^2 \right].$$

This contains a dimensionful gauge coupling constant e^2 , which defines the scale of the system, and the two dimensionless parameters

$$x = \frac{\lambda}{e^2} \text{ and } y = \frac{m^2}{e^4}.$$

The continuum action can be discretised using either the compact or the non-compact formalism for the gauge field – in the following we shall use exclusively the non-compact formalism, which has several advantages over the compact one.

2. Classical solutions

The three-dimensional U(1)+Higgs theory contains classical, cylindrically symmetric solutions of the equations of motion – strings. These topological objects have been studied in great detail in connection with superconductivity and cosmic

strings. They can be characterized by

$$\oint dx \nabla \varphi = 2\pi n_C,$$

where n_C is the *winding number*. This can be defined on the lattice [1,2] by computing the integer n_C for a closed loop C by

$$\sum_{l \in C} Y_l \equiv 2\pi n_C,$$

where

$$Y_{(\mathbf{x}, \mathbf{x}+\hat{i})} = [\alpha_i(\mathbf{x}) + \varphi(\mathbf{x} + \hat{i}) - \varphi(\mathbf{x})]_{\pi} - \alpha_i(\mathbf{x}).$$

3. External magnetic field

The phase structure of the theory at zero external magnetic field is strictly speaking only known in the compact theory [3]. The study of the non-compact theory is in progress. However, it is known that at small values of the scalar self-coupling x , perturbation theory works and the theory has a first order phase transition from the symmetric to the broken phase. The transition weakens as x is increased. At some critical value $x_c \sim 0.5$ it is expected that the transition becomes continuous. The continuous transition can be observed by measuring the photon mass. We suggest that in the non-compact theory the string free energy could be used as an order parameter, as well.

*Poster presented by J. Peisa.

The situation changes dramatically if an external magnetic field is present. Depending on x , the system can have either two or four phases. If x is small, there is just one transition from the symmetric to the broken phase (from normal to superconducting phase in condensed matter terminology), while for large x the system has also a vortex phase. This vortex phase can be divided to two further phases, a vortex lattice phase and a vortex liquid phase. The transition between these two vortex phases has been observed to be first order in real high T_c superconductors [4], and it would be interesting to see if this is predicted already by the Ginzburg-Landau theory. At the moment, the computer simulations needed to answer this question would be extremely costly.

A constant magnetic field in the z -direction corresponds to a background gauge field, for instance $\alpha_i^{bg} = ae\delta_{i2}Bx$. However, the dynamics of the system may prefer to distribute the flux of the magnetic field in an inhomogeneous way. In fact, the only thing which remains constant and can be fixed is the total flux through a given surface (e.g., the whole lattice). On a lattice with strictly periodic boundaries, there is no magnetic flux going through the lattice. However, by choosing the boundary conditions to be, for example,

$$\alpha_2(N_x, 0, n_z) = \alpha_2(0, 0, n_z) + ea^2BN_xN_y,$$

one forces a magnetic flux $\Phi_0 = ea^2BN_xN_y$ to go through the xy -plane. Note that this is only possible if one uses a non-compact gauge field. Also, even though all values of the magnetic field are possible in principle, values which lead to a non-periodicity in the hopping term cause large inhomogeneities, so that in a finite volume one can only use discrete values, namely $ea^2BN_xN_y = 2\pi m$ for the magnetic field. Thus the allowed values of the flux are $\Phi_0 = 2\pi m$.

Even though in principle the magnetic field cannot penetrate the type I superconductor (x small), the lattice construction above forces a flux through the lattice. What happens is that a macroscopic volume of the system remains in the symmetric phase, allowing the flux to penetrate the lattice. In Type II superconductors it is, however, expected that the magnetic field penetrates the system forming real vortices.

4. String density

Even though the classical string solutions have a higher energy than the true vacuum, it is possible that string-like objects are generated by quantum or thermal fluctuations. This possibility has been studied in both the U(1)+Higgs theory [2] and the SU(2)+Higgs theory [5].

In Ref. [2] the behavior of the string density – number of strings passing through a closed loop – was studied. It was found that at finite lattice spacing the 1×1 loop shows clear dependence of the parameters x and y . However, at the continuum limit the result is independent of the parameters (and can in fact be obtained from a theory containing just a free massless scalar field) – the result is pure UV noise.

It was also shown that the $\beta_G \times \beta_G$ -loop, which has a constant physical size as the lattice spacing is decreased, can contain divergent parts or can get a contribution from the UV-noise. The easiest way to remove unphysical contributions is to study the differences of string densities at various y , which should be free of the UV-noise.

5. Vortex Free Energy

We present a study of the vortex free energy T per unit length at $\beta_G = 4$ and at $x = 2$. This observable is an order parameter, as its value is finite in the Higgs phase and zero in the Coulomb phase.

A quantized magnetic flux of magnitude $\Phi = 2\pi$ in a finite box is forced into the system in one of the arbitrary chosen lattice directions, as described above. The gauge part of the action on the lattice then has the form

$$S_{\text{gauge}} = \frac{\beta_G}{2} \sum_P (\alpha_P - 2\pi\bar{\alpha}_P)^2,$$

otherwise the theory remains unchanged. Here the $\bar{\alpha}_P$ have nonvanishing values $\bar{\alpha}_P = +1, -1$ only, if their dual coincides with a closed loop, which on the torus is closed by the boundary.

In the Higgs phase (at small values of y) the vortex free energy T per unit length is finite,

$$T = \frac{1}{L} \ln Z(\Phi = 0)/Z(\Phi = 2\pi).$$

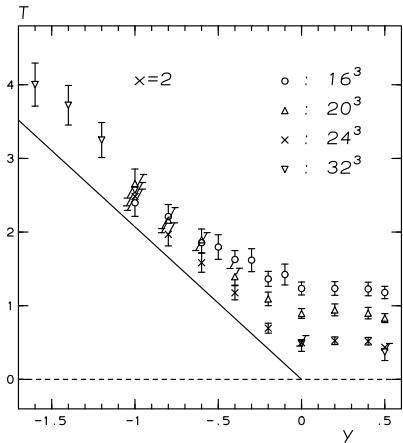


Figure 1. The vortex free energy T per unit length plotted as a function of y at $x = 2$. The solid line corresponds to the mean field result.

In this phase the magnetic flux is confined into a string and a dislocation of linear dimension L is formed in the L^3 boxes considered.

In the Coulomb phase (large values of y) the vortex free energy T vanishes. Here the theory exhibits long range order as characterized by a massless excitation. Large finite size effects within T signal the existence of photons.

In Figures 1 and 2 we present Monte Carlo measurements of the quantity T . It can be seen that T is finite in the small y -region of the theory. Finite size effects in the quantity T are under control at $y = -1.0$, as can be seen from Figure 2.

There are large finite size effects in the quantity T for values of $y \geq 0$ (see Fig. 1). These finite volume values of T extrapolate to the value 0 at positive values of y . The vanishing of T there and the observation of large finite size effects is consistent with the presence of a massless mode.

Our data in the Higgs region are close to the mean field result. Mean field scaling is expected to be valid away from the critical point. It is an open question, whether the critical singularity is dominated by mean field exponents or not. A more conclusive study is currently being prepared [6].

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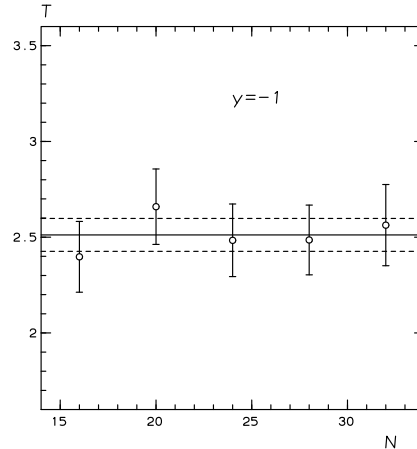


Figure 2. The vortex free energy for $x = 2$, $y = -1$ as a function of the linear extent $L = Na$. There are no visible finite size effects.

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