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What's new with the electroweak phase transition?*

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We review the status of non-perturbative lattice studies of the electroweak phase transition. In the Standard Model, the complete phase diagram has been reliably determined, and the conclusion is that there is no phase transition at all for the experimentally allowed Higgs masses. In the Minimal Supersymmetric Standard Model (MSSM), in contrast, there can be a strong first order transition allowing for baryogenesis. Finally, we point out possibilities for future simulations, such as the problem of CP-violation at the MSSM electroweak phase boundary.

1. INTRODUCTION

Primordial nucleosynthesis computations tell that the net baryon to photon number ratio η in the Early Universe is a definite non-vanishing number, $\eta = (1...9) \times 10^{-10}$. On the other hand, it is natural to assume that after inflation, $\eta = 0$. The last instance in the post-inflationary history of the Universe during which $\eta > 0$ could have been generated, is the electroweak phase transition, at $T_c \sim 100$ GeV [1]. Thus, electroweak baryogenesis is in a way the most conservative scenario of baryon number generation, and at the same time, the only scenario which is experimentally testable in existing collider experiments.

The scenario of electroweak baryogenesis has quite a few different ingredients. To generate a baryon number, one needs anomalous baryon number violating processes in the symmetric high temperature phase, microscopic C- and CP-violation, and a thermal non-equilibrium (for a review, see [2]). It is perhaps surprising that many of these ingredients can be studied non-perturbatively with lattice simulations. Indeed, baryon number violation has been studied both in the symmetric and broken phases of the theory ([3,4] and references therein). Something can perhaps also be said about CP-violation in the MSSM (Sec. 5). Finally, whether there is non-

equilibrium or not, depends on the order and strength of the transition.

The main interest here will be on the last of these questions. In addition to what kind of non-equilibrium phenomena there can be, this determines whether the baryon number violating processes are sufficiently switched off after the transition for any baryon number possibly produced to remain there. Indeed, while the baryon number violating rate is assumed to be only parametrically suppressed before the transition, $\dot{B} \sim \alpha_W^5 \ln(C/\alpha_W) T^4$ [5], it is exponentially suppressed after the transition, $\dot{B} \sim \exp[-45(v_H/T)] T^4$ [1]. The general constraint for the baryon number generated to remain there after the transition is $v_H/T_c \gtrsim 1$ [1,2], and we would hence like to compute this ratio.

2. PERTURBATION THEORY

In principle, v_H/T can be computed in perturbation theory. So why does one need simulations? This may seem like quite a relevant question, as we are studying the electroweak sector of the Standard Model, for which perturbation theory works perfectly at zero temperature.

However, things are different at finite temperatures. This is due to the so-called infrared problem. The point is simply that there is a new scale T , and there can thus be new expansion parameters such as $g^2 T/(4\pi m)$, where g^2 represents the couplings of the theory and m the masses. In practice, this kind of an expansion parameter can

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emerge from the Bose-Einstein distribution n_b :

$$g^2 n_b(m) = \frac{g^2}{e^{m/T} - 1} \stackrel{m \ll T}{\approx} \frac{g^2 T}{m}. \quad (1)$$

Thus, it is the bosonic degrees of freedom which are particularly problematic. Now, if m represents, e.g., the W mass $m_W = gv_H/2$, then $m \approx 0$ in the symmetric phase, and perturbation theory need not work at all. The phase transition takes place between the symmetric and broken phases, and thus its characteristics may also be unreliably described. Thus we need lattice simulations.

3. NON-PERTURBATIVE METHODS

3.1. 4d simulations

In principal, the most straightforward way to attack the problem is to do standard four-dimensional (4d) finite temperature lattice simulations. However, in the present context 4d simulations turn out to be quite demanding. This is, somewhat surprisingly, due to the fact that the coupling is weak. A weak coupling makes the system have multiple scales, and if a lattice with spacing a and extent N is to describe the infinite volume and continuum limits, one must require

$$a \ll \frac{1}{\pi T} \ll \frac{1}{\sqrt{2}gT} \ll \frac{1}{g^2 T} \ll Na. \quad (2)$$

For small g , lattices thus need to be very large.

It is quite remarkable that in spite of this severe requirement, a continuum extrapolation can sometimes be carried out (see below). In particular, one can employ an asymmetric lattice spacing [6], which should essentially relax the leftmost inequality in Eq. (2).

The second problem with the 4d simulations is that only the bosonic sector of the Standard Model can be studied, since chiral quarks (especially the top) cannot be put on the lattice.

3.2. 3d simulations

Another possibility is simulations in a three-dimensional (3d) effective theory. The main idea of this approach is to combine the best parts of perturbation theory and simulations: one can integrate out massive modes perturbatively, which works well since the couplings are small, and then

study light modes non-perturbatively. In the first step, the original 4d theory reduces to a 3d one.

The 3d approach allows to overcome the two problems of 4d simulations mentioned above. Indeed, since the large mass scales ($\pi T, \sqrt{2}gT$) are removed, it is easier to satisfy Eq. (2). Consequently, an infinite volume and continuum extrapolation with a relative error of, say, 5%, can be obtained with rather moderate computer resources (for a review, see [7]). Moreover, one can study the full theory with chiral fermions [8], for realistic (small) gauge couplings. In fact, the same simulation results tell about the infrared properties of many other theories, as well, such as the MSSM in a part of its parameter space [9].

The main question concerning dimensional reduction is, of course, how accurate the effective theory constructed really is. Parametrically, the error δG for static Green's functions G is $\delta G/G \lesssim O(g^3)$ [8], but an essential point is to convert this parametric estimate into a numerical one. What one can do is to compute some particular higher dimensional operators, and see what kind of an effect they give in different infrared Green's functions. This leads to errors on the $\sim 1\%$ level, and often even smaller in theories without the top quark [8]. A conservative error estimate is then $\lesssim 5\%$ in the Standard Model (for $m_H \lesssim 250$ GeV). In the MSSM the errors can be a bit larger, since the strongly interacting squarks may play a significant role, and since there are more mass parameters which may compromise the high temperature expansion. It is perhaps worth stressing that perturbative dimensional reduction does not work at all for the QCD phase transition, where the coupling is large.

4. SIMULATION RESULTS

We now review the main recent physics results obtained with the two approaches.

4.1. The Standard Model

In the 3d approach, the Lagrangian relevant for the Standard Model is

$$\mathcal{L}_{3d} = \frac{1}{4} F_{ij}^a F_{ij}^a + (D_i \phi)^\dagger D_i \phi + m_3^2 \phi^\dagger \phi + \lambda_3 (\phi^\dagger \phi)^2. \quad (3)$$

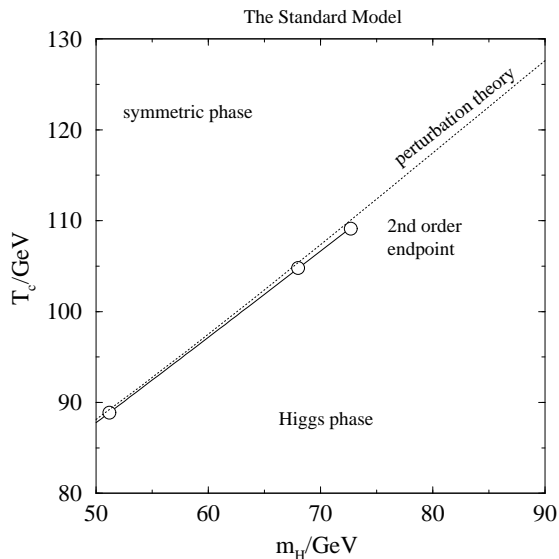


Figure 1. The phase diagram of the Standard Model. The non-perturbative endpoint location has been studied with 3d simulations in [11–14] and with 4d simulations in [15–18]. In perturbation theory (dotted line), the transition is always of the first order.

The $U(1)$ group has here been neglected (i.e., $\sin^2 \theta_W = 0$), since its effects are small [10]. Let us denote

$$x = \lambda_3/g_3^2, \quad y = m_3^2(g_3^2)/g_3^4. \quad (4)$$

In the 4d simulations, one studies the $SU(2)+\text{Higgs}$ theory, whose Lagrangian is precisely Eq. (3) but in 4d.

The theory in Eq. (3) has a first order phase transition for small Higgs masses (small values of x) [7]. The transition gets weaker for larger Higgs masses, and ends at $m_H \sim 80$ GeV [11], see Fig. 1. Recently, the interest has been in studying the endpoint region in some detail. Here, perturbation theory does not work at all and the dynamics is completely non-perturbative.

The fact that there is an endpoint, was first reliably demonstrated in [11,12]. The endpoint location was determined more precisely in [13]. A continuum extrapolation of the endpoint location was made in [14], employing improvement

formulas derived in [19]:

$$x_c = 0.0983(15), \quad y_c = -0.0175(13). \quad (5)$$

In [14], it was also shown that the endpoint belongs to the 3d Ising universality class.

The values in Eq. (5) can be converted to the endpoint locations in different 4d physical theories, using the relations derived in [8]. Some values are given in Table 1. The errors here represent the errors in Eq. (5): no additional errors have been added from dimensional reduction.

With 4d simulations, the endpoint location in the $SU(2)+\text{Higgs}$ model has been studied at a fixed (symmetric) lattice spacing in [15,16], and with an asymmetric lattice spacing in [17,18]. A continuum extrapolation has been carried out in [18], and that result is shown in Table 1. It should be noted that the exact $\overline{\text{MS}}$ gauge coupling to which the 4d simulations correspond, is not known. This affects strongly the critical temperature ($T_c \propto m_H/g$), while the endpoint location itself is not that sensitive.

We can now compare the 3d and 4d results for $SU(2)+\text{Higgs}$. Clearly, they are completely compatible.

Finally, consider the effect of $\sin^2 \theta_W$. In general, the hypercharge $U(1)$ group makes the transition slightly stronger, though not by very much [10]. Thus one might also expect that the endpoint location changes to somewhat larger x than in Eq. (5). The infinite volume and continuum extrapolation of the endpoint location has not been determined with $\sin^2 \theta_W = 0.23$, but finite volumes have been studied in [20]. On a lattice with $4/(g_3^2 a) = 8$ and volume = 32^3 , we get

$$\begin{aligned} x_c^0 &= 0.1043(22), & y_c^0 &= -0.02860(99), \\ x_c^1 &= 0.1045(14), & y_c^1 &= -0.02125(76), \end{aligned} \quad (6)$$

where $(^0)$ refers to $\sin^2 \theta_W = 0$ and $(^1)$ to $\sin^2 \theta_W = 0.23$. Hence x_c does not appear to depend significantly on $\sin^2 \theta_W$, while y_c changes a bit. Assuming that the same pattern remains there at the infinite volume and continuum limits, the endpoint location in physical units is given in Table 1 also for $\sin^2 \theta_W = 0.23$.

Recent topics of interest, other than the endpoint location, include the excitation spectrum

Table 1

The endpoint location in different theories. No dimensional reduction error estimates have been added to the 3d results here. The values cited for $\sin^2 \theta_W = 0.23$ assume that the finite volume and lattice spacing effects do not depend on $\sin^2 \theta_W$, see the text. The comparison of critical temperatures between 4d and 3d is sensitive to the relation of $g_{\overline{\text{MS}}}^2$ and g_R^2 , which is not known, while $m_{H,c}$ is much less sensitive.

Method	Theory	Couplings		Endpoint location	
		$\sin^2 \theta_W$	$g_{\overline{\text{MS}}}^2(m_W)$	$m_{H,c}/\text{GeV}$	$T_c(m_{H,c})/\text{GeV}$
3d	Standard Model	0	0.426	72.2(7)	110.1(8)
	Standard Model	0.23	0.426	72.3(7)	109.2(8)
	SU(2)+Higgs	0	0.57(2)	65.9(7)	133(4)
4d [18]	SU(2)+Higgs	0	$g_R^2 \approx 0.57(2)$	66.5(14)	128(6)

around the endpoint [21], and the behaviour of some topology related observables [22].

All in all, we can summarize the main lattice results for the Standard Model as follows:

Perturbation theory: The non-perturbative transition is weaker than in perturbation theory and ends at $m_H = 72(2)$ GeV.

Cosmology: Although in principle all the ingredients for baryogenesis are there already in the Standard Model, in practice one needs something more, to have a first order transition.

Dimensional reduction: In the first order regime, the results from 4d and 3d have been observed to be completely compatible [23]. Now a similar agreement has been demonstrated in the non-perturbative endpoint regime [18]. Thus we can be confident that the accuracy estimates of dimensional reduction are reliable. This is important since theories with chiral fermions can presently only be studied with the 3d approach.

4.2. MSSM

In contrast to the Standard Model, baryogenesis could in principle work in the MSSM. This is because (1) there can be more CP-violation in the MSSM, due to new complex phases in the scalar sector (see Sec. 5), and because (2) there can be a stronger transition in the MSSM, due to a larger number of light bosonic degrees of freedom, *viz.* the squarks. Thus MSSM is a natural candidate for electroweak baryogenesis.

The parameter regime relevant for MSSM baryogenesis has been studied extensively in perturbation theory. The relevant case has been found to be that the right-handed stops are light,

$m_{\bar{t}_R} < m_{\text{top}}$, and the Higgs mass is anything allowed by experiment and by the MSSM, $m_H = 90\dots 110$ GeV [24–27].

As was the case for the Standard Model, perturbative estimates are nevertheless not necessarily reliable. Thus, the transition has been studied with 3d simulations [28].

The light degrees of freedom appearing in the 3d effective theory are now the spatial components of the SU(2) and SU(3) gauge fields, the right-handed stop U , and the combination H of the two Higgses H_1, \bar{H}_2 which is light at the phase transition point. The corresponding action is

$$\begin{aligned} \mathcal{L}_{3d} = & \frac{1}{4} F_{ij}^a F_{ij}^a + \frac{1}{4} G_{ij}^A G_{ij}^A + \gamma_3 H^\dagger H U^\dagger U \quad (7) \\ & + (D_i^w H)^\dagger (D_i^w H) + m_{H3}^2 H^\dagger H + \lambda_{H3} (H^\dagger H)^2 \\ & + (D_i^s U)^\dagger (D_i^s U) + m_{U3}^2 U^\dagger U + \lambda_{U3} (U^\dagger U)^2. \end{aligned}$$

Here D_i^w, D_i^s are the SU(2) and SU(3) covariant derivatives, and F_{ij}^a, G_{ij}^A are the corresponding field strengths. The parameters of this action can be determined in the standard way [28].

To simulate the theory in Eq. (7), is straightforward but technically demanding. For details, we refer to [28].

The basic result of the simulations is shown in Fig. 2. There we show the Higgs field expectation value after the transition, for $m_H = 95$ GeV, as a function of a parameter \tilde{m}_U which determines the zero temperature right-handed stop mass through $m_{\bar{t}_R} \approx (m_{\text{top}}^2 - \tilde{m}_U^2)^{1/2} \approx 160\dots 150$ GeV. The lattice results are compared with 2-loop perturbation theory (the solid line).

Based on Fig. 2, we can summarize the lattice

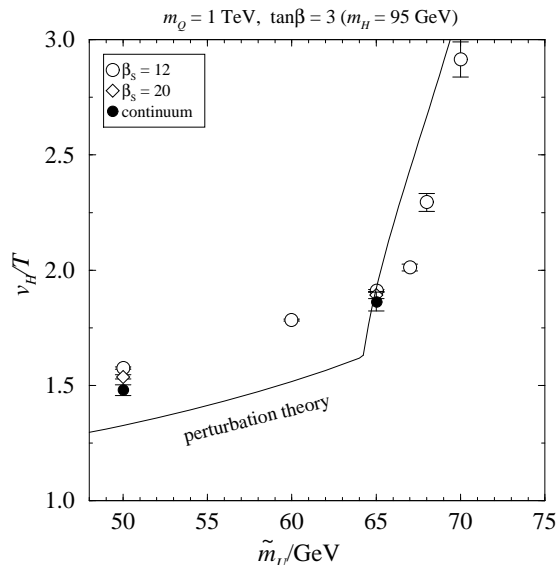


Figure 2. The Higgs field expectation value in the MSSM [28], compared with 2-loop perturbation theory. For $\tilde{m}_U \gtrsim 64\dots 67$ GeV, there is a two-stage transition, and the values of v_H/T shown represent the situation after the latter stage.

results for the MSSM electroweak phase transition as follows:

Perturbation theory: In contrast to the Standard Model, the electroweak phase transition in the MSSM can be stronger than in 2-loop perturbation theory, even for large m_H (although the difference is not very large). Thus perturbation theory gives a conservative estimate.

Cosmology: If the non-perturbative strengthening effect remains there also for larger Higgs masses than shown in Fig. 2, then Higgs masses up to 105...110 GeV are allowed for baryogenesis, provided that $m_{\tilde{t}_R} < m_{t_{\text{op}}}$ [25,26].

5. FUTURE PROSPECTS

As we have seen, lattice simulations have been quite successful in solving the problem of the electroweak phase transition: the case of the Standard Model is now completely understood, and similar techniques have been applied also in the MSSM. One has also been able to demonstrate

with 4d simulations that the dimensionally reduced 3d theory works for non-perturbative quantities as well as for perturbative ones: the non-perturbative infrared dynamics is really three-dimensional, as it has to be (see also [29]).

What is it then that still remains to be done? We list here a few open questions, related in particular to the MSSM:

1. What happens for other parameter values in the MSSM (a larger m_H , non-vanishing squark mixing, etc), when the transition gets weaker?

2. Could 4d simulations be used to estimate non-perturbatively the accuracy of dimensional reduction in a theory similar to the MSSM? Could 4d simulations be used in some regions of the parameter space [30] where the theory is not weakly coupled at zero temperature, and thus dimensional reduction does not work?

3. Finally, can one say something about CP-violation with simulations? The existence of CP-violation is one of the ingredients for electroweak baryogenesis: otherwise, one produces the same amounts of baryons and anti-baryons, and no net asymmetry arises. Let us discuss the last question in some more detail.

To get enough CP-violation for producing the observed baryon asymmetry, turns out to be a non-trivial requirement [2]. It is hence an interesting prospect that there are new sources of CP-violation in the MSSM, related to the trilinear couplings of Higgses and the squarks.

An intriguing observation is now that since one has two Higgses in the MSSM, the effect of the new CP-violating parameters can also propagate to a CP-violating phase between the Higgses. Thus, the idea arises that maybe there is a non-trivial profile of the CP-violating phase at the phase boundary between the broken and symmetric phases [31–33]. The phase boundary is the region relevant for baryogenesis, and this scenario might allow for sufficient CP-violation, without violating the constraints that have to be satisfied in the broken phase.

It appears that whether such a phenomenon takes place, can again be studied non-perturbatively with a simple 3d theory. The theory in Eq. (7) is not enough, though: in that case, we were interested in the strength of the tran-

sition, and it was sufficient to study a particular linear combination of H_1, \tilde{H}_2 . Now we are interested in a question for which both of the Higgses H_1, \tilde{H}_2 should be kept in the effective theory. The effective theory will then be more complicated than in Eq. (7), but can be derived with precisely the same methods. In such a theory, there are new CP-violating operators, e.g. $\text{Im} H_1^\dagger \tilde{H}_2$, which could have a non-trivial profile at the phase boundary between the symmetric and broken phases. This problem could, in principle, be solved with lattice simulations.

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