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HARD THERMAL LOOPS IN CHIRAL PERTURBATION THEORY

Cristina Manuel

*Theory Division, CERN**CH-1211, Geneva 23, Switzerland**(Talk given at the 5th International Workshop on**Thermal Field Theories and their Applications, Regensburg (Germany), August 1998)*

It is shown how the hard thermal loop approximation can be used in chiral perturbation theory to study some thermal properties of Goldstone bosons. Hard thermal effects are first studied in the non-linear sigma model. Then those results are used to obtain the thermal corrections to the transverse and longitudinal gauge field masses in the electroweak theory in the limit of a strongly interacting Higgs boson.

I. INTRODUCTION

This talk is devoted to give a brief account on the emergence of hard thermal loops [1] in the framework of chiral perturbation theory (χ PT) [2–4]. First, the one-loop thermal effects arising in the non-linear sigma model will be discussed [5,6]. Second, it will be shown how to use the previous results in the electroweak model in the limit where the Higgs boson becomes strongly interacting [7].

The goal of this talk is showing how at a very “cheap” price one can obtain the one-loop thermal effective action for soft fields in the two above mentioned theories. In order to do so one only needs to understand the symmetry principles which lie behind hard thermal loops, and how they were discovered in the context of the high temperature T phase of QCD.

Let us first review the high T phase of QCD and part of the progress the community achieved in the last years [8]. The motivation which led to the discovery of HTL’s was the failure of the naive one-loop perturbative analysis in that regime of the theory. This problem was solved by the now classical works of Braaten and Pisarski [1], with also important contribution of other groups. As we learned from those works, the naive one-loop computations at finite T are not complete, since there are one-loop Feynman diagrams, the *hard thermal loops* (HTL’s), which are as important as the tree amplitudes, and therefore they

have to be included consistently in all contributions to non-trivial order in the gauge coupling constant.

Since the pioneering work on HTL’s, it is much what we have learned about them: their interesting symmetry properties [9], the construction of effective actions $\Gamma_{HTL}[A]$ from different approaches [10], and the success, as well as limitations, of the resummation techniques [11].

In this talk it will be shown how the same HTL’s give account of thermal properties of Goldstone bosons.

II. QCD AT LOW ENERGIES AND TEMPERATURES

If quarks were massless then the QCD Lagrangian would have an exact global symmetry $SU_R(N_f) \times SU_L(N_f)$, where N_f is the number of quark flavors¹. The QCD spectrum of particles indicates that this global symmetry is spontaneously broken down to $SU_{R+L}(N_f)$. Associated to the spontaneous breaking of chiral symmetry there are $N_f^2 - 1$ Goldstone bosons. For $N_f = 3$ those are the octet of pseudoscalar mesons (π ’s, K ’s, η). The above picture is only approximately valid, since quarks have a non-vanishing mass, and thus the (pseudo) Goldstone bosons are not massless.

It is possible to study the low energy physics of QCD in the framework of chiral perturbation theory (χ PT) [2–4]. At low energies the physics of the strong interactions is dominated by the lightest particles of the QCD spectrum, the (pseudo) Goldstone bosons.

In this talk we will mainly be concerned in studying thermal effects in χ PT [12] in the chiral limit, that is,

¹Actually, the global symmetry is larger, but it is broken by quantum effects that will not be discussed here.

in the limit of massless quarks. This will allow us to understand the properties of a thermal gas of Goldstone bosons. The validity of this analysis will be restricted to low T . The reason for this being so is that at low T the strong interactions should be dominated by the lightest particles, that is, the Goldstone bosons. At higher T the contribution in the partition function of heavier particles of the QCD spectrum would start to become relevant.

A chiral Lagrangian is expanded in derivatives of the Goldstone fields and also in the explicit chiral symmetry breaking parameters, such as the masses of the three light quarks. The perturbative series in χ PT is not written in terms of a coupling constant, but rather on the energies and masses of the pseudoscalar mesons.

The lowest order chiral Lagrangian is [3]

$$\mathcal{L}_2 = \frac{f_\pi^2}{4} (Tr(\nabla_\mu \Sigma^\dagger \nabla_\mu \Sigma) + Tr(\chi^\dagger \Sigma + \chi \Sigma^\dagger)) , \quad (2.1)$$

where Σ is a $SU(N_f)$ matrix, which is written in term of the pseudoscalar mesons ϕ as $\Sigma = \exp(i\phi/f_\pi)$, and $f_\pi = 92.4$ MeV can be identified, to first order, with the pion decay constant. The covariant derivative is defined as

$$\nabla_\mu \Sigma = \partial_\mu \Sigma - i(v_\mu + a_\mu)\Sigma + i\Sigma(v_\mu - a_\mu) , \quad (2.2)$$

v_μ and a_μ being external vector and axial vector sources, respectively. The field $\chi = B(s + ip)$, where B is related to the quark condensate and s and p are scalar and pseudoscalar external sources, respectively.

For the time being all the external sources will be taken as $v_\mu = a_\mu = s = p = 0$. The generalization of the present analysis in the presence of external background sources is rather straightforward, as we will see later on.

In the absence of external sources the Lagrangian (2.1) reduces to that of a non-linear sigma model. The non-linear sigma model has a global $SU_R(N_f) \times SU_L(N_f)$ symmetry, where the field Σ transform as $\Sigma'(x) = U_R \Sigma(x) U_L^\dagger$, and $U_{R,L} \in SU_{R,L}(N_f)$.

To obtain the one-loop effective action the background field method (BFM) will be used. The BFM is a standard technique to evaluate the loop effects generated by a Lagrangian and consists in expanding it around the solution of the classical equations of motion. The one-loop effective action is then obtained after integrating out the quantum fluctuations.

In our case one defines

$$\Sigma(x) = \xi(x)h(x)\xi(x) , \quad (2.3)$$

where $\bar{\Sigma} = \xi^2$ is the classical solution to the equations of motion and h is the quantum field. At this point one writes $h = \exp(i\tilde{\phi}/f_\pi)$, where $\tilde{\phi}$ is a traceless and hermitian matrix, and expands the exponentials, keeping only terms up and including the quadratic in $\tilde{\phi}$ in the Lagrangian $\mathcal{L}_2 = \mathcal{L}_2^{(0)} + \mathcal{L}_2^{(2)} + \dots$. It is possible to express the above terms as [3]

$$\mathcal{L}_2^{(0)} = -f_\pi^2 Tr(\Delta_\mu)^2 \quad (2.4)$$

$$\mathcal{L}_2^{(2)} = \frac{1}{4} Tr(d_\mu \tilde{\phi})^2 - \frac{1}{4} Tr([\Delta_\mu, \tilde{\phi}]^2) , \quad (2.5)$$

where

$$d_\mu \tilde{\phi} = \partial_\mu \tilde{\phi} + [\Gamma_\mu, \tilde{\phi}] , \quad (2.6a)$$

$$\Gamma_\mu = \frac{1}{2} (\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger) , \quad (2.6b)$$

$$\Delta_\mu = \frac{1}{2} (\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger) , \quad (2.6c)$$

The transformation properties of ξ and h under the global $SU_R(N_f) \times SU_L(N_f)$ symmetry are

$$\xi'(x) = U_R \xi(x) U_L^\dagger(x) = U(x) \xi(x) U_L^\dagger , \quad (2.7a)$$

$$h'(x) = U(x) h(x) U^\dagger(x) , \quad (2.7b)$$

where U is a unitary matrix which depends on $\xi(x)$, U_R and U_L^\dagger . The transformation rules obeyed by the new fields are then deduced from (2.7a) - (2.7b)

$$\tilde{\phi}'(x) = U(x) \tilde{\phi}(x) U^\dagger(x) , \quad (2.8a)$$

$$\Gamma'_\mu(x) = U(x) \Gamma_\mu(x) U^\dagger(x) + U(x) \partial_\mu U^\dagger(x) , \quad (2.8b)$$

$$\Delta'_\mu(x) = U(x) \Delta_\mu(x) U^\dagger(x) . \quad (2.8c)$$

The field Γ_μ transforms like a connection, while Δ_μ and $\tilde{\phi}$ transforms covariantly. Thus, it can be immediately checked that under the above symmetry each term in (2.5) remains invariant. The Lagrangian (2.5) looks formally as the bosonic matter part of the Lagrangian of a non-Abelian gauge theory, Γ_μ being the corresponding vector gauge field, and $\tilde{\phi}$ being the bosonic field. There is also an additional coupling between the Δ_μ and $\tilde{\phi}$ fields, but apart from that, things look the same as in a non-Abelian gauge theory. This parallelism with a gauge field

theory is just formal, since there is not a kinetic term for Γ_μ , neither for Δ_μ , and thus those fields do not propagate. However, this parallelism will prove to be very useful to find the one-loop thermal effective action for soft background fields.

The one-loop effective action of the non-linear sigma model is obtained by integrating out the $\tilde{\phi}$ fields. At zero temperature it can be done by evaluating the determinant of a differential operator, since the action is quadratic in the $\tilde{\phi}$ fields². We will not consider here the $T = 0$ one-loop effective action (see Ref. [3] for that), and concentrate only on the thermal part.

At this point it seems very obvious that for external background fields with soft momenta, that is $\ll T$, the one-loop thermal effective action generated after integrating out the $\tilde{\phi}$ fields in the first term in (2.5) is *exactly the same* as the one that would emerge in a real gauge field theory, if Γ_μ were a real background gauge field. In this way, it is very easy to understand that also HTL's appear in this model, and that the effective action $\Gamma_{HTL}[\Gamma_\mu]$ also arises naturally. There is also a tadpole diagram generated by the last term of (2.5). Finally, one finds [6]

$$S_2 + \delta S_{2,T} = -f_\pi^2(T) \int d^4x Tr(\Delta_\mu^2(x)) \quad (2.9)$$

$$- \frac{NT^2}{12} \int \frac{d\Omega_{\mathbf{q}}}{4\pi} \int d^4x d^4y Tr \left(\Gamma_{\mu\lambda} \frac{Q^\mu Q_\nu}{-(Q \cdot d)^2} \Gamma^{\nu\lambda} \right),$$

where

$$\Gamma_{\mu\nu} = \partial_\mu \Gamma_\nu - \partial_\nu \Gamma_\mu + [\Gamma_\mu, \Gamma_\nu], \quad (2.10)$$

and $Q = (i, \mathbf{q})$ is a null vector $Q^2 = 0$. The angular integral in (3.13) is done over all directions of the three dimensional unit vector \mathbf{q} . The second term of the r.h.s. of Eq. (2.9) it is just $\Gamma_{HTL}[\Gamma_\mu]$ as written in Ref. [9].

Hard thermal effects change f_π into $f_\pi(T)$ as

$$f_\pi(T) = f_\pi \left(1 - \frac{N T^2}{24 f_\pi^2} \right) \quad (2.11)$$

which agrees with the result computed in the literature (see Ref. [12,13,5]).

²Note also that the jacobian of the change of variables is one at one-loop order.

Although the same one-loop thermal effective action appears in two rather different models, it should be clear that their respectively physical meanings are completely different. In a gauge field theory, $\Gamma_{HTL}[A_\mu]$ is a gauge invariant Debye mass term for the chromoelectric fields. In the non-linear sigma model $\Gamma_{HTL}[\Gamma_\mu]$ describes thermal scattering among Goldstone bosons, respectful with the symmetries of the model. This can be checked by writing $\xi = \exp(i\phi/2f_\pi)$, and expanding the exponentials, so that

$$\Delta_\mu = \frac{i}{2f_\pi} \partial_\mu \phi + O\left(\frac{1}{f_\pi^3}\right) \quad (2.12)$$

$$\Gamma_\mu = \frac{1}{8f_\pi^2} [\phi, \partial_\mu \phi] + O\left(\frac{1}{f_\pi^4}\right). \quad (2.13)$$

When the above expressions are plugged into the effective action $\Gamma_{HTL}[\Gamma_\mu]$ one can then read the thermal corrections to the four point functions, six point functions, etc.

Finally, let us explain how one can generalize the above analysis in the presence of external vector v_μ and axial vector a_μ currents. Introducing the combinations

$$F_\mu^R = v_\mu + a_\mu, \quad F_\mu^L = v_\mu - a_\mu, \quad (2.14)$$

and changing the definitions of the Γ_μ and Δ_μ fields as follows

$$\Gamma_\mu = \frac{1}{2} (\xi^\dagger \nabla_\mu^R \xi + \xi \nabla_\mu^L \xi^\dagger), \quad (2.15a)$$

$$\Delta_\mu = \frac{1}{2} (\xi^\dagger \nabla_\mu^R \xi - \xi \nabla_\mu^L \xi^\dagger), \quad (2.15b)$$

$$\nabla_\mu^l = \partial_\mu - iF_\mu^l, \quad l = R, L, \quad (2.15c)$$

the same one-loop thermal effective action Eq. (2.9) for soft fields is found after integrating out the ϕ field.

III. THE ELECTROWEAK MODEL IN THE STRONGLY INTERACTING HIGGS BOSON LIMIT

In the previous Section the one-loop thermal effective action in a theory describing the interactions of soft Goldstone bosons has been computed. In this Section we will consider a theory where the Goldstone bosons are eaten by gauge fields to become massive: we will consider the $SU_W(2) \times U_Y(1)$ electroweak model.

The bosonic sector of the electroweak model can be written as [14]

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} \text{Tr}(W_{\mu\nu}W^{\mu\nu}) - \frac{1}{4} B_{\mu\nu}B^{\mu\nu} \\ & + \frac{1}{4} \text{Tr}(D_\mu M D^\mu M^\dagger) - \frac{\lambda}{4} \left(\frac{1}{2} \text{Tr}(MM^\dagger) - \frac{\mu^2}{\lambda} \right)^2, \end{aligned} \quad (3.1)$$

The covariant derivative acting on the matrix M is

$$D_\mu M = \partial_\mu M + ig W_\mu M - i \frac{g'}{2} M B_\mu \tau^3. \quad (3.2)$$

The matrix M is written in terms of the physical Higgs field H and the would-be Goldstone bosons ϕ^a as

$$M(x) = (v + H(x)) \Sigma(x), \quad \Sigma(x) = \exp(i \frac{\vec{\phi} \cdot \vec{\tau}}{v}) \quad (3.3)$$

where $v = \sqrt{\mu^2/\lambda}$ is the vacuum expectation value.

This non-linear representation of the Higgs sector is very suited to study the model in the strongly interacting limit $\lambda \rightarrow \infty$. In that limit the Higgs mass, $M_H = \sqrt{2\lambda v^2}$, becomes large, and the Higgs field can be integrated out. Then the effective Lagrangian of the electroweak theory reduces at tree level to [14]

$$\begin{aligned} \mathcal{L}_{eff} = & -\frac{1}{2} \text{Tr}(W_{\mu\nu}W^{\mu\nu}) - \frac{1}{4} B_{\mu\nu}B^{\mu\nu} \\ & + \frac{v^2}{4} \text{Tr}(D_\mu \Sigma D^\mu \Sigma^\dagger). \end{aligned} \quad (3.4)$$

That is, the low energy effective theory for the bosonic sector of the electroweak model is a gauged $N = 2$ non-linear sigma model.

In the unitary gauge, that is, in the gauge where all the would-be Goldstone bosons are eaten by the gauge fields (i.e. where $\Sigma = 1$), one can read off the masses of the physical gauge fields from Eq. (3.4). The fields W_μ^+ , Z_μ and A_μ are defined as

$$W_\mu^\mp = \frac{1}{\sqrt{2}} (W_\mu^1 \pm iW_\mu^2), \quad (3.5a)$$

$$Z_\mu = \cos \theta_W W_\mu^3 - \sin \theta_W B_\mu, \quad (3.5b)$$

$$A_\mu = \sin \theta_W W_\mu^3 + \cos \theta_W B_\mu, \quad (3.5c)$$

where θ_W is the Weinberg angle, $\tan \theta_W = g'/g$. The masses of those fields are

$$M_W = \frac{vg}{2}, \quad M_Z = \frac{v}{2} \sqrt{g^2 + g'^2}, \quad (3.6)$$

while the mass of the photon is $M_\gamma = 0$. Recall that the electric charge is defined as

$$e = \frac{gg'}{\sqrt{g^2 + g'^2}}. \quad (3.7)$$

The one-loop effective action associated to the Lagrangian (3.4) can be computed with the help of the background field method. The gauge fields are split into background and quantum gauge fields as follows

$$W_\mu(x) = \bar{W}_\mu(x) + w_\mu(x), \quad (3.8a)$$

$$B_\mu(x) = \bar{B}_\mu(x) + b_\mu(x). \quad (3.8b)$$

The background gauge fields have been represented by upper case letters with a bar, while the quantum ones are denoted by lower case letters. The matrix Σ is split multiplicatively, exactly as it was done in Eq. (2.3).

After the splitting of fields is done, the Lagrangian (3.4) is separately invariant under background and quantum gauge transformations.

In the spirit of the BFM the Lagrangian \mathcal{L}_{eff} is expanded around the classical fields, keeping terms which are quadratic in the quantum fluctuations. To derive the one-loop effective action one has to integrate out the quantum fields, adding before the corresponding quantum gauge-fixing and quantum Faddeev-Popov terms.

In the BFM it is possible to fix the background and quantum gauges independently. In our case, and to simplify the computations, it is convenient to choose the unitary gauge for the background fields. Then the background Goldstone fields disappear completely from the Lagrangian, since they are eaten by the background gauge fields to become massive. In order to do so it is convenient to use the Stueckelberg formalism [15].

If one performs the Stueckelberg transformation

$$\bar{W}'_\mu = \xi^\dagger \bar{W}_\mu \xi - \frac{i}{g} \xi^\dagger \partial_\mu \xi, \quad (3.9a)$$

$$\bar{B}'_\mu \frac{\tau^3}{2} = \xi (\bar{B}_\mu \frac{\tau^3}{2}) \xi^\dagger - \frac{i}{g'} \xi \partial_\mu \xi^\dagger, \quad (3.9b)$$

$$w'_\mu = \xi^\dagger w_\mu \xi, \quad b'_\mu \frac{\tau^3}{2} = \xi (b_\mu \frac{\tau^3}{2}) \xi^\dagger, \quad (3.9c)$$

and one writes the Lagrangian in terms of the primed fields, all the background fields ξ disappear completely!

The Stueckelberg transformation simplifies drastically the one-loop computations. Once the computation is finished, the Stueckelberg transformation has to be inverted to recover the presence of the background Goldstone bosons in the final one-loop effective action.

In order to simplify the notation from now on I will omit the primes in the fields, keeping in mind that the transformation has to be inverted at the end of the computation.

To integrate out the quantum fields a quantum gauge fixing condition invariant under the background gauge transformation has to be given. In the unitary background gauge the gauge fixing condition for the quantum fields is chosen as

$$\mathcal{L}_{gf}^{(2)} = -\frac{1}{a_w} \text{Tr} \left(\bar{D}_W^\mu w_\mu - \frac{1}{4} a_w g v \phi \right)^2 \quad (3.10)$$

$$- \frac{1}{2a_b} \left(\partial^\mu b_\mu + \frac{1}{2} a_b g' v \phi_3 \right)^2,$$

where a_w and a_b are the gauge fixing parameters. These gauge fixing terms are chosen such as to cancel the unwanted pieces $\partial^\mu b_\mu \phi_3$ and $\text{Tr}(\partial^\mu w_\mu \phi)$ in $\mathcal{L}_{eff}^{(2)}$. The form of the gauge fixing term in an arbitrary background gauge can be obtained by inverting the Stueckelberg transformation.

The Faddeev-Popov terms associated to the gauge fixing (3.10) are computed as usual. Finally, the complete one-loop quantum Lagrangian reads in Minkowski space

$$\begin{aligned} & \mathcal{L}_{eff}^{(2)} + \mathcal{L}_{gf}^{(2)} + \mathcal{L}_{FP}^{(2)} \quad (3.11) \\ &= \text{Tr} \left(w_\mu (g^{\mu\nu} \bar{D}_W^2 + \frac{1-a_w}{a_w} \bar{D}_W^\mu \bar{D}_W^\nu + 2ig\bar{W}^{\mu\nu}) w_\nu \right) \\ &+ \frac{1}{2} b_\mu \left(g^{\mu\nu} \partial^2 + \frac{1-a_b}{a_b} \partial^\mu \partial^\nu \right) b_\nu \\ &+ M_W^2 \text{Tr}(w_\mu w^\mu) + \frac{M_B^2}{2} b_\mu b^\mu - gg' v^2 w_\mu^3 b^\mu \\ &+ \frac{1}{4} \text{Tr}(\bar{d}_\mu \phi)^2 - \frac{1}{4} \text{Tr}[\bar{\Delta}_\mu, \phi]^2 - \frac{a_w M_W^2}{4} \text{Tr} \phi^2 - \frac{a_b M_B^2}{2} \phi_3^2 \\ &+ 2v \text{Tr} \left((g w_\mu - g' b_\mu \frac{\tau^3}{2}) \bar{\Gamma}^\mu \phi \right) \\ &- \eta_a^\dagger (\delta^{ab} \bar{D}_W^2 + \delta^{ab} a_w M_W^2) \eta_b \end{aligned}$$

where $M_B^2 = g'^2 v^2 / 4$ and

$$\bar{D}_W^\mu = \partial^\mu + ig[\bar{W}^\mu, \cdot], \quad (3.12a)$$

$$\bar{d}_\mu \phi = \partial_\mu \phi + [\bar{\Gamma}_\mu, \phi], \quad (3.12b)$$

$$\bar{\Gamma}_\mu = \frac{i}{2} \left(g \bar{W}_\mu + g' \bar{B}_\mu \frac{\tau^3}{2} \right), \quad (3.12c)$$

$$\bar{\Delta}_\mu = \frac{i}{2} \left(g \bar{W}_\mu - g' \bar{B}_\mu \frac{\tau^3}{2} \right). \quad (3.12d)$$

The ghost fields η_a are associated to the w_μ^a quantum fields. Since the ghost associated to the b_μ field does not couple to any background external field, it has been omitted in Eq. (3.11).

The one-loop Lagrangian (3.11) is written in the unitary background gauge. It can be obtained in an arbitrary background gauge by inverting the Stueckelberg transformation. However, it is much simpler to integrate out the quantum fields first, and invert the transformation afterwards to obtain the one-loop effective action in a general background gauge.

For soft background fields the leading thermal corrections arise when the internal quantum fields are hard [1], that is, of energy $\sim T$. If one neglects corrections of order M_W/T and M_Z/T in the final answers, then it is possible to neglect those masses for the quantum fields. In other words, for hard quantum fields the terms ∂^2 of the Lagrangian are of the order T^2 , which are dominant as compared to the terms $M_{W,B}^2$, which therefore will be neglected.

The computation simplifies once the masses of the quantum fields are neglected. One encounters here the same one-loop thermal amplitudes, the HTL's, which appear in the BFM of Yang-Mills theories [1], as well as in the non-linear sigma model in the presence of external background sources [6]. There are also new types of vertices in (3.11), which do not appear in the BFM studies of the previous mentioned theories: those which couple quantum gauge fields and quantum Goldstone bosons. However, a power counting analysis shows that the one-loop thermal corrections generated by those vertices are subleading as compared to the HTL's, and therefore they will be neglected.

The one-loop thermal effective action for soft background gauge fields is then a combination of the one which appears in a Yang-Mills theory and the one in the non-linear sigma model in the presence of external sources. By translating those results to our case one finds the following one-loop thermal effective action [7]

$$\begin{aligned} & S_{eff} + \delta S_{eff,T} = \quad (3.13) \\ & \int d^4x \left\{ -\frac{1}{2} \text{Tr}(\bar{W}_{\mu\nu} \bar{W}^{\mu\nu}) - \frac{1}{4} \bar{B}_{\mu\nu} \bar{B}^{\mu\nu} \right. \\ & \left. + \frac{v^2(T)}{4} \text{Tr} \left(g \bar{W}^\mu - g' \bar{B}^\mu \frac{\tau^3}{2} \right)^2 \right\} \\ & - \frac{T^2}{6} \int \frac{d\Omega_{\mathbf{q}}}{4\pi} \int d^4x d^4y \text{Tr} \left(\bar{\Gamma}_{\mu\lambda} \frac{Q^\mu Q_\nu}{-(Q \cdot \bar{d})^2} \bar{\Gamma}^{\nu\lambda} \right) \\ & + \frac{g^2 T^2}{3} \int \frac{d\Omega_{\mathbf{q}}}{4\pi} \int d^4x d^4y \text{Tr} \left(\bar{W}_{\mu\lambda} \frac{Q^\mu Q_\nu}{-(Q \cdot \bar{D}_W)^2} \bar{W}^{\nu\lambda} \right) \end{aligned}$$

where $\bar{W}_{\mu\nu}, \bar{B}_{\mu\nu}$ are the field strengths of the corresponding background gauge fields, and

$$v(T) = v \left(1 - \frac{1}{12} \frac{T^2}{v^2} \right), \quad (3.14)$$

$$\bar{\Gamma}_{\mu\nu} = \partial_\mu \bar{\Gamma}_\nu - \partial_\nu \bar{\Gamma}_\mu + [\bar{\Gamma}_\mu, \bar{\Gamma}_\nu], \quad (3.15)$$

Let us remind the meaning of each term of Eq. (3.13). The two first terms are the kinetic pieces for the soft background gauge fields. The last piece in Eq. (3.13) is the HTL effective action for the non-Abelian gauge field \bar{W}_μ , and it is generated by considering the one-loop thermal effects of the hard quantum gauge field w_μ^a , and the quantum ghosts η^a [1]. The third and fourth terms in Eq. (3.13) arise after considering the one-loop thermal effects of the hard quantum Goldstone bosons ϕ^a , (see the previous Section with the following identifications: $F_\mu^R = -g\bar{W}_\mu$, $F_\mu^L = -g'\bar{B}_\mu \frac{\tau^3}{2}$, and $\xi = \xi^\dagger = 1$).

In the static limit the non-local terms of Eq. (3.13) become local, and then apart from the kinetic terms for the gauge fields, one has

$$\begin{aligned} \delta\mathcal{L}_{eff,T}^{static} &= \frac{v^2(T)}{4} \text{Tr} \left(g\bar{W}^\mu - g'\bar{B}^\mu \frac{\tau^3}{2} \right)^2 \\ &+ \frac{T^2}{12} \text{Tr} \left(g\bar{W}_0 + g'\bar{B}_0 \frac{\tau^3}{2} \right)^2 + \frac{2g^2T^2}{3} \text{Tr}(\bar{W}_0)^2. \end{aligned} \quad (3.16)$$

If one expresses Eq. (3.16) in terms of the physical fields \bar{W}^+ , \bar{Z}_μ and \bar{A}_μ , one obtains the corrections to their longitudinal and transverse masses, plus couplings of the \bar{Z}^0 with \bar{A}^0 fields.

The longitudinal and transverse gauge modes get different thermal corrections to their masses. The thermal masses for the transverse modes are

$$M_{W,t}^2(T) = \frac{g^2v^2(T)}{4}, \quad (3.17)$$

$$M_{Z,t}^2(T) = \frac{(g^2 + g'^2)v^2(T)}{4}, \quad (3.18)$$

$$M_{\gamma,t}^2(T) = 0, \quad (3.19)$$

while for the longitudinal ones are

$$M_{W,l}^2(T) = \frac{g^2v^2(T)}{4} + \frac{3g^2T^2}{4}, \quad (3.20)$$

$$M_{Z,l}^2(T) = \frac{(g^2 + g'^2)v^2(T)}{4} + \frac{(g^2 + g'^2)T^2}{12} \quad (3.21)$$

$$\begin{aligned} &\times (\cos^2 \theta_W - \sin^2 \theta_W)^2 + \frac{2g^2T^2}{3} \cos^2 \theta_W, \\ M_{\gamma,l}^2(T) &= e^2T^2. \end{aligned} \quad (3.22)$$

To express the electric thermal mass of the photon in terms of the electric charge e , use of the relation (3.7) has been made. The above results agree were first obtained in Ref. [16].

IV. CONCLUSIONS

It has been shown how in the chiral limit hard thermal loops appear in the framework of χ PT. This fact allows us to study certain thermal properties of Goldstone bosons with ease, both in a theory where those are real particles, or where those are unphysical since they are eaten by the gauge fields to become massive.

Some other applications of the HTL's techniques in χ PT have already been exploited. In the literature HTL's have been used to compute thermal corrections to the anomalous decay $\pi^0 \rightarrow \gamma\gamma$ [17], to the Wess-Zumino-Witten action [6], or to the electromagnetic mass difference of pions [18]. Further applications of HTL's in χ PT will surely be found.

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