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WHAT IS THE UPPER LIMIT ON THE LIGHTEST SUPERSYMMETRIC HIGGS MASS?*

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Abstract

In this talk the question of *what is the upper bound on the lightest supersymmetric Higgs mass, m_h* is addressed. This question is relevant since experimental lower bounds on m_h might implement, in the near future, exclusion of supersymmetry. By imposing (perturbative) unification of the gauge couplings at some high scale $\gtrsim 10^{17}$ GeV, we have found that for a top-quark mass $M_t = 175$ GeV, and depending on the supersymmetric parameters, this bound can be as high as 205 GeV.

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1 Introduction: MSSM and gauge unification

Low-energy supersymmetry (MSSM) [1] is a key ingredient in the best-qualified candidate models to supersede the Standard Model (SM) at energies beyond the TeV range. The MSSM is supported both by theoretical arguments and experimental indirect hints. From the theoretical point of view supersymmetry achieves cancellation of ultraviolet quadratic divergences which is welcome to alleviate the hierarchy problem. From the experimental side, using LEP electroweak precision data, the MSSM achieves gauge coupling unification at a value and scale given by

$$\alpha_{\text{GUT}} \sim 1/25, \quad M_{\text{GUT}} \sim 2 \times 10^{16} \text{ GeV} . \quad (1)$$

On the other hand, the extensive experimental search of the (super)-partners of SM elementary particles predicted by supersymmetry (SUSY) has been unsuccessful so far, challenging [2], with the rise of experimental mass limits, the naturalness and relevance of SUSY for electroweak-scale physics.

In this context, the sector of the theory responsible for electroweak symmetry breaking has a special status. While all superpartners of the known Standard Model particles can be made heavy by simply rising soft supersymmetry-breaking mass parameters in the model, the Higgs sector necessarily contains a physical Higgs scalar whose mass does not depend sensitively on the details of soft masses but is fixed by the scale of the electroweak symmetry breaking¹. More in detail, in the MSSM, unlike in the SM, the quartic coupling is not an independent parameter but

$$\lambda = \frac{1}{2}(g_2^2 + \frac{3}{5}g_1^2) \cos^2 2\beta, \quad (2)$$

where $\tan \beta = \langle H_2^0 \rangle / \langle H_1^0 \rangle$ and g_2 and $\sqrt{3/5}g_1$ are the $SU(2)_L \times U(1)_Y$ gauge couplings.

Equation (2) has the important consequence that the value, and renormalization, of the quartic coupling in the MSSM is related to that of $g_2^2 + 3g_1^2/5$. This means in turn that the mass of one (usually the lightest one) of the CP-even scalar mass eigenvalues is not controlled by the SUSY breaking parameters, but by the electroweak breaking ($v = 174.1 \text{ GeV}$). The existence of the SM-like Higgs is a generic feature of all Supersymmetric Standard Models (SSM). In the MSSM the bound

$$m_h^2 \leq M_Z^2 \cos^2 2\beta \leq M_Z^2, \quad (3)$$

follows directly from (2). Eq. (3) seems an indication that non-discovery of the Higgs boson at LEP could rule out the MSSM independently of any direct SUSY searches. In fact, since present experimental bounds at LEP already set fairly stringent bounds on the Higgs mass

$$m_h \gtrsim 90 \text{ GeV}, \quad (4)$$

does it mean that the MSSM is already ruled out?

2 Bounds in the MSSM

Were it not for radiative corrections [4, 5, 6] the MSSM would have already been excluded! The leading effects are included in radiative corrections from the top-stop sector, with $\tilde{t}_{L,R}$

¹Here and throughout we assume symmetry breaking is achieved in a weakly interacting Higgs sector. This is specially well motivated and natural in SUSY models. The reader should be aware of other more complicated possibilities [3].

mixing parameterized by

$$\tilde{A}_t \equiv \frac{A_t - \mu/\tan\beta}{M_{\text{SUSY}}}. \quad (5)$$

In the absence of mixing, leading higher-order corrections [7, 8, 9] are resummed by shifting the energy scale ($M_{\text{SUSY}} \rightarrow \sqrt{M_{\text{SUSY}}m_t}$) in the one-loop result²,

$$\Delta m_{h,\text{rad}}^2 = \frac{3}{4\pi^2 v^2} m_t^4 \left(\sqrt{M_{\text{SUSY}}m_t} \right) \log \frac{M_{\text{SUSY}}^2}{m_t^2}. \quad (6)$$

In the presence of mixing there is an important (threshold) contribution as

$$\Delta m_{h,\text{th}}^2 = \frac{3}{8\pi^2} \frac{m_t^4}{v^2} \tilde{A}_t^2 \left(2 - \frac{1}{6} \tilde{A}_t^2 \right). \quad (7)$$

These corrections push the upper mass limit for the lightest Higgs boson of the MSSM up to 125 GeV (for a top-quark mass $M_t = 175$ GeV and $M_{\text{SUSY}} < 1$ TeV), as can be seen from the lowest curve in the right panel of Fig. 1. So there appears an immediate question: what would happen if (when) experimental bounds on the Higgs mass reach (and overcome) ~ 125 GeV? Should we give up SSMS?

3 Bounds in the MSSM with a gauge singlet

A singlet field S added to the MSSM does not spoil unification and from this point of view it is as legitimate as the MSSM itself. But the S field can couple in the superpotential W to the Higgs sector as

$$W = \lambda_1 S H_1 \cdot H_2, \quad (8)$$

leading to a contribution to the Higgs quartic coupling as

$$\Delta\lambda = \lambda_1^2 \sin^2 2\beta. \quad (9)$$

The contribution (9) can be sizable for small values of $\tan\beta$. An upper triviality limit on λ_1 (and thus on $\lambda(M_Z)$) exists if one requires perturbativity of all parameters of the theory in the energy range [M_Z, M_{GUT}]. In fact the absolute limit is reached when $\lambda_1(Q)$ goes non-perturbative, i.e. when³

$$\frac{\lambda_1^2(M_{\text{GUT}})}{4\pi} = \mathcal{O}(1). \quad (10)$$

This constraint is very strong, since λ_1 increases rapidly with the scale, and the gain in m_h with respect to the MSSM value is modest [10, 11, 12].

Inspection of the renormalization group equations [11]

$$\begin{aligned} 8\pi^2 \frac{d\lambda_1}{dt} &= \left[-\frac{3}{2}g_2^2 - \frac{3}{10}g_1^2 + \frac{3}{2}(h_t^2 + h_b^2) + 2\lambda_1^2 \right] \lambda_1 \\ 8\pi^2 \frac{dh_t}{dt} &= \left[-\frac{8}{3}g_3^2 - \frac{3}{2}g_2^2 - \frac{13}{30}g_1^2 + 3h_t^2 + \frac{1}{2}h_b^2 + \frac{3}{4}\lambda_1^2 \right] h_t \\ 8\pi^2 \frac{dh_b}{dt} &= \left[-\frac{8}{3}g_3^2 - \frac{3}{2}g_2^2 - \frac{7}{30}g_1^2 + 3h_b^2 + \frac{1}{2}h_t^2 + \frac{3}{4}\lambda_1^2 \right] h_b \end{aligned} \quad (11)$$

²The physical top-quark mass M_t is related to the running $\overline{\text{MS}}$ top-quark mass m_t by the relation $m_t = M_t/[1 + 4\alpha_3(M_t)/3\pi]$, where g_3 is the $SU(3)$ coupling constant.

³The actual value on the right hand side of Eq. (10) does not matter for the precise value of the bound since the transition to the non-perturbative regime is very abrupt.

shows that the presence of gauge couplings slows down the evolution of the couplings $\lambda_1, h_{t,b}$ with the renormalization scale. Therefore, the stronger the gauge couplings, the slower the evolution of λ_1 , the larger values of $\lambda_1(M_Z)$ are consistent with perturbativity, and finally the higher values of m_h can be reached. Gauge couplings can be strengthened, without spoiling gauge unification, by adding complete (anomaly free) $SU(5)$ representations [13]. The simplest possibility is adding a number N_5 of pairs $(\mathbf{5} + \bar{\mathbf{5}})$. In fact for $N_5 = 4$ perturbativity is almost saturated ($\alpha_{\text{GUT}} \sim 1/5$) if $M_{5\bar{5}} \sim M_{\text{SUSY}}$ ⁴.

The unification of the gauge couplings and the upper bounds on m_h for the MSSM plus a singlet and $N_5 = 4$ are shown in the left and right panels, respectively, of Fig. 1. We observe

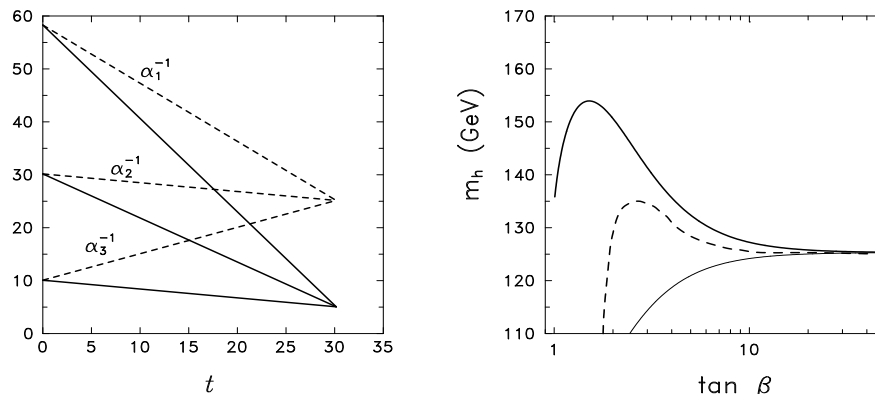


Figure 1: Left panel: Running gauge couplings for the MSSM with a gauge singlet (dashed lines) and the MSSM+4($\mathbf{5} + \bar{\mathbf{5}}$) (solid lines) with $t = \log(Q/M_{\text{SUSY}})$. Right panel: upper limit on m_h as a function of $\tan \beta$ for the MSSM (thin solid curve), the MSSM with a gauge singlet (thick dashed curve) and the MSSM with a gauge singlet and 4($\mathbf{5} + \bar{\mathbf{5}}$) (thick solid line).

from Fig. 1 that the maximum value of m_h is at $\tan \beta \sim 1.8$ and corresponds to $m_h \sim 155$ GeV, which is a more sizable increase with respect to the MSSM.

Suppose again that our experimental colleagues exclude, in a few years from now (LHC) a Higgs as light as ~ 155 GeV! Should we (once for all) exclude the SSM? We will next describe what we consider to be the absolute upper bound on the lightest Higgs mass in SSM consistent with gauge unification, or even with perturbativity for scales below the Planck mass.

4 Absolute bound and gauge unification

To maximize the upper bound on m_h we next assume that the model also contains extra chiral multiplets with the appropriate quantum numbers to give couplings of the form $W = h_X X H_i H_j$. Thus, X can only be a singlet (S) or a $Y = 0, \pm 1$ triplet (T_Y) [11, 14]. Such terms modify the quartic Higgs coupling via F -terms. From the gauge-invariant trilinear superpotential

$$W = \lambda_1 H_1 \cdot H_2 S + \lambda_2 H_1 \cdot T_0 H_2 + \chi_1 H_1 \cdot T_1 H_1 + \chi_2 H_2 \cdot T_{-1} H_2, \quad (12)$$

⁴Of course other possibilities are accessible by introducing intermediate scales, but the previous one will lead to the highest upper bounds.

the tree-level mass bound follows:

$$\begin{aligned}
m_h^2/v^2 &\leq \frac{1}{2}(g_2^2 + \frac{3}{5}g_1^2) \cos^2 2\beta + \left(\lambda_1^2 + \frac{1}{2}\lambda_2^2\right) \sin^2 2\beta \\
&+ 4\chi_1^2 \cos^4 \beta + 4\chi_2^2 \sin^4 \beta.
\end{aligned}
\tag{13}$$

Different terms have different $\tan \beta$ dependence and are important in different regimes. For example, the χ_2 contribution will be crucial for the upper limit in the large- $\tan \beta$ region. S and T_0 induce the same $\tan \beta$ dependence but the λ_1 correction can be more important than that of λ_2 . For this reason we do not take into account the possible effect of T_0 representations.

To achieve unification with only one scale M_{SUSY} ($= 1$ TeV) is not completely trivial. When the MSSM is enlarged by one singlet S and a pair $\{T_1, T_{-1}\}$ (to cancel anomalies) the running g_1^2 and g_2^2 meet at $M_X \sim 10^{17}$ GeV. Interestingly enough, this is closer to the heterotic string scale than the MSSM unification scale. Of course, g_3^2 fails to unify unless extra matter is added. This can be achieved, for example, by adding 4 $(3 + \bar{3})$ [$SU(2)_L \times U(1)_Y$ ‘singlet quark’ chiral multiplets] or $(3 + \bar{3})$ plus one $SU(3)_c$ octet. In addition to this, we can still have a $(5 + \bar{5})$ $SU(5)$ pair, which will not change the unification scale. The unification of the couplings is shown in Fig. 2 (left panel, solid lines).

For comparison, dashed lines show the running couplings when their RG beta functions are fixed in such a way that all couplings reach a Landau pole at the unification scale. In this case the low-energy couplings are fully determined by the ‘light’ matter content of the model (which fixes the RG beta functions). The dashed lines can be considered as the perturbative upper limit on the gauge couplings, and comparison with the solid lines shows that our model is close to saturation and represents a concrete realization of the most extreme scenario to maximize the m_h bound. The emphasis here should lie in this fact rather than in the plausibility or physics motivation of the model *per se*.

Having optimized in this way the most appropriate running gauge couplings, we turn to the running of λ_1 and $\chi_{1,2}$. The relevant RG equations are [10, 11]:

$$\begin{aligned}
8\pi^2 \frac{d\lambda_1}{dt} &= \left[-\frac{3}{2}g_2^2 - \frac{3}{10}g_1^2 + \frac{3}{2}(h_t^2 + h_b^2) + 3(\chi_1^2 + \chi_2^2) + 2\lambda_1^2 \right] \lambda_1 \\
8\pi^2 \frac{d\chi_{1,2}}{dt} &= \left[-\frac{7}{2}g_2^2 - \frac{9}{10}g_1^2 + 3h_{b,t}^2 + 7\chi_{1,2}^2 + \lambda_1^2 \right] \chi_{1,2} \\
8\pi^2 \frac{dh_t}{dt} &= \left[-\frac{8}{3}g_3^2 - \frac{3}{2}g_2^2 - \frac{13}{30}g_1^2 + 3h_t^2 + \frac{1}{2}h_b^2 + 3\chi_2^2 + \frac{1}{2}\lambda_1^2 \right] h_t \\
8\pi^2 \frac{dh_b}{dt} &= \left[-\frac{8}{3}g_3^2 - \frac{3}{2}g_2^2 - \frac{7}{30}g_1^2 + 3h_b^2 + \frac{1}{2}h_t^2 + 3\chi_2^2 + \frac{1}{2}\lambda_1^2 \right] h_b.
\end{aligned}
\tag{14}$$

For a given value of $\tan \beta$ (which influences the top and bottom Yukawa couplings entering the RGs) we find the maximum value of the particular combination of Yukawa couplings that enters the bound (13), compatible with perturbativity up to M_S . To add the important radiative corrections we follow Refs. [7, 8, 9], which include two-loop RG improvement and stop-mixing effects. We fix $M_t = 175$ GeV.

Before presenting our results, it is worth discussing in more detail the bound presented in Eq. (13). If the extra fields responsible for the enhancement of m_h sit at 1 TeV, should their effect not decouple from the low-energy effective theory? Indeed, in a simple toy model with

$$W = mS^2 + \lambda_1 H_1 \cdot H_2 S,
\tag{15}$$

the limit $m \gg M_Z, M_{SUSY}$ leads to an effective superpotential in which no renormalizable coupling of S to $H_1 \cdot H_2$ exists [technically, the F -term contribution ($\sim \lambda_1^2$) to the Higgs

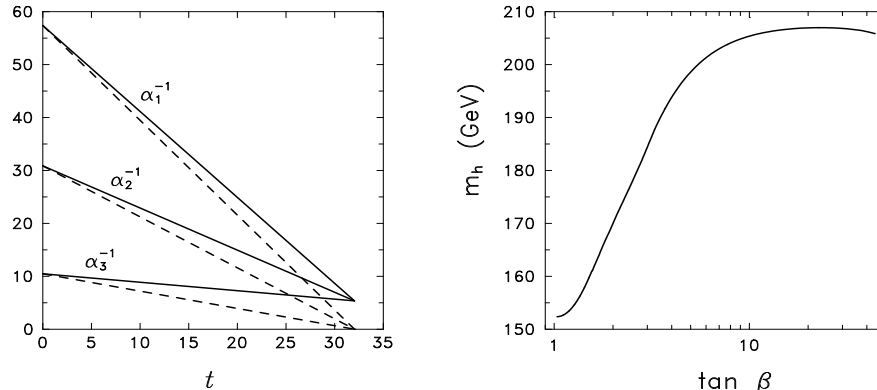


Figure 2: Left panel: running gauge couplings for the model discussed in the text (solid lines) and its upper perturbative limit (dashed lines). Right panel: Upper bound on m_h for both models.

doublet self-interactions is cancelled by a tree diagram that interchanges the heavy singlet, thus realizing decoupling]. If, to avoid this, we take $m \sim M_Z$, more than one light Higgs will appear in the spectrum. A complicated mixed squared-mass matrix results whose lightest eigenvalue does not saturate the bound (13). Is then this mass limit simply too conservative an overestimate of the real upper limit? It is easy to convince oneself that, in the presence of soft breaking masses, the perfect decoupling cancellation obtained in the large SUSY mass limit does not take place (we assume $m \lesssim M_{\text{SUSY}}$) and the final lightest Higgs mass depends in a complicated way on these soft mass parameters. The interesting outcome is that soft masses can be adjusted in order to saturate the bound (13); the numbers we will present can thus be reached in particular models and no limits lower than these can be given without additional assumptions (which we will not make here, in the interest of generality).

The final bound, with radiative corrections included, is presented in Fig. 2 (right panel). For $\tan \beta \lesssim 2$ the main contribution to the m_h value comes from the λ_1 coupling. In particular for $\tan \beta \simeq 1.8$ the effect of λ_1 is maximized and yields a bound on $m_h \sim 155$ GeV, as in the model presented in the previous section [13]. For larger values of $\tan \beta$ the effect of λ_1 drops off and χ_2 dominates. We also see that the bound can reach values as large as 205-210 GeV, that we should consider as the absolute bound in supersymmetric models, which are perturbative up to the high scale and without intermediate scales. Finally we can notice that the χ_1 coupling does play a role only for $\tan \beta \sim 1$ [14] where the bound is ~ 140 GeV.

5 Conclusions

In conclusion, we calculate a numerical absolute upper limit on the mass of the lightest supersymmetric Higgs boson for any model with arbitrary matter content compatible with gauge coupling unification around (and perturbativity up to) the string scale. With this assumption, we show that this light Higgs mass can be as high as ~ 205 GeV. The model saturating this bound has asymptotically divergent gauge couplings and points toward non-perturbative unification. Besides being of obvious interest to the experimentalists, this result has interest for theorists too. If Higgs searches reach the MSSM bounds without finding a signal for a Higgs boson, this could be taken, if one is willing to stick to (perturbative) low-energy supersymmetry, as evidence for additional matter beyond the minimal model.

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