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# Nonperturbative three-point functions of the $O(N)$ sigma model in the $1/N$ expansion at NLO

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## Abstract

We present a calculation of the three-point functions of the  $O(N)$ -symmetric sigma model. The calculation is done nonperturbatively by means of a higher-order  $1/N$  expansion combined with a tachyonic regularization which we proposed in previous publications. We use the results for calculating the standard model process  $f\bar{f} \rightarrow H \rightarrow WW$  nonperturbatively in the quartic coupling of the scalar sector.

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# Nonperturbative three-point functions of the $O(N)$ sigma model in the $1/N$ expansion at NLO

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## Abstract

We present a calculation of the three-point functions of the  $O(N)$ -symmetric sigma model. The calculation is done nonperturbatively by means of a higher-order  $1/N$  expansion combined with a tachyonic regularization which we proposed in previous publications. We use the results for calculating the standard model process  $f\bar{f} \rightarrow H \rightarrow WW$  nonperturbatively in the quartic coupling of the scalar sector.

In previous publications [1] we developed a nonperturbative approach for calculating processes where the quartic self-coupling of the scalar sector of the standard model becomes large, and therefore usual perturbation theory becomes unreliable. This nonperturbative approach is based on extending the standard Higgs sector to an  $O(N)$ -symmetric sigma model, calculating the scattering amplitudes nonperturbatively as a power series in  $1/N$ , and recovering the standard model in the limit  $N = 4$ . The connection to the physics of electroweak vector bosons is provided by the equivalence theorem.

The idea of expanding in the number of degrees of freedom of the theory under consideration instead of the coupling constant is rather old [2, 3]. This is in principle a very attractive idea which attempts to find a solution which is valid at strong coupling as well as at weak coupling, and which is free of the renormalization scheme ambiguities associated with conventional perturbation theory.

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However, perturbation theory was by far wider used than the  $1/N$  expansion for phenomenological purposes. An enormous amount of work in the recent years resulted in powerful tools for the computation of higher-loop Feynman graphs. The success of perturbation theory is due to the fact that Feynman diagrams can be calculated for any theory, and — with increasing difficulty though — in higher loop-orders.

These are precisely the issues which limited the applicability of the nonperturbative methods based on the  $1/N$  expansion. First, the precise structure of the  $1/N$  coefficients depends on the theory under consideration. This structure can become so complicated as to make a direct computation of all the graphs prohibitive even for the first term of the  $1/N$  expansion. A classical example of this type is the planar QCD in four dimensions [4], where the topological structure of the graphs is known, but they could not be actually calculated so far. Second, in most cases of physical interest the actual value of  $N$  is not that large as to make the leading order a good approximation. Such an example is the Higgs physics with which we deal in this letter. Another example is the ordinary QED treated as the  $N_e = 1$  limit of an Abelian theory coupled to  $N_e$  species of electrons. When  $N$  is not large enough, the leading order solution in  $1/N$ , although simple, is numerically not an approximation of acceptable accuracy for phenomenological purposes. Thus the inclusion of higher-order corrections is mandatory. Higher-order contributions in  $1/N$  are however difficult to calculate. This is because of technical problems of combinatorial nature, and also because of the lack of techniques to calculate certain classes of multiloop graphs. On the fundamental side there is the question of treating renormalon-type chains inside higher-order  $1/N$  graphs.

For the case of the  $O(N)$ -symmetric sigma model, we have shown that the problems enumerated above can be dealt with, and higher-order calculations in the  $1/N$  expansion are feasible [1, 5]. The problem of casting all higher-loop graphs into a manageable form is solved by using the auxiliary field formalism due to Coleman, Jackiw and Politzer [2]. The tachyon problem is dealt with by means of a minimal tachyonic subtraction [1]. The problem of evaluating the necessary multiloop graphs is solved numerically, by adapting a numerical three-loop technique for massive diagrams [1, 6].

The two-point functions of the  $O(N)$  sigma model were calculated in refs. [1]. This leads to a nonperturbative relation between the Higgs mass and width at next-to-leading order in  $1/N$ . Since in the scalar sector of the standard model perturbation theory has already been extended at two-loop level [7, 8, 9], this allows a strong test of the nonperturbative  $1/N$  result at weak coupling. Indeed, at weak coupling there is an impressive numerical agreement between two-loop perturbation theory and the next-to-leading order  $1/N$  expansion. It is the purpose of this letter to extend these results for the three-point functions of this theory.

As in the case of two-loop functions, we start with the ordinary Lagrangian of an  $O(N)$ -symmetric sigma model. Following ref. [2], we modify this Lagrangian by introducing a non-dynamical auxiliary field  $\chi$ :

$$\begin{aligned}
i\frac{v}{\sqrt{N}}\hat{E}_1(s) &= \text{Diagram 1} - \text{Diagram 2} \\
i\frac{v}{\sqrt{N}}\hat{F}_1(s) &= \text{Diagram 3} - \text{Diagram 4} \\
i\frac{v}{\sqrt{N}}\hat{F}_2(s) &= \text{Diagram 5} - \text{Diagram 6} \\
i\frac{v}{\sqrt{N}}\hat{F}_3(s) &= \text{Diagram 7} - \text{Diagram 8} + \text{Diagram 9}
\end{aligned}$$

Figure 1: *The definition of the subtracted vertex graphs. The blob on internal lines indicates the dressed nonperturbative propagators at LO in  $1/N$ , which result upon summation of infinite chains of one-loop bubble diagrams. The wavy line is the  $\sigma$  field, the solid line is the  $\pi$  field, and the dashed line is the  $\chi$  field. The boxes indicate internal and overall ultraviolet subtractions performed at an arbitrary subtraction scale  $\mu$ .*

$$\begin{aligned}
\mathcal{L} &= \frac{1}{2}\partial_\nu\Phi_0\partial^\nu\Phi_0 - \frac{\mu_0^2}{2}\Phi_0^2 - \frac{\lambda_0}{4!N}\Phi_0^4 + \frac{3N}{2\lambda_0}\left(\chi_0 - \frac{\lambda_0}{6N}\Phi_0^2 - \mu_0^2\right)^2 \\
&\equiv \frac{1}{2}\partial_\nu\Phi_0\partial^\nu\Phi_0 - \frac{1}{2}\chi_0\Phi_0^2 + \frac{3N}{2\lambda_0}\chi_0^2 - \frac{3\mu_0^2N}{\lambda_0}\chi_0 \quad , \quad \Phi_0 \equiv (\phi_0^1, \phi_0^2, \dots, \phi_0^N) \quad (1)
\end{aligned}$$

This does not change the physical content of the sigma model, because if one eliminates  $\chi$  by using its equation of motion, one recovers the original Lagrangian. The advantage of the auxiliary field formalism is that the quartic coupling of the  $\Phi$  field disappears, being replaced by trilinear vertices which involve the  $\chi$  field. This results into an enormous simplification of the possible topologies of multiloop diagrams which may appear in higher orders of  $1/N$ .

The multiloop Feynman diagrams which contribute to the three-point functions of this theory at NLO in  $1/N$  are shown in figure 1. Actually we define in this picture the subtracted  $1/N$  graphs  $\hat{E}_1$ ,  $\hat{F}_1$ ,  $\hat{F}_2$  and  $\hat{F}_3$ . These are the ultraviolet finite combinations which actually appear in the expressions of observable physical quantities upon inclusion of the necessary  $1/N$  counterterms.

The subtracted three-point graphs of figure 1 are calculated numerically with the

same methods as the two-point graphs [1]. The main difference is that, because of the different kinematic combination of the external momenta, the rotation of the spatial component of the loop momentum must be done on a more complicated complex path for avoiding the singularities of the integrands. The technical aspects of this procedure are discussed in detail in ref. [5].

Once the subtracted  $1/N$  graphs are calculated, they can be used for deriving physical amplitudes. Here we will consider the Higgs resonance shape in the scattering process  $f\bar{f} \rightarrow H \rightarrow WW$ . This process is for instance relevant for direct searches at a possible muon collider. Also it is related to the corrections of enhanced electroweak strength to the Higgs production mechanism by gluon fusion at hadron colliders [11].

At NLO in the  $1/N$  expansion, by including the relevant  $1/N$  graphs of all loop-orders and the corresponding counterterms derived from the Lagrangian of eq. 1, one obtains the following nonperturbative expression for the  $f\bar{f} \rightarrow H \rightarrow WW$  scattering amplitude (the overall factor from the tree level Yukawa coupling not included):

$$\begin{aligned}
\mathcal{M}_{WW} &= \frac{m^2(s)}{\sqrt{N}v} \frac{1 - \frac{1}{N}f_2}{s - m^2(s) \left[1 - \frac{1}{N}f_1(s)\right]} \\
f_1(s) &= \frac{m^2(s)}{v^2} \hat{\alpha}(s) + 2\hat{\gamma}(s) + \frac{v^2}{m^2(s)} \left[ \hat{\beta}(s) - 2\frac{s - m^2(s)}{v^2} (\delta Z_\sigma - \delta Z_\pi) \right] \\
f_2(s) &= \frac{m^2(s)}{v^2} \hat{\alpha}(s) + \hat{\gamma}(s) - \hat{\phi}(s) - \frac{v^2}{m^2(s)} \hat{\eta}(s)
\end{aligned} \tag{2}$$

Note that this expression is renormalization scheme independent. Here we used the notation  $\hat{\eta}(s) = \hat{E}_1(s)$  and  $\hat{\phi}(s) = \hat{F}_1(s) + \hat{F}_2(s) + \hat{F}_3(s)$ . Similarly,  $\hat{\alpha}(s)$ ,  $\hat{\beta}(s)$  and  $\hat{\gamma}(s)$  are the subtracted two-point functions of the model, and  $\delta Z_\sigma$  and  $\delta Z_\pi$  are the wave function renormalization constants of order  $1/N$  of the Higgs and Goldstone fields, which were defined and calculated in ref. [1].  $m^2(s)$  is the LO self-energy (see the notations of ref. [1]), and  $\sqrt{N}v = 246$  GeV is the vacuum expectation value of the Higgs field.

We plot in figure 2 a set of nonperturbative line shapes of the Higgs resonance as it appears in this scattering process. We also plot the location of the peaks of these line shapes. For comparison we show also the corresponding location of the peaks in perturbation theory, calculated in two-loop order [10, 11]. Just as in the case of the  $f\bar{f} \rightarrow H \rightarrow f'\bar{f}'$  scattering process [1], the NLO  $1/N$  solution agrees well with two-loop perturbation theory at weak coupling. As the coupling increases, a saturation effect sets in [1], and the width of the resonance increases without the position of the peak increasing at the same time. Nonperturbatively, the saturation value of the position of the peak is at about 980 GeV. This is not far from the 930 GeV value found in the fermion scattering process [1]. Of course, the shape of the resonance is process dependent, as opposed to the position of the complex pole of the Higgs particle, which is universal.

To conclude, we calculated nonperturbatively the three-point functions of the scalar sector of the standard model by means of a  $1/N$  expansion at next-to-leading order. Combining the three-point function with the already available two-point functions, we derived the nonperturbative amplitude of the scattering process  $f\bar{f} \rightarrow H \rightarrow ZZ, WW$ . Similarly to the already known  $f\bar{f} \rightarrow H \rightarrow f'f'$  scattering, the NLO  $1/N$  solution agrees very well with two-loop perturbation theory at weak coupling. At strong coupling we confirm the existence of a Higgs mass saturation effect. In this process the saturation value is about 980 GeV. This is comparable with the value of 930 GeV obtained from fermion scattering.

## References

- [1] A. Ghinculov, T. Binoth and J.J. van der Bij, *Phys. Rev.* **D57** (1998) 1487; T. Binoth, A. Ghinculov, J.J. van der Bij *Phys. Lett.* **B417** (1998) 343.
- [2] S. Coleman, R. Jackiw and H.D. Politzer, *Phys. Rev.* **D10** (1974) 2491.
- [3] H.J. Schnitzer, *Phys. Rev.* **D10** (1974) 1800; L. Dolan and R. Jackiw, *Phys. Rev.* **D9** (1974) 3320; R. Casalbuoni, D. Dominici and R. Gatto, *Phys. Lett.* **B147** (1984) 419; M.B. Einhorn, *Nucl. Phys.* **B246** (1984) 75.
- [4] G. 't Hooft, *Nucl. Phys.* **B72** (1974) 461.
- [5] T. Binoth and A. Ghinculov, *CERN-TH/98-244* (1998), *hep-ph/9808393*.
- [6] A. Ghinculov, *Phys. Lett.* **B385** (1996) 279.
- [7] A. Ghinculov and J.J. van der Bij, *Nucl. Phys.* **B436** (1995) 30; A. Ghinculov, *Phys. Lett.* **B337** (1994) 137; (E) **B346** (1995) 426; *Nucl. Phys.* **B455** (1995) 21;
- [8] P.N. Maher, L. Durand and K. Riesselmann, *Phys. Rev.* **D48** (1993) 1061; (E) **D52** (1995) 553; A. Frink, B.A. Kniehl, D. Kreimer, K. Riesselmann, *Phys. Rev.* **D54** (1996) 4548.
- [9] V. Borodulin and G. Jikia, *Phys. Lett.* **B391** (1997) 434.
- [10] T. Binoth and A. Ghinculov, *Phys. Rev.* **D56** (1997) 3147.
- [11] A. Ghinculov and J.J. van der Bij, *Nucl. Phys.* **B482** (1996) 59; A. Ghinculov, T. Binoth and J.J. van der Bij, *Phys. Lett.* **B427** (1998) 343.

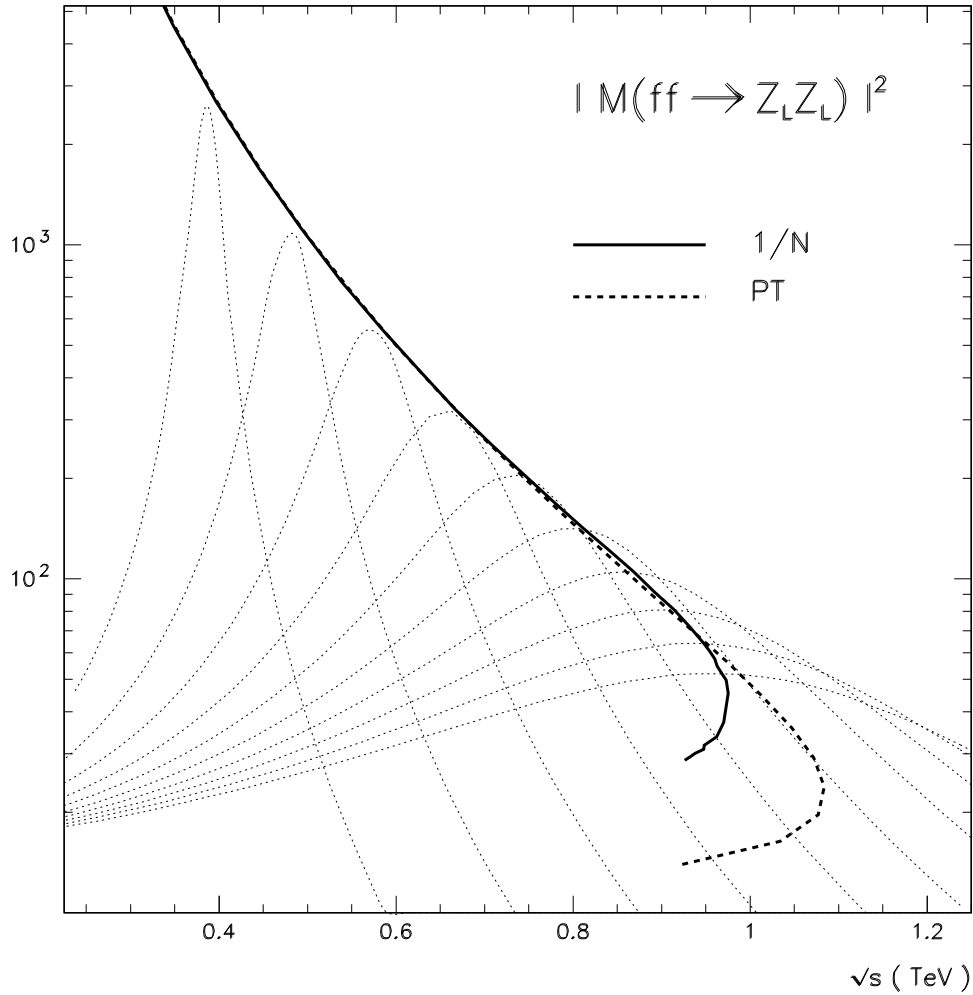


Figure 2: *The line shape of the Higgs resonance in the scattering process  $f\bar{f} \rightarrow H \rightarrow ZZ, WW$  for different values of the quartic coupling. The solid line indicates the position of the maxima of the resonances of the nonperturbative  $1/N$  expansion when the coupling is increased. The dotted line corresponds to the two-loop perturbative scattering amplitude.*