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ZZ' Mixing in Presence of Standard Weak Loop Corrections

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Abstract

We derive a method for a common treatment of Z' exchange, QED corrections, and weak loops. It is based on the form factor approach to the description of weak loop corrections to partial Z widths and cross sections. Problems connected with Z,Z' mixing are discussed with special care. Our theoretical results are applied to the package `ZFITTER`. We demonstrate two different ways to the data analysis - one based on an extension of the standard model cross sections, the other on model-independent formulae together with the Z width calculations in presence of a Z'.

With the resulting package `ZEFIT`$\oplus$`ZFITTER`, LEP1 data from fermion pair production, including Bhabha scattering, can be analysed *on*, but also *off* the Z peak. Further, the code may be used at very high energies, e.g. in the region of a possible future linear e^+e^--collider.

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1 Introduction

Although the Standard Model [1] has been verified with a precision including one loop corrections, there is a general consensus that we are far away from a final understanding of the elementary particle world. A unification of forces seems to happen at much higher mass scales than are directly accessible by present accelerators. Candidates for a truly unifying theory usually predict additional gauge degrees of freedom, thus leading in a natural way to the existence of new, heavy neutral gauge bosons besides the photon and the Z boson of the standard theory (see e.g. [2]).

Since γ, Z, Z' are neutral particles with vector and axial-vector couplings, a search for a Z' is complicated by the fact that there is no special final state signature. Below the production threshold, theoretical predictions consist of minor quantitative modifications of the neutral current cross sections. As a consequence, one needs very precise predictions for cross sections and asymmetries. Important reactions for a dedicated search are:

$$e^+e^- \longrightarrow (\gamma, Z, Z') \longrightarrow f^+f^-(\gamma), \tag{1}$$

$$e^+e^- \longrightarrow (\gamma, Z, Z') \longrightarrow e^+e^-(\gamma), \tag{2}$$

$$ep \longrightarrow (\gamma, Z, Z') \longrightarrow eX(\gamma). \tag{3}$$

In earlier studies, we investigated QED (i.e. pure photonic) corrections together with Z' exchange both in e^+e^--annihilation into fermions [3] and in ep-scattering [4]. In the latter reaction, we applied also part of the material presented here (Z' effects in presence of weak loops without Z,Z' mixing).

In principle, Z' effects can be searched for by three different effects:
• via virtual Z' exchange (at sufficient energy, also present without Z,Z' mixing)
• via modification of the mass of the standard Z boson seen at LEP1 due to Z, Z' mixing
• via modification of the couplings of the standard Z boson seen at LEP1 due to Z, Z' mixing; this in fact concerns two different, although related observables - the Z width [$\sim$ peak height] and cross sections [$\sim$ line shape].

For large enough Z' masses, the direct cross section contributions due to Z' exchange may be neglected at LEP1 energies completely. Then, one can concentrate on the consequences of the Z,Z' mixing. *In fact, LEP1 is the ideal place to search for this phenomenon.*

From existing measurements it is known that the mixing is very small if not vanishing; see e.g. [5] and references quoted therein. In such a situation, one has to study carefully the interplay of the weak standard theory loop effects with the Z' influence via particle mixing.

Often the LEP1 data are analysed *after* a model-independent interpretation of the line shape (and of asymmetries) in terms of e.g. partial widths (or effective couplings), confronting those as 'observed quantities' with theoretical expectations. Our approach will also allow to do so. In addition, we derived formulae which open the possibility of a direct interpretation of e.g. the line shape as function of the energy in terms of standard model parameters and, in parallel, of the Z, Z' mixing. (Not mentioning for a moment the direct Z' cross section part.) This is done in a specific way following the form factor notation of weak loop effects on which the

Dubna/Zeuthen approach (and e.g. the code ZFITTER which has been supplemented here) so heavily relies.

We remind the reader that for each scattering process, there are four complex-valued form factors $\rho, \kappa_e, \kappa_f, \kappa_{ef}$, which allow an exact description of the weak (non-photonic) contributions to the scattering process [6, 7]. For an exact description of Z,Z' mixing, one has to understand how these form factors become modified.
Since the partial and total widths of the Z boson [8] contain similar form factors ρ_Z, κ_Z, a similar procedure has to be applied there, too.

We will also give a short comment on the relation of the formulae derived here to those used by other authors.
This concerns the change of form factors by the Z,Z' mixing.
Further, we define the weak mixing angle $\sin^2\theta_W$ from the unmixed mass parameters also in presence of Z,Z' mixing. As a result, the calculation of the standard weak loop corrections remains unchanged, too. Many other authors prefer to redefine the weak mixing angle as being derived from the physical mass of the Z mass eigenstate [9]. This leads to the introduction of a ρ parameter in the definition of the weak mixing angle. The difference between the two procedures is essentially a different book keeping and does not influence the experimental determination of e.g. the t-quark mass or the Z,Z' mixing angle θ_M. Although, a direct comparison of the form factors of the different approaches may be meaningless.

The present article contains a precise formulation of the common treatment of standard weak loops and of Z' effects together with their realisation in the code ZEFIT$\oplus$ZFITTER. Since QED corrections are model-independent in the sense that they are well-defined if vector and axial vector couplings, mass and width of the Z' are known, the flexible, semi-analytic multi-purpose code ZFITTER [10] can be used after some modifications for the calculation of QED corrections. Thus, we do not discuss explicit formulae on QED corrections in presence of a Z'.
In section 3, we comment on the Z width. It is shown how the weak form factors which renormalise the Fermi constant (ρ_Z) and the weak mixing angle (κ_Z) are influenced by a mixing of Z, Z'. In section 4, the same is done for the corresponding form factors of the differential cross sections $\rho(s, Q^2)$ and $\kappa_e(s, Q^2), \kappa_f(s, Q^2), \kappa_{ef}(s, Q^2)$. Section 5 contains some explicit cross section formulae. In section 6, the structure of the package ZEFIT and its interplay with ZFITTER is described. It contains the above mentioned changes of form factors together with Born cross section parts containing Z' exchange [for applications at energies beyond the LEP1 region]. Further, in appendix A the explicit vector and axial vector couplings of the Z' in two important classes of extended gauge models, the E_6 based and left-right symmetric models, are determined as functions of one free parameter, θ_{E6} or α_{LR}, respectively. Appendix B contains the output of a test program for the use of ZEFIT at LEP1 energies.
At the end of this introduction to the subject we would like to stress that the code originally is intended for LEP1 physics, but may also be used at higher energy, e.g. for Z' searches at LEP 200 or LINAC 500. An application of this kind and a comprehensive list of references may be found in [11].

2 Gauge Boson Mixing

The coupling constants are defined for the symmetry eigenstates Z, Z'. The Lagrangian

$$\mathcal{L} = eA_\mu J^\mu_\gamma + gZ_\mu J^\mu_Z + g' Z'_\mu J^\mu_{Z'} \tag{4}$$

contains currents of the form

$$J^\mu_n = \sum_f \bar{f}\gamma^\mu \left[v_f(n) + \gamma_5 a_f(n)\right] f, \quad n = \gamma, Z, Z'. \tag{5}$$

In the following, complications will arise from a mixing of Z,Z' since our renormalisation is performed on mass shell, i.e. for mass eigenstates Z_1, Z_2. In this chapter, we distinguish between these states:

$$\left(\begin{array}{c} Z_1 \\ Z_2 \end{array} \right) = \left(\begin{array}{cc} \cos\theta_M & \sin\theta_M \\ -\sin\theta_M & \cos\theta_M \end{array} \right) \left(\begin{array}{c} Z \\ Z' \end{array} \right). \tag{6}$$

The weak mixing angle θ_W is related to the gauge boson masses and to the gauge boson mixing angle θ_M as follows:

$$t_M^2 = \tan^2\theta_M = \frac{M_Z^2 - M_1^2}{M_2^2 - M_Z^2}, \tag{7}$$

$$M_Z \equiv \frac{M_W}{\cos\theta_W}. \tag{8}$$

These equations correspond to [9]:

$$\rho_{mix} \;=\; \frac{M_W^2}{M_1^2\cos^2\theta_W} = \frac{M_Z^2}{M_1^2} = \frac{1 + t_M^2\, M_2^2/M_1^2}{1+t_M^2} = 1 + \sin^2\theta_M \left(\frac{M_2^2}{M_1^2} - 1 \right). \tag{9}$$

The Z boson mass measured at LEP1 is $M_1 = 91.177$ GeV. The couplings of the mass eigenstates to fermions are :

$$v_f(1) = \cos\theta_M v_f + \frac{g'}{g}\sin\theta_M v'_f, \tag{10}$$

$$v_f(2) = \cos\theta_M v'_f - \frac{g}{g'}\sin\theta_M v_f, \tag{11}$$

$$g = (\sqrt{2}G_\mu M_1^2)^{1/2}, \qquad v_f = a_f(1 - 4|Q_f|\sin^2\theta_W), \qquad a_f = I_3^L(f), \tag{12}$$

with analogue definitions for the axial couplings.
The photon couplings are defined such that $Q_e = -1$.

3 Partial Z widths in Presence of Z, Z' Mixing

Without Z, Z' mixing, the matrix element for the decay of the Z boson into a fermion pair may be written as follows [8]:

$$\begin{aligned} \bar{\mathcal{M}}_f \;&\sim\; \sqrt{\frac{G_\mu}{\sqrt{2}}M_Z^2}\epsilon^\alpha \sqrt{\rho_f} a_f \bar{u} \left[\gamma_\alpha(1+\gamma_5) - 4\sin^2\theta_W \kappa_f\right] u \\ &\sim\; \sqrt{\frac{G_\mu}{\sqrt{2}}M_Z^2}\epsilon^\alpha \bar{a}_f \bar{u} \left[\gamma_\alpha\gamma_5 + \gamma_\alpha \frac{\bar{v}_f}{\bar{a}_f}\right] u, \end{aligned} \tag{13}$$

where ρ and κ contain the weak loop corrections as determined in the on mass shell renormalisation scheme, after the replacement of the coupling constant α by the muon decay constant. They are related by:

$$\frac{\pi\alpha}{2\sin^2\theta_W\cos^2\theta_W} = \frac{G_\mu}{\sqrt{2}}M_Z^2(1-\Delta r). \tag{14}$$

The factor $(1-\Delta r)$ becomes part of the form factor ρ [8].
The resulting decay width is:

$$\begin{aligned}\bar{\Gamma}_f &= \frac{G_\mu}{\sqrt{2}}\frac{M_Z^3}{12\pi}c_f\rho_f\left[1-4|Q_f|\sin^2\theta_W\kappa_f+8(|Q_f|\sin^2\theta_W\kappa_f)^2\right] \\ &= \frac{G_\mu}{\sqrt{2}}\frac{M_Z^3}{6\pi}c_f\left[\bar{v}^2+\bar{a}^2\right],\end{aligned} \tag{15}$$

where c_f is a color factor in case of quarks. Further, we used that

$$\frac{\bar{v}_f}{\bar{a}_f} = 1-4|Q_f|\sin^2\theta_W\kappa_f. \tag{16}$$

The effective vector and axial vector couplings are defined as follows:

$$\bar{a}_f = \sqrt{\rho_f}\,I_3^L(f), \tag{17}$$

$$\bar{v}_f = \bar{a}_f\left[1-4|Q_f|\sin^2\theta_W\kappa_f\right], \tag{18}$$

where $I_3^L(f)$ is the weak isospin of fermion f. It has to be stressed here that the weak form factors are introduced originally for the overall normalisation of the matrix element and for the weak mixing angle, but not for the axial and vector couplings.

If now Z and Z' mix, the above formulae must be written down for the mass eigenstate Z_1 with mass $M_1 = 91.177$ GeV and in terms of the couplings of the 'physical Z boson' Z_1. One can rewrite the Born couplings after mixing as follows:

$$a_f(1) = c_M\,a_f + s_M\frac{g'}{g}\,a'_f = (c_M + s_M\frac{g'a'_f}{ga_f})a_f, \tag{19}$$

$$a_f(1) = (1-y_f)a_f, \tag{20}$$

$$y_f = -s_M\frac{g'a'_f}{ga_f}+(1-c_M)\sim -s_M\frac{g'a'_f}{ga_f}. \tag{21}$$

Similarly, we make the ansatz:

$$\frac{v_f(1)}{a_f(1)} = \frac{v_f+t_Mv'_fg'/g}{a_f+t_Ma'_fg'/g} \equiv 1-4|Q_f|\sin^2\theta_W(1-x_f), \tag{22}$$

from which we derive:

$$x_f = (1-v_f/a_f)^{-1}\left(\frac{v_f+t_Mv'_fg'/g}{a_f+t_Ma'_fg'/g}-\frac{v_f}{a_f}\right), \tag{23}$$

$$x_f \approx s_M \frac{g'}{g}\frac{a'_f}{a_f}\frac{v'_f/a'_f - v_f/a_f}{1 - v_f/a_f}. \tag{24}$$

In terms of these variables, the Born matrix element after mixing is:

$$\begin{aligned}
\mathcal{M}_f(1) &\sim \sqrt{\frac{G_\mu}{\sqrt{2}}M_1^2}\epsilon^\alpha \bar{a}_f(1)\bar{u}\left[\gamma_\alpha\gamma_5 + \gamma_\alpha\frac{\bar{v}_f(1)}{\bar{a}_f(1)}\right]u \\
&\sim \sqrt{\frac{G_\mu}{\sqrt{2}}M_1^2}\epsilon^\alpha(1-y_f)a_f\bar{u}\left[\gamma_\alpha(1+\gamma_5) - 4\sin^2\theta_W(1-x_f)\gamma_\alpha\right]u.
\end{aligned}\tag{25}$$

Starting from this expression, it is evident how to take into account the weak loop corrections of the (unmixed) standard theory. The following replacements have to be performed:

$$\rho_f \to \rho_f^M = \rho_{mix}(1-y_f)^2\rho_f, \tag{26}$$

$$\kappa_f \to \kappa_f^M = (1-x_f)\kappa_f, \tag{27}$$

$$\bar{a}_f(1) = \sqrt{\rho_f^M}\, I_3^L(f), \tag{28}$$

$$\bar{v}_f(1) = \bar{a}_f\left[1 - 4|Q_f|\sin^2\theta_W\kappa_f^M\right]. \tag{29}$$

The width of the 'physical Z' in presence of standard weak loops and mixing now is:

$$\begin{aligned}
\bar{\Gamma}(1)_f &= \frac{G_\mu}{\sqrt{2}}\frac{M_1^3}{12\pi}c_f\rho_f^M\left[1 - 4|Q_f|\sin^2\theta_W\kappa_f^M + 8(|Q_f|\sin^2\theta_W\kappa_f^M)^2\right] \\
&= \frac{G_\mu}{\sqrt{2}}\frac{M_1^3}{6\pi}c_f\left[\bar{v}_f(1)^2 + \bar{a}_f(1)^2\right].
\end{aligned}\tag{30}$$

This is the expression for a partial width of the Z boson studied at LEP1 as they are used in the `ZEFIT` package for fits including the Z, Z' mixing.

One now can study the expressions which result in case of both small mixing and weak loop corrections:

$$\bar{a}_f(1) \equiv a_f^{eff} \equiv (1-y_f)\sqrt{\rho_f}\, a_f = \sqrt{\rho_f}(c_M\, a_f + s_M\, a'_f). \tag{31}$$

A naive ansatz for the same effective coupling could be:

$$a_f^{eff} = c_M(\sqrt{\rho_f}\, a_f) + s_M a'_f. \tag{32}$$

A direct comparison shows that one should not expect a more than tiny difference.
For the resulting effective vector coupling, one gets similarly:

$$\bar{v}_f(1) = \rho_f(c_M a_f + s_M a'_f)[1 - 4\sin^2\theta_W(1-x_f)\kappa_f]. \tag{33}$$

A naive ansatz could be here:

$$v_f^{eff} = c_M[\sqrt{\rho_f}\, a_f(1 - 4\sin^2\theta_W|Q_f|\kappa_f)] + s_M v'_f. \tag{34}$$

4 Weak Form Factors of the Scattering Matrix Element in Presence of Z, Z' Mixing

The Born matrix element for the scattering through the mass eigenstate Z_1 (being observed at LEP1) is:

$$\mathcal{M}_1(s, \cos\vartheta) \sim \frac{1}{s-m_1^2} a_e(1) a_f(1) \left[\frac{G_\mu}{\sqrt{2}} M_Z^2\right] \left[\gamma_\mu \left(\frac{v_e(1)}{a_e(1)} + \gamma_5\right)\right] \otimes \left[\gamma^\mu \left(\frac{v_f(1)}{a_f(1)} + \gamma_5\right)\right], \tag{35}$$

where m_1^2 denotes the complex mass parameter including finite width effects[1]. The following short notation is used:

$$A_\gamma \otimes B^\gamma = [\bar{u}_e A_\gamma u_e] \cdot [\bar{u}_f B^\gamma u_f]\,. \tag{36}$$

A similar ansatz may be written in the t-channel. The matrix element may be rewritten in terms of standard theory (unmixed) variables $a_{e,f}, \sin^2\theta_W$:

$$\begin{aligned}
&\mathcal{M}_1(s, \cos\vartheta) \sim \\
&\frac{1}{s-m_1^2} a_e a_f \left[\frac{G_\mu}{\sqrt{2}} M_Z^2\right] (1-y_e)(1-y_f)[L_\mu \otimes L^\mu - 4|Q_e|\sin^2\theta_W(1-x_e)\gamma_\mu \otimes L^\mu \\
&-4|Q_f|\sin^2\theta_W(1-x_f)L_\mu \otimes \gamma^\mu + 16|Q_eQ_f|\sin^4\theta_W(1-x_e)(1-x_f)\gamma_\mu \otimes \gamma^\mu],
\end{aligned} \tag{37}$$

$$L_\mu = \gamma_\mu(1+\gamma_5), \tag{38}$$

where again we used that one can write

$$\frac{v_f(1)}{a_f(1)} = 1 - 4|Q_f|\sin^2\theta_W(1-x_f), \tag{39}$$

$$a_f(1) = (1-y_f)a_f, \tag{40}$$

with the same definitions of x_f, y_f as in the case of the Z width.

Without mixing, the weak loop corrections influence the matrix element as follows:

$$\begin{aligned}
&\bar{\mathcal{M}}_Z(s, \cos\vartheta) \sim \\
&\frac{1}{s-m_Z^2} \left[\frac{G_\mu}{\sqrt{2}} M_Z^2\right] a_e a_f \rho(s,\cos\vartheta)[L_\mu \otimes L^\mu - 4|Q_e|\sin^2\theta_W \kappa_e(s,\cos\vartheta)\gamma_\mu \otimes L^\mu \\
&-4|Q_f|\sin^2\theta_W\kappa_f(s,\cos\vartheta)L_\mu \otimes \gamma^\mu + 16|Q_eQ_f|\sin^4\theta_W\kappa_{ef}(s,\cos\vartheta)\gamma_\mu \otimes \gamma^\mu].
\end{aligned} \tag{41}$$

For massless fermions, the four form factors $\rho, \kappa_e, \kappa_f, \kappa_{ef}$ are the most general ansatz for the weak radiative corrections. In Born approximation, $\rho = \kappa = 1$.

In case of Z, Z' mixing, the matrix element may be written in a similar form:

$$\begin{aligned}
&\bar{\mathcal{M}}_1(s, \cos\vartheta) \sim \\
&\frac{1}{s-m_1^2} \left[\frac{G_\mu}{\sqrt{2}} M_1^2\right] a_e(1) a_f(1) \rho^M(s,\cos\vartheta) \\
&[L_\mu \otimes L^\mu - 4|Q_e|\sin^2\theta_W \kappa_e^M(s,\cos\vartheta)\gamma_\mu \otimes L^\mu \\
&-4|Q_f|\sin^2\theta_W\kappa_f^M(s,\cos\vartheta)L_\mu \otimes \gamma^\mu + 16|Q_eQ_f|\sin^4\theta_W\kappa_{ef}^M(s,\cos\vartheta)\gamma_\mu \otimes \gamma^\mu],
\end{aligned} \tag{42}$$

[1] We do not discuss here problems connected with the definition of gauge boson masses in dependence on the handling of the energy dependence of the width.

where it is:

$$\rho^M = \rho_{mix}(1-y_e)(1-y_f)\rho, \tag{43}$$
$$\kappa_f^M = (1-x_f)\kappa_f, \tag{44}$$
$$\kappa_{ef}^M = (1-x_e)(1-x_f)\kappa_{ef}. \tag{45}$$

The matrix element may be rewritten in terms of effective weak neutral vector and axial vector couplings:

$$\bar{\mathcal{M}}_1(s,\cos\vartheta) \sim \frac{1}{s-m_1^2}\left[\frac{G_\mu}{\sqrt{2}}M_1^2\right][\bar{a}_e\bar{a}_f\gamma_\mu\gamma_5\otimes\gamma_\mu\gamma_5 + \bar{v}_e\bar{a}_f\gamma_\mu\otimes\gamma_\mu\gamma_5 + \bar{a}_e\bar{v}_f\gamma_\mu\gamma_5\otimes\gamma^\mu + \bar{v}_{ef}\gamma_\mu\otimes\gamma^\mu], \tag{46}$$

$$\bar{a}_f(1) = \sqrt{\rho^M(s,\cos\vartheta)}\, I_3^L(f), \tag{47}$$
$$\bar{v}_f(1) = \bar{a}_f(1)\left[1-4|Q_f|\sin^2\theta_W\kappa_f^M(s,\cos\vartheta)\right], \tag{48}$$
$$\bar{v}_{ef}(1) = \bar{a}_e(1)\bar{v}_f(1)+\bar{v}_e(1)\bar{a}_f(1)-\bar{a}_e(1)\bar{a}_f(1)\left[1-16|Q_eQ_f|\sin^4\theta_W\kappa_{ef}^M(s,\cos\vartheta)\right]. \tag{49}$$

An equivalent notation is (with and without mixing):

$$\bar{a}_f = \sqrt{\rho}a_f, \tag{50}$$
$$\bar{v}_f = \sqrt{\rho}\kappa_f v_f + (1-\kappa_f)\bar{a}_f, \tag{51}$$
$$\bar{v}_{ef} = \rho\kappa_{ef}v_ev_f + (\bar{v}_e - \sqrt{\rho}\kappa_{ef}v_e)\bar{a}_f + \bar{a}_e(\bar{v}_f - \sqrt{\rho}\kappa_{ef}v_f) - (1-\kappa_{ef})\bar{a}_e\bar{a}_f. \tag{52}$$

Alternatively, one could define the axial vector couplings to be unchanged by the radiative corrections. Then, the Fermi constant absorbs the weak form factor $\rho(s,\cos\vartheta)$ and becomes dependent on the process and its kinematics:

$$G_\mu \to \bar{G}_\mu^M = G_\mu\rho^M(s,\cos\vartheta). \tag{53}$$

The other form factors renormalise the weak mixing angle $\sin^2\theta_W = 1 - M_W^2/M_Z^2$:

$$\sin^2\theta_W \to \left\{\begin{array}{l}\sin^2\theta_W\kappa_e^M(s,\cos\vartheta)\\ \sin^2\theta_W\kappa_f^M(s,\cos\vartheta)\\ \sin^2\theta_W\sqrt{\kappa_{ef}^M(s,\cos\vartheta)}\end{array}\right. . \tag{54}$$

Finally, one should mention that the mixing of gauge bosons Z, Z' as discussed here could be mimicked by a mixing of standard fermions with exotic fermions [2], [12][2] with quite similar influence on (4,5).

In order to simplify a comparison of the present approach to that of other groups, we now give the explicit leading top quark mass m_t dependence of the form factors and Δr [7, 13, 14, 15] in case of mixing:

$$\sin^2\theta_W M_W^2 = \frac{\pi\alpha/(\sqrt{2}G_\mu)}{1-\Delta r}, \tag{55}$$
$$\sin^2\theta_W = 1-\frac{M_W^2}{M_Z^2} = 1-\frac{M_W^2}{M_1^2\rho_{mix}}, \tag{56}$$

[2] In [12], the common influence of exotic fermions and a Z' is discussed.

and ρ_{mix} is defined in (9). Further[3],

$$\Delta r = 1 + \Delta r^{rem} - (1 + \frac{\cos^2\theta_W}{\sin^2\theta_W}\delta\bar{\rho})(1 - \Delta\alpha), \tag{57}$$

$$\delta\bar{\rho} = 3\mathcal{T}\left[1 - (2\pi^2 - 19)\mathcal{T} - \frac{\alpha_s}{\pi}\frac{2}{3}(1 + \frac{\pi^2}{3})\right], \tag{58}$$

$$\mathcal{T} = \frac{G_\mu}{\sqrt{2}}\frac{m_t^2}{8\pi^2}. \tag{59}$$

Here, α_s is the strong interaction coupling constant, and Δr^{rem} contains the $O(\alpha)$ corrections to Δr without the contribution $\mathcal{T}$. The $\Delta\alpha$ contains the fermionic one-loop insertions to α. For the cross section form factors, analogue formulae hold:

$$\rho^M = \rho_{mix}(1 - y_e)(1 - y_f)\frac{1 + \Delta\rho_f^{rem}}{1 - \delta\bar{\rho}}, \tag{60}$$

$$\kappa_f^M = (1 - x_f)(1 + \Delta\kappa_f^{rem})(1 + \frac{\cos^2\theta_W}{\sin^2\theta_W}\delta\bar{\rho}), \tag{61}$$

$$\kappa_{ef}^M = (1 - x_e)(1 - x_f)(1 + \Delta\kappa_{ef}^{rem})(1 + \frac{\cos^2\theta_W}{\sin^2\theta_W}\delta\bar{\rho})^2. \tag{62}$$

For the partial Z widths,

$$\rho_f^M = \rho_{mix}(1 - y_f)^2\frac{1 + \Delta\rho_f^{rem}}{1 - \delta\bar{\rho}}, \tag{63}$$

$$\kappa_f^M = (1 - x_f)(1 + \Delta\kappa_f^{rem})(1 + \frac{\cos^2\theta_W}{\sin^2\theta_W}\delta\bar{\rho}). \tag{64}$$

The cross section form factors and those of the partial Z widths differ by their remnant parts and, in case of the ρ, by their mixing factors. At very high energy, the remnant parts of form factors become substantial and both the approximate flavor independance, the similarity of width and cross section form factors, and the factorization property of κ_{ef} are lost.

We remind the reader here that it was not our intention to discuss the various weak mixing angle definitions which allow an aproximate short hand notation for partial Z widths and cross sections. We introduced the weak corrections such that they are exact to weak one loop order after introduction of the form factors.
A good approximation to the so-called effective weak mixing angle which often is used for a description of LEP1 data is

$$\sin^2\theta_{W,eff} = \kappa\ \sin^2\theta_W, \tag{65}$$

where one can take any one of the (real part of) form factors κ_f, calculated at $s = M_Z^2, \cos\vartheta = 0$ or the corresponding form factor from a partial width, e.g. Γ_e. For some details see [6] and for a comparative discussion [16].

At the end of this section, we should mention that all the above derivations are valid for Bhabha scattering (2), too. This is due to the fact that the discussion has been based on the matrix elements $\mathcal{M}$ before building cross sections out of them.

[3]We follow the conventions of `ZFITTER4` [10] with the following setting of flags: IAMT4=2, IQCD $\neq 0$. The actual version of `ZEFIT` is to be used together with `ZFITTER` v.3.05 until the official release of version 4. The corresponding flag setting in version 3 is: IAMT4=1. The resummation of the higher order QCD terms is not realized there.

5 Improved Born Approximation in Presence of Gauge Boson Mixing

The matrix element $\mathcal{M}_{Z'}$ with Z' exchange is in case of mixing:

$$\begin{aligned}
\mathcal{M}_2(s,\cos\vartheta) \quad \sim \quad & \frac{{g'}^2}{s-m_2^2}[\bar{a}_e(2)\bar{a}_f(2)\gamma_\mu\gamma_5\otimes\gamma_\mu\gamma_5+\bar{v}_e(2)\bar{a}_f(2)\gamma_\mu\otimes\gamma_\mu\gamma_5 \\
& + \bar{a}_e(2)\bar{v}_f(2)\gamma_\mu\gamma_5\otimes\gamma^\mu+\bar{v}_{ef}(2)\gamma_\mu\otimes\gamma^\mu], \qquad (66)
\end{aligned}$$

where $\bar{a}_f(2),\bar{v}_f(2)$ are the renormalized vector and axial vector couplings of the Z_2.
As long as we search for a Z' well below the production threshold, we can safely neglect here all radiative corrections to this amplitude,

$$\bar{a}_f(2)=a_f(2), \qquad \bar{v}_f(2)=v_f(2), \qquad \bar{v}_{ef}(2)=v_e(2)v_f(2). \qquad (67)$$

In sum, our discussion leads to the following net matrix element, where we also add up the photon exchange diagram with running QED coupling:

$$\mathcal{M}=\mathcal{M}_\gamma+\mathcal{M}_1+\mathcal{M}_2. \qquad (68)$$

Let us now have a look at the squared matrix elements. In case of massless fermion production, four different combinations of coupling constants may occur in reactions (1,3) from the interference of the vector bosons m and n:

$$\begin{aligned}
\bar{C}_1(m,n) &= \bar{a}_e(m)\bar{a}_e^*(n)\bar{a}_f(m)\bar{a}_f^*(n)+\bar{a}_e(m)\bar{a}_e^*(n)\bar{v}_f(m)\bar{v}_f^*(n) \qquad (69)\\
&\quad + \bar{v}_e(m)\bar{v}_e^*(n)\bar{a}_f(m)\bar{a}_f^*(n)+\bar{v}_{ef}(m)\bar{v}_{ef}^*(n),\\
\bar{C}_2(m,n) &= \bar{a}_e(m)\bar{v}_e^*(n)\bar{v}_f(m)\bar{a}_f^*(n)+\bar{v}_e(m)\bar{a}_e^*(n)\bar{a}_f(m)\bar{v}_f^*(n) \qquad (70)\\
&\quad + \bar{a}_e(m)\bar{v}_{ef}^*(n)\bar{a}_f(m)+\bar{a}_e^*(n)\bar{v}_{ef}(m)\bar{a}_f^*(n),\\
\bar{C}_3(m,n) &= \bar{a}_e(m)\bar{v}_e^*(n)\bar{a}_f(m)\bar{a}_f^*(n)+\bar{v}_e(m)\bar{a}_e^*(n)\bar{a}_f(m)\bar{a}_f^*(n) \qquad (71)\\
&\quad + \bar{a}_e(m)\bar{v}_{ef}^*(n)\bar{v}_f(m)+\bar{a}_e^*(n)\bar{v}_{ef}(m)\bar{v}_f^*(n),\\
\bar{C}_4(m,n) &= \bar{a}_e(m)\bar{a}_e^*(n)\bar{a}_f(m)\bar{v}_f^*(n)+\bar{a}_e(m)\bar{a}_e^*(n)\bar{v}_f(m)\bar{a}_f^*(n) \qquad (72)\\
&\quad + \bar{v}_e(m)\bar{v}_{ef}^*(n)\bar{a}_f(m)+\bar{v}_e^*(n)\bar{v}_{ef}(m)\bar{a}_f^*(n).
\end{aligned}$$

The starred couplings of vector boson n and the corresponding propagators are complex conjugated; a procedure which is necessary only in the s-channel.

Now we have all formulae needed to write down the improved Born cross section. For initial state radiation, they read[4]:

$$\begin{aligned}
\sigma_T^0 \quad = \quad & \frac{\pi\alpha^2}{2s'}\Re e\sum_{m,n=0}^{N}\chi_m(s')\chi_n^*(s')\Big[\bar{C}_1(m,n)L_1H_1 \qquad (73)\\
& + \bar{C}_3(m,n)L_2H_1+\bar{C}_4(m,n)L_1H_2+\bar{C}_2(m,n)L_2H_2\Big],
\end{aligned}$$

[4] While for final state radiation the s' is replaced by s, the s-dependence is more complicated for the initial-final state interference [10, 17]. We further remind the reader that ZFITTER doesn't return differential cross sections at all; (76) is shown for illustrational purposes.

$$\sigma^0_{FB} = \frac{\pi\alpha^2}{2s'}\Re e \sum_{m,n=0}^{N} \chi_m(s')\chi_n^*(s')\Big[\bar{C}_2(m,n)L_1H_1 \tag{74}$$

$$+ \bar{C}_4(m,n)L_2H_1 + \bar{C}_3(m,n)L_1H_2 + \bar{C}_1(m,n)L_2H_2\Big],$$

$$\chi_n(s) = \frac{g_n^2}{4\pi\alpha}\frac{s}{s-m_n^2}, \tag{75}$$

$$\frac{d\sigma}{d\cos\vartheta} = (1+\cos^2\vartheta)\sigma^0_T + (2\cos\vartheta)\sigma^0_{FB}. \tag{76}$$

The quantities $L_1,\ L_2,\ H_1$ and H_2 are combinations of the polarizations of the beams (e^-, e^+) and of the helicities of the final fermions:

$$L_1 = 1 - \lambda_+\lambda_-, \quad L_2 = \lambda_+ - \lambda_-, \quad H_1 = \frac{1}{4}(1 - h_+h_-), \quad H_2 = \frac{1}{4}(h_+ - h_-), \tag{77}$$

with λ_- and λ_+ (h_- and h_+) being the polarizations of electron and positron (fermion and antifermion). The masses in the propagator $\chi_n(s)$ are:

$$\begin{array}{rcl} m_0^2 = & 0 & \gamma \\ m_1^2 = & M_1^2 - iM_1\Gamma_1 & Z \\ m_2^2 = & M_2^2 - iM_2\Gamma_2 & Z'. \end{array}$$

In case of a (Z,Z) Born cross section with a polarization of the electron beam, the following combinations correspond to the well-known coupling factors:

$$C_1(Z,Z) + \lambda_+ C_3(Z,Z) = (a_e^2 + v_e^2 + 2\lambda_+ a_e v_e)(a_f^2 + v_f^2), \tag{78}$$

$$C_2(Z,Z) + \lambda_+ C_4(Z,Z) = [2a_e v_e + \lambda_+(a_e^2 + v_e^2)](2a_f v_f). \tag{79}$$

With the above definitions, we have all the necessary prerogatives to calculate cross sections with both weak loop effects and Z' exchange. The resulting Born formulae can be used as input for a QED calculation.

The above expressions for σ^0_T and σ^0_{FB} are used in the redefinitions to be performed in the subroutine `BORN` of `ZFITTER`.

6 Structure of ZEFIT

The package `ZEFIT` should be run together with `ZFITTER`. For execution it has to be loaded before the package `ZFITTER` since it contains subroutines which originate from `ZFITTER` but are modified for the description of the Z'.
Searching for a Z', we assume that the Z boson interactions are correctly described by the Standard Model. Then there are two ways to use the package. *Either* one performs a model-independent fit to data with `ZFITTER` and searches with the result - partial widths or effective couplings - for a Z', using basically the `ZWRATE` or `ROKANC` subroutines of `DIZET`[5], *or* one tries an immediate fit to the cross sections, using basically subroutine `ZCUT` calling `EWINIT`

[5] `ZWRATE` calls `ROKAPP` to calculate the weak form factors of the widths.

of ZFITTER[6]. The common package ZEFIT$\oplus$ZFITTER is prepared for both applications. The second approach, within the (possibly extended) Standard Model terminology, is technically more involved. So, we will use for the present purpose of demonstration only the corresponding interface and branch with subroutine ZANALY of ZFITTER v.3.05.

Input arguments:

AMZ is the mass of the Z boson,

AMH is the mass of the Higgs boson in GeV,

AMT is the top mass in GeV,

INDF selects the final state fermion type, see also the table in the test example in Appendix B,

SQS is the centre of mass energy in GeV,

IFAST $= 1$ allows a fast calculation without geometrical and kinematical cuts. It must be **IFAST** $= 0$ if cuts are required,

IRCUT $= 0, 1$ chooses between an acollinearity cut and a cut on the photon energy.

New additional input parameters coming from a Z':

AMZE is the mass of the Z' in GeV,

ZMIX is the mixing angle between Z and Z',

IZE $= 0, 1, 2$ chooses the model (Standard Model, E_6 model, left-right model).

As examples, we have foreseen a Z' coming from an E_6-GUT and from a left-right symmetric model. The model chosen must be specified by the flag **IZE**. The Standard Model is realised for **IZE** $= \mathbf{0}$, the E_6 model is chosen with **IZE** $= \mathbf{1}$ and the LR model with **IZE** $= \mathbf{2}$. Inside the E_6 model the parameter **TETAE6** must be set as the mixing angle of the two extra Z generators. For the left-right symmetric model the parameter **ANGLR** has to be defined. **ANGLR** is limited, i.e. $\sqrt{\frac{2}{3}} \leq$ **ANGLR** $\leq \sqrt{2}$. The unit for the parameters **TETAE6** and **ANGLR** in ZEFIT is radian.

Additional flags of ZFITTER The flags for weak loop and QED corrections are set in the subroutine ZINITF [10] :

IWEAK $= 0$ or 1, switches the $O(\alpha)$ weak loops.

IHVP $= 1$, 2, 3 characterises the vacuum polarization parametrization. The best choice is **IHVP** $= 3$.

IQCD $= 0$, 1, 2, 3, 4 gives the QCD corrections to the vector boson self energies in Δr, the widths and the cross section. **IQCD** $= 3$ is recommended for LEP applications.

[6] EWINIT uses the subroutine ROKAP in ZFITTER v.3.05 (ROKANC in ZFITTER v.4) to calculate the form factors of the cross sections.

IAMT4 $= 0$, 1; the leading two loop effects of the type $O(\alpha^2 m_t^4)$ can be included.

IBOX $= 0$, 1; WW and ZZ box corrections may be taken into account.

IFINAL $= 0$, 1 chooses between the approximate final state correction by the factor $[1 + 3\pi Q_f^2/(4\pi)]$ and the correct $O(\alpha)$ final state correction including soft photon exponentiation.

INTERF $= 0$, 1 switches the $O(\alpha)$ initial - final state interference.

IPHOT2 $= 0$, 1; second order QED leading logs may be taken into account.

Additionally to these flags of `ZFITTER`, we introduced
IALFRUN = 0,1; which allows an independent switching off and on of running α_{QED}.

<u>**Output parameters:**</u>

SBORN - Born cross section in nb.

STOT - total cross section in nb.

ABORN - Born forward-backward asymmetry.

ATOT - forward-backward asymmetry.

In fig. 1 the block diagram of `ZEFIT` is shown. The subroutines in the dashed boxes remained unchanged.

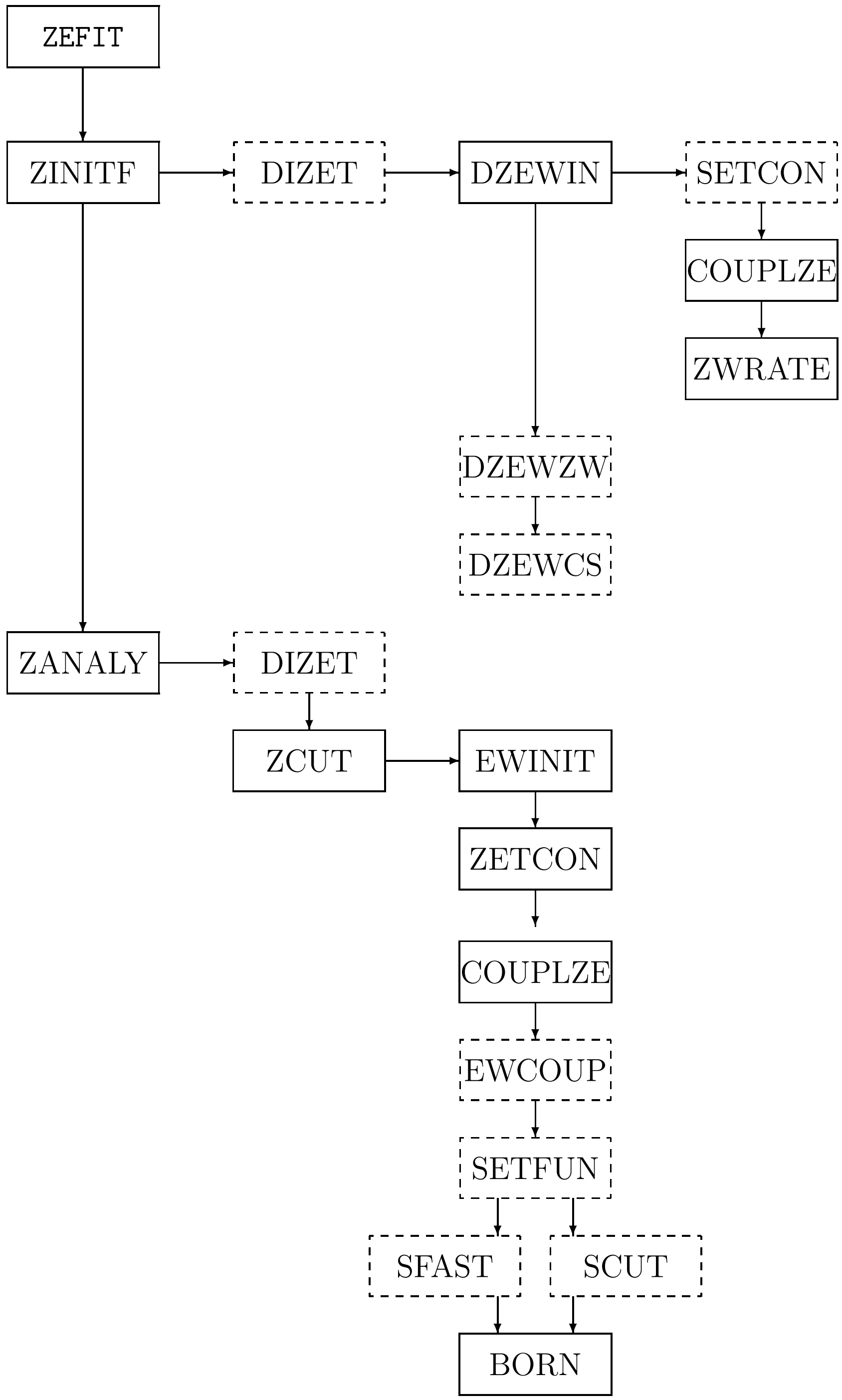

Figure 1: **Structur of the package `ZEFIT`.** Subroutines in solid boxes are rewritten and replace the corresponding subroutines of `ZFITTER` in case of Z, Z' mixing. The subroutines in dashed boxes remain unchanged. They are shown here in order to illustrate the interplay of `ZEFIT` with `ZFITTER`.

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Appendix A
Additional parameters in theories with extra Z bosons

The couplings of Z_1 and Z_2 to fermions are in general a result of a mixing between the Standard Model Z and the Z' as introduced in (6):

$$\begin{aligned}
v_f(1) &= \cos\theta_M \; v_f + \; \sin\theta_M \; \frac{g'}{g} v'_f, \\
a_f(1) &= \cos\theta_M \; a_f + \; \sin\theta_M \; \frac{g'}{g} a'_f, \\
v_f(2) &= \cos\theta_M \; v'_f - \; \sin\theta_M \; \frac{g}{g'} v_f, \\
a_f(2) &= \cos\theta_M \; a'_f - \; \sin\theta_M \; \frac{g}{g'} a_f.
\end{aligned} \tag{A.1}$$

The couplings of the extra Z boson to fermions a'_f and v'_f depend on the particular model. In `ZEFIT` we have implemented extra Z bosons coming from an E_6 GUT or from a left-right model.

In the E_6 GUT the E_6 group [18] is assumed to be broken to the standard model group structure in the following way [19]:

$$\begin{aligned}
E_6 \longrightarrow SO(10) \times U(1)_\psi \longrightarrow SU(5) \times U(1)_\chi \times U(1)_\psi \longrightarrow \\
\longrightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_\chi \times U(1)_\psi.
\end{aligned} \tag{A.2}$$

We further assume that the following linear combination of the two extra Z bosons Z_χ of $U(1)_\chi$ and Z_ψ of $U(1)_\psi$ is light and can be detected at future colliders [20]:

$$Z' = \cos\theta_E \; Z_\chi + \sin\theta_E \; Z_\psi. \tag{A.3}$$

The couplings between the Z' and fermion f are [21]:

$$\begin{aligned}
a'_\nu &= \tfrac{3}{2} Q_\chi \cos\theta_E + \tfrac{1}{2} Q_\Psi \sin\theta, & v'_\nu &= \tfrac{3}{2} Q_\chi \cos\theta_E + \tfrac{1}{2} Q_\Psi \sin\theta, \\
a'_e &= Q_\chi \cos\theta_E + Q_\Psi \sin\theta, & v'_e &= 2Q_\chi \cos\theta_E, \\
a'_u &= -Q_\chi \cos\theta_E + Q_\Psi \sin\theta, & v'_u &= 0, \\
a'_d &= Q_\chi \cos\theta_E + Q_\Psi \sin\theta, & v'_d &= -2Q_\chi \cos\theta_E,
\end{aligned} \tag{A.4}$$

$$Q_\chi = \frac{1}{\sqrt{10}}, \qquad Q_\psi = \frac{1}{\sqrt{6}}. \tag{A.5}$$

The Z' coupling constant g' may be determined by the assumption that the renormalisation group evolution of g' is the same as that of g [22],

$$g' = \sqrt{\frac{5}{3}} \; \sin\theta_W \; g. \tag{A.6}$$

Of particular interest for applications is the completely specified case $Z' \equiv Z_\eta = \sqrt{3/8}\ Z_\chi - \sqrt{5/8}\ Z_\psi$ as suggested by superstring theories [23, 24]. The most general Z' in a E_6 GUT can therefore be described by three additional parameters: θ_M =ZMIX, θ_E =TETAE6 and $M(Z_2)$ =AMZE.
Another origin of a Z' could be a left-right symmetric model [25]. As in the E_6 case, we have θ_M =ZMIX and $M(Z_2)$ =AMZE and one additional parameter α =ANGLR. In this model we have the following couplings of the Z' to the fermions:

$$\begin{aligned}
a'_\nu &= \frac{1}{2\alpha}, & v'_\nu &= a'_\nu, \\
a'_e &= \frac{1}{2}\alpha, & v'_e &= \frac{1}{\alpha} - a'_e, \\
a'_u &= -\frac{1}{2}\alpha, & v'_u &= -\frac{1}{3\alpha} - a'_u, \\
a'_d &= \frac{1}{2}\alpha, & v'_d &= -\frac{1}{3\alpha} - a'_d.
\end{aligned} \tag{A.7}$$

The relation between g and g' is:

$$g' = \sin\theta_W\ g. \tag{A.8}$$

Concerning the width of the Z', we will assume that it can decay only into particles of the known three fermion generations including the top-quark.

Of course, one also can make no model assumptions about the Z' at all. Then, one has to specify arbitrarily all needed couplings of the Z' to fermions and the width of the Z'. In addition, the Z, Z' mixing angle and the Z' mass have to be given.

Appendix B
The Main Routine of the Test Program

```
      PROGRAM ZEFIT
c
c
*********************************************************************
**                                                                 **
**     ZEFIT  -  a package for extra Z searches at LEP             **
**                                                                 **
**                                                                 **
**  A package [1] ZPRIME physics in presence of QED and weak       **
**  loop corrections as realised  in ZFITTER [2]                   **
**  A possible mixing of Z and Z' is taken into account.           **
**                                                                 **
**  Version:       2.00                                            **
**  Release:       November 25,1991                                **
**  Authors:       A. Leike, S. Riemann, T. Riemann                **
**                 Inst. for High Energy Physics, Zeuthen          **
**  Contact in L3: S. Riemann, riemanns@cernvm                     **
**                                                                 **
**  [1]  A.Leike, S.Riemann, T. Riemann:                           **
**       Z.Z' Mixing                                               **
**       in Presence of Standard Weak Loop Corrections,            **
**       Munich Univ. prepr. LMU-91/06 (Dec 1991)                  **
**  [2]  D.Bardin et.al.:                                          **
**       A Users Guide to ZFITTER: An Analytical Program for       **
**       Fermion Pair Production in e^+ e^- Annihilation,          **
**       in preparation.                                           **
**                                                                 **
*********************************************************************
*
      IMPLICIT REAL*8(A-H,O-W,Z)
      IMPLICIT COMPLEX * 16 (X,Y)
      REAL*4 TIME1,TIME2
      PARAMETER (NSARR=10,NMODEL=1)
*
*  NSARR  - NUMBER OF SQRTS POINTS TO BE CALCULATED
*  NMODEL - NUMBER OF POINTS SPECIFIED WITHIN AN EXTRA Z MODEL
*
      COMMON/ROVEFZ/ARROFZ(0:10),ARVEFZ(0:10)
      COMMON/FRINIT/ NPAR(30),ZPAR(30)
      COMMON/ZE/TETAMD,AMZE,GAMZP,ZMIX,CHIE
     +,         VEEZE,VEFZE,AEFZE,VQEZE(6),VQFZE(6)
     +,         XVEEZE,XVEFZE,XAEFZE,XVEFGZ,XAEFGZ
     +,         XVQEZE(6),XVQFZE(6),XVQFGZ(6)
```

```
       COMMON /IZE/ IZE
*  ARROFZ(0:9) - CALCULATED EFFECTIVE RO'S FOR EACH CHANNEL
*  ARVEVZ(0:9) - CALCULATED EFFECTIVE VECTOR COUPLINGS FOR EACH CHANNEL
*  ARROFZ(0:10) AND ARVEFZ(0:10) ARE UNDEFINED AND NOT USED
*  ARVEVZ      = 1 - 4 * ABS(QF) * SINTW**2 ( SINTW**2 IS EFFECTIVE )
      DIMENSION WIDTHS(0:10)
      DIMENSION TETAE6(NMODEL),ANGLR(NMODEL)
      DIMENSION SARR(NSARR)
* ARRAY OF CMS ENERGIES
      DATA SARR /87.d0,88.D0,89.0D0,90.d0,91.d0,91.17d0,92.d0,93.d0
     +,          94.d0,95.d0/
**
* CHOICE OF E_6 ZPRIME MODEL (TETA6 in RADIAN)
C     DATA TETAE6 /0.D0, 1.5708D0, -.91174D0/
      DATA TETAE6 /0.D0/
* CHOICE OF LEFT-RIGHT EXTRA Z MODEL (ANGLR in RADIAN)
      DATA ANGLR /0.81649658D0, 1.D0, 1.4142135D0/
C                  sqrt(2/3)          sqrt(2)
**
* INPUT FOR INITIALIZATION SUBROUTINE ZINITF:
* Z-MASS (ALL INPUT MASSES IN GEV)
      AMZ=91.180D0
      AMZLEP=AMZ
* TOP-MASS
      AMT=150D0
* HIGGS-MASS
      AMH=3D2
* ZPRIME MASS (NOT RELEVANT IF IZE = 0)
      AMZE= 5000.D0
* TWO QCD-CORRECTION FACTORS (THE SECOND ONE FOR B-BBAR CHANNEL)
      QCDCOR=1.040D0
      QCDCOB=1.045D0
*
* IZE = 1,2 (0) EXTRA Z CONTRIBUTIONS ARE (NOT) INCLUDED
*     = 0   STANDARD MODEL
*     = 1   E_6 MODEL (TETAE6 MUST BE SPECIFIED)
*     = 2   LEFT-RIGHT MODEL (ANGLR MUST BE SPECIFIED)
      IZE=1
* ZMIX - Z, ZPRIME MIXING ANGLE
      ZMIX = 0.01D0
* IF IFAST=1       WITHOUT CUTS
* IF IFAST=0       CUTS ARE POSSIBLE
      IFAST=1
* PLEASE, LOOK INTO ZINITF IN ORDER TO INITIALIZE ALL FLAGS AND
* PARAMETERS WHICH COULD BE (AND SHOULD BE) INITIALIZED FOR A SPECIFIC
* TASK.
*
```

```
      IF(IZE.EQ.0)  THEN
        INMOD=1
      ELSE
        INMOD=NMODEL
      ENDI
* SPECIFY THE MODEL DEPENDENT ANGLE
      DO 20  IZP=1,INMOD
* EXTRA Z FROM E_6 MODEL
      IF(IZE.EQ.1) TETAMD=TETAE6(IZP)
* EXTRA Z FROM LEFT-RIGHT MODEL
      IF(IZE.EQ.2) THEN
        ANGMIN = DSQRT(2.D0/3.D0)-1.D-5
        ANGMAX = DSQRT(2.D0)+1.D-5
        TETAMD=ANGLR(IZP)
        IF(TETAMD.LT.ANGMIN .OR. TETAMD.GT.ANGMAX) THEN
          PRINT*,' LEFT-RIGHT PARAMETER OUT OF RANGE'
          STOP
        ENDIF
      ENDIF
*
      CALL ZINITF(AMZ,AMT,AMH,QCDCOR,QCDCOB,SW2,WIDTHS)
* OUTPUT OF ZINITF:
* SW2 - CALCULATED QUANTITY = 1-AMW**2/AMZ**2
* WIDTHS(INDF) - CALCULATED PARTIAL CHANNEL Z-WITDHS (IN MEV)
*               FOR 0<INDF<9 AS DESCRIBED ABOVE
*               FOR   INDF=10 IT CONTAINS CALCULATED TOTAL Z-WIDTH
*
      IF(IZE.EQ.0)PRINT'(/,'' STANDARD MODEL, NO EXTRA Z EXTENSION'',/)'
      IF(IZE.EQ.1)PRINT'(/,'' EXTRA Z FROM E6 GUT'',/)
      IF(IZE.EQ.2)PRINT'(/,'' EXTRA Z FROM LEFT-RIGHT MODEL'',/)'
      PRINT 1000,AMZLEP,AMT,AMH
      IF(IZE.EQ.1)  PRINT 1100,AMZE,ZMIX,TETAMD
      IF(IZE.EQ.2)  PRINT 1200,AMZE,ZMIX,TETAMD
      PRINT 1300,QCDCOR,QCDCOB,SW2
      PRINT 1500,WIDTHS
      PRINT 1550,NPAR(1),NPAR(2),NPAR( 3),NPAR( 4)
     &,          NPAR(8),NPAR(9),NPAR(10),NPAR(16)
*
* GEOMETRICAL CUTS OVER THE ANGLE BETWEEN E+ AND OUTGOING ANTI-FERMION
      ANG1=140D0
      ANG2=40D0
      IF (IFAST .NE. 1) PRINT 1700,ANG2,ANG1
*
*  KINEMATICAL CUTS:
*  IF IRCUT=1 THEN SPRIME (IN GEV**2)
*  IF IRCUT=0 THEN ACOL+EMIN (IN DEGREES AND GEV, RESPECTIVELY)
*
```

```
      IRCUT = 0
      SPRIME= 1D-6
      ACOL  = 25.D0
      EMIN  = .0D0
      IF(IFAST.EQ.0) THEN
         IF(IRCUT.EQ.1) THEN
            PRINT 1800,SPRIME
         ELSE
            PRINT 1900,ACOL,EMIN
         ENDIF
      ELSEIF(IFAST.EQ.1) THEN
         PRINT 1600
      ELSE
         IFAST = 1
         PRINT 1600
      ENDIF
*
      DO 11 INDF=2,2
*  INDF=0 FOR NEUTRINO
*      =1 FOR ELECTRON
*      =2 FOR MUON
*      =3 FOR TAU
*      =4 FOR UP
*      =5 FOR DOWN
*      =6 FOR CHARM
*      =7 FOR STRANGE
*      =8 FOR TOP ( RETURNS ZERO )
*      =9 FOR BOTTOM
*     =10 FOR HADRONS
      PRINT 2000,INDF
      GAMEE=WIDTHS(1)/1D3
      GAMZ=WIDTHS(10)/1D3
      IF(INDF.NE.10) THEN
         GAMFI=WIDTHS(INDF)/1D3
      ELSE
         GAMFI=0D0
         DO 4 IH=4,9
 4       GAMFI=GAMFI+WIDTHS(IH)/1D3
      ENDIF
      IF(INDF .NE. 10) PRINT 2500
      IF(INDF .EQ. 10) PRINT 2600
* DO LOOP OVER THE SQRT(S)=SQS
      DO 10 IS=1,NSARR
      SQS=SARR(IS)
*
* NOW EVERYTHING WILL BE CALCULATED IN THE FRAMEWORK OF THE STANDARD
* MODEL BY ZANALY- SUBROUTINE.
```

```
*
      IF(INDF.NE.10) THEN
         CALL ZANALY(IFAST,INDF,SQS,IRCUT,SPRIME,ACOL,EMIN,ANG1,ANG2,
     &               AMZ,AMT,AMH,QCDCOR,QCDCOB,SBORN,STOT,ABORN,ATOT)
         CS =STOT
         AFB=ATOT
         IF(INDF.EQ.0) THEN
            PRINT 3000,SQS,CS
         ELSE
            PRINT 3100,SQS,CS,AFB
         ENDIF
      ELSE
         CS =0D0
         DO 53 INDFH=4,9
         CALL ZANALY(IFAST,INDFH,SQS,IRCUT,SPRIME,ACOL,EMIN,ANG1,ANG2,
     &               AMZ,AMT,AMH,QCDCOR,QCDCOB,SBORN,STOT,ABORN,ATOT)
         CS =CS +STOT
 53      CONTINUE
         PRINT 3000,SQS,CS
      ENDIF
 10   CONTINUE
 11   CONTINUE
 20   CONTINUE
1000 FORMAT(1X,/1X,'INPUT :  MZ    = ',F6.3,' GEV'
    &     ,'    MT    = ',F6.2,' GEV'
    &,     '    MH  = ',F8.2,' GEV'
1100 FORMAT ('          AMZE  = ',F6.1,' GeV'
    &,              '    ZMIX  = ',F6.4,'     TETAE6 = ',F8.4,/)
1200 FORMAT ('          AMZE  = ',F6.1,' GeV'
    &,              '    ZMIX  = ',F6.4,'      ANGLR = ',F8.4,/)
1300 FORMAT (' QCD CORRECTION FACTORS = ',2(F7.5,2X),/
    &       ,' OUTPUT:    SW2         = ',F5.4)
1500 FORMAT (1X,'PARTIAL AND TOTAL Z-WIDTHS IN MEV',/
    &,5x,'nu,nubar  =',F7.1,8x,'e+,e-  =',F7.1,6x,'mu,mubar =',F7.1
    &,/,5x,'tau+,tau- =',F7.1,8x,'u,ubar =',F7.1,8x,'d,dbar =',F7.1
    &,/,5x,'c,cbar    =',F7.1,8x,'s,sbar =',F7.1
    &,/,5x,'t,tbar    =',F7.1,8x,'b,bbar =',F7.1,8x,'TOTAL  =',F7.1
    &,/)
1550 FORMAT (/,' FLAGS IN INITIALISATION ROUTINE ZINITF:',
    &          /,' IWEAK  =',I2,'  IHVP   =',I2,'  IQCD   =',I2,
    &          '  IBOX    =',I2,/,' INTERF =',I2,'  IFINAL =',I2,
    &           '  IPHOT2 =',I2,'  IAMT4   =',I2)
1700 FORMAT (1X,'GEOMETRICAL ACCEPTANCE CUTS: ANG1,ANG2=',2(F5.1,2X))
2500 FORMAT (/,1X,'SQRT(S) [GeV]',6X,'CROSS SECTION [nb]',4X,
    &         'ASYMMETRY')
2600 FORMAT (/,1X,'SQRT(S) [GeV]',6X,'CROSS SECTION [nb]' )
1800 FORMAT (/,1X,'KINEMATICAL CUT: SPRIME=',E6.1,1X,'GEV')
```

```
1900 FORMAT (/,1X,'KINEMATICAL CUTS: ACOL=',F5.1,1X,'DEGREES',2X,
   &            'EMIN=',F5.1,1X,'GEV')
1600 FORMAT (/,1X,'NO KINEMATICAL CUT (IFAST=1)')
2000 FORMAT (1X,/,' CHOSEN FERMION CHANNEL INDEX - INDF=',I2)
3000 FORMAT (7X,F7.3,11X, F13.4)
3100 FORMAT (7X,F7.3,11X,2F13.4)
      END
```

Output of the Test Example

```
 EXTRA Z FROM E6 GUT



 INPUT :  MZ    = 91.180 GEV    MT    = 150.00 GEV    MH  =  300.00 GEV
          AMZE  =  500.0 GEV    ZMIX  = 0.0100     TETAE6 =  0.0000

 QCD CORRECTION FACTORS = 1.04000  1.04500
 OUTPUT:    SW2         = .2260
 PARTIAL AND TOTAL Z-WIDTHS IN MEV
     nu,nubar  =  169.2        e+,e-  =   83.2      mu,mubar =   83.2
     tau+,tau- =   83.0        u,ubar =  296.5        d,dbar =  386.0
     c,cbar    =  296.1        s,sbar =  386.0
     t,tbar    =    0.0        b,bbar =  379.8        TOTAL  = 2501.6


 FLAGS IN INITIALISATION ROUTINE ZINITF:
 IWEAK  = 1  IHVP   = 3  IQCD   = 3  IBOX    = 1
 INTERF = 1  IFINAL = 1  IPHOT2 = 1  IAMT4   = 1

 NO KINEMATICAL CUT (IFAST=1)

 CHOSEN FERMION CHANNEL INDEX - INDF= 2

 SQRT(S) [GeV]      CROSS SECTION [nb]    ASYMMETRY
        87.000                  0.1393      -0.3713
        88.000                  0.2109      -0.2950
        89.000                  0.3671      -0.2071
        90.000                  0.7519      -0.1122
        91.000                  1.4042      -0.0192
        91.170                  1.4533      -0.0047
        92.000                  1.1713       0.0566
        93.000                  0.6813       0.1090
        94.000                  0.4338       0.1448
        95.000                  0.3076       0.1703
```