Bi-Maximal Neutrino Mixing in the MSSM with a Single Right-Handed Neutrino

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Abstract

We discuss neutrino masses in the framework of a minimal extension of the minimal supersymmetric standard model (MSSM) consisting of an additional single right-handed neutrino superfield N with a heavy Majorana mass M, which induces a single light see-saw mass m_{ν_3} leaving two neutrinos massless at tree-level. This trivial extension to the MSSM may account for the atmospheric neutrino data via $\nu_\mu \to \nu_\tau$ oscillations by assuming a near maximal mixing angle $\theta_{23} \sim \pi/4$ and taking $\Delta m_{23}^2 \sim m_{\nu_3}^2 \sim 2.5 \times 10^{-3}\ eV^2$. In order to account for the solar neutrino data we appeal to one-loop radiative corrections involving internal loops of SUSY particles, which we show can naturally generate an additional light neutrino mass $m_{\nu_2} \sim 10^{-5}\ eV$ again with near maximal mixing angle $\theta_{12} \sim \pi/4$. The resulting scheme corresponds to so-called "bi-maximal" neutrino mixing involving "just-so" solar oscillations.

Atmospheric neutrino data from Super-Kamiokande [1] and SOUDAN [2], when combined with the recent CHOOZ data [3], are consistent with $\nu_\mu \to \nu_\tau$ oscillations with near maximal mixing and $\Delta m_{23}^2 \sim 2.5 \times 10^{-3}~eV^2$. [1] A critical appraisal of current neutrino data can be found in [5, 6]. In practice a common standard approach to neutrino masses is to introduce three right-handed neutrinos with a heavy Majorana mass matrix. When the usual Dirac mass matrices of neutrinos and charged leptons are taken into account, the see-saw mechanism then leads to the physical light effective Majorana neutrino masses and mixing angles relevant for experiment. There is a huge literature concerning this and other kinds of approach to neutrino masses which is impossible to do justice to. In ref.[7] we merely list a few recent papers, from which which the full recent literature may be reconstructed. For older work on neutrino masses see for instance [8], where extensive references to earlier work may be found.

In a recent paper [9] one of us followed a minimalistic approach and introduced below the GUT scale only a single "right-handed neutrino" [2] N into the standard model (or supersymmetric standard model) with a heavy Majorana mass $\frac{1}{2}M\bar{N}N^c$ where $M < M_{GUT}$. With only a single right-handed neutrino the low energy neutrino spectrum consists of a light neutrino $\nu_3 \approx s_{23}\nu_\mu + c_{23}\nu_\tau$ with mass m_{ν_3}, plus two massless neutrinos consisting of the orthogonal combination $\nu_0 \approx c_{23}\nu_\mu - s_{23}\nu_\tau$ together with ν_e which we have here assumed for simplicity has a zero Yukawa coupling λ_e to N. [3] By contrast the Yukawa couplings λ_μ and λ_τ of ν_μ and ν_τ to N are non-zero and determine the 23 mixing angle via $t_{23} = \lambda_\mu/\lambda_\tau$. Maximal mixing corresponds to $\lambda_\mu = \lambda_\tau$, with $\lambda_e = 0$ and $\Delta m_{23}^2 = m_{\nu_3}^2$ may be chosen to be in the correct range to

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[1] The data are equally consistent if ν_τ is replaced by a light sterile neutrino, and indeed many authors have considered adding an extra ν_τ light singlet neutrino state [4], although we shall not do so here.

[2] Since CP conjugation converts a right-handed neutrino into a left handed anti-neutrino, it should strictly be called a gauge singlet.

[3] We shall use notation such as $s_{23} \equiv \sin\theta_{23}$, $c_{23} \equiv \cos\theta_{23}$, $t_{23} \equiv \tan\theta_{23}$, extensively throughout this paper.

account for the atmospheric neutrino data.

In order to account for the solar neutrino [10] data via the MSW effect [11] we were forced to depart from minimality by introducing a small additional tau neutrino mass coming from GUT scale physics $m_{\nu_\tau} \sim \frac{m_t^2}{M_{GUT}} \sim few \times 10^{-3} eV$ [9]. The tau neutrino mass is clearly of the correct order of magnitude to lift the degeneracy of the two previously massless neutrinos by just the right amount in order explain the solar neutrino data via the MSW effect [11] since $\Delta m_{12}^2 \sim m_{\nu_\tau}^2 \sim 10^{-5} \ eV^2$. In addition we needed to allow $\lambda_e \neq 0$ in order to generate a small mixing angle $\theta_{12} \approx \theta_{13}$ (where CHOOZ requires the small angle MSW solution for the higher end of the Super-Kamiokande neutrino mass range.) In order to account for such a tau neutrino mass perturbation we suggested that in addition to the single right-handed neutrino N there are three "conventional" right-handed neutrinos with GUT scale masses which, together with quark-lepton unification à la minimal $SO(10)$, give rise to a hierarchy of neutrino masses: $m_{\nu_\tau} : m_{\nu_\mu} : m_{\nu_e} \approx m_t^2 : m_c^2 : m_u^2$ (up to Clebsch relations, RG running effects, and heavy Majorana textures). In this scenario the tau neutrino mass dominates and gives the desired perturbation. The original single right-handed neutrino N is then identified as belonging to an extra multiplet below the GUT scale.

In the present paper we wish to return to minimality by assuming there is only a single right-handed neutrino N below the GUT scale with *no* additional right-handed neutrinos at the GUT scale. More generally we shall assume *no* additional operators arising from the GUT or string scale which give rise to neutrino masses. Our starting point is then the tree-level spectrum described in the second paragraph above involving one massive plus two massless neutrinos. Our basic observation is that in general there is no symmetry which protects the masslessness of the two neutrinos, so they are expected to acquire masses beyond the tree-level. To be definite we shall show that masses for ν_0 and ν_e can be generated from one-loop SUSY corrections and

lead to a neutrino mass $m_{\nu_2} \sim 10^{-5}$ eV, corresponding to $\nu_2 \approx s_{12}\nu_e + c_{12}\nu_0$, while the orthogonal combination $\nu_1 \approx c_{12}\nu_e - s_{12}\nu_0$ remains approximately massless. The value of the 12 mixing angle is controlled by a ratio of soft flavour-violating parameters, $t_{12} = \Delta_{13}/\Delta_{23}$ which could be large, and $\Delta m_{12}^2 \sim m_{\nu_2}^2$. In this case we arrive at approximate "bi-maximal" flavour mixing in both the $(\nu_e - \nu_0)$ and $(\nu_\mu - \nu_\tau)$ sectors [12]. This may be compared to bi-maximal mixing in the $(\nu_e - \nu_\mu)$ and $(\nu_\mu - \nu_\tau)$ sectors [13]. Both forms of bi-maximal mixing rely on the "just-so" [14, 15] vacuum oscillation description of the solar neutrino data. For example a recent best-fit to solar neutrino data in the vacuum oscillation region requires [6] $\Delta m_{12}^2 \simeq 6.5 \times 10^{-11}$ eV^2 and a mixing angle $\sin^2 2\theta_{12} \sim 0.75$. Following [16], we assume that this is not in conflict with the spectrum of electron neutrinos detected from SN1987A [17]. The main point of this paper is to show that bi-maximal mixing can arise in a natural way from a bare minimum of ingredients: the addition of a single right-handed neutrino to the MSSM.

The MSSM plus one heavy right-handed neutrino N has a superpotential which contains, in the addition to the usual terms such as $\mu H_1 H_2$ plus the quark and charged lepton Yukawa couplings, the following new terms involving the new N superfield,

$$W_{new} = \lambda_\alpha L_\alpha N H_2 + \frac{1}{2} M N N \qquad (1)$$

where the subscript $\alpha = e, \mu, \tau$ indicates that we are in the charged lepton mass eigenstate basis, e.g. $L_e = (\nu_{eL}, e_L^-)$ where e^- is the electron mass eigenstate and ν_e is the associated neutrino weak eigenstate, and λ_e is a Yukawa coupling to the singlet N in this basis. In this basis the soft terms involving sneutrinos are

$$\begin{aligned} V_{soft} &= m_{\tilde{L}_\alpha \tilde{L}_\beta^*}^2 \tilde{L}_\alpha \tilde{L}_\beta^* + m_{\tilde{N}\tilde{N}^*}^2 \tilde{N}\tilde{N}^* \\ &+ (\lambda_\alpha A_\alpha H_2 \tilde{L}_\alpha \tilde{N} + M B_N \tilde{N}\tilde{N} + h.c.) \end{aligned} \qquad (2)$$

The potential involving sneutrinos, including F-terms, D-terms and soft terms is, in the usual notation where $\tan\beta = v_2/v_1$ is the ratio of Higgs vacuum expectation

3

values,

$$V = (m^2_{\tilde{L}_\alpha \tilde{L}^*_\beta} + \lambda_\alpha \lambda_\beta v^2_2 + \frac{1}{2} m^2_Z \cos 2\beta \delta_{\alpha\beta})\tilde{\nu}_\alpha \tilde{\nu}^*_\beta + (M^2 + m^2_{\tilde{N}\tilde{N}^*})\tilde{N}\tilde{N}^*$$
$$+ [(v_2 A_\alpha \lambda_\alpha - \lambda_\alpha \mu v_2 \cot \beta)\tilde{\nu}_\alpha \tilde{N} + (\lambda_\alpha v_2 M)\tilde{\nu}_i \tilde{N}^* + MB_N \tilde{N}\tilde{N} + h.c.] \quad (3)$$

We neglect $m^2_{\tilde{N}\tilde{N}^*}$ with respect to M^2 for the remainder of the paper.

The tree-level neutrino spectrum was discussed in ref.[9] and is summarised below. The usual fermionic see-saw mechanism which is responsible for the light effective Majorana mass $m_{\nu_\alpha \nu_\beta}\nu_\alpha \nu_\beta$ can be represented by the diagram in Fig.1.
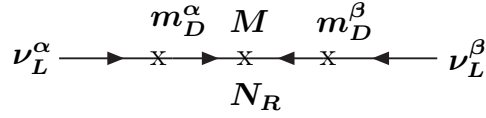

Figure 1: A diagrammatic representation of the see-saw mechanism. M is the singlet Majorana mass, and we have defined $m^\alpha_D \equiv \lambda_\alpha v_2$.

The see-saw mechanism represented by Fig.1 leads to the light effective Majorana matrix in the $\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}$ basis:

$$m_{\nu_\alpha \nu_\beta} = \begin{pmatrix} \lambda^2_e & \lambda_e \lambda_\mu & \lambda_e \lambda_\tau \\ \lambda_e \lambda_\mu & \lambda^2_\mu & \lambda_\mu \lambda_\tau \\ \lambda_e \lambda_\tau & \lambda_\mu \lambda_\tau & \lambda^2_\tau \end{pmatrix} \frac{v^2_2}{M} \quad (4)$$

with phase choices such that the Yukawa couplings are real. The matrix in Eq.4 has two zero eigenvalues and one non-zero eigenvalue $m_{\nu_3} = \sum_\alpha |\lambda_\alpha|^2 v^2_2/M$ corresponding to the eigenvector $\nu_3 = \lambda_\alpha \nu_\alpha / \sqrt{\sum_\beta |\lambda_\beta|^2}$. This can easily be understood from Eq.1 where it is clear that only the combination ν_3 couples to N with a Yukawa coupling $\lambda_3 = \sqrt{\lambda^2_\beta}$. In the $\lambda_e = 0$ limit ν_e is massless and the other two eigenvectors are simply

$$\begin{pmatrix} \nu_0 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} c_{23} & -s_{23} \\ s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} \quad (5)$$

where $t_{23} = \lambda_\mu/\lambda_\tau$ and the ν_0 is massless at tree-level. If we allow a small but non-zero λ_e then we denote the massless states by primes. In this case the weak eigenstates

4

are related to the mass eigenstates by [9]

$$
\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L = U_0 \begin{pmatrix} \nu'_e \\ \nu'_0 \\ \nu_3 \end{pmatrix}_L \tag{6}
$$

where

$$
U_0 = \begin{pmatrix} 1 & \left(\frac{c_{23}}{s_{23}}\right)\theta_1 & \theta_1 \\ -\frac{\theta_1}{s_{23}} & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \tag{7}
$$

where $\theta_1 \approx \lambda_e/\sqrt{\lambda_\mu^2 + \lambda_\tau^2}$. The unitary matrix U_0 is the tree-level neutrino mixing matrix (the analogue of the CKM matrix). This follows since the charged weak currents are given by:

$$
W_\mu^-(\bar{e}, \bar{\mu}, \bar{\tau})_L \gamma^\mu \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L + h.c. \tag{8}
$$

where $\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}$ are neutrino weak eigenstates which couple with unit strength to e, μ, τ, respectively. Note that due to the massless degeneracy of ν_0 and ν_e the choice of basis is not unique.

The MSSM generalisation of the see-saw mechanism was recently discussed by Grossman and Haber [18]. The SUSY analogue of the Majorana neutrino mass $m_{\nu_\alpha \nu_\beta} \bar{\nu}_\alpha{}^c \nu_\beta$ is the "Majorana" sneutrino mass $m_{\tilde{\nu}_\alpha \tilde{\nu}_\beta}^2 \tilde{\nu}_\alpha \tilde{\nu}_\beta$. Such masses, which perhaps should more properly be referred to as lepton number violating masses, do not appear directly in the potential in Eq.3, but can be generated by the scalar analogue of the see-saw mechanism. One way [18] to understand the origin of a "Majorana" sneutrino mass is to separate each sneutrino into real and imaginary components $\tilde{\nu} = 2^{-1/2}(\tilde{\nu}_R + i\tilde{\nu}_I)$ and $\tilde{N} = 2^{-1/2}(\tilde{N}_R + i\tilde{N}_I)$ then observe that a lepton number conserving ("Dirac") sneutrino mass is the same for real and imaginary parts of the field:

$$
m_{\tilde{\nu}\tilde{\nu}^*}^2 \tilde{\nu}\tilde{\nu}^* = \frac{m_{\tilde{\nu}\tilde{\nu}^*}^2}{2}(\tilde{\nu}_R^2 + \tilde{\nu}_I^2) \tag{9}
$$

whereas a lepton number violating ("Majorana") mass contributes with opposite sign

to the mass of the real and imaginary components:

$$(m_{\tilde{\nu}\tilde{\nu}}^2 \tilde{\nu}\tilde{\nu} + h.c.) = m_{\tilde{\nu}\tilde{\nu}}^2 (\tilde{\nu}_R^2 - \tilde{\nu}_I^2) \tag{10}$$

Grossman and Haber studied the 4×4 matrix for the one-family case and showed that it separates into two independent 2×2 matrices corresponding to the CP even and CP odd states. To calculate the sneutrino Majorana mass they find eigenvalues of the 2×2 matrices and take mass differences. However one can equally well think of the Majorana sneutrino masses diagramatically, as the scalar analogue of the diagrammatic representation of the usual fermionic see-saw mechanism discussed above, as follows.

As in the fermionic see-saw mechanism the light effective Majorana sneutrino masses come from integrating out a heavy state, in this case $\tilde{N}$ which has the lepton number violating interactions. Thus the scalar see-saw diagrams involve an $\tilde{N}$ propagator. To get a contribution to $m_{\tilde{\nu}_\alpha \tilde{\nu}_\beta}^2 \tilde{\nu}_\alpha \tilde{\nu}_\beta$ that is $O(1/M)$, we need at least one power of M on top to cancel the $1/M^2$ from the propagator. There are two possible diagrams, illustrated in Figs. 2 and 3.
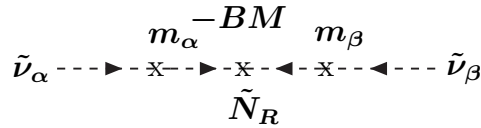

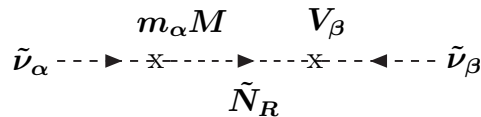Figure 2: A scalar see-saw diagram. We have defined $m_\alpha \equiv \lambda_\alpha v_2 M$.

Figure 3: Another scalar see-saw diagram. We have defined $V_\beta \equiv v_2 A_\beta \lambda_\beta - \lambda_\beta \mu v_2 \cot \beta$.

We estimate the contribution from Fig.2 to be $\simeq -B\lambda_\alpha\lambda_\beta v_2^2/M$, and from Fig. 3 to be $\simeq (A_\beta - \mu\cot\beta)\lambda_\alpha\lambda_\beta v_2^2/M$. The total contribution to the light effective Majorana (= lepton number violating) sneutrino mass is then

$$m^2_{\tilde{\nu}_\alpha\tilde{\nu}_\beta} \simeq (A_\beta - \mu\cot\beta - B)m_{\nu_\alpha\nu_\beta} \tag{11}$$

which consists of a product of SUSY masses times the tree-level Majorana neutrino mass. This result agrees with the result in ref.[18].

Having generated a *sneutrino* Majorana mass it is straightforward to see that such a mass will lead to one-loop radiative corrections to *neutrino* Majorana masses [18] via the self-energy diagram in Fig. 4.
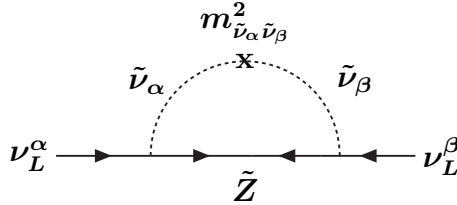

Figure 4: One-loop diagram generating a Majorana neutrino mass. The lepton number violation comes from the effective sneutrino "Majorana" (lepton number violating) mass $m^2_{\tilde{\nu}_\alpha\tilde{\nu}_\beta}$ arising from Figs.2,3 which we have condensed into a single "x" here.

The diagram involves an internal loop of neutralinos and sneutrinos, with the lepton number violating Majorana sneutrino mass at the heart of the diagram. This diagram applies to an arbitrary basis. In particular it applies to the basis in which the tree-level Majorana neutrino masses are diagonal, which is the most convenient basis for calculating loop corrections to neutrino masses. In this basis the Majorana sneutrino masses are also diagonal (assuming that $A_\alpha\lambda_\alpha$ is aligned with λ_α), as is clear from Eq.11, and consist of two zero Majorana mass sneutrinos [4] plus a non-zero

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[4]The usual lepton number conserving sneutrino masses are of course non-zero.

Majorana mass given by Eq.11 in this basis:

$$m_{\tilde{\nu}_3 \tilde{\nu}_3}^2 \approx (A_3 - \mu \cot \beta - B) m_{\nu_3} \tag{12}$$

where A_3 is the trilinear soft parameter associated with the Yukawa coupling eigenvalue λ_3 (ignoring flavour violation.) According to Grossman and Haber [18] the one-loop correction to the tree-level neutrino mass is given by:

$$\delta m_{\nu_3} \approx \frac{g^2}{32\pi^2 \cos^2 \theta_W} \frac{m_{\tilde{\nu}_3 \tilde{\nu}_3}^2}{\bar{m}} \sum_j f(y_j) |Z_{jZ}|^2 \equiv \epsilon m_{\nu_3} \tag{13}$$

where the $y_j = \bar{m}^2 / m_{\tilde{\chi}_j^0}^2$, $\tilde{\chi}_j^0$ is the j-th neutralino, the function $f(y_j) \sim 0.25 - 0.57$ and $\bar{m}^2$ is an average sneutrino mass (see Eq.14, 15, and 16.) We have defined a quantity ϵ whose value is typically $\epsilon \sim 10^{-3}$, assuming that all soft SUSY breaking parameters and μ (including those associated with the right-handed neutrino N) are of a similar order of magnitude. Grossman and Haber actually consider the case that $B \gg 1$ TeV but we shall assume here that $B \sim 1$ TeV. Thus we conclude that the correction to the tree-level neutrino mass is of order 0.1% and so is utterly negligible.

Since $\delta m_{\nu_3} \propto m_{\nu_3}$ one might be tempted to conclude that the two neutrino states which are massless at tree-level remain massless at one-loop. However this conclusion would be incorrect due to the effects of flavour violation which we have so far ignored. There are two possible origins of flavour violation that can be relevant for neutrino masses:

(i) Soft lepton number conserving sneutrino masses which are flavour-violating;

(ii) Trilinear parameters A_β which are misaligned with the Yukawa couplings λ_β.

In the present paper we shall focus on case (i) only since we shall see later that the mechanism responsible for generating small non-zero neutrino masses has two possible sources: (i) or (i)+(ii). There is no significant source of small neutrino masses coming from (ii) alone, and so for simplicity we shall ignore the effect of (ii) in making our estimates.

We shall need to be able to move from one basis to another and be able to deal with flavour-violating effects in any given basis. The tool for doing this which we shall use is the "mass-insertion approximation". According to the mass insertion approach in changing basis we must rotate the superfield as a whole, so that the gauge couplings do not violate flavour. This means that in the fermion mass eigenstate basis the sfermions have off-diagonal masses, and flavour-violation is dealt with by sfermion propagator mass insertions. To begin with we shall develop the mass-insertion approximation in the charged lepton mass eigenstate basis, then change basis to the tree-level neutrino mass eigenstate basis where we shall actually perform our estimates.

In the charged lepton mass eigenstate basis for simplicity we assume that the lepton number conserving sneutrino masses are approximately proportional to the unit matrix:

$$M^2_{\tilde{L}_\alpha \tilde{L}^*_\beta} = m^2_{\tilde{L}_\alpha \tilde{L}^*_\beta} + \lambda_\alpha \lambda_\beta v_2^2 + \frac{1}{2} m_Z^2 \cos 2\beta \delta_{\alpha\beta} = \bar{m}^2 (\delta_{\alpha\beta} + \Delta_{\alpha\beta}) \tag{14}$$

where the $\Delta_{\alpha\beta}$ are small. Note that Δ is dimensionless. The inverse propagator in this basis is given by

$$p^2 - \bar{m}^2(\delta_{\alpha\beta} + \Delta_{\alpha\beta}) = (p^2 - \bar{m}^2) \left(\delta_{\alpha\beta} - \frac{\bar{m}^2 \Delta_{\alpha\beta}}{p^2 - \bar{m}^2} \right) \tag{15}$$

so to linear order in Δ, the propagator becomes

$$\frac{\delta_{\alpha\beta}}{p^2 - \bar{m}^2} + \frac{1}{p^2 - \bar{m}^2} \bar{m}^2 \Delta_{\alpha\beta} \frac{1}{p^2 - \bar{m}^2} \tag{16}$$

where $\bar{m}$ is some average sneutrino (lepton number conserving) mass.

Now we need to relate the lepton number conserving sneutrino mass matrix in the charged lepton mass eigenstate basis to that in the tree-level neutrino mass eigenstate basis. Since in the mass insertion approach we rotate each component of the superfields equally, the rotation on the sneutrinos is the same as that for the neutrinos and is given by the unitary matrix U_0 in Eq.7. Thus the relation between the sneutrinos

in the two bases is the scalar version of Eq.6

$$\begin{pmatrix} \tilde{\nu}_e \\ \tilde{\nu}_\mu \\ \tilde{\nu}_\tau \end{pmatrix} = U_0 \begin{pmatrix} \tilde{\nu}'_e \\ \tilde{\nu}'_0 \\ \tilde{\nu}_3 \end{pmatrix} \tag{17}$$

(Of course the rotation acts on complete $SU(2)_L$ doublets even though we only exhibit it for the neutrinos and sneutrinos.) The relation between the soft masses in the two bases is then given by:

$$(\tilde{\nu}_e^*, \tilde{\nu}_\mu^*, \tilde{\nu}_\tau^*) M^2_{\tilde{L}_\alpha \tilde{L}_\beta^*} \begin{pmatrix} \tilde{\nu}_e \\ \tilde{\nu}_\mu \\ \tilde{\nu}_\tau \end{pmatrix} = (\tilde{\nu}'^*_e, \tilde{\nu}'^*_0, \tilde{\nu}_3^*) M'^2_{\tilde{L}_i \tilde{L}_j^*} \begin{pmatrix} \tilde{\nu}'_e \\ \tilde{\nu}'_0 \\ \tilde{\nu}_3 \end{pmatrix} \tag{18}$$

where the soft masses are related by

$$M'^2_{\tilde{L}_i \tilde{L}_j^*} = U_0^\dagger M^2_{\tilde{L}_\alpha \tilde{L}_\beta^*} U_0 \tag{19}$$

Thus the soft masses in the tree-level neutrino mass eigenstate basis can be written as

$$M'^2_{\tilde{L}_i \tilde{L}_j^*} = \bar{m}^2 (\delta_{ij} - \Delta'_{ij}) \tag{20}$$

where the Δ's are related by

$$\Delta'_{ij} = U_0^\dagger \Delta_{\alpha\beta} U_0 \tag{21}$$

We now proceed to discuss the loop contributions to the neutrino masses which are zero at tree-level, again working in the basis $\nu^i = (\nu'_e, \nu'_0, \nu_3)$ in which the tree-level neutrino masses are diagonal. As discussed such corrections rely on flavour-violating effects, and non-zero masses develop at one loop via the neutralino exchange diagram with flavour-violating mass insertions along the sneutrino propagator. In the basis in which we are working the only non-zero Majorana sneutrino mass is $m_{\tilde{\nu}_3 \tilde{\nu}_3}$, and so the diagram must always have this single Majorana mass insertion at its centre. The flavour-violating mass insertions therefore must connect the neutrino of interest to $\tilde{\nu}_3$, and so the mass insertions which are relevant are of the form Δ'_{i3}. See Fig.5.
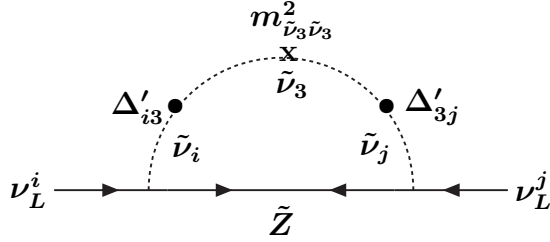
Figure 5: One-loop diagram generating a Majorana neutrino mass that is not aligned with the tree level mass. The lepton number violation comes from the effective sneutrino "Majorana" (lepton number violating) mass $m^2_{\tilde{\nu}_3 \tilde{\nu}_3}$, and the misalignment from off diagonal slepton masses Δ'_{k3}, treated here in the mass insertion approximation. This diagram is in the tree level neutrino mass eigenstate basis.

The one-loop corrected neutrino mass matrix, to lowest non-zero order in the Δ'_{i3}s, is given by:

$$m^{(1)}_{\nu_i \nu_j} = \begin{pmatrix} \Delta'^2_{13} & \Delta'_{13}\Delta'_{23} & \Delta'_{13} \\ \Delta'_{32}\Delta'_{31} & \Delta'^2_{23} & \Delta'_{23} \\ \Delta'_{31} & \Delta'_{32} & 1 \end{pmatrix} \epsilon m_{\nu_3} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_{\nu_3} \end{pmatrix} \tag{22}$$

The upper 2×2 block of the matrix clearly requires two mass insertions and so may be thought to be negligible compared to the other elements of the matrix. However on the contrary this block actually gives the dominant contribution to the neutrinos which are massless at tree-level. The reason is that the contributions to these masses from the third row and column of the matrix are suppressed by a sort of see-saw mechanism. Thus if one only includes Δ to linear order, then the neutrinos which are massless at tree-level will get see-saw masses $\sim \Delta^2 \epsilon^2 m_{\nu_3}$, which are to small to be interesting (suppressed by two powers of ϵ). Therefore to excellent approximation we may just consider the upper 2×2 block of the matrix involving two mass insertions. In the basis $\nu^i = (\nu'_e, \nu'_0)$ this is of the form

$$m^{(1)}_{\nu_i \nu_j} = \begin{pmatrix} \Delta'^2_{13} & \Delta'_{13}\Delta'_{23} \\ \Delta'_{32}\Delta'_{31} & \Delta'^2_{23} \end{pmatrix} \epsilon m_{\nu_3} \tag{23}$$

which has one zero eigenvalue $m_{\nu_1} = 0$ and one non-zero eigenvalue

$$m_{\nu_2} = (\Delta'^2_{13} + \Delta'^2_{23})\epsilon m_{\nu_3} \tag{24}$$

The corresponding eigenvectors are related to the tree-level massless eigenstates ν'_e, ν'_0 by

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} c_{12} & -s_{12} \\ s_{12} & c_{12} \end{pmatrix} \begin{pmatrix} \nu'_e \\ \nu'_0 \end{pmatrix} \qquad (25)$$

where the mixing angle is given by

$$t_{12} = \Delta'_{13}/\Delta'_{23} \qquad (26)$$

Thus the SUSY radiative corrections induce a further two-state remixing of ν'_e, ν'_0. The final one-loop corrected neutrino mixing matrix is obtained from Eqns. 6,7,25,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L = U' \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L \qquad (27)$$

where

$$U' = \begin{pmatrix} c_{12} - \frac{s_{12}}{t_{23}}\theta_1 & s_{12} + \frac{c_{12}}{t_{23}}\theta_1 & \theta_1 \\ -s_{12}c_{23} - \frac{c_{12}}{s_{23}}\theta_1 & c_{12}c_{23} - \frac{s_{12}}{s_{23}}\theta_1 & s_{23} \\ s_{12}s_{23} & -c_{12}s_{23} & c_{23} \end{pmatrix} \qquad (28)$$

In order to obtain "just so" vacuum oscillations we must be able to achieve two things:

(i) We need the mass m_{ν_2} to be of order 10^{-5} eV. For $\epsilon \simeq 10^{-3}$ and $m_{\nu_3} \simeq 5 \times 10^{-2}$ eV, we need $\Delta'^2_{13} + \Delta'^2_{23} \simeq .2$.

(ii) We need a large mixing angle $\theta_{12} \sim \pi/4$ which corresponds to $t_{12} = \Delta'_{13}/\Delta'_{23} \sim 1$.

Using Eq.21 we can relate Δ'_{ij} to the $\Delta_{\alpha\beta}$ quantities in the charged lepton mass eigenstate basis where the phenomenological bounds from lepton flavour violating processes appear. For example there is an extremely strong model independent limit from $\mu \to e\gamma$ on $\Delta_{e\mu} < 7.7 \times 10^{-3}$ [19] (for 100 GeV sleptons). Since the limit on this quantity is so much stronger than on the other entries, we may assume it is zero. If we also set $\theta_1 = 0$ then we find

$$\Delta'_{13} = \Delta'_{31} = c_{23}\Delta_{e\tau}$$

$$\Delta'_{23} = \Delta'_{32} = s_{23}c_{23}\Delta_{\mu\mu} + (2c_{23}^2 - 1)\Delta_{\mu\tau} - s_{23}c_{23}\Delta_{\tau\tau} \qquad (29)$$

Assuming maximal mixing in the 23 sector, the condition for maximal mixing in the 12 sector is then

$$\Delta_{\tau e} \simeq \frac{1}{\sqrt{2}}(\Delta_{\mu\mu} - \Delta_{\tau\tau}) \tag{30}$$

Assuming maximal mixing in the 12 and 23 sectors ($c_{12} = s_{12} = c_{23} = s_{23} = 1/\sqrt{2}$), we find $\Delta'^2_{13} + \Delta'^2_{23} = \Delta^2_{\tau e}$. To get $m_{\nu_2} \sim m_{\nu_3}\epsilon\Delta^2 \sim 10^{-5}$ eV, we need $\Delta_{e\tau} \sim .4$. The experimental bound on $\Delta_{e\tau}$ is 29, so there is phenomenologically nothing wrong with such a large Δ. It is also (just) small enough for the mass insertion approximation to be applicable, and since here we only interested in the order of magnitude of the effects our approximations are adequate. In any case the Δs may be reduced slightly if ϵ is somewhat larger than we have estimated.

The mechanism described here is in fact basis independent. A compact notation which illustrates this is briefly developed in the Appendix.

Finally we briefly discuss the contribution of the second source of flavour violation, coming from the trilinear parameters, to the masses of ν'_e, ν'_0. The basic effect comes from the fact that in the charged lepton mass eigenstate basis the trilinear parameters associated with the Yukawa couplings $\lambda_e, \lambda_\mu, \lambda_\tau$ may be unequal $A_e \neq A_\mu \neq A_\tau$. This would imply that in the basis in which the Yukawa couplings are diagonal apart from the trilinear mass A_3 associated with the coupling $v_2 A_3 \lambda_3 \tilde{\nu}_3 \tilde{N}$ there will be further (small) trilinear parameters a_1, a_2 associated with the couplings $v_2 a_1 \tilde{\nu}'_e \tilde{N}, v_2 a_2 \tilde{\nu}'_0 \tilde{N}$. These couplings generate small off-diagonal sneutrino Majorana masses via the mechanism in Fig.3. However the Yukawa couplings λ_1, λ_2 are zero in this basis so sneutrino masses are only generated in the third row and column of the sneutrino Majorana matrix. If the same were true for the corresponding neutrino masses it would lead to negligible contributions to the masses of ν'_e, ν'_0 due to the see-saw type suppression discussed above. However if we focus on the relevant upper 2×2 block of the neutrino Majorana mass matrix we can see that there is a one-loop

diagram similar to Fig.5 but now involving both an off-diagonal sneutrino Majorana mass $m^2_{\tilde{\nu}_i\tilde{\nu}_3}$ *and* a mass insertion Δ'_{3j}. This will lead to additional contributions to the 2×2 neutrino matrix which for simplicity we have not included in our estimates.

To summarise, we have shown that both the atmospheric and solar neutrino data may be accounted for by a remarkably simple model: the MSSM with the addition of a single right-handed neutrino N. In the $\lambda_e = 0$ limit when $\theta_1 = 0$ the physics of atmospheric and solar neutrinos can be described very simply. From the point of view of atmospheric oscillations the mass splitting between the two lightest neutrinos is too small to be important and the physics is described by the two state mixing of Eq.5, maintaining the $\nu_\mu \leftrightarrow \nu_\tau$ tree-level prediction. From the point of view of solar oscillations since $\lambda_e = 0$ the electron neutrino contains no component of ν_3 and so the physics is described by two state mixing in Eq.25 (dropping the primes) induced by the radiative corrections. The neutrino mixing matrix in Eq.28 becomes:

$$U = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}c_{23} & c_{12}c_{23} & s_{23} \\ s_{12}s_{23} & -c_{12}s_{23} & c_{23} \end{pmatrix} \tag{31}$$

When $\theta_{12} = \theta_{23} = \pi/4$ it corresponds to bi-maximal mixing in the $(\nu_e - \nu_0)$ and $(\nu_\mu - \nu_\tau)$ sectors [12]. More generally at tree-level the spectrum is controlled by the 4 parameters $\lambda_e, \lambda_\mu, \lambda_\tau, M$ where we choose $\lambda_e \ll \lambda_\mu \approx \lambda_\tau$ to obtain $\theta_{23} \approx \pi/4$, and the eigenvalue $m_{\nu_3} = \lambda^2_\alpha v^2_2/M \sim 5 \times 10^{-2}$ eV to obtain $\Delta m^2_{23} = m^2_{\nu_3} \sim 2.5 \times 10^{-3}$ eV^2 as suggested by the recent atmospheric data. The effect of radiative corrections is model dependent since it depends on the SUSY masses, and crucially on the soft flavour-violating parameters. To illustrate the mechanism we have made some simple estimates based on flavour violation due to the soft scalar masses only. We have seen that it is quite reasonable to obtain $\Delta m^2_{12} \sim 10^{-10}$ eV^2 and a large mixing angle θ_{12} from such effects. Thus the origin of the tiny neutrino mass $m_{\nu_2} \sim 10^{-5}$ eV required for "just so" neutrino oscillations may simply be due to the SUSY corrections arising from the atmospheric neutrino mass in the single right-handed neutrino model.

Appendix

In equation (22), we wrote the neutrino mass matrix as an expansion in ϵ (the loop parameter) and the Δs. To linear order in Δ and ϵ, the mass matrix is a seesaw, with ν_3 playing the role of the heavy neutrino. The massless-at-tree-level neutrinos ν_0 and ν_e acquire seesaw masses of order $\Delta^2 \epsilon^2 m_{\nu_3}$, which we neglect because $\epsilon \sim 10^{-3}$. This means that we only need to consider the $O(\Delta^2 \epsilon)$ submatrix involving ν_e and ν_0. This is of the form

$$[m_\nu^{loop}]_{ij} = \frac{\epsilon}{M}(\Delta_{ik} m_{Dk} m_{Dl} \Delta_{lj}) \qquad (32)$$

in an arbitrary basis. Note that $(\nu_3)_i \propto m_{Di}$; ie the direction of the massive-at-tree-level neutrino is in the direction of the Dirac mass, so I can write

$$m_{Dk} m_{Dl} = |m_D|^2 [\nu_3 \nu_3^T]_{kl} \qquad (33)$$

The 2×2 matrix we are interested in is therefore

$$\epsilon m_{\nu_3} \begin{bmatrix} (\nu_e \cdot \Delta \cdot \nu_3)^2 & (\nu_e \cdot \Delta \cdot \nu_3)(\nu_3 \cdot \Delta \cdot \nu_0) \\ (\nu_e \cdot \Delta \cdot \nu_3)(\nu_3 \cdot \Delta \cdot \nu_0) & (\nu_0 \cdot \Delta \cdot \nu_3)^2 \end{bmatrix} \qquad (34)$$

in (ν_e, ν_0) basis. But we know ν_3, ν_e and ν_0 in the charged lepton mass eigenstate basis, from equation (5), so we can evaluate the matrix elements in the charged lepton mass eigenstate basis where we know the experimental bounds on the Δs. This gives us the previously discussed results.

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