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HANDBOOK OF HIGHER TWIST DISTRIBUTION AMPLITUDES OF VECTOR MESONS IN QCD

PATRICIA BALL
CERN–TH, CH–1211 Genève 23, Switzerland

V.M. BRAUN
NORDITA, Blegdamsvej 17, DK–2100 Copenhagen Ø, Denmark

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We give a summary of existing results on higher twist distribution amplitudes of vector mesons in QCD. Special attention is payed to meson mass corrections which turn out to be large. A "shopping list" is presented of most important nonperturbative parameters which enter distribution amplitudes.

1 General framework

The notion of distribution amplitudes refers to momentum fraction distributions of partons in the meson in a particular Fock state with fixed number of components. For the minimal number of constituents, the distribution amplitude ϕ is related to the Bethe-Salpeter wave function ϕ_{BS} by

$$\phi(x) \sim \int^{|k_\perp|<\mu} d^2k_\perp\, \phi_{BS}(x,k_\perp). \tag{1}$$

The standard approach to distribution amplitudes, which is due to Brodsky and Lepage [1], considers the hadron's parton decomposition in the infinite momentum frame. A conceptually different, but mathematically equivalent formalism is the light-cone quantization [2]. Either way, power suppressed contributions to exclusive processes in QCD, which are commonly referred to as higher twist corrections, are thought to originate from three different sources:

- contributions of "bad" components in the wave function and in particular of those with "wrong" spin projection;
- contributions of transverse motion of quarks (antiquarks) in the leading twist components, given for instance by integrals as above with additional factors of $k_\perp^2$;

- contributions of higher Fock states, with additional gluons and/or quark-antiquark pairs.

We take a somewhat different point of view and define light-cone distribution amplitudes as meson-to-vacuum transition matrix elements of nonlocal gauge-invariant light-cone operators. This formalism is convenient for the study of higher-twist distributions thanks to its gauge and Lorentz invariance and allows to solve all equations of motion explicitly, relating different higher-twist distributions to each other. We will find that all dynamical degrees of freedom are those describing contributions of higher Fock states, while all other higher-twist effects are given in terms of the latter without any free parameters.

The report is divided into three sections, the first of which is introductory and the last two present the summary of distribution amplitudes up to twist 4. The expressions collected in these sections are principally the result of recent studies reported in Refs. [3,4,5] which considerably extend the earlier analysis in Ref. [6]. We use a simplified version of the set of twist-4 distributions derived in [5], taking into account only contributions of the lowest conformal partial-waves, and for consistency discard contributions of higher partial-waves in twist-3 distributions in cases where they enter physical amplitudes multiplied by additional powers of m_ρ. Four-particle distributions of twist 4 start with higher conformal spin and must be put to zero to the present accuracy. The SU(3)-breaking effects are taken into account in leading-twist distributions and partially for twist-3, but neglected for twist-4. Explicit expressions are given for a (charged) ρ-meson. Distribution amplitudes for other vector-mesons are obtained by trivial substitutions.

Throughout this report we denote the meson momentum by P_μ and introduce the light-like vectors p and z such that

$$p_\mu = P_\mu - \frac{1}{2} z_\mu \frac{m_\rho^2}{pz}. \tag{2}$$

The meson polarization vector $e_\mu^{(\lambda)}$ is decomposed in projections onto the two light-like vectors and the orthogonal plane as

$$e_\mu^{(\lambda)} = \frac{(e^{(\lambda)} \cdot z)}{pz} \left(p_\mu - \frac{m_\rho^2}{2pz} z_\mu \right) + e_{\perp\mu}^{(\lambda)}. \tag{3}$$

We use the standard Bjorken-Drell convention [7] for the metric and the Dirac matrices; in particular $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, and the Levi-Civita tensor $\epsilon_{\mu\nu\lambda\sigma}$ is defined as the totally antisymmetric tensor with $\epsilon_{0123} = 1$. The covariant derivative is defined as $D_\mu \equiv \overrightarrow{D}_\mu = \partial_\mu - igA_\mu$, and we also use the notation

$\overleftarrow{D}_\mu = \overleftarrow{\partial}_\mu + igA_\mu$ in later sections. The dual gluon field strength tensor is defined as $\widetilde{G}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}G^{\rho\sigma}$.

1.1 Conformal partial wave expansion

Conformal partial wave expansion in QCD [8,9,10,11,12,13] parallels the partial wave expansion of wave functions in standard quantum mechanics, which allows to separate the dependence on angular coordinates from radial ones. The basic idea is to write down distribution amplitudes as a sum of contributions from different conformal spins. For a given spin, the dependence on the momentum fractions is fixed by the symmetry. To specify the function, one has to fix the coefficients in this expansion at some scale; conformal invariance of the QCD Lagrangian then guarantees that there is no mixing between contributions of different spin to leading logarithmic accuracy. For leading twist distributions the mixing matrix becomes diagonal in the conformal basis and the anomalous dimensions are ordered with spin. Thus, only the first few "harmonics" contribute at sufficiently large scales (for sufficiently hard processes).

For higher twist distributions, the use of the conformal basis offers a crucial advantage of "diagonalizing" the equations of motion: since conformal transformations commute with the QCD equations of motion, the corresponding constraints can be solved order by order in the conformal expansion. Note that relations between different distributions obtained in this way are exact: despite the fact that conformal symmetry is broken by quantum corrections, equations of motion are not renormalized and remain the same as in free theory.

The general procedure to construct the conformal expansion for arbitrary multiparticle distributions was developed in [10,12]. To this end each constituent field has to be decomposed (using projection operators, if necessary) in components with fixed (Lorentz) spin projection onto the light-cone.

Each such component corresponds to a so-called quasiprimary field in the language of conformal field theories, and has conformal spin

$$j = \frac{1}{2}\,(l + s), \tag{4}$$

where l is the canonical dimension and s the (Lorentz) spin projection.[a]

Multi-particle states built of quasiprimary fields can be expanded in irreducible unitary representations with increasing conformal spin. An explicit

[a] $l = 3/2$ for quarks and $l = 2$ for gluons; the quark field is decomposed as $\psi = \psi_+ + \psi_- \equiv (1/2)\,\not{z}\not{p}\psi + (1/2)\,\not{p}\not{z}\psi$ with spin projections $s = +1/2$ and $s = -1/2$, respectively. For the gluon field strength there are three possibilities: $z_\mu G_{\mu\perp}$ corresponds to $s = +1$, $p_\mu G_{\mu\perp}$ has $s = -1$ and $G_{\perp\perp}$, $z_\mu p_\nu G_{\mu\nu}$ correspond both to $s = 0$.

expression for the distribution amplitude of a multi-particle state with the lowest conformal spin $j = j_1 + \ldots + j_m$ built of m primary fields with the spins j_k is

$$\phi_{as}(\alpha_1, \alpha_2, \ldots, \alpha_m) = \frac{\Gamma[2j_1 + \ldots + 2j_m]}{\Gamma[2j_1]\ldots\Gamma[2j_m]} \alpha_1^{2j_1-1}\alpha_2^{2j_2-1}\ldots\alpha_m^{2j_m-1}. \qquad (5)$$

Here α_k are the corresponding momentum fractions. This state is nondegenerate and cannot mix with other states because of conformal symmetry. Multi-particle irreducible representations with higher spin $j+n, n=1,2,\ldots$, are given by polynomials of m variables (with the constraint $\sum_{k=1}^m \alpha_k = 1$) which are orthogonal over the weight function (5).

1.2 Equations of motion

We collect here exact operator identities which can be derived using the approach of[14] and which present a nonlocal equivalent of the equations of motion for Wilson local operators. Taking suitable matrix elements one derives a set of relations between distribution amplitudes, which generally allow to express two-particle distributions of higher twist in terms of three-particle distributions. The corresponding relations are taken into account in the results given in later sections. The rationale for presenting the operator relations themselves is that we found them useful in practical calculations:

$$\partial^\mu \bar{u}(x)\gamma_\mu d(-x) = -i\int_{-1}^{1} dv\, \bar{u}(x)x^\nu gG_{\nu\mu}(vx)\gamma^\mu d(-x), \qquad (6)$$

$$\frac{\partial}{\partial x_\mu}\bar{u}(x)\gamma_\mu d(-x) = -i\int_{-1}^{1} dv\, v\bar{u}(x)x^\nu gG_{\nu\mu}(vx)\gamma^\mu d(-x), \qquad (7)$$

$$\begin{aligned}\partial^\mu \bar{u}(x)\sigma_{\mu\nu} d(-x) = &-i\frac{\partial}{\partial x_\nu}\bar{u}(x)d(-x) + \int_{-1}^{1} dv\, v\bar{u}(x)x^\rho gG_{\rho\nu}(vx)d(-x)\\ &- i\int_{-1}^{1} dv\, \bar{u}(x)x^\rho gG_{\rho\mu}(vx)\sigma^\mu{}_\nu d(-x),\end{aligned} \qquad (8)$$

$$\begin{aligned}\frac{\partial}{\partial x_\mu}\bar{u}(x)\sigma_{\mu\nu} d(-x) = &-i\partial_\nu\bar{u}(x)d(-x) + \int_{-1}^{1} dv\, \bar{u}(x)x^\rho gG_{\rho\nu}(vx)d(-x)\\ &- i\int_{-1}^{1} dv\, v\bar{u}(x)x^\rho gG_{\rho\mu}(vx)\sigma^\mu{}_\nu d(-x),\end{aligned} \qquad (9)$$

$$\bar{u}(x)d(-x) - \bar{u}(0)d(0) = \int_0^1 dt\int_{-t}^{t} dv\, \bar{u}(tx)x^\alpha\sigma_{\alpha\beta}x^\mu gG_{\mu\beta}(vx)d(-tx)$$

$$+ i \int_0^1 dt \, \partial^\alpha \left\{ \bar{u}(tx) \sigma_{\alpha\beta} x^\beta d(-tx) \right\} . \tag{10}$$

In all cases gauge factors are implied in between the constituent fields,

$$[x, y] = \mathrm{Pexp}[ig \int_0^1 dt \, (x - y)_\mu A^\mu (tx + (1 - t)y)], \tag{11}$$

and we introduced a shorthand notation for the derivative over the total translation:

$$\partial_\alpha \left\{ \bar{u}(tx) \Gamma d(-tx) \right\} \equiv \frac{\partial}{\partial y^\alpha} \left\{ \bar{u}(tx + y) \Gamma d(-tx + y) \right\} \Big|_{y \to 0} \tag{12}$$

with the generic Dirac matrix structure Γ. For simplicity, we omit operators involving quark masses, see [4,5].

Two more relations are:

$$\begin{aligned}
\bar{u}(x) \gamma_\mu d(-x) &= \int_0^1 \, dt \, \frac{\partial}{\partial x_\mu} \, \bar{u}(tx) \not{x} d(-tx) \\
&- \int_0^1 dt \, \int_{-t}^t dv \, \bar{u}(tx) \Big\{ t g \tilde{G}_{\mu\nu}(vx) x^\nu \not{x} \gamma_5 + i v g G_{\mu\nu}(vx) x^\nu \not{x} \Big\} d(-tx) \\
&+ \int_0^1 dt \, t \, \int_{-t}^t dv \, \bar{u}(tx) \Big\{ x^2 g \tilde{G}_{\mu\nu}(vx) \gamma^\nu \gamma_5 - x_\mu x^\nu g \tilde{G}_{\nu\rho}(vx) \gamma^\rho \gamma_5 \Big\} d(-tx) \\
&- i \epsilon_{\mu\nu\alpha\beta} \int_0^1 dt \, t \, x^\nu \partial^\alpha \left[\bar{u}(tx) \gamma^\beta \gamma_5 d(-tx) \right] ,
\end{aligned} \tag{13}$$

and similarly with an additional γ_5:

$$\begin{aligned}
\bar{u}(x) \gamma_\mu \gamma_5 d(-x) &= \int_0^1 \, dt \, \frac{\partial}{\partial x_\mu} \, \bar{u}(tx) \not{x} \gamma_5 d(-tx) \\
&- \int_0^1 dt \, \int_{-t}^t dv \, \bar{u}(tx) \Big\{ t g \tilde{G}_{\mu\nu}(vx) x^\nu \not{x} + i v g G_{\mu\nu}(vx) x^\nu \not{x} \gamma_5 \Big\} d(-tx) \\
&+ \int_0^1 dt \, t \, \int_{-t}^t dv \, \bar{u}(tx) \Big\{ x^2 g \tilde{G}_{\mu\nu}(vx) \gamma^\nu - x_\mu x^\nu g \tilde{G}_{\nu\rho}(vx) \gamma^\rho \Big\} d(-tx) \\
&- i \epsilon_{\mu\nu\alpha\beta} \int_0^1 dt \, t \, x^\nu \partial^\alpha \left[\bar{u}(tx) \gamma^\beta d(-tx) \right] .
\end{aligned} \tag{14}$$

Finally, the following formula is sometimes useful:

$$\frac{\partial^2}{\partial x_\alpha \partial x^\alpha} \bar{u}(x) \Gamma \, d(-x) = -\partial^2 \bar{u}(x) \Gamma \, d(-x) + \bar{u}(x) [\Gamma \sigma G + \sigma G \, \Gamma] d(-x)$$

$$
\begin{aligned}
&- 2ix^\nu \frac{\partial}{\partial x_\mu}\int_{-1}^{1} dv\, v\, \bar{u}(x)\Gamma G_{\nu\mu}(vx)d(-x) - 2ix^\nu \partial_\mu \int_{-1}^{1} dv\, \bar{u}(x)\Gamma G_{\nu\mu}(vx)d(-x) \\
&+ 2\int_{-1}^{1} dv \int_{-1}^{v} dt\, (1+vt)\bar{u}(x)\Gamma x^\mu x^\nu G_{\mu\rho}(vx)G^\rho{}_\nu(tx)d(-x) \\
&+ ix^\nu \int_{-1}^{1} dv\, (1+v^2)\, \bar{u}(x)\Gamma[D_\mu, G^\mu{}_\nu](vx)d(-x)
\end{aligned} \tag{15}
$$

where $[D_\mu, G^\mu{}_\nu] = -t^A(\bar{\psi}\gamma_\nu t^A\psi)$ assuming summation over light flavors ψ.

1.3 Meson mass corrections

The structure of meson mass corrections in exclusive processes is in general more complicated than of target mass corrections in deep inelastic scattering in which case they can be resummed using the Nachtmann variable [15]. For illustration, consider the simplest matrix element

$$
\begin{aligned}
&\langle 0|\bar{u}(x)\not{x}d(-x)|\rho^-(P,\lambda)\rangle = \\
&\qquad = f_\rho m_\rho (e^{(\lambda)}x)\int_0^1 du\, e^{i(2u-1)Px}\Big[\phi(u) + \frac{x^2}{4}\Phi(u) + O(x^4)\Big].
\end{aligned} \tag{16}
$$

We assume that $x^2 \ll \Lambda_{\rm QCD}^{-2}$, but nonzero, $\phi(u)$ is the twist-2 distribution amplitude and $\Phi(u)$ describes higher-twist corrections in which we want to calculate "kinematic" contributions due to nonzero ρ-meson mass.

A common wisdom tells that meson mass corrections are related to contributions of leading twist operators. Indeed, conditions of symmetry and zero traces for twist-2 local operators imply

$$
\begin{aligned}
&\langle 0|\Big[\bar{u}\not{x}(i \overleftrightarrow{D} x)^n d\Big]_{\rm tw.2}|\rho^-(P,\lambda)\rangle = \\
&\qquad = f_\rho m_\rho (e^{(\lambda)}x)\left[(Px)^n - \frac{x^2 m_\rho^2}{4}\frac{n(n-1)}{n+1}(Px)^{n-2}\right]\langle\!\langle O_n\rangle\!\rangle,
\end{aligned} \tag{17}
$$

where $[\ldots]_{\rm tw.2}$ denotes taking the leading twist part (subtraction of traces, in this case) and $\langle\!\langle O_n\rangle\!\rangle$ is the reduced matrix element related to the n-th moment of the leading twist distribution

$$
M_n^{(\phi)} \equiv \int_0^1 du\, (2u-1)^n\phi(u) = \langle\!\langle O_n\rangle\!\rangle. \tag{18}
$$

Expanding (16) at short distances $x \to 0$ and comparing with (17), we find that the same reduced matrix element gives a contribution to the twist 4 distribution amplitude

$$M_n^{(\Phi)} \equiv \int_0^1 du\,(2u-1)^n \Phi(u) = \frac{1}{n+3} m_\rho^2 \langle\!\langle O_{n+2} \rangle\!\rangle, \tag{19}$$

which is the direct analogue of Nachtmann's correction for deep inelastic scattering.

The result in (19) is, however, incomplete. The reason is that in exclusive processes one has to take into account higher-twist operators containing full derivatives, and vacuum-to-meson matrix elements of such operators reduce, in certain cases, to powers of the meson mass times reduced matrix elements of leading twist operators. For the case at hand, write[14]

$$\begin{aligned}\bar u(x) \not x d(-x) &= \Big[\bar u(x) \not x d(-x)\Big]_{\mathrm{tw.2}} + \frac{x^2}{4}\int_0^1 dt\, \frac{\partial^2}{\partial x_\alpha \partial x^\alpha} \bar u(tx) \not x d(-tx) + O(x^4) \\ &= \Big[\bar u(x) \not x d(-x)\Big]_{\mathrm{tw.2}} - \frac{x^2}{4}\int_0^1 dt\, t^2\, \partial^2 [\bar u(tx) \not x d(-tx)] \\ &\qquad + \text{contributions of operators with gluons} + O(x^4)\end{aligned} \tag{20}$$

where we used Eq. (15) to arrive at the last line. In the matrix element one can make the substitution $\partial^2 \to -m_\rho^2$. Expanding, again, at short distances, and comparing with the similar expansion of (16) we get an additional contribution to $M_n^{(\Phi)}$ so that the corrected version of (19) becomes

$$M_n^{(\Phi)} = \frac{1}{n+3} m_\rho^2 \big[\langle\!\langle O_{n+2} \rangle\!\rangle + \langle\!\langle O_n \rangle\!\rangle\big] + \text{ gluons}. \tag{21}$$

Assuming the asymtotic form of the leading-twist distribution amplitude ϕ, $\phi(u) = 6u(1-u)$, so that $\langle\!\langle O_n \rangle\!\rangle = 3/[2(n+1)(n+3)]$, this equation for moments is easily solved and gives

$$\Phi(u) = 30u^2(1-u)^2 \left[\frac{2}{5} m_\rho^2 + \frac{4}{3} m_\rho^2 \zeta_4\right], \tag{22}$$

where we have included the "genuine" twist 4 correction (term in ζ_4) due to the twist 4 quark-gluon operator, see definition in Eq. (43). The QCD sum rule estimate is $\zeta_4 \sim 0.15$ [17], so that the meson mass effect onto the twist 4 distribution function is by a factor two larger than the "genuine" twist 4 correction. This is an important difference to deep inelastic scattering, where the target mass corrections are small.

The present discussion is still oversimplified and does not provide with a complete separation of meson mass effects. The major complication arises because of contributions of operators of the type

$$\partial_{\mu_1}\left[\bar{u}\gamma_{\mu_1}(i\overleftrightarrow{D}_{\mu_2})\ldots(i\overleftrightarrow{D}_{\mu_n})d\right]_{\rm tw.2}. \tag{23}$$

Such operators can be expressed in terms of operators with extra gluon fields, which means, conversely, that certain combinations of $\bar{q}Gq$ operators reduce to divergences of leading twist operators and give rize to extra meson mass correction terms. The corresponding corrections to twist 4 distributions involve, however, higher-order contributions in the conformal expansion of the distribution amplitudes of leading twist and do not affect the result in (22), which is to leading conformal spin accuracy. A detailed analysis will be presented in [5].

2 Summary of chiral-even distributions

Two-particle quark-antiquark distribution amplitudes are defined as matrix elements of non-local operators on the light-cone [4]:

$$\langle 0|\bar{u}(z)\gamma_\mu d(-z)|\rho^-(P,\lambda)\rangle = f_\rho m_\rho\left[p_\mu\frac{e^{(\lambda)}\cdot z}{p\cdot z}\int_0^1 du\, e^{i\xi p\cdot z}\phi_\parallel(u,\mu^2)\right.$$
$$\left.+e^{(\lambda)}_{\perp\mu}\int_0^1 du\, e^{i\xi p\cdot z}g_\perp^{(v)}(u,\mu^2)-\frac{1}{2}z_\mu\frac{e^{(\lambda)}\cdot z}{(p\cdot z)^2}m_\rho^2\int_0^1 du\, e^{i\xi p\cdot z}g_3(u,\mu^2)\right] \tag{24}$$

and

$$\langle 0|\bar{u}(z)\gamma_\mu\gamma_5 d(-z)|\rho^-(P,\lambda)\rangle =$$
$$=\frac{1}{2}\left(f_\rho - f_\rho^T\frac{m_u+m_d}{m_\rho}\right)m_\rho\epsilon_\mu{}^{\nu\alpha\beta}e^{(\lambda)}_{\perp\nu}p_\alpha z_\beta\int_0^1 du\, e^{i\xi p\cdot z}g_\perp^{(a)}(u,\mu^2). \tag{25}$$

For brevity, here and below we do not show the gauge factors between the quark and the antiquark fields and use the short–hand notation

$$\xi = u-(1-u) = 2u-1.$$

The vector and tensor decay constants f_ρ and f_ρ^T are defined, as usual, as

$$\langle 0|\bar{u}(0)\gamma_\mu d(0)|\rho^-(P,\lambda)\rangle = f_\rho m_\rho e^{(\lambda)}_\mu, \tag{26}$$
$$\langle 0|\bar{u}(0)\sigma_{\mu\nu} d(0)|\rho^-(P,\lambda)\rangle = if_\rho^T(e^{(\lambda)}_\mu P_\nu - e^{(\lambda)}_\nu P_\mu). \tag{27}$$

The distribution amplitude $\phi_\parallel$ is of twist-2, $g_\perp^{(v)}$ and $g_\perp^{(a)}$ are twist-3 and g_3 is twist-4. All four functions $\phi=\{\phi_\parallel, g_\perp^{(v)}, g_\perp^{(a)}, g_3\}$ are normalized as

$$\int_0^1 du\,\phi(u) = 1, \tag{28}$$

which can be checked by comparing the two sides of the defining equations in the limit $z_\mu \to 0$ and using the equations of motion. We keep the (tiny) corrections proportional to the u and d quark-masses m_u and m_d to indicate the SU(3)-breaking corrections for K^*- and ϕ-mesons.

In addition, we have to define three-particle distributions:

$$\begin{aligned}\langle 0|\bar u(z) g\widetilde G_{\mu\nu}(vz)\gamma_\alpha\gamma_5 d(-z)|\rho^-(P,\lambda)\rangle &= f_\rho m_\rho p_\alpha[p_\nu e^{(\lambda)}_{\perp\mu} - p_\mu e^{(\lambda)}_{\perp\nu}]\mathcal{A}(v,pz)\\ &+ f_\rho m_\rho^3\frac{e^{(\lambda)}\cdot z}{pz}[p_\mu g^\perp_{\alpha\nu} - p_\nu g^\perp_{\alpha\mu}]\widetilde\Phi(v,pz)\\ &+ f_\rho m_\rho^3\frac{e^{(\lambda)}\cdot z}{(pz)^2}p_\alpha[p_\mu z_\nu - p_\nu z_\mu]\widetilde\Psi(v,pz), \qquad (29)\end{aligned}$$

$$\begin{aligned}\langle 0|\bar u(z) g G_{\mu\nu}(vz) i\gamma_\alpha d(-z)|\rho^-(P)\rangle &= f_\rho m_\rho p_\alpha[p_\nu e^{(\lambda)}_{\perp\mu} - p_\mu e^{(\lambda)}_{\perp\nu}]\mathcal{V}(v,pz)\\ &+ f_\rho m_\rho^3\frac{e^{(\lambda)}\cdot z}{pz}[p_\mu g^\perp_{\alpha\nu} - p_\nu g^\perp_{\alpha\mu}]\Phi(v,pz)\\ &+ f_\rho m_\rho^3\frac{e^{(\lambda)}\cdot z}{(pz)^2}p_\alpha[p_\mu z_\nu - p_\nu z_\mu]\Psi(v,pz), \qquad (30)\end{aligned}$$

where

$$\mathcal{A}(v,pz) = \int \mathcal{D}\underline{\alpha}\, e^{-ipz(\alpha_u-\alpha_d+v\alpha_g)}\mathcal{A}(\underline{\alpha}), \qquad (31)$$

etc., and $\underline{\alpha}$ is the set of three momentum fractions $\underline{\alpha} = \{\alpha_d, \alpha_u, \alpha_g\}$. The integration measure is defined as

$$\int \mathcal{D}\underline{\alpha} \equiv \int_0^1 d\alpha_d \int_0^1 d\alpha_u \int_0^1 d\alpha_g\, \delta(1-\sum \alpha_i). \qquad (32)$$

The distribution amplitudes $\mathcal{V}$ and $\mathcal{A}$ are of twist-3, while the rest is twist-4 and we have not shown further Lorentz structures corresponding to twist-5 contributions[b].

Calculation of exclusive amplitudes involving a large momentum-transfer reduces to evaluation of meson-to-vacuum transition matrix elements of non-local operators, which can be expanded in powers of the deviation from the light-cone. To twist-4 accuracy one can use the expression for the axial-vector matrix element in (25) as it stands, replacing the light-cone vector z_μ by the actual quark-antiquark separation x_μ. For the vector operator, the light-cone expansion to twist-4 accuracy reads:

$$\langle 0|\bar u(x)\gamma_\mu d(-x)|\rho^-(P,\lambda)\rangle =$$

[b] We use a different normalization of three-particle twist-3 distributions compared to [4].

$$
\begin{aligned}
&= f_\rho m_\rho \left\{ \frac{e^{(\lambda)}x}{Px} P_\mu \int_0^1 du\, e^{i\xi Px} \Big[\phi_\parallel(u,\mu) + \frac{m_\rho^2 x^2}{4} \mathbb{A}(u,\mu)\Big] \right. \\
&\quad + \left(e_\mu^{(\lambda)} - P_\mu \frac{e^{(\lambda)}x}{Px}\right) \int_0^1 du\, e^{i\xi Px}\, g_\perp^{(v)}(u,\mu) \\
&\quad \left. - \frac{1}{2} x_\mu \frac{e^{(\lambda)}x}{(Px)^2} m_\rho^2 \int_0^1 du\, e^{i\xi Px} \mathbb{C}(u,\mu) \right\}, \qquad (33)
\end{aligned}
$$

where

$$
\mathbb{C}(u) = g_3(u) + \phi_\parallel(u) - 2 g_\perp^{(v)}(u), \qquad (34)
$$

and $\mathbb{A}(u)$ can be related to integrals of three-particle distributions using the equations of motion. All distribution functions in (33) are assumed to be normalized at the scale $\mu^2 \sim x^{-2}$ (to leading-logarithmic accuracy).

For the leading twist-2 distribution amplitude $\phi_\parallel$ we use

$$
\phi_\parallel(u) = 6u\bar{u} \left[1 + 3a_1^\parallel\, \xi + a_2^\parallel\, \frac{3}{2}(5\xi^2 - 1)\right]. \qquad (35)
$$

The parameters $a_{1,2}^\parallel$ are defined as the local matrix elements

$$
\begin{aligned}
\langle 0|\bar{u} \not{z} (i \overleftrightarrow{D} z) d|\rho^-(P,\lambda)\rangle &= (e^{(\lambda)}z)(pz) f_\rho m_\rho\, \frac{3}{5}\, a_1^\parallel, \\
\langle 0|\bar{u} \not{z} (i \overleftrightarrow{D} z)^2 d|\rho^-(P,\lambda)\rangle &= (e^{(\lambda)}z)(pz)^2 f_\rho m_\rho \left\{\frac{1}{5} + \frac{12}{35}\, a_2^\parallel\right\}. \qquad (36)
\end{aligned}
$$

The numerical values are specified in Table 1. The expressions for higher-twist distributions given below correspond to the simplest self-consistent approximation that satisfies the QCD equations of motion [4,5]:

- Three-particle distributions of twist-3:

$$
\begin{aligned}
\mathcal{V}(\underline{\alpha}) &= 540\, \zeta_3 \omega_3^V (\alpha_d - \alpha_u) \alpha_d \alpha_u \alpha_g^2, \qquad (37) \\
\mathcal{A}(\underline{\alpha}) &= 360\, \zeta_3 \alpha_d \alpha_u \alpha_g^2 \Big[1 + \omega_3^A \frac{1}{2}(7\alpha_g - 3)\Big]. \qquad (38)
\end{aligned}
$$

- Two-particle distributions of twist-3:

$$
\begin{aligned}
g_\perp^{(a)}(u) &= 6u\bar{u}\left[1 + a_1^\parallel \xi + \left\{\frac{1}{4} a_2^\parallel + \frac{5}{3}\zeta_3 \left(1 - \frac{3}{16}\,\omega_3^A + \frac{9}{16}\omega_3^V\right)\right\}(5\xi^2 - 1)\right] \\
&\quad + 6\,\widetilde{\delta}_+\,(3u\bar{u} + \bar{u}\ln\bar{u} + u\ln u) + 6\,\widetilde{\delta}_-\,(\bar{u}\ln\bar{u} - u\ln u), \qquad (39)
\end{aligned}
$$

$$g_\perp^{(v)}(u) = \frac{3}{4}(1+\xi^2) + a_1^\parallel\,\frac{3}{2}\,\xi^3 + \left(\frac{3}{7}\,a_2^\parallel + 5\zeta_3\right)\left(3\xi^2-1\right)$$
$$+ \left[\frac{9}{112}\,a_2^\parallel + \frac{15}{64}\,\zeta_3\Big(3\,\omega_3^V - \omega_3^A\Big)\right]\left(3-30\xi^2+35\xi^4\right)$$
$$+ \frac{3}{2}\,\widetilde{\delta}_+\,(2+\ln u + \ln\bar u) + \frac{3}{2}\,\widetilde{\delta}_-\,(2\xi + \ln\bar u - \ln u). \qquad (40)$$

- Three-particle distributions of twist-4:

$$\widetilde{\Phi}(\underline{\alpha}) = \Big[-\frac{1}{3}\zeta_3 + \frac{1}{3}\zeta_4\Big]30(1-\alpha_g)\alpha_g^2,$$
$$\Phi(\underline{\alpha}) = \Big[-\frac{1}{3}\zeta_3 + \frac{1}{3}\zeta_4\Big]30(\alpha_u-\alpha_d)\alpha_g^2,$$
$$\widetilde{\Psi}(\underline{\alpha}) = \Big[\frac{2}{3}\zeta_3 + \frac{1}{3}\zeta_4\Big]120\alpha_u\alpha_d\alpha_g,$$
$$\Psi(\underline{\alpha}) = 0. \qquad (41)$$

- Two-particle distributions of twist-4:

$$\mathbb{A}(u) = \left[\frac{4}{5} + \frac{4}{105}\,a_2^\parallel + \frac{20}{9}\zeta_4 + \frac{8}{9}\zeta_3\right]30u^2(1-u)^2,$$
$$g_3(u) = 6u(1-u) + \left[\frac{1}{7}\,a_2^\parallel + \frac{10}{3}\zeta_4 - \frac{20}{3}\zeta_3\right](1-3\xi^2),$$
$$\mathbb{C}(u) = \left[\frac{3}{2} - \frac{2}{7}\,a_2^\parallel + \frac{10}{3}\zeta_4 + \frac{10}{3}\zeta_3\right](1-3\xi^2), \qquad (42)$$

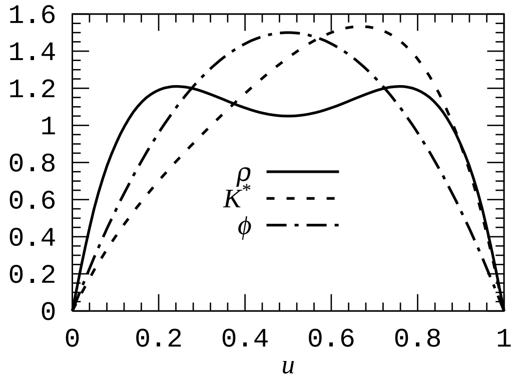
where the dimensionless couplings ζ_3 and ζ_4 are defined as local matrix elements

$$\langle 0|\bar u g\tilde G_{\mu\nu}\gamma_\alpha\gamma_5 d|\rho^-(P,\lambda)\rangle = f_\rho m_\rho\zeta_3\left[e_\mu^{(\lambda)}\Big(P_\alpha P_\nu - \frac{1}{3}m_\rho^2\,g_{\alpha\nu}\Big)\right.$$
$$\left. - e_\nu^{(\lambda)}\Big(P_\alpha P_\mu - \frac{1}{3}m_\rho^2\,g_{\alpha\mu}\Big)\right] + \frac{1}{3}f_\rho m_\rho^3\zeta_4\left[e_\mu^{(\lambda)}g_{\alpha\nu} - e_\nu^{(\lambda)}g_{\alpha\mu}\right], \qquad (43)$$

and have been estimated from QCD sum-rules[16,17]. ω_3^V is defined as

$$\langle 0|\bar u \not{z}(gG_{\alpha\beta}z^\alpha i(\overrightarrow{D}\,z) - (i\,\overleftarrow{D}\,z)gG_{\alpha\beta}z^\alpha)d|\rho^-(P,\lambda)\rangle =$$
$$= i(pz)^3 e_{\perp\beta}^{(\lambda)} m_\rho f_\rho\,\frac{3}{28}\,\zeta_3\omega_3^V + O(z_\beta) \qquad (44)$$

Figure 1: The twist-2 distribution amplitude $\phi_{\|}(u)$ for ρ, K^* and ϕ mesons. Renormalization point is $\mu = 1\,\mathrm{GeV}$.



and ω_3^A is defined as

$$\langle 0|\bar{u}\not{z}\gamma_5 \left[iDz, g\tilde{G}_{\mu\nu}z^\mu\right] d|\rho^-(P,\lambda)\rangle = -(pz)^3 e^{(\lambda)}_{\perp\nu} m_\rho f_\rho \zeta_3 \left(\frac{3}{7} + \frac{3}{28}\omega_3^A\right) + O(z_\beta). \tag{45}$$

The terms in $\delta_\pm$ and $\widetilde{\delta}_\pm$ specify quark-mass corrections in twist-3 distributions induced by the equations of motion. The numerical values of these and other coefficients are listed in Tables 1 and 2[c]. Note that we neglect SU(3)-breaking effects in twist-4 distributions and in gluonic parts of twist-3 distributions. In Fig. 1 we plot the leading-twist distribution amplitude $\phi_{\|}$ for ρ, K^* and ϕ mesons.

3 Summary of chiral-odd distributions

There exist four different two-particle chiral-odd distributions [4] defined as

$$\begin{aligned}\langle 0|\bar{u}(z)\sigma_{\mu\nu}d(-z)|\rho^-(P,\lambda)\rangle = i f_\rho^T \Bigg[&(e^{(\lambda)}_{\perp\mu}p_\nu - e^{(\lambda)}_{\perp\nu}p_\mu)\int_0^1 du\, e^{i\xi p\cdot z}\phi_\perp(u,\mu^2)\\ &+ (p_\mu z_\nu - p_\nu z_\mu)\frac{e^{(\lambda)}\cdot z}{(p\cdot z)^2} m_\rho^2 \int_0^1 du\, e^{i\xi p\cdot z} h_{\|}^{(t)}(u,\mu^2)\\ &+ \frac{1}{2}(e^{(\lambda)}_{\perp\mu}z_\nu - e^{(\lambda)}_{\perp\nu}z_\mu)\frac{m_\rho^2}{p\cdot z}\int_0^1 du\, e^{i\xi p\cdot z} h_3(u,\mu^2)\Bigg], \end{aligned}\tag{46}$$

[c] In the notation of Ref. [4], $\omega^A_{1,0} \equiv \omega_3^A$, $\zeta_3^A \equiv \zeta_3$, and $\zeta_3^V \equiv (3/28)\zeta_3\omega_3^V$.

Table 1: Masses and couplings of vector-meson distribution amplitudes, including SU(3)-breaking. In cases where two values are given, the upper one corresponds to the scale $\mu^2 = 1\,\mathrm{GeV}^2$ and the lower one to $\mu^2 = 5\,\mathrm{GeV}^2$, respectively. We use $m_s(1\,\mathrm{GeV}) = 150\,\mathrm{MeV}$ and put the u and d quark mass to zero.

V	$\rho^\pm$	$K^*_{u,d}$	$\bar{K}^*_{u,d}$	ϕ
f_V[MeV]	198 ± 7	226 ± 28	226 ± 28	254 ± 3
f_V^T[MeV]	160 ± 10	185 ± 10	185 ± 10	215 ± 15
	152 ± 9	175 ± 9	175 ± 9	204 ± 14
$a_1^\parallel$	0	0.19 ± 0.05	-0.19 ± 0.05	0
		0.17 ± 0.04	-0.17 ± 0.04	
$a_2^\parallel$	0.18 ± 0.10	0.06 ± 0.06	0.06 ± 0.06	0 ± 0.1
	0.16 ± 0.09	0.05 ± 0.05	0.05 ± 0.05	
$a_1^\perp$	0	0.20 ± 0.05	-0.20 ± 0.05	0
		0.18 ± 0.05	-0.18 ± 0.05	
$a_2^\perp$	0.20 ± 0.10	0.04 ± 0.04	0.04 ± 0.04	0 ± 0.1
	0.17 ± 0.09	0.03 ± 0.03	0.03 ± 0.03	
δ_+	0	0.24	0.24	0.46
		0.22	0.22	0.41
δ_-	0	-0.24	0.24	0
		-0.22	0.22	
$\widetilde{\delta}_+$	0	0.16	0.16	0.33
		0.13	0.13	0.27
$\widetilde{\delta}_-$	0	-0.16	0.16	0
		-0.13	0.13	

Table 2: Couplings for twist-3 and 4 distribution amplitudes for which we do not include SU(3)-breaking. Renormalization scale as in the previous table.

	ζ_3	ω_3^A	ω_3^V	ω_3^T	ζ_4	ζ_4^T	$\tilde{\zeta}_4^T$
V	0.032	-2.1	3.8	7.0	0.15	0.10	-0.10
	0.023	-1.8	3.7	7.5	0.13	0.07	-0.07

$$\langle 0|\bar{u}(z)d(-z)|\rho^-(P,\lambda)\rangle = \\ -i\left(f_\rho^T - f_\rho\frac{m_u+m_d}{m_\rho}\right)(e^{(\lambda)}\cdot z)m_\rho^2\int_0^1 du\, e^{i\xi p\cdot z}h_\parallel^{(s)}(u,\mu^2). \quad (47)$$

The distribution amplitude $\phi_\perp$ is twist-2, $h_\parallel^{(s)}$ and $h_\parallel^{(t)}$ are twist-3 and h_3 is twist-4. All four functions $\phi = \{\phi_\perp, h_\parallel^{(s)}, h_\parallel^{(t)}, h_3\}$ are normalized to

$$\int_0^1 du\,\phi(u) = 1.$$

Three-particle chiral-odd distributions are defined to twist-4 accuracy as

$$\begin{aligned}
&\langle 0|\bar{u}(z)\sigma_{\alpha\beta}gG_{\mu\nu}(vz)d(-z)|\rho^-(P,\lambda)\rangle = \\
&= f_\rho^T m_\rho^2\frac{e^{(\lambda)}\cdot z}{2(p\cdot z)}[p_\alpha p_\mu g_{\beta\nu}^\perp - p_\beta p_\mu g_{\alpha\nu}^\perp - p_\alpha p_\nu g_{\beta\mu}^\perp + p_\beta p_\nu g_{\alpha\mu}^\perp]\mathcal{T}(v,pz) \\
&+ f_\rho^T m_\rho^2[p_\alpha e_{\perp\mu}^{(\lambda)}g_{\beta\nu}^\perp - p_\beta e_{\perp\mu}^{(\lambda)}g_{\alpha\nu}^\perp - p_\alpha e_{\perp\nu}^{(\lambda)}g_{\beta\mu}^\perp + p_\beta e_{\perp\nu}^{(\lambda)}g_{\alpha\mu}^\perp]T_1^{(4)}(v,pz) \\
&+ f_\rho^T m_\rho^2[p_\mu e_{\perp\alpha}^{(\lambda)}g_{\beta\nu}^\perp - p_\mu e_{\perp\beta}^{(\lambda)}g_{\alpha\nu}^\perp - p_\nu e_{\perp\alpha}^{(\lambda)}g_{\beta\mu}^\perp + p_\nu e_{\perp\beta}^{(\lambda)}g_{\alpha\mu}^\perp]T_2^{(4)}(v,pz) \\
&+ \frac{f_\rho^T m_\rho^2}{pz}[p_\alpha p_\mu e_{\perp\beta}^{(\lambda)}z_\nu - p_\beta p_\mu e_{\perp\alpha}^{(\lambda)}z_\nu - p_\alpha p_\nu e_{\perp\beta}^{(\lambda)}z_\mu + p_\beta p_\nu e_{\perp\alpha}^{(\lambda)}z_\mu]T_3^{(4)}(v,pz) \\
&+ \frac{f_\rho^T m_\rho^2}{pz}[p_\alpha p_\mu e_{\perp\nu}^{(\lambda)}z_\beta - p_\beta p_\mu e_{\perp\nu}^{(\lambda)}z_\alpha - p_\alpha p_\nu e_{\perp\mu}^{(\lambda)}z_\beta + p_\beta p_\nu e_{\perp\mu}^{(\lambda)}z_\alpha]T_4^{(4)}(v,pz)
\end{aligned} \quad (48)$$

and

$$\begin{aligned}
\langle 0|\bar{u}(z)gG_{\mu\nu}(vz)d(-z)|\rho^-(P,\lambda)\rangle &= if_\rho^T m_\rho^2[e_{\perp\mu}^{(\lambda)}p_\nu - e_{\perp\nu}^{(\lambda)}p_\mu]S(v,pz), \\
\langle 0|\bar{u}(z)ig\widetilde{G}_{\mu\nu}(vz)\gamma_5 d(-z)|\rho^-(P,\lambda)\rangle &= if_\rho^T m_\rho^2[e_{\perp\mu}^{(\lambda)}p_\nu - e_{\perp\nu}^{(\lambda)}p_\mu]\widetilde{S}(v,pz). \quad (49)
\end{aligned}$$

Of these seven amplitudes, $\mathcal{T}$ is twist-3 and the other six are twist-4.

The light-cone expansion of the non-local tensor operator can be written to twist-4 accuracy as

$$
\begin{aligned}
\langle 0|\bar{u}(x)\sigma_{\mu\nu}d(-x)|\rho^-(P,\lambda)\rangle = \\
= i f_\rho^T \Bigg[(e_\mu^{(\lambda)}P_\nu - e_\nu^{(\lambda)}P_\mu)\int_0^1 du\, e^{i\xi Px}\left[\phi_\perp(u) + \frac{m_\rho^2 x^2}{4}\mathbb{A}_T(u)\right] \\
+ (P_\mu x_\nu - P_\nu x_\mu)\frac{e^{(\lambda)}\cdot x}{(Px)^2} m_\rho^2 \int_0^1 du\, e^{i\xi Px}\mathbb{B}_T(u) \\
+ \frac{1}{2}(e_\mu^{(\lambda)}x_\nu - e_\nu^{(\lambda)}x_\mu)\frac{m_\rho^2}{Px}\int_0^1 du\, e^{i\xi Px}\mathbb{C}_T(u)\Bigg], \qquad (50)
\end{aligned}
$$

where $\mathbb{B}_T$ and $\mathbb{C}_T$ are expressed in terms of the distribution amplitudes defined above as

$$
\begin{aligned}
\mathbb{B}_T(u) &= h_\parallel^{(t)}(u) - \frac{1}{2}\phi_\perp(u) - \frac{1}{2}h_3(u), \\
\mathbb{C}_T(u) &= h_3(u) - \phi_\perp(u), \qquad (51)
\end{aligned}
$$

and $\mathbb{A}_T$ can be related to integrals over three-particle distribution functions using the equations of motion.

For the leading twist-2 distribution amplitude $\phi_\perp$ we use

$$
\phi_\perp(u) = 6u\bar{u}\left[1 + 3a_1^\perp\,\xi + a_2^\perp\,\frac{3}{2}(5\xi^2-1)\right], \qquad (52)
$$

with parameter values as specified in Table 1; the definitions of $a_{1,2}^\perp$ are analaguous to Eqs. (36). The expressions for higher-twist distributions given below correspond to the simplest self-consistent approximation that satisfies all QCD equations of motion [4,5]:

- Three-particle distribution of twist-3:

$$
\mathcal{T}(\underline{\alpha}) = 540\,\zeta_3\omega_3^T(\alpha_d-\alpha_u)\alpha_d\alpha_u\alpha_g^2. \qquad (53)
$$

- Two-particle distributions of twist-3:

$$
\begin{aligned}
h_\parallel^{(s)}(u) &= 6u\bar{u}\left[1 + a_1^\perp\xi + \left(\frac{1}{4}a_2^\perp + \frac{5}{8}\,\zeta_3\omega_3^T\right)(5\xi^2-1)\right] \\
&\quad + 3\,\delta_+\,(3u\bar{u} + \bar{u}\ln\bar{u} + u\ln u) + 3\,\delta_-\,(\bar{u}\ln\bar{u} - u\ln u), \qquad (54) \\
h_\parallel^{(t)}(u) &= 3\xi^2 + \frac{3}{2}\,a_1^\perp\,\xi(3\xi^2-1) + \frac{3}{2}a_2^\perp\,\xi^2\,(5\xi^2-3)
\end{aligned}
$$

$$
+\frac{15}{16}\zeta_3\omega_3^T(3-30\xi^2+35\xi^4)+\frac{3}{2}\,\delta_+\,(1+\xi\,\ln\bar{u}/u)
+\frac{3}{2}\,\delta_-\,\xi\,(2+\ln u+\ln\bar{u}). \qquad (55)
$$

- Three-particle distributions of twist-4:

$$
\begin{aligned}
T_1^{(4)}(\underline{\alpha}) &= T_3^{(4)}(\underline{\alpha}) = 0,\\
T_2^{(4)}(\underline{\alpha}) &= 30\widetilde{\zeta}_4^T(\alpha_d-\alpha_u)\alpha_g^2,\\
T_4^{(4)}(\underline{\alpha}) &= -30\zeta_4^T(\alpha_d-\alpha_u)\alpha_g^2,\\
S(\underline{\alpha}) &= 30\zeta_4^T(1-\alpha_g)\alpha_g^2,\\
\widetilde{S}(\underline{\alpha}) &= 30\widetilde{\zeta}_4^T(1-\alpha_g)\alpha_g^2.
\end{aligned} \qquad (56)
$$

- Two-particle distributions of twist-4:

$$
\begin{aligned}
h_3(u) &= 6u(1-u)+5\left[\zeta_4^T+\widetilde{\zeta}_4^T-\frac{3}{70}a_2^\perp\right](1-3\xi^2),\\
\mathbb{A}_T(u) &= 30u^2(1-u)^2\left[\frac{2}{5}+\frac{4}{35}a_2^\perp+\frac{4}{3}\zeta_4^T-\frac{8}{3}\widetilde{\zeta}_4^T\right].
\end{aligned} \qquad (57)
$$

The constant ω_3^T is defined as

$$
\begin{aligned}
\langle 0|\bar{u}\sigma_{\mu\nu}z^\nu(gG^{\mu\beta}z_\beta(i\stackrel{\rightarrow}{D}z)-(i\stackrel{\leftarrow}{D}z)gG^{\mu\beta}z_\beta)d|\rho^-(P,\lambda)\rangle &=\\
&= (pz)^2(e^{(\lambda)}z)m_\rho^2 f_\rho^T\frac{3}{28}\,\zeta_3\omega_3^T.
\end{aligned} \qquad (58)
$$

The constants ζ_4^T and $\widetilde{\zeta}_4^T$ are defined as

$$
\begin{aligned}
\langle 0|\bar{u}gG_{\mu\nu}d|\rho^-(P,\lambda)\rangle &= if_\rho^T m_\rho^3\zeta_4^T(e_\mu^{(\lambda)}P_\nu-e_\nu^{(\lambda)}P_\mu),\\
\langle 0|\bar{u}g\widetilde{G}_{\mu\nu}i\gamma_5 d|\rho^-(P,\lambda)\rangle &= if_\rho^T m_\rho^3\widetilde{\zeta}_4^T(e_\mu^{(\lambda)}P_\nu-e_\nu^{(\lambda)}P_\mu).
\end{aligned} \qquad (59)
$$

Numerical values for all parameters are given in Table 1[d]. As in the chiral-even case, we neglect SU(3)-breaking corrections in twist-4 distributions. In Fig. 2, we plot the leading-twist distribution amplitude $\phi_\perp$ for ρ, K^* and ϕ mesons.

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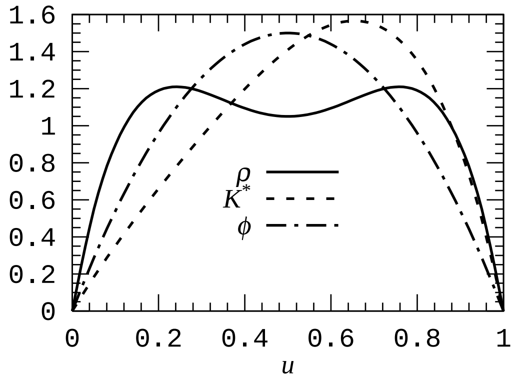
[d]In notations of Ref.[4] $\zeta_3^T \equiv (3/28)\zeta_3\omega_3^T$.

Figure 2: Twist-2 distribution amplitude $\phi_\perp(u, \mu = 1\,\mathrm{GeV})$ for ρ, K^* and ϕ mesons.



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