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Estimating the Parameters of Bose-Einstein Correlations from the Two-Particle Correlation Function in Multihadronic Final States

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Abstract

To estimate the strength of the Bose-Einstein correlations and the radius of the hadronization region in multiparticle production, the two-particle correlation functions R for identical pairs is adjusted to a parametric function describing the enhancement at small momentum differences. This is usually done by means of a binned uncorrelated least squares fit. This article demonstrates that this procedure underestimates the statistical errors. A recipe is given to construct from the data the covariance matrix between the bins of the histogram of the two-particle correlation function.

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1 Introduction

An enhancement in the production of pairs of pions of the same charge and similar momenta produced in high energy collisions was first observed in antiproton annihilations and attributed to Bose-Einstein statistics appropriate to identical pion pairs [1].

Bose-Einstein Correlations (BEC) between pion pairs can be used to study the space-time structure of the hadronization source [2]. This has been done for hadron-hadron, heavy ion, muon-hadron and e^+e^- collisions (see for example Ref. [3] for reviews).

The precise measurement of BEC parameters is especially important for LEP 2 physics. Interference due to Bose-Einstein correlations in hadronic decays of WW pairs has been discussed on a theoretical basis, in the framework of the measurement of the W mass [4]: this interference could induce a systematic uncertainty on the W mass measurement in the 4-jet mode which is of the order of 40 MeV, i.e., comparable with the expected accuracy of the measurement. For a review of measurements of BEC in WW pairs, see for example [5].

2 Determination of BEC parameters

To study the enhanced probability for the emission of two identical bosons, the correlation function R is used as a probe. For pairs of particles, it is defined as

$$R(p_1, p_2) = \frac{P(p_1, p_2)}{P_0(p_1, p_2)}, \quad (1)$$

where $P(p_1, p_2)$ is the two-particle probability density, subject to Bose-Einstein symmetrization, p_i is the four-momentum of particle i , and $P_0(p_1, p_2)$ is a reference two-particle distribution which, ideally, resembles $P(p_1, p_2)$ in all respects, apart from the lack of Bose-Einstein symmetrization.

If $d(x)$ is the space-time distribution of the source, $R(p_1, p_2)$ takes the form

$$R(p_1, p_2) = 1 + |G[d(x)]|^2,$$

where $G[d(x)] = \int d(x)e^{-i(p_1-p_2)\cdot x} dx$ is the Fourier transform of $d(x)$. Thus, by studying the correlations between the momenta of pion pairs, one can determine the distribution of the points of origin of the pions. Experimentally, the effect is often described in terms of the Lorentz-invariant variable Q , defined by $Q^2 = (p_1 - p_2)^2 = M^2(\pi\pi) - 4m_\pi^2$, where M is the invariant mass of the two pions. The correlation function can then be written as

$$R(Q) = \frac{P(Q)}{P_0(Q)}, \quad (2)$$

which is usually parametrized by the function

$$R(Q) = N \left(1 + \lambda e^{-r^2 Q^2} \right). \quad (3)$$

In the above equation, the pion source is spherically symmetric and gaussian, the parameter r gives the RMS source radius, λ is the strength of the correlation between the pions and N an overall normalization factor. The data from e^+e^- annihilations from PEP energies to LEP show values of r around 0.6 fm; the value of λ strongly depends on the analysis technique.

It can be understood from what said above that the main problems in the study of BEC are given by a good choice of the reference sample, and by the definition of the normalization N .

Most of the experimental studies are done inclusively, using all the observed charged tracks measured. Electrons, charged kaons and protons do not correlate with pions and clearly reduce the experimental correlation function. Another reduction of $R(Q)$ is due to non-prompt pions, i.e., pions from decays of particles with lifetime larger than the hadronization scale of around 1 fm/c (like K^0 , Λ and b and c hadrons). These pions are not expected to correlate with those from the primary hadronization.

3 Analysis

The analysis performed in this paper was based on the JETSET simulation [6]. JETSET has shown to reproduce well the experimental inclusive distributions measured by LEP [7].

BEC were described by the Bose-Einstein simulation algorithm LUBOEI, fully integrated in the JETSET simulation. The values generated for the pion momenta are modified by this algorithm, which reduces the differences for pairs of like-sign particles. This code has been shown [8] to reproduce well the two particle correlation functions measured in Z decays if Bose-Einstein correlations are switched on with a Gaussian parametrization for pions that are produced either promptly or as decay products of short-lived resonances (resonances with lifetime longer than the $K^*(890)$ lifetime were considered long-lived) and if the parameter values $\lambda = 1$ and $r = 0.5$ fm are used as input³. The value $\lambda=1$ for direct pions corresponds to $\lambda \sim 0.35$ for all pions or $\lambda \sim 0.25$ for all particles [8]. The fitted value of the radius r depends on the choice of the reference sample. Using a Monte Carlo reference sample changes typically r from the input value of 0.5 fm to a 20% higher value.

The use of simulated samples gives the possibility of knowing the normalization and of defining an unbiased reference sample simply by “switching off” the Bose-Einstein correlations.

4 The naive approach to the fit gives wrong results

We choose as a case study the hadronic decay of WW pairs with full Bose-Einstein effect and the BEC parameters set to $\lambda = 1$ and $r = 0.5$ fm respectively. 400 samples of 2,000 events each were simulated. For the reference sample P_0 , 40,000 events were simulated without BEC, in such a way that the error on P_0 gave a negligible contribution to the error on R .

For each sample a histogram of the correlation function was built, using 40 bins of 50 MeV each from 0 to 2 GeV. The normalization factor N was fixed by imposing that the average value of R between 1 GeV and 2 GeV was equal to unity.

For each of the 400 samples we performed a χ^2 fit to the form (3). The average values of λ and r from the 400 fits were:

$$\lambda = 0.509 \pm 0.016,$$

³The measured values of the parameters with a mixing reference sample for such “direct” pions in Z decays were $\lambda = 1.06 \pm 0.17$, $r = 0.49 \pm 0.05$ fm [8].

$$r = 0.569 \pm 0.012 \text{ fm}$$

(the statistical error corresponds to the average of the errors).

The “pull” of the fitted values of λ and r is shown in Figure 1. A Gaussian fit gives $\sigma_{(\lambda-\langle\lambda\rangle)/\sigma_\lambda} = 1.35$ with $\chi^2/\text{DoF} = 35/37$, and $\sigma_{(r-\langle r\rangle)/\sigma_r} = 1.75$ with $\chi^2/\text{DoF} = 30/38$.

The error on r is thus underestimated by a factor 1.75 while the error on λ is underestimated by a factor 1.35. The fact that these factors are different indicates that the “naive” fitting procedure described in this section could give possible biases in the determination of the parameters.

When WW pairs in which one W decays hadronically and the other leptonically are simulated, the pulls are smaller (1.30 for r and 1.24 for λ). This can be explained by the lower multiplicity, which gives smaller correlations. The case of the Z particle is similar to the semileptonic W.

5 Improved technique for the evaluation

The presence of bin-to-bin correlations in R is unavoidable: if there are, say, M positive tracks, the same positive track enters $(M - 1)$ times in the two-particle density P . We build the generic nondiagonal term C_{ij} ($i \neq j$) of the covariance matrix by assuming that it is given by the number of times that a track entering in the i th-bin enters also in the j th-bin.

However, this is not the only statistical correlation effect, since the same track can also enter several times in the same bin. The latter effect can be accounted for by rescaling the error of each bin, $\sigma(b_i)$, to the expected relative error for the true independent counts. If we call r_i the number of extra counts due to tracks already present once in the bin i , then the number of independent counts d_i for the bin i is $d_i = b_i - r_i$ and

$$\frac{\sigma(b_i)}{b_i} = \frac{1}{\sqrt{d_i}}. \quad (4)$$

The diagonal terms C_{ii} of the covariance matrix are assumed to be the squares of $\sigma(b_i)$.

The correlation matrix $\rho_{ij} = C_{ij}/(\sigma_i\sigma_j)$ looks like in Figure 2a. In Figure 2b, the ratio between the errors on the bin contents estimated from Eq. (4) and the square root of the number of entries b_i .

For each of the 400 samples we performed a χ^2 fit to the form (3), this time by using the covariance matrix described above. The average values of λ and r from the 400 fits were:

$$\begin{aligned} \lambda &= 0.499 \pm 0.021, \\ r &= 0.581 \pm 0.017 \text{ fm} \end{aligned}$$

(the statistical error corresponds to the average of the errors).

The “pull” of the fitted values of λ and r is shown in Figure 3. A Gaussian fit gives $\sigma_{(\lambda-\langle\lambda\rangle)/\sigma_\lambda} = 1.02$ with $\chi^2/\text{DoF} = 28/26$, and $\sigma_{(r-\langle r\rangle)/\sigma_r} = 1.09$ with $\chi^2/\text{DoF} = 35/31$.

The error on r and λ are thus correctly estimated (at better than 10%). A residual discrepancy can be attributed to the fact that, due to a Q -dependence of the fraction of particles which do not display BEC, the fitting function (3) does not describe the data perfectly.

As an example, in Figure 4 the correlation functions $R(Q)$ for like-sign pairs (closed circles) in one of the 400 simulated samples is shown together with the diagonal errors from the naive approach and from the approach proposed in this paper, and with the fitting function corresponding to the average results of the fit in the two approaches.

6 Conclusions

Using a binned uncorrelated least squares fit to evaluate the parameters λ and r of Bose-Einstein correlations underestimates the statistical errors and might bias the result.

The error underestimate has been shown to be of the order of 30% in the case of single W and Z and of about 50% in the case of the hadronic decay of W pairs. A recipe is given to construct from the data a heuristic covariance matrix between the bins of the histogram of the two-particle correlation function.

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References

- [1] G. Goldhaber et al., Phys. Rev. **120** (1960) 300.
- [2] G.I. Kopylov and M.I. Podgoretskii, Sov. J. Nucl. Phys. **19** (1974) 215; G.I. Kopylov, Phys. Lett. **B50** (1974) 472;
G. Cocconi, Phys. Lett. **B49** (1974) 459;
M. Deutschmann et al., Nucl. Phys. **B204** (1982) 333;
A. Giovannini and G. Veneziano, Nucl Phys. **B130** (1977) 61;
M. G. Bowler, Z. Phys. **C29** (1985) 617.
- [3] B. Lörstad, Int. J. Mod. Phys. **A4** (1989) 2861;
W. A. Zajc, in “Hadronic Multiparticle Production”, P. Carruthers ed., World Scientific (1988) 235.
- [4] T. Sjöstrand and V.A. Khoze, Z. Phys. **C62** (1994) 281;
L. Lönnblad and T. Sjöstrand, Phys. Lett. **B351** (1995) 293;
“Physics at LEP2”, eds. G. Altarelli, T. Sjöstrand and F. Zwirner, CERN 96-01 (1996);
A. Ballestrero et al., “Report of the Working Group on W Mass and QCD”, hep-ph/9709283 (September 1997);
L. Lönnblad and T. Sjöstrand, Eur. Phys. J. **C2** (1998) 165.
- [5] A. De Angelis, “Interconnection Effects in Multiparticle Production from WW Events at LEP”, CERN-OPEN/97-024 and hep/ex/9805005, to be published in Proc. XXVIII Int. Symposium on Multiparticle Dynamics, Frascati, September 1997.
- [6] T. Sjöstrand, “PYTHIA 5.7 / JETSET 7.4”, CERN-TH.7112/93 (1993);
T. Sjöstrand, Comp. Phys. Comm. **82** (1994) 74.
- [7] DELPHI Coll., P. Abreu et al., Z. Phys. **C77** (1996) 11.
- [8] DELPHI Coll., P. Abreu et al., Z. Phys. **C63** (1994) 17.

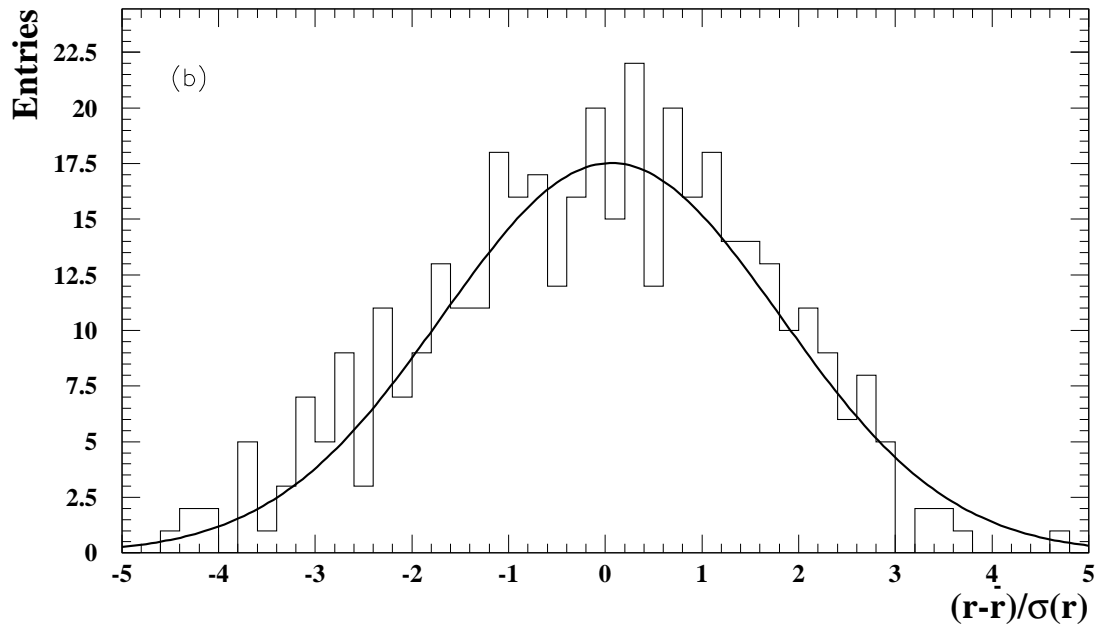
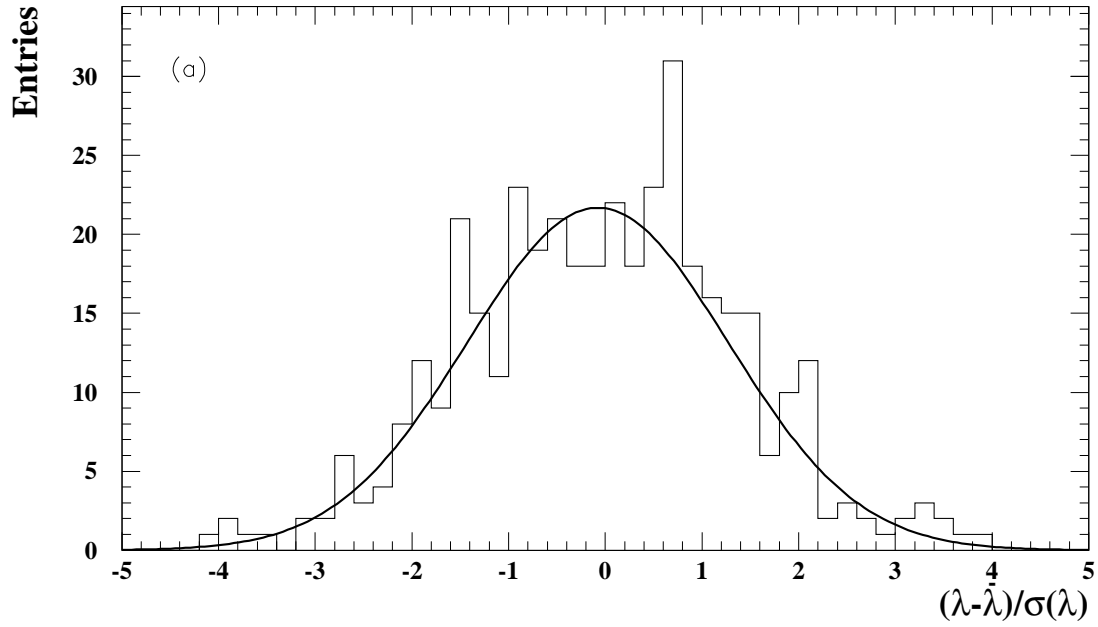


Figure 1: (a) Pull function for the fitted parameter λ using a binned uncorrelated least squares fit in the simulated samples. A Gaussian fit is superimposed as a solid line. (b) Same as (a) but for the parameter r .

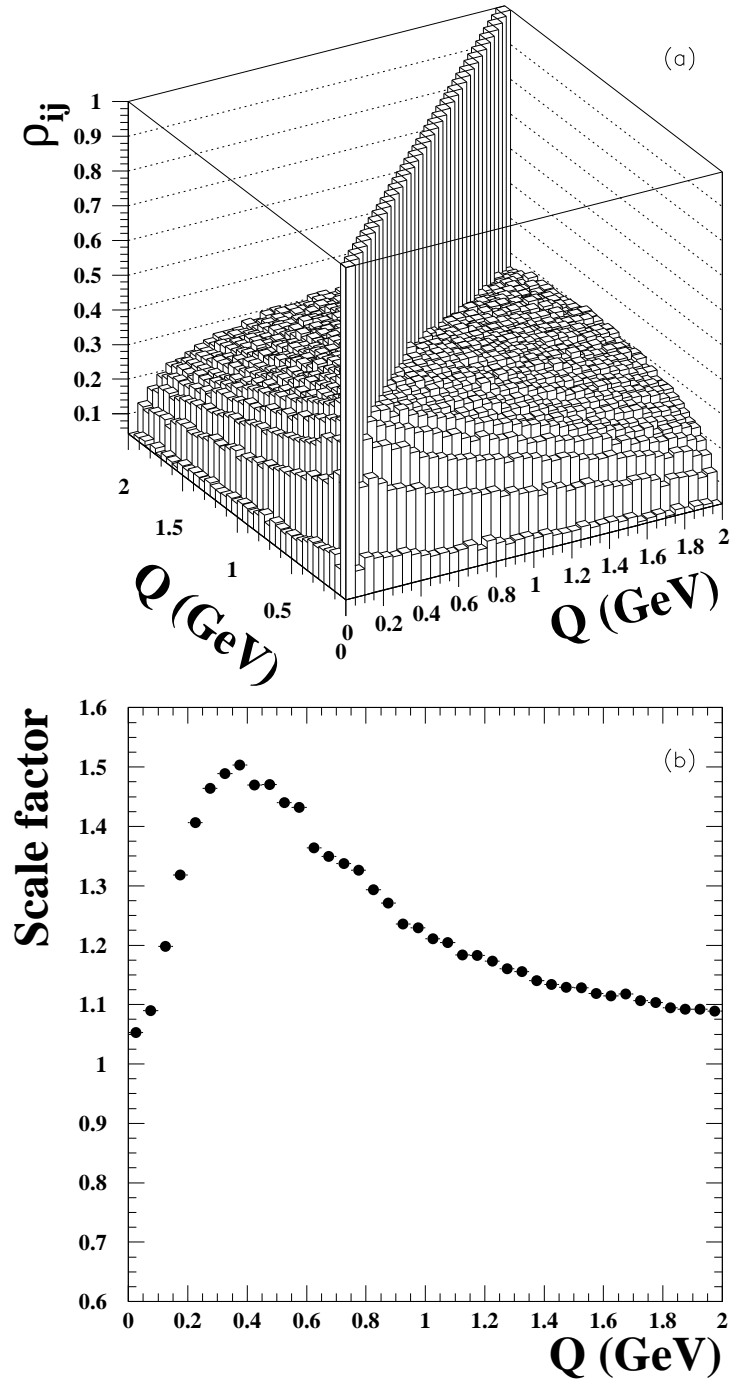


Figure 2: (a) Correlation matrix (see text). (b) Ratio between the errors on the bin contents estimated with the procedure described in this paper and the errors computed from the naive approach.

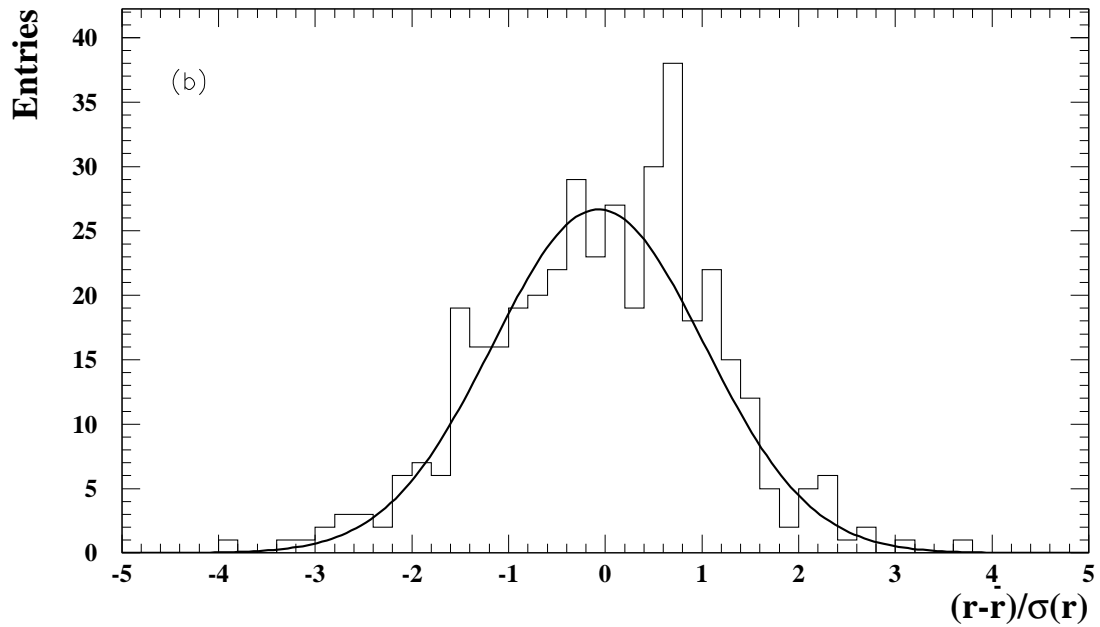
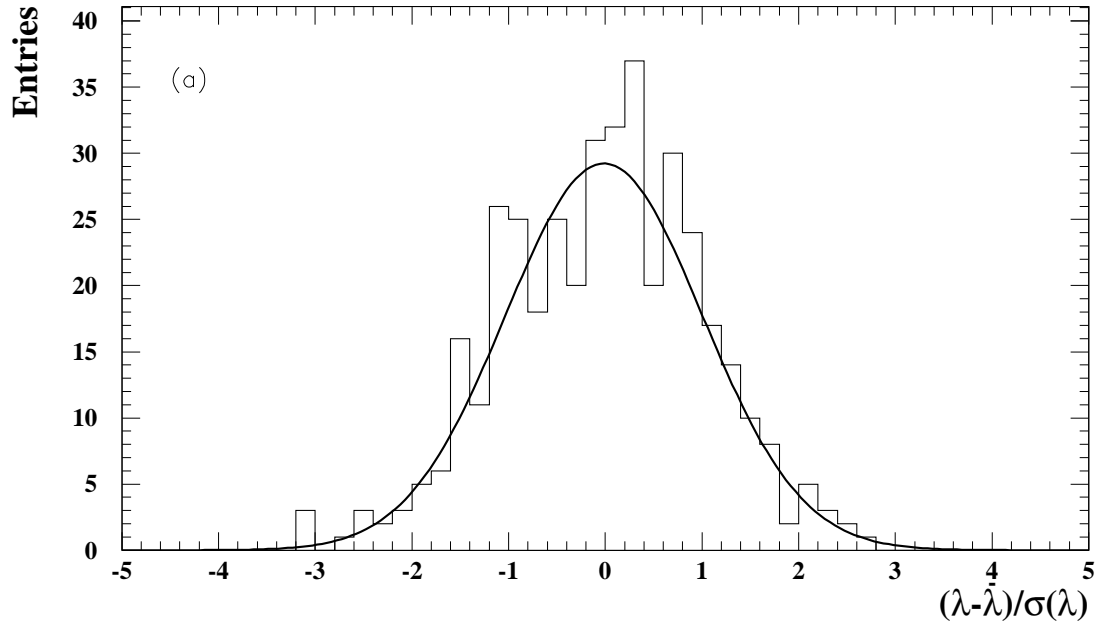


Figure 3: (a) Pull function for the fitted parameter λ in the simulated samples using the technique for the error estimate described in this paper. A Gaussian fit is superimposed as a solid line. (b) Same as (a) but for the parameter r .

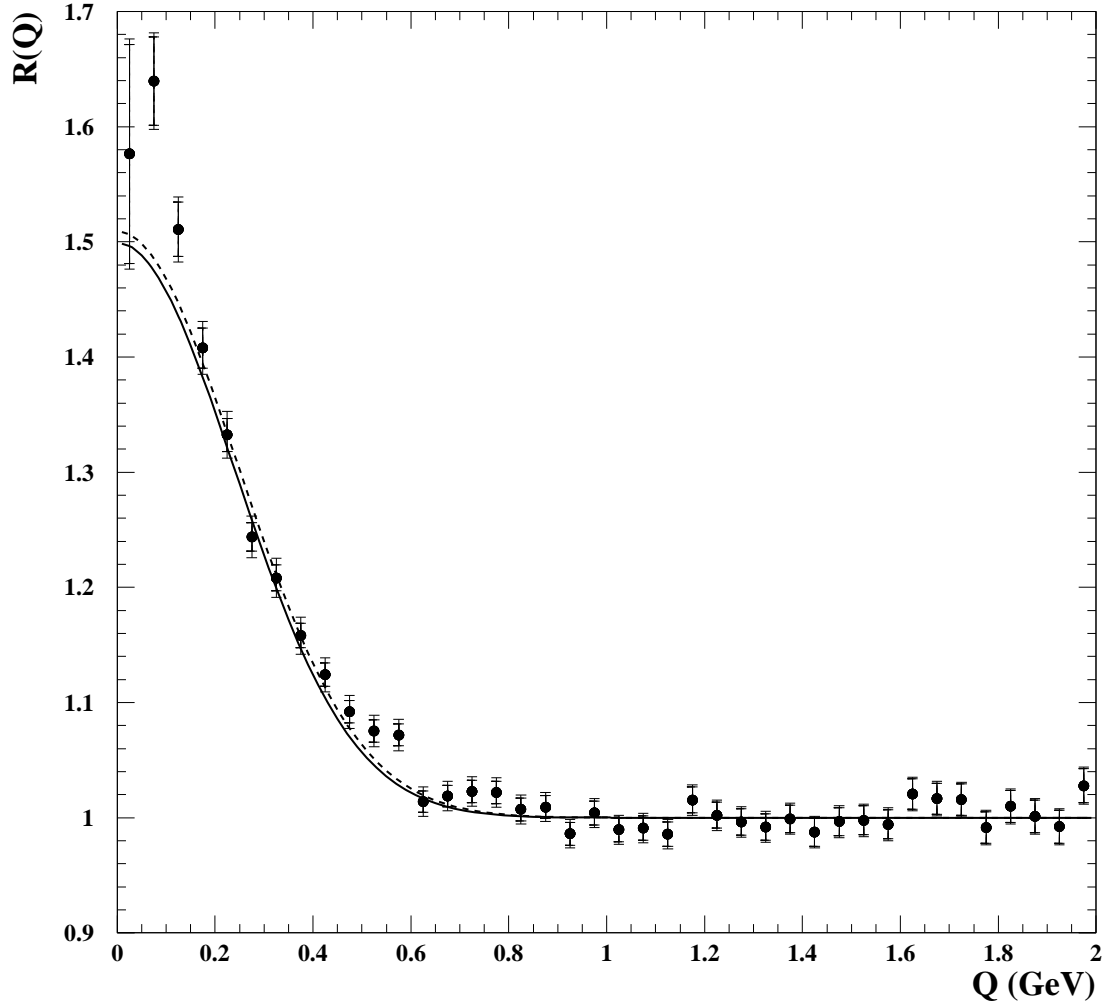


Figure 4: The correlation function $R(Q)$ for like-sign pairs (closed circles) in one of the simulated samples. The error bars shown are the square roots of the diagonal elements of the covariance matrix. The inner error bars correspond to the error computed from the naive approach. The dashed line shows the fit to Eq. (3) where λ and r are the averages of the fitted values using the naive approach. The solid line corresponds to the approach proposed in this article.