MEASUREMENT OF THE OPTICAL PARAMETERS OF A TRANSFER LINE USING MULTI-PROFILE ANALYSIS

G. Arduini, M. Giovannozzi, D. Manglunki, M. Martini, CERN, Geneva, Switzerland

Abstract

The standard approach to measure the optical parameters and the emittance in a transfer line is based on the analysis of the profiles measured by three monitors. This is feasible, provided the dispersion function is known a priori. In this paper we propose to measure the complete set of five parameters (the two independent Twiss parameters, the emittance, the dispersion function and its derivative) by using five monitors with one bending magnet interleaved. The results of some measurements carried out in the transfer line connecting the CERN PS and SPS rings are presented.

1 THEORY

The standard way to measure the optical parameters α, β and the emittance ϵ in a transfer line is based on the use of three monitors [1, 2]. Under the following assumptions:

- The dispersion function D is zero or well-known all along the section where the monitors are installed.
- The transfer matrices of the beam line sections between the monitors are known.
- The particle motion is fully decoupled between the monitors.

One can compute the optical parameter from the measurements of beam profiles.

If σ_i indicates the profile half-width measured in the ith monitor, then

$$\sigma_i^2 = \beta_i \, \epsilon, \qquad i = 1, 2, 3 \tag{1}$$

where the beam emittance is defined as the width of a Gaussian-distributed beam at one sigma. Using the fact that the motion is fully decoupled, it is possible to use the invariance of the emittance to express the Twiss parameters at the locations of the monitors 2 and 3 in terms of those at the monitor 1, namely:

$$\beta_i = C_i^2 \beta_1 - 2 C_i S_i \alpha_1 + S_i^2 \gamma_1, \qquad i = 2, 3. \tag{2}$$

Here the transfer matrix between monitor 1 and i is given by

$$\mathcal{T}_i = \begin{pmatrix} C_i & S_i \\ C_i' & S_i' \end{pmatrix} \tag{3}$$

and the parameter $\gamma = (1 + \alpha^2)/\beta$ has been introduced. The set of Eqs. (1) can be solved using the relations (2), giving

$$\epsilon = \sigma_1^2 \Lambda \qquad \beta_1 = \frac{1}{\Lambda} \qquad \alpha_1 = \frac{\Gamma}{2\Lambda}.$$

Here Γ, Λ are given in terms of the measured profile widths

$$\Gamma = \frac{[(\sigma_3/\sigma_1)^2 - C_3^2]/S_3^2 - [(\sigma_2/\sigma_1)^2 - C_2^2]/S_2^2}{(C_2/S_2) - (C_3/S_3)}$$

$$\Lambda^2 = (\sigma_2/\sigma_1)^2/S_2^2 - (C_2/S_2)^2 + (C_2/S_2)\Gamma - \Gamma^2/4$$

In the case where the hypothesis on the dispersion function is not satisfied, the approach to be used in order to measure the Twiss parameters and the beam emittance has to be modified. The starting point is a different version of Eq. (1)

$$\sigma_i^2 = \beta_i \, \epsilon + D_i^2 \delta^2, \tag{4}$$

where δ represents the relative momentum spread of the beam at one sigma. It is customary to measure the dispersion function independently. A shift of the beam energy is performed and the corresponding variation of the average position on the monitors is recorded. A linear fit allows the determination of the dispersion at the location of the monitors. On the other hand, the energy spread can be easily measured by using a longitudinal pick-up. Hence, once the D_is and δ are known, it is possible to apply the method previously described using Eq. (4) instead of Eq. (1).

Although the measurement of the dispersion is conceptually rather simple, in practice it perturbs the standard operation in order to vary the beam energy. Hence it is not advisable to apply this method routinely. Furthermore, in some cases the extraction process is so complex that the energy shift would completely destroy the ejected beam or alter the beam parameters with respect to the nominal situation.

For these reasons it is feasible to have an alternative method which measures the whole set of beam parameters α, β, D, D' and ϵ at the same time without varying any machine parameter. From simple arguments, the information extracted from five monitors should be enough to compute the five unknowns. It turns out that this is the case, provided at least one dipole is installed in the section of the beam line delimited by the first and last monitor.

Once again the starting point is Eq. (4), connecting the optical parameters and the measured beam profile in the presence of dispersion, and the transformation law (2) used to propagate the Twiss parameters. In this case it is also necessary to consider the relation between the value of the dispersion function at the location of the monitors. The propagation of D is performed by using the same transfer matrix used to transform the optical parameters

$$D_i = C_i D_1 + S_i D_1' + \xi_i \qquad i = 2, 3, 4, 5 \tag{5}$$

where ξ is the contribution to the dispersion due to a dipole magnet between the first and the ith monitor. By using Eqs. (2) and (5), (4) can be cast in the form of a system of five equations. It can be written in matrix form as

$$\boldsymbol{\Sigma} = \mathcal{M}\boldsymbol{\Pi}, \tag{6}$$

where

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_1^2 \\ \sigma_2^2 - \xi_2\delta^2 \\ \sigma_3^2 - \xi_3\delta^2 \\ \sigma_4^2 - \xi_4\delta^2 \\ \sigma_5^2 - \xi_5\delta^2 \end{pmatrix} \qquad \boldsymbol{\Pi} = \begin{pmatrix} \beta_1\,\epsilon + D_1^2\delta^2 \\ \alpha_1\,\epsilon - D_1 D_1'\delta^2 \\ \gamma_1\,\epsilon + D'^2_1\delta^2 \\ D_1\delta^2 \\ D_1'\delta^2 \end{pmatrix}$$

and

$$\mathcal{M} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ C_2^2 & -2C_2\,S_2 & S_2^2 & 2C_2\,\xi_2 & 2S_2\,\xi_2 \\ C_3^2 & -2C_3\,S_3 & S_3^2 & 2C_3\,\xi_3 & 2S_3\,\xi_3 \\ C_4^2 & -2C_4\,S_4 & S_4^2 & 2C_4\,\xi_4 & 2S_4\,\xi_4 \\ C_5^2 & -2C_5\,S_5 & S_5^2 & 2C_5\,\xi_5 & 2S_5\,\xi_5 \end{pmatrix}$$

In order to invert the matrix $\mathcal{M}$ it is sufficient that two ξ are different from zero. This is certainly the case if a dipole magnet is located upstream of the fourth monitor. In this case, from the solution given by $\mathcal{M}^{-1}\boldsymbol{\Sigma} = \boldsymbol{\Sigma'}$ one can determine the Twiss parameters as follows

$$\begin{array}{rclcrcl} D_1 & = & \Sigma_4'/\delta^2 & & & & \\ D_1' & = & \Sigma_5'/\delta^2 & & & & \\ \beta_1 & = & A/\sqrt{AC-B^2} & & A & = & \Sigma_1' - \Sigma'^2_4/\delta^2 \\ \alpha_1 & = & B/\sqrt{AC-B^2} & \text{with} & B & = & \Sigma_2' + \Sigma_4'\Sigma_5'/\delta^2 \\ \epsilon & = & \sqrt{AC-B^2} & & C & = & \Sigma_3' - \Sigma'^2_5/\delta^2 \end{array}$$

2 THE MODEL

The approach described in the previous section has been applied to the transfer line between the CERN Proton Synchrotron and the Super Proton Synchrotron. This line is divided into two parts: the TT2 line and the TT10 line. TT2 transports the beam from the extraction point of the PS machine to the TT10 part, which, in turn, connects the transfer line to the SPS injection point. At the junction of the two lines, the beam is deflected ≈ 81 mrad to the right. Due to the difference in height between the PS and SPS, a vertical deflection angle of ≈ 60 mrad is imposed at the entrance of TT10 and then cancelled before injection in the SPS.

Three Secondary Emission Monitors are installed both in TT2 and in the TT10 section. These two sets of monitors are routinely used to perform emittance and Twiss parameter measurement in both lines. For this purpose, the standard method based on Eq. (1) is used, with the dispersion measured by performing an energy shift.

In Fig. (1) the horizontal and vertical β-functions (upper part) are shown, together with the horizontal and vertical dispersion (lower part), for the TT2 and TT10 lines. The TT2-TT10 transfer line has an important role in the LHC era. In fact, to achieve the LHC top performance [3], it is

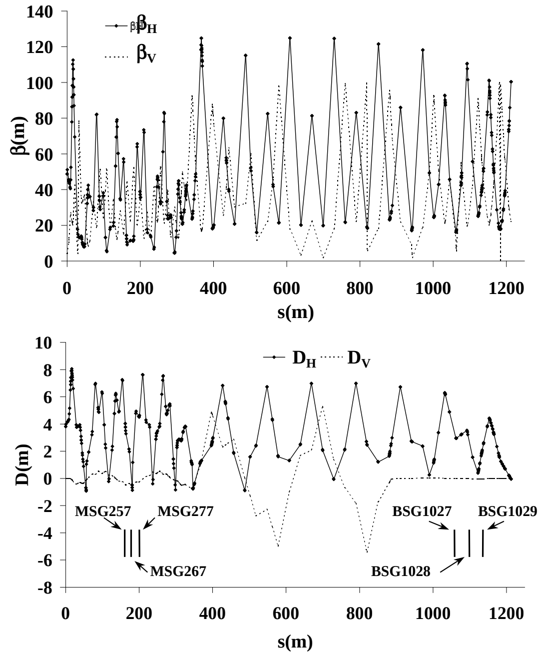

Figure 1: Horizontal and vertical β-functions (upper part) and dispersion (lower part) for the TT2 and TT10 lines. The position of the beam profile monitors installed in the lines is also shown.

important to avoid emittance blow-up along the injection chain [4, 5]. Although a transfer line cannot generate such a blow-up, nevertheless it can be a source of transverse or longitudinal mismatch at injection in the subsequent machine. Therefore it is important to be able to measure the optical parameters to detect a potential mismatch. In this context, the proposed approach represents a robust method to measure the optical parameters during normal operation.

3 MEASUREMENTS

In order to test the proposed method, a comparison has been carried out between a set of measurements performed in the standard way, with a second set obtained by applying our approach.

At high energy, 26 GeV/c, the proton beam is extracted from the PS machine using a kicker magnet and delivered to the SPS through the TT2-TT10 line. The beam intensity is about 1.1×10^{12} ppp, distributed into 20 bunches 6.1 ns long with a momentum spread δ of about 0.26×10^{-3}.

3.1 *Standard method*

In order to apply Eq. (1), it is necessary to measure the horizontal and vertical dispersion at the monitor's location. This can be done as described above. For each value of the energy five measurements of the beam centroid have been taken. The average of these values is used in the analysis, while the variance of the five measurements is used to estimate the error associated with the measurement. The results are reported in Table 1. A new set of five different measurements of the beam width for each of the monitors has been taken. The variance of the five measurements has been used to estimate the error. The emittance and the Twiss parameters are then computed using Eqs. (1). The results are reported in Table 2.

Table 1: Horizontal and vertical dispersion at the monitor's location in TT2 and TT10 lines.

		D_H [m]	D_V [m]
TT2	MSG257	3.03± 0.04	-0.56± 0.02
	MSG267	-0.50± 0.04	-0.547± 0.008
	MSG277	-3.56± 0.06	-0.200± 0.008
TT10	BSG1027	-1.72± 0.06	0.00 ± 0.02
	BSG1028	-0.32± 0.09	0.25 ± 0.02
	BSG1029	4.03± 0.09	0.27 ± 0.01

Table 2: Horizontal and vertical Twiss parameters and emittance on the first monitor of the TT2 and TT10 lines. The values shown are computed using the standard method.

Beam parameters	TT2 MSG257	TT10 BSG1027
β_H [m]	13.0±0.2	39.6±0.6
α_H	1.71±0.04	-1.15±0.04
ϵ_H [mm mrad]	0.66±0.01	0.72±0.01
β_V [m]	52.7±0.5	53.1±0.7
α_V	-2.63±0.05	1.53±0.03
ϵ_V [mm mrad]	0.44±0.01	0.54±0.01

3.2 Multi-profile method

In this case, the computation of the five optical parameters is performed without any change in the beam parameters, simply measuring the profile width on five different monitors. The geometry of the TT2-TT10 does not allow the application of this method to measure the vertical optical parameters. In fact, two vertical dipoles are installed between the three monitors in TT2 and those in TT10: the first one bends the beam down towards the SPS and the second one cancels the deflection. Therefore, the global effect is approximately equivalent to having no bending magnet at all.

In the horizontal plane, the proposed technique can be applied to measure the Twiss parameters. As only five monitors are needed, it is possible to obtain different sets of measured parameters according to the various combinations of five monitors out of the six available. In Table 3 the results for three different combinations of monitors are shown. In the last two columns the average μ of the three sets of values and the standard deviation $\sigma_{\rm s.d.}$, used as an estimate of the error associated with this method are reported. The combinations shown in Table 3 are made by the three monitors in TT2 and two monitors in TT10, namely

Table 3: Horizontal Twiss parameters, emittance and dispersion on the first monitor of TT2 computed using the multi-profile method.

Beam parameters	Comb. 1	Comb. 2	Comb. 3	μ	$\sigma_{\rm s.d.}$
β_H [m]	13.90	13.39	12.36	13.2	0.8
α_H	1.89	1.70	1.47	1.7	0.2
ϵ_H [mm mrad]	0.63	0.66	0.58	0.62	0.01
D_H [m]	2.68	2.49	5.50	3.6	1.7
D'_H	-0.07	-0.47	-0.92	-0.5	0.4

Comb. 1: MSG257, MSG267, MSG277, BSG1027, BSG1028
Comb. 2: MSG257, MSG267, MSG277, BSG1027, BSG1029
Comb. 3: MSG257, MSG267, MSG277, BSG1028, BSG1029
The other possibilities, combining two monitors of TT2 and three of TT10, provided results in poor agreement with those obtained using the standard method. A possible explanation could be the different resolution of the monitors installed in the two sections (0.5 mm for those in TT2 and 2.5 mm for those in TT10).

The results obtained are encouraging, showing a rather good agreement with the outcome of the standard method. It is worth noting that although the average dispersion value agrees within only 20 %, the average Twiss parameters and the emittance show a better agreement (less than 10 %).

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