TeV-Scale GUTs [a]

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In this talk, we summarize our recent proposal for lowering the scale of grand unification to the TeV range though the appearance of extra spacetime dimensions. Particular emphasis is placed on the perturbativity and predictivity of our scenario, as well as its sensitivity to unification-scale effects.

1   Introduction

The possibility of extra large spacetime dimensions has recently received considerable attention [1,2,3,4,5,6,7,8]. One of the first serious investigations of the phenomenological properties of theories with extra large dimensions was made in Ref.[1], where the lightness of the supersymmetry-breaking scale was related to the largeness of a string-theoretic compactification radius. More recently, in Ref.[2], extra large dimensions figured prominently in a radical new proposal for avoiding the gauge hierarchy problem by lowering the Planck scale to the TeV scale. The extra dimensions required to achieve this are in the (sub-)millimeter range, and imply a profound change in Newton's gravitational force law at such distances. Likewise, in Refs.[4,5,6,8], it was shown how extra large dimensions could also be used to lower the fundamental string scale to the TeV scale. The idea of taking the string scale in the TeV range originates in Ref.[9], and makes use of special features of open-string theory first pointed out in Ref.[10].

Besides the Planck and string scales, there also exists one additional high fundamental scale in physics: the GUT scale. Indeed, while the Planck and (perturbative heterotic) string scales are related directly to each other, the GUT scale stands independently. In this talk, therefore, we will concentrate on our complementary proposal [3,6] of lowering the fundamental GUT scale to the

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TeV range. By definition, one of the unique issues faced in attempting to lower the GUT scale to the TeV range is that the scale of gauge coupling unification must be substantially shifted from its usual MSSM value near $M_{\mathrm{GUT}} \approx 2 \times 10^{16}$ GeV. This in turn requires that the usual logarithmic running of the MSSM gauge couplings must somehow be altered. Remarkably, however, as demonstrated in Refs.[3,6], extra spacetime dimensions have precisely the effect we want: they modify the running of the gauge couplings in such a way that not only is the unification preserved, but in fact it occurs more rapidly. This then leads to a reduction in the unification scale. Thus, through extra dimensions, it becomes possible for the first time to contemplate grand unification occurring at an intermediate scale — indeed, as low as the TeV scale. In other words, we now see that it becomes possible to replace a four-dimensional GUT at 10^{16} GeV with a higher-dimensional GUT at the TeV scale.

There is one aspect of this proposal that deserves special attention. By its very nature, such a radical change in the GUT scale requires that the running of the gauge couplings fully experience the effects of the extra dimensions. This in turn requires that the Standard Model fields actually propagate in the extra dimensions, and moreover that the effects of these extra dimensions not be arranged to cancel as the result of the effects of other symmetries. In this respect, the extra dimensions required in this proposal are fundamentally different from those of Ref.[1] (in which the effects of the extra dimensions on the gauge couplings are arranged to cancel as the result of $N = 4$ supersymmetry); from those of Ref.[2] (with respect to which the Standard Model particles are trapped on an effective brane); and also from those of Ref.[8] (in which the effects of the extra dimensions are restricted to certain open-string sectors). As far as we are aware, therefore, the effects of extra dimensions on the running of the gauge couplings had not been investigated prior to Refs.[3,6], and as we shall see, this leads to special subtleties that we shall discuss below.

2 Gauge Coupling Unification in the Presence of Extra Dimensions

We begin by quickly recalling the usual four-dimensional result. In the usual MSSM calculation, one derives the one-loop running of the $SU(3) \times SU(2) \times U(1)$ gauge couplings by evaluating one-loop wavefunction renormalization diagrams, where all MSSM states can propagate in the loop. This then leads to the logarithmic running equation

$$\alpha_i^{-1}(\mu) \;=\; \alpha_i^{-1}(M_Z) \;-\; \frac{b_i}{2\pi} \ln \frac{\mu}{M_Z} \;. \tag{1}$$

where the one-loop MSSM beta-function coefficients are $(b_1, b_2, b_3) = (33/5, 1, -3)$. Given the experimentally measured gauge couplings at the Z-scale, we can use Eq. (1) to extrapolate upwards in energy. This then leads to the conventional unification near $M_{\mathrm{GUT}} \approx 2 \times 10^{16}$ GeV.

How do we extend this calculation into higher dimensions? The basic idea is relatively simple. First, we imagine that there exist $\delta \equiv D-4$ extra spacetime dimensions, each compactified on a circle of radius R. Here $\delta = 1, 2, ...$ can take any integer value, and likewise $\mu_0 \equiv R^{-1}$, which sets the energy threshold for the extra dimensions, can range anywhere from several hundred GeV all the way up to the usual GUT scale $M_{\mathrm{GUT}} \approx 2 \times 10^{16}$ GeV. Corresponding to each MSSM field, there will be an infinite tower of Kaluza-Klein states whose masses are separated by μ_0. For various technical reasons discussed in Refs. [3,6], we actually must compactify on orbifolds rather than circles, and likewise there are some subtleties involved in arranging the Kaluza-Klein towers corresponding to the MSSM states. The upshot is that the states at the excited levels of the Kaluza-Klein towers must fall into $N = 2$ supermultiplets (even though the ground-state zero-mode MSSM states are only $N = 1$ supersymmetric). It also turns out that one has the freedom to choose whether or not to have Kaluza-Klein towers for the MSSM fermions. We shall therefore denote by the additional free parameter $\eta = 0, 1, 2, 3$ the number of MSSM chiral generations which we shall assume to have Kaluza-Klein towers.

Given this setup, it is then relatively straightforward to calculate the running of the gauge couplings: we simply re-evaluate the standard one-loop wave-function renormalization diagrams with the usual MSSM states as well as their corresponding Kaluza-Klein excitations in the loop. Of course, strictly speaking, such a theory is non-renormalizable due to the infinite towers of Kaluza-Klein states, but we are free to truncate these towers at some arbitrarily high excitation level without affecting our results. Therefore, for the purposes of calculating gauge coupling renormalization effects, we may consider this to be a fully renormalizable field theory. Evaluating the one-loop diagrams, we then find the general result [6]

$$\alpha_i^{-1}(\Lambda) \;=\; \alpha_i^{-1}(\mu) \;-\; \frac{b_i - \tilde{b}_i}{2\pi} \ln \frac{\Lambda}{\mu} \;-\; \frac{\tilde{b}_i}{4\pi} \int_{r\Lambda^{-2}}^{r\mu^{-2}} \frac{dt}{t} \left\{ \vartheta_3 \left(\frac{it}{\pi R^2} \right) \right\}^{\delta} \quad (2)$$

where the Jacobi theta-function $\vartheta_3(\tau) \equiv \sum_{n_i=-\infty}^{\infty} e^{\pi i \tau n^2}$ reflects the sum over the Kaluza-Klein states. Here the new beta-function coefficients $\tilde{b}_i$ corresponding to the excited Kaluza-Klein levels are given by

$$(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) \;=\; (3/5, -3, -6) \;+\; \eta \, (4, 4, 4) \, , \quad (3)$$

and the numerical coefficient r is given by $r \equiv \pi(X_\delta)^{-2/\delta}$ where $X_\delta \equiv 2\pi^{\delta/2}/\delta\Gamma(\delta/2)$. While the final term in Eq. (2) reflects the contributions from the Kaluza-Klein towers, the penultimate term reflects the fact that the zero-mode MSSM states and their Kaluza-Klein towers are not identical. [6] Note that all of the terms in Eq. (2) together form the one-loop result, and thus this expression is renormalization-group scheme-independent.

For many practical purposes, it is possible to *approximate* the result (2) in the following way: for $\mu \leq \mu_0$, we may replace Eq. (2) by Eq. (1), while for $\mu \geq \mu_0$, we may replace Eq. (2) by the power-law expression

$$\alpha_i^{-1}(\mu) \; = \; \alpha_i^{-1}(\mu_0) \; - \; \frac{b_i - \tilde{b}_i}{2\pi} \ln \frac{\mu}{\mu_0} \; - \; \frac{\tilde{b}_i X_\delta}{2\pi\delta} \left[\left(\frac{\mu}{\mu_0} \right)^\delta - 1 \right] \; . \qquad (4)$$

However, for certain precision calculations (as will be discussed below), it may be necessary to use the full one-loop result given in Eq. (2).
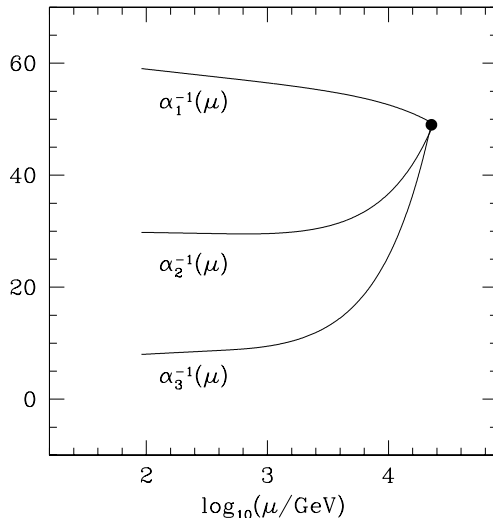

Figure 1: Unification of gauge couplings at the new unification scale $M'_{\mathrm{GUT}} \approx 20$ TeV, assuming the appearance of a single extra spacetime dimension of radius $R^{-1} = 1$ TeV.

The remarkable feature of this higher-dimensional running is that gauge coupling unification is nevertheless preserved. As the most interesting case, let us consider $\mu_0 = 1$ TeV, $\delta = 1$, and $\eta = 0$. We then find the unification

shown in Fig. 1. Thus, we see that extra dimensions are consistent with the emergence of a grand unified theory in the TeV range — *i.e.*, the GUT scale has been lowered all the way to the TeV scale! However, this is not the only possibility, and it is shown in Refs. [3,6] that such a unification with a reduced unification scale occurs regardless of the values of $\mu_0 \equiv R^{-1}$, δ, or η.

3 Perturbativity, Higher-Loop Corrections, and Sensitivity to Unification-Scale Thresholds

Before proceeding further, it is important to discuss this unification in some detail. Specifically, the power-law running raises a number of important questions. Why does the unification occur? How robust is it against higher-order corrections? How perturbative is the resulting theory? How exact is the unification? How predictive is it? And most importantly, how sensitive is it to unification-scale effects?

First, we must discuss *why* the unification occurs. It is clear that if it had been the case that $b_i = \tilde{b}_i$ for all i, then unification would have occurred as a direct consequence of the usual unification in the MSSM, for the appearance of the extra Kaluza-Klein towers with beta-functions $\tilde{b}_i$ would have simply resembled additional copies of the MSSM matter content. In our case, by contrast, we actually have $b_i \neq \tilde{b}_i$. However, continued unification does not require $b_i = \tilde{b}_i$. Instead, all that is required is that $B_{ij} \equiv (\tilde{b}_i - \tilde{b}_j)/(b_i - b_j)$ be independent if (i,j), or equivalently that $B_{12}/B_{13} = 1$ and $B_{13}/B_{23} = 1$. We find that in our scenario, these relations hold *approximately*: $B_{12}/B_{13} \approx 0.94$ and $B_{13}/B_{23} \approx 0.92$. Indeed, it is apparent from Fig. 1 that this leads to a fairly precise unification, and increasing the inverse radius R^{-1} only makes the unification more precise. It is this numerical coincidence which underlies our unification.

Given that this (approximate) one-loop unification occurs, it is natural to ask about higher-loop corrections. *A priori*, they might be expected to be particularly substantial in our case, since they would also seem to have power-law behavior. However, in our scenario, the excited Kaluza-Klein states are always in $N = 2$ supermultiplets. This means that all higher-order power-law contributions vanish identically, which in turn implies that one-loop power-law behavior is exact. Thus, all that remains are the usual MSSM higher-order logarithmic effects, which are strongly suppressed.

Next, let us discuss the perturbativity of our scenario. It is apparent from Fig. 1 that the unification is very *weakly coupled* (even more so than within the MSSM). This is true even if we change the values of R and δ. However, strictly speaking, this does not imply that the unification is *perturbative*. Indeed, even

though our new unified coupling $\alpha'_{\rm GUT}$ satisfies $\alpha'_{\rm GUT} \ll 4\pi$, the perturbation-theory expansion parameter is really $N\alpha'_{\rm GUT}$ where $N \sim (M'_{\rm GUT}/\mu_0)^\delta$ is the number of Kaluza-Klein states propagating in the loops. Thus, for true perturbativity, we must really require $N\alpha'_{\rm GUT} \ll 4\pi$. However, in Fig. 2, we show the value of $N\alpha'_{\rm GUT}$ that arises in our scenario for different values of radii μ_0, δ, and η. We see that the constraint $N\alpha'_{\rm GUT} \ll 4\pi$ is easily satisfied. Thus, our unification is not only weakly coupled, but also perturbative.
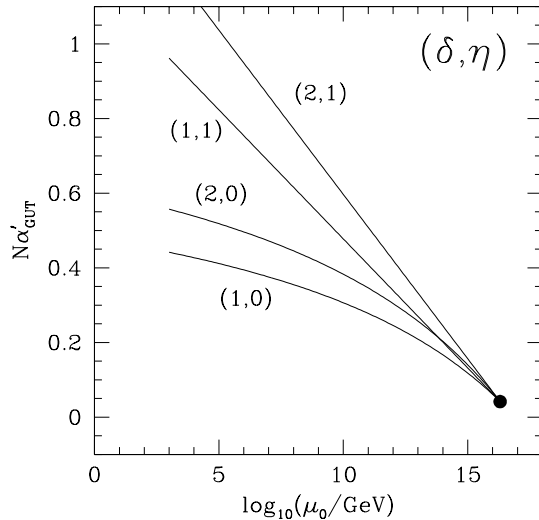

Figure 2: Values of $N\alpha'_{\rm GUT}$ for different values of (δ, η), as a function of $\mu_0 \equiv R^{-1}$. In all cases, the perturbativity constraint $N\alpha'_{\rm GUT} \ll 4\pi$ is easily satisfied.

Finally, let us now discuss issues pertaining to the *precision* of our unification. We have already remarked above that the unification is only approximate because the Kaluza-Klein beta-functions $\tilde{b}_i$ do not have differences whose ratios exactly match those of the MSSM beta-functions. How serious a problem is this?

One immediate observation suggests that this problem is severe. Let us suppose that we run α_1 and α_2 up to their unification point, demand exact unification with α_3 at that point, and then run α_3 back down to the Z-scale. Carrying out this calculation using the *full* result in Eq. (2), we find $\alpha_3(M_Z) \approx 0.174$, which is many standard deviations away from the experimental result. However, in our scenario, it does not take a significant threshold

effect at the unification scale to remedy the situation. For example, it is easy to show that a mere 6% threshold effect at the unification scale can eliminate this discrepancy. Such threshold effects can easily arise from SUSY thresholds, GUT thresholds, MSSM higher-order logarithmic corrections, and holomorphic anomaly contributions. They can also arise in string theory due to the contributions of heavy string states. Of course, these corrections can be calculated only when a complete theory at M'_{GUT} is specified.

However, this then raises a new question. Because of the steepness of the slopes of the curves in Fig. 1 near the unification point, it is natural to worry that our scenario might be unreasonably *sensitive* to unification-scale effects. For example, if we wish to ensure that $\alpha_3(M_Z)$ remains within its experimental uncertainties, how fine-tuned must these threshold effects be? Clearly a fine-tuning factor of 10^{-6} or more would be unacceptable. However, once again, it is straightforward to show that no major fine-tuning is involved. Specifically, in order to quantify any potential fine-tuning parameter that might be involved in our scenario due to the steepness of the curves in Fig. 1, we can consider how much of a shift ΔM in the unification scale can be tolerated if $\alpha_3(M_Z)$ is to remain within acceptable limits. Taking $\Delta\alpha_3(M_Z) \approx 0.003$ as the experimental uncertainty around the central value, we find a corresponding unification-scale sensitivity $\Delta M \approx 0.13$ TeV for $\mu_0 = 1$ TeV. This amounts to a fine-tuning of the order of one part in ten.

Thus, we conclude that our unification scenario is predictive, perturbative, and not unreasonably sensitive to unification-scale effects.

4   New Questions Posed by TeV-Scale GUTs

Given this TeV-scale unification, a number of questions immediately arise. We shall not have space here to discuss these questions in detail, so we shall merely summarize some of the results that can be found in Refs. [3,6].

4.1   Proton Decay

First, if we contemplate the appearance of a TeV-scale GUT, we immediately face the problem of proton-decay mediated by light X and Y bosons and Higgs(ino) triplets. However, these contributions can be cancelled[6] to all orders in perturbation theory as a result of the symmetry properties of the higher dimensions (essentially a parity argument for the fifth dimension). This is then an intrinsically higher-dimensional solution to the proton-decay problem.

4.2 Fermion Masses and Soft Masses

Just like the gauge couplings, the Yukawa couplings and soft masses will now also experience power-law behavior. Can this be used to explain, for example, the fermion mass hierarchy? Of course, extra dimensions are universal, and will not introduce a flavor dependence by themselves. Thus, some flavor dependence still must be introduced (*e.g.*, in the unification-scale theory). But the important point is that this flavor dependence need not be large, because the power-law running due to the extra spacetime dimensions can *amplify* the effects of even small flavor-dependent couplings. This is discussed in more detail in Ref. [6].

4.3 Need for Supersymmetry?

Given that new dimensions could appear at a TeV (thereby eliminating the obvious gauge hierarchy problem), one might also question the need for supersymmetry. Can the gauge couplings also unify at the TeV scale without supersymmetry? It is found that this can indeed occur in some circumstances[6]. This then suggests the possibility of *non*-supersymmetric TeV-scale GUTs.

4.4 Embeddings into String Theory

Given gauge coupling unification in the TeV range, it is natural to consider embedding this scenario into a string theory whose fundamental string scale is also reduced to the TeV scale. For technical reasons, such a string would have to be an open string.[b] The possibility of such TeV-scale open strings is discussed in Refs. [9,4,5,6], and a specific embedding of our scenario into TeV-scale strings is discussed in Ref. [6].

4.5 Next Directions

Clearly, this is only the tip of the iceberg. On the theoretical side, many aspects of physics beyond the Standard Model must now be considered in a new light. These include issues pertaining to supersymmetry, supersymmetry-breaking, GUT physics, and string theory. For each, we must now investigate the role of extra large dimensions, and the effects of the drastically altered energy scales. Even more excitingly, on the experimental side, many striking

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[b] In Ref. [8], it is pointed out that reduced-scale open strings do not necessarily imply power-law running for the gauge couplings. However, our point is the converse: power-law running for the gauge couplings — which is the only way to lower the GUT scale — requires reduced-scale open string theories, and is in fact the generic situation for such strings.

signals (such as the production, detection, decays, and indirect effects of TeV-scale Kaluza-Klein states) would be observable at future colliders. Many of these signals are discussed in Ref. [1]. Thus, it becomes possible to consider probing the properties of GUTs and strings *experimentally*, thereby giving rise to a possible new *experimental* direction for string phenomenology! Finally, because the fundamental energy scales of physics have changed, there will also be profound effects for cosmology.

As we have said, this is only the tip of the iceberg. However, as is evident from our results as well as those of Refs. [2,4], the important point is that extra spacetime dimensions provide a natural way of bringing fundamental physics down to low (and perhaps even accessible) energy scales. This titanic shift in thinking will have many consequences as yet unseen. Let us hope that our ship can explore this vast iceberg without sinking.

Acknowledgments

References

1. I. Antoniadis, *Phys. Lett.* **B246** (1990) 377; I. Antoniadis, C. Muñoz, and M. Quirós, *Nucl. Phys.* **B397** (1993) 515; I. Antoniadis, K. Benakli, and M. Quirós, *Phys. Lett.* **B331** (1994) 313.
2. N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, `hep-ph/9803315`.
3. K.R. Dienes, E. Dudas, and T. Gherghetta, `hep-ph/9803466` (to appear in *Phys. Lett.* **B**).
4. I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, `hep-ph/9804398`.
5. G. Shiu and S.-H.H. Tye, `hep-th/9805157`.
6. K.R. Dienes, E. Dudas, and T. Gherghetta, `hep-ph/9806292`.
7. N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, `hep-ph/9807344`.
8. C. Bachas, `hep-ph/9807415`.
9. J.D. Lykken, *Phys. Rev.* **D54** (1996) 3693.
10. E. Witten, *Nucl. Phys.* **B471** (1996) 135.