Universal Seesaw Mechanism with Universal Strength for Yukawa Couplings

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Abstract

Hypotheses of the universal seesaw mechanism and the *universal strength for Yukawa couplings* are applied to explain one possible origin of quasi-democratic mass matrices of a special type in a left-right symmetric model with the gauge group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)$. Two kinds of Higgs doublets are postulated to mediate scalar interactions between the i-th generation of light fermion doublets and the j-th generation of heavy fermion singlets with relative Yukawa coupling constants of the exponential form $e^{i\phi_{ij}}$, where ϕ_{ij} are real phase constants. The lowest seesaw approximation results effectively in self-adjoint mass matrices which are quasi-democratic and have the same diagonal elements. A set of values for the parameters ϕ_{ij} is found which reproduces the present experimental data for the absolute values of the CKM matrix elements, the Jarlskog parameter and the Wolfenstein parameters.

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1 Introduction

Among various forms of mass matrices [1–6], quasi-democratic mass matrices explain hierarchical structures of quark mass spectra and the CKM weak-mixing matrix in a simple and systematic way [7–21]. In a previous article [21] we investigated the eigenvalue problem of quasi-democratic mass matrices of special type,

$$\mathcal{M}_q = M_q \tilde{\Omega}_q, \qquad (q = u, d) \tag{1}$$

where M_q is a mass scale for the q-sector and $\tilde{\Omega}_q$ is the Hermitian matrix

$$\tilde{\Omega}_q = \frac{1}{3} \begin{pmatrix} 1 & a_3^q\, e^{i\delta_{12}^q} & a_2^q\, e^{-i\delta_{31}^q} \\ a_3^q\, e^{-i\delta_{12}^q} & 1 & a_1^q\, e^{i\delta_{23}^q} \\ a_2^q\, e^{i\delta_{31}^q} & a_1^q\, e^{-i\delta_{23}^q} & 1 \end{pmatrix} \tag{2}$$

with phases satisfying the restriction

$$\delta_{12}^q = 0, \quad \delta_{23}^q = -\delta_{31}^q = \phi_q. \tag{3}$$

Solving the mass eigenvalue problem in the first order perturbation approximation with respect to small deviations around the democratic limit $a_j^q = 1$ and $\phi_q = 0$, we found sum rules for the absolute values of the CKM matrix [22] elements and approximate expressions for the Jarlskog parameter [23] and the Wolfenstein parameters [24], all of which are consistent with the present experimental data. The purpose of this article is to derive effectively the above specific mass matrices from the hypothesis of the *universal strength for Yukawa couplings* (USY) through the universal seesaw mechanism.

Branco, Silva and Rebelo [15] first formulated the hypothesis of the USY which results in pure phase mass matrices of the form

$$\mathcal{M}_q = c_q \left[e^{i\phi_{ij}^q} \right], \qquad (q = u, d) \tag{4}$$

and there followed many similar investigations [16–19]. Fishbane and Hung [19] gave an example of Higgs field interactions that can produce the pure phase mass matrices. Since these pure phase mass matrices are not necessarily Hermitian, it is usual to introduce the dimensionless Hermitian matrices $H_q = \frac{1}{3c_q^2} \mathcal{M}_q \mathcal{M}_q^\dagger$ and solve the eigenvalue problem for H_q. Note that our

quasi-democratic mass matrices $\tilde{\Omega}_q$ have the same form as the Hermitian matrices H_q.

The universal seesaw mechanism (USM) [25–30] was invented to explain the smallness of the charged fermion masses relative to the electroweak scale by postulating the existence of exotic fermions belonging to electroweak singlets. In Ref. [28] we considered the model based on the left-right-symmetric gauge group [31]

$$G \equiv SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_y,$$

where a group $U(1)_y$ is generated by a new charge y. In the model, the chiral charges are assigned to the fermion and Higgs fields to distinguish generations and to restrict the pattern of the Yukawa interactions leading to mass matrices of the extended Fritzsch type [5] in the lowest seesaw approximation.

In this article the universal seesaw mechanism is implemented with the above mentioned left-right symmetric gauge group G [31]. We choose the simplest Higgs structure with a pair of Higgs doublets χ_L and χ_R and postulate that these two doublets interact with the fermions with the universal strength Y of the Yukawa couplings. Namely, the Higgs fields are assumed to mediate scalar interactions between the i-th generation of the light fermion doublets and the j-th generation of the heavy fermion singlets with Yukawa coupling constants of form $Ye^{i\phi_{ij}}$, where the ϕ_{ij} are real phase constants. Note here that the universality of the strength for Yukawa couplings is formulated more stringently than that of Branco et al. [15], since Y is common to the up- and down-quark sectors. The mass m_Q $(Q = U, D)$ is assigned to the up- and down-heavy fermion singlets. Through the breakdown of the symmetry G the Higgs doublets χ_L and χ_R acquire, respectively, the vacuum expectation values w_L and w_R. Under the first seesaw approximation, effective Hermitian mass matrices of the quasi-democratic form in Eqs. (1) and (2) are obtained. To take the first seesaw approximation corresponds just to the product operation $\mathcal{M}_q\mathcal{M}_q^\dagger$, as shown by Branco et al. The mass scale M_q in Eq. (1) is determined by

$$M_q = m_1^q + m_2^q + m_3^q \simeq 9Y^2 \frac{w_L w_R}{m_Q}. \tag{5}$$

Thus the scale difference for the up- and down-quark sectors, M_u and M_d, is reduced to that of heavy quark masses, m_U and m_D. The departure of M_u and M_d from the electroweak scale w_L is explained by the factor $9Y^2 w_R/m_Q$.

Note that all results in the previous article [21] hold here, since the effective mass matrices coincides with the quasi-democratic mass matrix of the specific form in Eqs. (1) and (2). Namely, the present model can inherit all the results obtained there.

This article is organized as follows. The model is formulated in §2. We derive the seesaw mass matrix in §3 and parametrize the effective mass matrices in §4. Exact solutions of the mass eigenvalue problems are given, and their relations with observable quantities are explained in §5. Results of numerical analyses are given in §6, and discussion is given in §7.

2 Model

Fundamental quarks in the model are classified with respect to the underlying gauge group G. The ordinary quarks belonging to the i-th generation ($i = 1, 2, 3$) have the transformation properties

$$q_{iL} = \begin{pmatrix} u^r & u^g & u^b \\ d^r & d^g & d^b \end{pmatrix}_{iL} \sim (3, 2, 1; \frac{1}{3}), \tag{6}$$

$$q_{iR} = \begin{pmatrix} u^r & u^g & u^b \\ d^r & d^g & d^b \end{pmatrix}_{iR} \sim (3, 1, 2; \frac{1}{3}), \tag{7}$$

where the fourth entry in the parentheses is the y charge generating $U(1)_y$ group. To implement USM, electroweak singlets of exotic quarks U_i and D_i, which have chiral projections transforming as

$$U_{ih} = (U^r \quad U^g \quad U^b)_{ih} \sim (3, 1, 1; \frac{4}{3}), \qquad (h = L, R) \tag{8}$$

$$D_{ih} = (D^r \quad D^g \quad D^b)_{ih} \sim (3, 1, 1; -\frac{2}{3}) \qquad (h = L, R) \tag{9}$$

are introduced as the seesaw partners for each generation.

We introduce $SU(2)_h$ ($h = L, R$) doublets of Higgs fields as

$$\chi_L \sim (1, 2, 1; -1), \qquad \chi_R \sim (1, 1, 2; -1), \tag{10}$$

which develop the vacuum expectation values

$$\langle \chi_L \rangle = \begin{pmatrix} w_L \\ 0 \end{pmatrix}, \qquad \langle \chi_R \rangle = \begin{pmatrix} w_R \\ 0 \end{pmatrix} \tag{11}$$

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to break the left-right symmetry and the Weinberg-Salam symmetry. Here, w_L and w_R are assumed to be real.

Our hypothesis of the USY asserts that the Higgs fields mediate scalar interactions between the i-th generation of the light fermion doublets and the j-th generation of the heavy fermion singlets with Yukawa coupling constants of the form $Y e^{i\phi_{ij}^f}$ $(f = u, d)$. The heavy fermion singlets U_{ih} (D_{ih}) are assumed to have a degenerate bare mass m_U (m_D). The most general form of the fermion Lagrangian density $\mathcal{L}_Y$ satisfying these requirements, and being invariant under the fundamental group G and the $L \leftrightarrow R$-symmetry, is given by

$$
\begin{aligned}
\mathcal{L}_Y =\ & Y \sum_{i,j} \{ e^{i\phi_{ij}^u} \bar{q}_{iL} \chi_L U_{jR} + e^{i\phi_{ij}^d} \bar{q}_{iL} \tilde{\chi}_L D_{jR} + e^{i\phi_{ij}^u} \bar{q}_{iR} \chi_R U_{jL} + e^{i\phi_{ij}^d} \bar{q}_{iR} \tilde{\chi}_R U_{jL} \} \\
& - \sum_i \{ m_U \bar{U}_{iL} U_{iR} + m_D \bar{D}_{iL} D_{iR} \} + \text{h.c.},
\end{aligned}
\tag{12}
$$

where $\tilde{\chi} = i\sigma_2 \chi^*$. It is worthwhile to emphasize that Y is the common strength of the Yukawa couplings for the up- and down-quark sectors. The minus sign for the bare mass terms of heavy quarks is introduced to adjust the sign of the effective quark mass matrices M_{eff}^q in §4.

3   Seesaw mass matrices

Spontaneous breakdowns of the underlying symmetry G induce 6×6 seesaw mass matrices for the up- and down-quark sectors. The mass matrices are expressed in seesaw block-matrix form as

$$
(\bar{f}_L \quad \bar{F}_L) \begin{pmatrix} 0 & M_L^f \\ M_R^f & M^F \end{pmatrix} \begin{pmatrix} f_R \\ F_R \end{pmatrix} + \text{h.c.},
\tag{13}
$$

where $f = (f_1, f_2, f_3)^{\mathrm{T}}$ are the column vectors of three ordinary quarks $(f = u, d)$ in the generation space, and $F = (F_1, F_2, F_3)^{\mathrm{T}}$ are the column vectors of exotic quarks $(F = U, D)$. The component mass matrices M_L^f and M_R^f are written

$$
M_L^f = w_L M^f, \quad M_R^f = w_R M^{f\dagger}
\tag{14}
$$

in terms of the pure phase matrix M^f defined by

$$
M^f \equiv Y \begin{pmatrix} e^{i\phi_{11}^f} & e^{i\phi_{12}^f} & e^{i\phi_{13}^f} \\ e^{i\phi_{21}^f} & e^{i\phi_{22}^f} & e^{i\phi_{23}^f} \\ e^{i\phi_{31}^f} & e^{i\phi_{32}^f} & e^{i\phi_{33}^f} \end{pmatrix}.
\tag{15}
$$

The submass matrix M^F has the diagonal structure

$$M^F = - \begin{pmatrix} m_F & 0 & 0 \\ 0 & m_F & 0 \\ 0 & 0 & m_F \end{pmatrix} = -m_F E, \qquad (16)$$

where E is 3×3 unit matrix.

On the assumption that

$$m_Q^2 \gg Y^2 w_R^2 \gg Y^2 w_L^2, \qquad (Q = U, D) \qquad (17)$$

we can apply the first seesaw approximation by ignoring $\mathcal{O}(Y^2 w_{L,R}^2 / m_F^2)$ in comparison with 1. Note that this seesaw condition is the special one induced by the hypothesis of USY. The seesaw mass matrices are block-diagonalized approximately by the bi-unitary transformations as

$$U_L^{f\dagger} \begin{pmatrix} 0 & M_L^f \\ M_R^f & M^F \end{pmatrix} U_R^f \simeq \begin{pmatrix} M_{\text{eff}}^f & 0 \\ 0 & M_{\text{eff}}^F \end{pmatrix} \qquad (18)$$

with

$$U_{L,R}^f = \begin{pmatrix} 1 - \frac{1}{2}\rho_{L,R}^{f\dagger}\rho_{L,R}^f & \rho_{L,R}^{f\dagger} \\ -\rho_{L,R}^f & 1 - \frac{1}{2}\rho_{L,R}^f\rho_{L,R}^{f\dagger} \end{pmatrix}. \qquad (19)$$

Here the component matrices ρ_L^f and ρ_R^f of the bi-unitary transformations are given by

$$\rho_L^{f\dagger} = M_L^f (M^F)^{-1}, \quad \rho_R^f = (M^F)^{-1} M_R^f. \qquad (20)$$

Therefore, the effective mass matrices for the ordinary and exotic quarks, M_{eff}^f and M_{eff}^F, are obtained in the forms

$$M_{\text{eff}}^f \equiv -M_L^f (M^F)^{-1} M_R^f, \quad M_{\text{eff}}^F \equiv M^F. \qquad (21)$$

4   Parametrization of effective mass matrices

Here let us use the suffices $q = u, d$ and $Q = U, D$, since our arguments are restricted to quarks.

Owing to the specific structures of the component matrices M_L^q, M_R^q and M^Q, the effective mass matrix M_{eff}^q for ordinary quarks is expressed by the Hermitian matrix

$$M_{\text{eff}}^q = -M_L^q (M^Q)^{-1} M_R^q = \frac{w_L w_R}{m_Q} M^q M^{q\dagger}. \qquad (22)$$

6

In order to cast the effective mass matrix into the specific form in Eqs. (1) and (2), we parametrize the Hermitian matrix $M^q M^{q\dagger}$ as

$$M^q M^{q\dagger} \equiv 3Y^2 \begin{pmatrix} 1 & a_3^q\, e^{i\delta_{12}^q} & a_2^q\, e^{-i\delta_{31}^q} \\ a_3^q\, e^{-i\delta_{12}^q} & 1 & a_1^q\, e^{i\delta_{23}^q} \\ a_2^q\, e^{i\delta_{31}^q} & a_1^q\, e^{-i\delta_{23}^q} & 1 \end{pmatrix}. \tag{23}$$

Here, a_1^q, a_2^q, a_3^q, δ_{12}^q, δ_{31}^q and δ_{23}^q are real parameters which are expressed in terms of the original phases ϕ_{ij}^q in Eq. (15) as follows:

$$\begin{aligned} a_1^q &= \frac{1}{3}[3 + 2\cos(\phi_{21}^q + \phi_{32}^q - \phi_{22}^q - \phi_{31}^q) + 2\cos(\phi_{21}^q + \phi_{33}^q - \phi_{31}^q - \phi_{23}^q) \\ &\quad + 2\cos(\phi_{32}^q + \phi_{23}^q - \phi_{22}^q - \phi_{33}^q)]^{1/2}, \end{aligned} \tag{24}$$

$$\begin{aligned} a_2^q &= \frac{1}{3}[3 + 2\cos(\phi_{11}^q + \phi_{32}^q - \phi_{12}^q - \phi_{31}^q) + 2\cos(\phi_{11}^q + \phi_{33}^q - \phi_{31}^q - \phi_{13}^q) \\ &\quad + 2\cos(\phi_{32}^q + \phi_{13}^q - \phi_{12}^q - \phi_{33}^q)]^{1/2}, \end{aligned} \tag{25}$$

$$\begin{aligned} a_3^q &= \frac{1}{3}[3 + 2\cos(\phi_{11}^q + \phi_{22}^q - \phi_{12}^q - \phi_{21}^q) + 2\cos(\phi_{11}^q + \phi_{23}^q - \phi_{21}^q - \phi_{13}^q) \\ &\quad + 2\cos(\phi_{22}^q + \phi_{13}^q - \phi_{12}^q - \phi_{23}^q)]^{1/2}, \end{aligned} \tag{26}$$

$$\delta_{12}^q = \tan^{-1}\left[\frac{\sin(\phi_{11}^q - \phi_{21}^q) + \sin(\phi_{12}^q - \phi_{22}^q) + \sin(\phi_{13}^q - \phi_{23}^q)}{\cos(\phi_{11}^q - \phi_{21}^q) + \cos(\phi_{12}^q - \phi_{22}^q) + \cos(\phi_{13}^q - \phi_{23}^q)}\right], \tag{27}$$

$$-\delta_{31}^q = \tan^{-1}\left[\frac{\sin(\phi_{11}^q - \phi_{31}^q) + \sin(\phi_{12}^q - \phi_{32}^q) + \sin(\phi_{13}^q - \phi_{33}^q)}{\cos(\phi_{11}^q - \phi_{31}^q) + \cos(\phi_{12}^q - \phi_{32}^q) + \cos(\phi_{13}^q - \phi_{33}^q)}\right], \tag{28}$$

and

$$\delta_{23}^q = \tan^{-1}\left[\frac{\sin(\phi_{21}^q - \phi_{31}^q) + \sin(\phi_{22}^q - \phi_{32}^q) + \sin(\phi_{23}^q - \phi_{33}^q)}{\cos(\phi_{21}^q - \phi_{31}^q) + \cos(\phi_{22}^q - \phi_{32}^q) + \cos(\phi_{23}^q - \phi_{33}^q)}\right]. \tag{29}$$

Note that, if the phases are redefined by making the replacement $\phi_{ij}^q - \phi_{jj}^q \Rightarrow \phi_{ij}^q$, it is possible to eliminate the terms ϕ_{ii} from all these expressions in Eqs. (24) $\sim$ (29). Therefore, without loss of generality, we can set $\phi_{11}^q = \phi_{22}^q = \phi_{33}^q = 0$.

5 Mass eigenvalue problems and observable quantities

In the manner described above the first seesaw approximation leads to the effective mass matrices

$$M_{\text{eff}}^q = M_q \tilde{\Omega}_q, \tag{30}$$

where $\tilde{\Omega}_q$ are the Hermitian matrices $\tilde{\Omega}_q$ in Eq. (2), and the mass scales M_q are fixed as

$$M_q = 9Y^2 \frac{w_L w_R}{m_Q}. \tag{31}$$

To solve the eigenvalue problems for effective mass matrices, it is convenient to simplify $\tilde{\Omega}_q$ further by the unitary (phase) transformation as

$$\tilde{\Omega}_q' \equiv \mathcal{P}_q^\dagger \tilde{\Omega}_q \mathcal{P}_q = \frac{1}{3} \begin{pmatrix} 1 & a_3^q & a_2^q \\ a_3^q & 1 & a_1^q e^{i\Delta_q} \\ a_2^q & a_1^q e^{-i\Delta_q} & 1 \end{pmatrix}, \tag{32}$$

with the unitary matrix given by

$$\mathcal{P}_q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\delta_{12}^q} & 0 \\ 0 & 0 & e^{i\delta_{31}^q} \end{pmatrix}, \tag{33}$$

where $\Delta_q = \delta_{12}^q + \delta_{23}^q + \delta_{31}^q$.

The eigenvalue problems

$$\tilde{\Omega}_q' \tilde{\boldsymbol{v}}_j^q = \tilde{\omega}_j^q \tilde{\boldsymbol{v}}_j^q \tag{34}$$

are solved with the eigenvectors $\tilde{\boldsymbol{v}}_j^q$ in the form

$$\tilde{\boldsymbol{v}}_j^q = N_j^q \begin{pmatrix} e^{i\Delta_q} a_1^q a_3^q + (3\tilde{\omega}_j^q - 1)a_2^q \\ e^{i\Delta_q}(3\tilde{\omega}_j^q - 1)a_1^q + a_2^q a_3^q \\ (3\tilde{\omega}_j^q - 1)^2 - (a_3^q)^2 \end{pmatrix}, \tag{35}$$

where N_j^q are the normalization constants given by

$$\begin{aligned} |N_j^q|^{-2} &= 3(3\tilde{\omega}_j^q - 1)^4 - [4(a_3^q)^2 + (a_2^q)^2 + (a_1^q)^2](3\tilde{\omega}_j^q - 1)^2 \\ &\quad + (a_3^q)^2[(a_3^q)^2 + (a_2^q)^2 + (a_1^q)^2]. \end{aligned} \tag{36}$$

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Accordingly, the Hermitian matrix $\tilde{\Omega}_q$ is diagonalized as

$$U_q^\dagger \mathcal{P}_q^\dagger \tilde{\Omega}_q \mathcal{P}_q U_q = U_q^\dagger \tilde{\Omega}_q' U_q = \begin{pmatrix} \tilde{\omega}_1^q & 0 & 0 \\ 0 & \tilde{\omega}_2^q & 0 \\ 0 & 0 & \tilde{\omega}_3^q \end{pmatrix}, \qquad (37)$$

where

$$U_q = (\, \tilde{\boldsymbol{v}}_1^q \quad \tilde{\boldsymbol{v}}_2^q \quad \tilde{\boldsymbol{v}}_3^q \,). \qquad (38)$$

The eigenvalues $\tilde{\omega}_j^q$ satisfy the following relations:

$$\begin{cases} \tilde{\omega}_1^q \tilde{\omega}_2^q \tilde{\omega}_3^q = \dfrac{1}{27} \left[1 - (a_3^q)^2 - (a_2^q)^2 - (a_1^q)^2 + 2a_3^q a_2^q a_1^q \cos \Delta_q \right], \\ \tilde{\omega}_1^q \tilde{\omega}_2^q + \tilde{\omega}_2^q \tilde{\omega}_3^q + \tilde{\omega}_3^q \tilde{\omega}_1^q = \dfrac{1}{9} \left[3 - (a_3^q)^2 - (a_2^q)^2 - (a_1^q)^2 \right], \\ \tilde{\omega}_1^q + \tilde{\omega}_2^q + \tilde{\omega}_3^q = 1. \end{cases} \qquad (39)$$

The masses m_j^q of ordinary quarks and the eigenvalues $\tilde{\omega}_j^q$ of the Hermitian matrix $\tilde{\Omega}_q$ are related by

$$m_j^q = M_q \tilde{\omega}_j^q, \qquad \tilde{\omega}_j^q = \frac{m_j^q}{m_1^q + m_2^q + m_3^q}. \qquad (40)$$

Eliminating the eigenvalues $\tilde{\omega}_j^q$ in Eqs. (31) and (40), we obtain

$$M_q = m_1^q + m_2^q + m_3^q = 9Y^2 \frac{w_L w_R}{m_Q}, \qquad \frac{m_1^u + m_2^u + m_3^u}{m_1^d + m_2^d + m_3^d} = \frac{m_D}{m_U}. \qquad (41)$$

Therefore, the different mass scales in the up- and down-quark sectors come from the difference of bare masses of the exotic quarks U and D. The gaps of the mass scales M_u and M_d from the electroweak scale w_L are suppressed by the seesaw factor $9Y^2 w_R/m_Q$. It has been shown [29, 30] that the seesaw approximation for the up-quark sector does not work effectively in the usual seesaw model. However, owing to the factor 9 in Eq. (31), which stems from the USY, the seesaw approximation is justified in the present model.

The gauge eigenstates f_L and F_L and the mass eigenstates $f_L^{(M)}$ and $F_L^{(M)}$ are related by

$$\begin{pmatrix} f_L^{(M)} \\ F_L^{(M)} \end{pmatrix} = \begin{pmatrix} U_f^\dagger & 0 \\ 0 & E \end{pmatrix} \begin{pmatrix} \mathcal{P}_f^\dagger & 0 \\ 0 & E \end{pmatrix} U_L^{f\dagger} \begin{pmatrix} f_L \\ F_L \end{pmatrix}, \qquad (42)$$

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where E is the 3×3 unit matrix, and $(f, F) = (u, U)$ and (d, D). In the first seesaw approximation, the mass eigenstate $f_L^{(M)}$ of left-handed ordinary quarks is given by

$$f_L^{(M)} \simeq U_f^\dagger \mathcal{P}_f^\dagger (f_L - \rho_L^{f\dagger} F_L) \tag{43}$$

in terms of the gauge eigenstates. Therefore the CKM matrix V [22] is constructed to be

$$V = U_u^\dagger \mathcal{P}_u^\dagger \mathcal{P}_d U_d. \tag{44}$$

In the case where $\Delta_u = \Delta_d = 0$, the eigenvalues $\tilde{\omega}_j^q$ and the eigenvectors $\tilde{\boldsymbol{v}}_j^q$ of matrices $\tilde{\Omega}_q$ lose dependence on δ_{ij}^q. Therefore, in such a case, the CKM matrix V depends only on the difference $\phi = \phi_u - \phi_d$, and it is allowed to impose the condition $\phi_d = 0$.

To realize the Wolfenstein parametrization [24] of the CKM matrix, which is convenient to determine the shape of the unitarity triangle, we carry out the phase transformation so that $V_{11}, V_{21}, V_{12}, V_{32}$ and V_{33} are real ($V_{11}, V_{12}, V_{33} > 0$ and $V_{21}, V_{32} < 0$). As a result, we can obtain the Wolfenstein parameters ρ and η as

$$\rho \simeq -\frac{\Re(V_{12}^* V_{32} V_{33}^* V_{13})}{|V_{12}||V_{21}||V_{23}||V_{32}|}, \qquad \eta \simeq \frac{J}{|V_{12}||V_{21}||V_{23}||V_{32}|}, \tag{45}$$

where J is the rephasing invariant Jarlskog parameter [23]:

$$J = \Im(V_{23} V_{12} V_{22}^* V_{13}^*). \tag{46}$$

In the next section, we report on the numerical analysis of observable quantities using the solutions in Eqs. (35), (36) and (39) for the mass eigenvalue problems. To extract physically meaningful analytical relations, these solutions are still too complicated. For this purpose it is therefore reasonable to use approximate solutions, as was done in the previous article [21]. Here it is worthwhile to emphasize that we can use all the results obtained there on the quark mass differences, the sum rules among the absolute values of the CKM matrix, the analytic expressions for the Jarlskog and Wolfenstein parameters in the situation considered presently.

6 Numerical analysis

The world averages of the absolute values of the CKM matrix elements are estimated by the particle data group [32] as follows:

$$
\begin{pmatrix}
0.9745 \sim 0.9760 & 0.217 \sim 0.224 & 0.0018 \sim 0.0045 \\
0.217 \sim 0.224 & 0.9737 \sim 0.9753 & 0.036 \sim 0.042 \\
0.004 \sim 0.013 & 0.035 \sim 0.042 & 0.9991 \sim 0.9994
\end{pmatrix}.
\tag{47}
$$

Because the observed CKM matrix elements are given at m_Z, it is necessary to know the values of running quark masses at m_Z. At the level of the 2-loop renormalization group, Fusaoka and Koide [33] obtained the values of running quark masses at m_Z as

$$
m_u = 2.33^{+0.42}_{-0.45}\,\mathrm{MeV}, \quad m_c = 677^{+56}_{-61}\,\mathrm{MeV}, \quad m_t = 181 \pm 13\,\mathrm{GeV},
$$

$$
m_d = 4.69^{+0.60}_{-0.66}\,\mathrm{MeV}, \quad m_s = 93.4^{+11.8}_{-13.0}\,\mathrm{MeV}, \quad m_b = 3.00 \pm 0.11\,\mathrm{GeV}.
\tag{48}
$$

Using the solutions in Eqs. (35), (36) and (39) for the mass eigenvalue problems, we proceeded with the numerical analysis and found a set of parameters which reproduces the experimental values of both the quark masses and the absolute values of CKM matrix elements. A typical solution obtained is as follows:

$$
\begin{cases}
\phi^u_{11} = 0, & \phi^u_{12} = 0.01455, & \phi^u_{13} = 0.07961, \\
\phi^u_{21} = 0.002182, & \phi^u_{22} = 0, & \phi^u_{23} = 0.09198, \\
\phi^u_{31} = 0.07509, & \phi^u_{32} = -0.2360, & \phi^u_{33} = 0
\end{cases}
\tag{49}
$$

and

$$
\begin{cases}
\phi^d_{11} = 0, & \phi^d_{12} = -0.1697, & \phi^d_{13} = 0.1970, \\
\phi^d_{21} = -0.06880, & \phi^d_{22} = 0, & \phi^d_{23} = 0.09669, \\
\phi^d_{31} = 0.4753, & \phi^d_{32} = -0.4347, & \phi^d_{33} = 0.
\end{cases}
\tag{50}
$$

Substituting these values into Eqs. (24) $\sim$ (29), the values of the quasi-democratic mass matrix elements are estimated to be

$$
\begin{cases}
a^u_3 = 0.9999, & \delta^u_{12} = 0, \\
a^u_2 = 0.9912, & \delta^u_{31} = -0.085, \\
a^u_1 = 0.9921, & \delta^u_{23} = 0.085,
\end{cases}
\tag{51}
$$

and

$$\begin{cases} a_3^d = 0.9927, & \delta_{12}^d = 0, \\ a_2^d = 0.9450, & \delta_{31}^d = 0, \\ a_1^d = 0.9193, & \delta_{23}^d = 0. \end{cases} \tag{52}$$

With this set of parameters, we obtain the estimates

$$|V| = \begin{pmatrix} 0.9753 & 0.2209 & 0.00357 \\ 0.2207 & 0.9745 & 0.04087 \\ 0.00869 & 0.04009 & 0.9992 \end{pmatrix} \tag{53}$$

for the absolute values of the CKM matrix elements,

$$\rho = 0.1213, \qquad \eta = 0.3765 \tag{54}$$

for the Wolfenstein parameters, and

$$J = 3.008 \times 10^{-5} \tag{55}$$

for the Jarlskog parameter.

Provided that $M_u = 1.817 \times 10^2$ GeV and $M_d = 3.098$ GeV, the values of the running quark masses at m_Z are obtained as

$$m_u = 2.33 \text{ MeV}, \quad m_c = 6.77 \times 10^2 \text{ MeV}, \quad m_t = 1.81 \times 10^2 \text{ GeV},$$

$$m_d = 4.69 \text{ MeV}, \quad m_s = 9.34 \times 10 \text{ MeV}, \quad m_b = 3.00 \text{ GeV}, \tag{56}$$

using Eq. (39). All these results are consistent with experimental results [32–34].

7   Discussion

We have formulated the universal seesaw mechanism with the universal strength for Yukawa couplings in the left-right symmetric gauge group G, obtaining effectively the quasi-democratic mass matrices of specific type. The left-right symmetric pairs of Higgs fields induce interactions between the i-th generation of the light fermion doublets and the j-th generation of the heavy fermion singlets with the coupling constants $Y e^{i\phi_{ij}^q}$. The universality of the strength

for Yukawa couplings is taken strictly here in the sense that Y is common to the up- and down-quark sectors in a left-right symmetric manner.

The scale difference for the up- and down-quark sectors, M_u and M_d, is ascribed to that of the heavy quark masses, m_U and m_D, as

$$M_u \; : \; M_d \;\; \simeq \;\; \frac{1}{m_U} \; : \; \frac{1}{m_D}. \tag{57}$$

The departure of M_u and M_d from the electroweak scale w_L is explained by the seesaw factor $9Y^2 w_R/m_Q$. It is the relative phases ϕ_{ij}^q of the Yukawa couplings that explain generational variations of masses in each quark sector. We found a set of values for the phase parameters ϕ_{ij}^q which reproduces the experimental values for the quark masses, the absolute values of CKM matrix elements, the Jarlskog parameter and the Wolfenstein parameters.

Using Eq. (41), we obtain

$$9Y^2 \frac{w_R}{m_U} \simeq \frac{m_t}{w_L} \sim 1, \qquad 9Y^2 \frac{w_R}{m_D} \simeq \frac{m_b}{w_L} \sim 0.02, \tag{58}$$

from which the ratio of the bare masses of exotic quarks is estimated as

$$\frac{m_D}{m_U} \sim 60. \tag{59}$$

Equations (17) and (58) impose the condition $81Y^2 \gg 1$ on the strength Y of the Yukawa coupling constants for the first seesaw approximation to hold. This implies that for Yukawa coupling constants with strength Y of the order, say, approximately $\frac{1}{2}$, the universal seesaw approximation is safely applicable to all the sectors in our model.

In Eqs. (49) $\sim$ (52) we have given explicitly a set of parameters which reproduces the experimental data. The values of the parameters in Eqs. (51) and (52) display systematic departures from the democratic limit $a_i^q = 1$ and $\delta_{ij}^q = 0$. It is not possible to find, however, any order in Eqs. (49) and (50) for the values of the phase parameters ϕ_{ij}^q. Therefore, as far as the present set of parameters is concerned, it is difficult to conclude that there is any indication of order or symmetry hidden in the relative phase couplings. Further numerical analysis must be done to look for all the possible sets of parameters which can reproduce the present experimental results.

In this article the arguments were restricted to the quark sector. The present formalism will be applied to the lepton sector, and the puzzles of solar and atomspheric neutrinos will be analized in a future publication.

References

[1] H. Fritzsch, Phys. Lett. **73B** (1978), 317; **85B** (1979), 81; Nucl. Phys. **B155** (1979), 189.
H. Fritzsch and Zhi-zhong Xing, Phys. Lett. **B353** (1995), 114.

[2] B. Stech, Phys. Lett. **130B** (1983), 189.

[3] M. Shin, Phys. Lett. **145B** (1984), 285.

[4] M. Gronau, R. Johnson and J. Schechter, Phys. Rev. Lett. **54** (1985), 2176.

[5] S. N. Gupta and J. M. Johnson, Phys. Rev. **D44** (1991), 2110.

[6] G. C. Branco, L. Lavoura and F. Mota, Phys. Rev. **D39** (1989), 3443.
G. C. Branco and J. I. Silva-Marcos, Phys. Lett. **B331** (1994), 390.
Y. Koide, Mod. Phys. Lett. **A12** (1997), 2655.

[7] H. Harari, H. Haut and J. Weyers, Phys. Lett. **78B** (1978), 459.

[8] T. Goldman and G. J. Stephenson, Jr., Phys. Rev. **D24** (1981), 236.

[9] Y. Koide, Phys. Rev. Lett. **47** (1981), 1241; Phys. Rev. **D28** (1983), 252; **D39** (1989), 1391.

[10] P. Kaus and S. Meshkov, Mod. Phys. Lett. **A3** (1988), 1251; Phys. Rev. **D42** (1990), 1863.

[11] L. Lavoura, Phys. Lett. **B228** (1989), 245.

[12] M. Tanimoto, Phys. Rev. **D41** (1990), 1586.

[13] H. Fritzsch and J. Plankl, Phys. Lett. **B237** (1990), 451.

[14] Y. Nambu, in *Proceedings of the International Workshop on Electroweak Symmetry Breaking*, Hiroshima (World Scientific, Singapore, 1992), p.1.

[15] G. C. Branco, J. I. Silva-Marcos and M. N. Rebelo, Phys. Lett. **B237** (1990), 446.

[16] G. C. Branco and J. I. Silva-Marcos, Phys. Lett. **B359** (1995), 166.
G. C. Branco, D. Emmanuel-Costa and J. I. Silva-Marcos, Phys. Rev. **D56** (1997), 107.

[17] J. Kalinowski and M. Olechowski, Phys. Lett. **B251** (1990), 584.

[18] P. M. Fishbane and P. Kaus, Phys. Rev. **D49** (1994), 3612; 4780; Z. Phys. **C75** (1997), 1.

[19] P. M. Fishbane and P. Q. Hung, Mod. Phys. Lett. **A12** (1997), 1737; Phys. Rev. **D57** (1998), 2743.

[20] T. Teshima and T. Sakai, Prog. Theor. Phys. **97** (1997), 653.

[21] I. S. Sogami, K. Nishida, H. Tanaka and T. Shinohara, Prog. Theor. Phys. **99** (1998), 281.

[22] N. Cabibbo, Phys. Rev. Lett. **10** (1963), 531.
M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49** (1973), 652.

[23] C. Jarlskog, Phys. Rev. Lett. **55** (1985), 1039; Z. Phys. **C29** (1985), 491.
I. Dunietz, O. W. Greenberg and Dan-di Wu, Phys. Rev. Lett. **55** (1985), 2935.

[24] L. Wolfenstein, Phys. Rev. Lett. **51** (1983),1945.

[25] Z. G. Berezhiani, Phys. Lett. **129B** (1983), 99; **150B** (1985), 177; Yadern. Fiz. **42** (1985), 1309.
D. Chang and R. N. Mohapatra, Phys. Rev. Lett. **58** (1987), 1600.
S. Rajpoot, Phys. Lett. **B191** (1987), 122; Phys. Rev. **D36** (1987), 1479.
A. Davidson and K. C. Wali, Phys. Rev. Lett. **59** (1987), 393; **60** (1988), 1813.
A. Davidson, S. Ranfone and K. C. Wali, Phys. Rev. **D41** (1990), 208.
S. Ranfone, Phys. Rev. **D42** (1990), 3819.

[26] I. S. Sogami, Prog. Theor. Phys. **85** (1991), 141.
I. S. Sogami and T. Shinohara, Prog. Theor. Phys. **86** (1991), 1031.
T. Shinohara, Prog. Theor. Phys. **90** (1993), 1057.

[27] I. S. Sogami, T. Shinohara and Y. Egawa, Prog. Theor. Phys. **87** (1992), 1005.

[28] I. S. Sogami and T. Shinohara, Phys. Rev. **D47** (1993), 2905.

[29] Y. Koide and H. Fusaoka, Z. Phys. **C71** (1996), 459.

[30] Y. Koide and M. Tanimoto, Z. Phys. **C72** (1996), 333.
Y. Koide and H. Fusaoka, Prog. Theor. Phys. **97** (1997), 459.
Y. Koide, Phys. Rev. **D56** (1997), 2656.
T. Morozumi, T. Satou, M. N. Rebelo and M. Tanimoto, Phys. Lett. **B410** (1997), 233.

[31] G. Senjanovic and R. N. Mohapatra, Phys. Rev. **D12** (1975), 1502.
G. Senjanović, Nucl. Phys. **B153** (1979), 334.
R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44** (1980), 912;
Phys. Rev. **D23** (1981), 165.

[32] Particle Data Group, C. Caso et al., Eur. Phys. J. **C3** (1998), 1.

[33] H. Fusaoka and Y. Koide, Phys. Rev. **D57** (1998), 3986.

[34] M. Neubert, Int. J. Mod. Phys. **A11** (1996), 4173.