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Experimental Insertions made of two symmetric triplets

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1 INTRODUCTION

This analysis relies on an analytical treatment of a schematic thin lens model of generic insertions based on two symmetric triplets separated by a drift. It allows to explore their capabilities and provides, without using a numerical search, the existing solutions which can easily be extended to the equivalent thick lens model. Such generic insertions have the general property to transfer the beam from a double waist or focus, with in general different horizontal and vertical beam dimensions, to a given point of a FODO lattice. The clue consists in asking the inner triplet to provide a condition of β-crossing and using the outer triplet as a "FODO transformer" from β-crossing to β-crossing with different amplitudes. This two-step adjustment provides the required flexibility in the choice of the overall insertion length and in the distance between the two triplets. The FODO transformer is a useful by-product which can be profitable in other situations.

A particular insertion with geometrical parameters close to those of an LHC [1] experimental insertion is used to illustrate how to apply the analysis. The domain of existence of solutions is given and the tunability of the insertion in terms of β-amplitude at the Interaction Point is explored, for the particular conditions retained.

Each symmetric (in its geometry) triplet is the combination of a doublet and its mirror image. The doublet consists of a first drift space of length L_1 followed by a quadrupole of magnetic gradient G_2 and length l_q followed by a second drift space of length L_3 which separates it from a second quadrupole of magnetic gradient G_4 and length l_q. In thin lens model, the drifts are replaced by

$$l_1 = L_1 + l_q/2$$
$$l_3 = L_3 + 3\, l_q/2$$

and the gradient by the normalised integrated strengths g_2 and g_4. The total number of parameters per triplet is four and the strength ratio $k = g_4/g_2$ is used preferably to g_4 itself (for $k = -1$, the triplet is 'canonical'). Fig. 1 shows the two thin-lens triplets using this notation with the indices i and o for the inner and outer triplet, respectively. The length of the drift between the inner triplet exit (ITE) and the outer triplet entrance (OTE) is noted l_5.

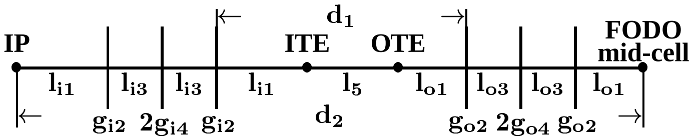

Figure 1: Two triplets model

2 THE INNER TRIPLET

The study of the inner triplet makes use of the expressions [2] derived to find the domain of the free parameter-values providing the Twiss functions required at OTE according to a specific criterion. The criterion retained is to obtain a symmetric β-crossing

$$\beta_x = \beta_y = \beta_o \quad , \quad \alpha_x = -\alpha_y = \alpha_o \qquad (1)$$

This choice allows for an easy matching to the outer symmetric triplet and is suggested on the analogy of the FODO structure we eventually want to connect with.

In an insertion, the distance l_{i1} is fixed and only three free parameters remain, i.e. l_{i3}, g_{i2} and k to specify the triplet. The study of the behaviour of the Twiss quantities at OTE (index o) as functions of these parameters requires the expressions of l_5, α_o and β_o in terms of the Twiss quantities at ITE (index i). Using the usual equations of transfer of the betatron functions and observing that a drift does not modify $\gamma = (1+\alpha^2)/\beta$, we get

$$l_5 = \frac{\alpha_{x,i} + \alpha_{y,i}}{2\gamma_i} \quad , \quad \alpha_o = \frac{\alpha_{x,i} - \alpha_{y,i}}{2}$$
$$\beta_o = \frac{1}{2}[\beta_{x,i} + \beta_{y,i} - l_5(\alpha_{x,i} + \alpha_{y,i})] \qquad (2)$$

The preservation of γ in a drift and the equality $\gamma_{x,o} = \gamma_{y,o}$ coming from (1) impose a constraint on the initial free parameters which are reduced to two. Writing it explicitely and using the expressions of the transfer matrices in terms of the triplet parameters give the following equation for the strength ratio k:

$$k^2(a_2 g_{i,2}^4 + b_2 g_{i,2}^2) + k(a_1 g_{i,2}^4 + b_1 g_{i,2}^2 + c_1) + b_0 g_{i,2}^2 + c_0 = 0 \quad (3)$$

where a_2,b_2,a_1,b_1,c_1,b_0,c_0 are functions of $l_{i,1}$,$l_{i,3}$ and β^* [3]. Both solutions of (3) when they exist are negative as expected and $-k$ is generally smaller than 1 (canonical triplets do not satisfy all the constraints, in particular an additional one on β_{max}). In the following, they are referred as the smallest or largest root in real value.

A program has been written to explore the values taken by k, $l_c = l_5 + 2(l_{i,1} + l_{i,3})$, β_o and α_o when varying $l_{i,3}$ and g_2 respectively.

The results show that the largest solution of the second order equation (3) is to be avoided, because it is associated with very high values of the α- and β-functions at OTE and the corresponding distance l_c from the IP is very sensitive to small changes of the gradient $g_{i,2}$.

3 THE OUTER TRIPLET

While the inner triplet (Section 2) transforms the double waist at the IP into a symmetric $\beta-$ crossing at OTE, the outer triplet is designed to match the β-functions from this symmetric crossing to the following symmetric crossing corresponding to the FODO mid-cell Point. For this precise reason, we can call it a “FODO transformer” which can be useful as a regular lattice adapter in all cases where a step-variation of the β-functions are required.

At both the entrance and the exit of the outer triplet, we assume by definition that

$$\begin{aligned} \beta_{x,1} = \beta_{y,1} = \beta_1 \quad &, \quad \alpha_{x,1} = -\alpha_{y,1} = \alpha_1 \\ \beta_{x,2} = \beta_{y,2} = \beta_2 \quad &, \quad \alpha_{x,2} = -\alpha_{y,2} = \alpha_2 \end{aligned} \quad (4)$$

and that in addition they satisfy the inequality $\alpha_1\beta_2 + \alpha_2\beta_1 \neq 0$. Using then the analysis of [3], based on the expressions of the transfer matrix coefficients as functions of the Twiss parameters at the entrance and exit, it is possible to rigorously demonstrate that there are two and only two symmetric triplets which match the betatron functions according to eq. (4). This assumes the absence of constraints on the phase advances, but includes the following conditions that are required for solutions to exist:

$$\begin{aligned} \beta_2 \neq \beta_1 \quad &, \quad \alpha_2 \neq -\alpha_1 \\ \gamma_2 \neq \gamma_1 \quad &, \quad \alpha_1\beta_2 + \alpha_2\beta_1 \neq 0 \end{aligned}$$

The parameters of the two corresponding symmetric triplets can be explicitely written in terms of α_1, α_2, γ_1, and γ_2. The first solution is:

$$\begin{aligned} l_1 &= \left| \frac{(\alpha_1+\alpha_2)z_{r,1} + \alpha_0}{\gamma_1 - \gamma_2} \right| \\ g_2 &= \frac{\gamma_1-\gamma_2}{(\alpha_1+\alpha_2)} \frac{1}{1 - z_{r,1}^2} \\ l_3 &= 2l_1/z_{r,1}^2 \\ g_4 &= \frac{(\alpha_1+\alpha_2)z_{r,1}^3 g_2}{2\alpha_0} \end{aligned}$$

where $z_{r,1}$ is the (only) real solution of the cubic equation $z^3 + z + 2\alpha_0/(\alpha_1+\alpha_2) = 0$, solution which can be explicitely written ([3], App.B) in terms of α_1, α_2 and α_0 defined hereafter

$$\alpha_0 = \frac{\sqrt{(\beta_2-\beta_1)^2 + (\alpha_1\beta_2+\alpha_2\beta_1)^2}}{\sqrt{\beta_1\beta_2}}.$$

The second solution is:

$$\begin{aligned} f &= 1/g_2 = \frac{\alpha_1+\alpha_2}{\gamma_1-\gamma_2} \\ l_1 &= |f/(\alpha_1+\alpha_2)|\sqrt{\alpha_0(\alpha_0 + |\alpha_1+\alpha_2|)} \\ l_3 &= |f|\sqrt{1 + |\alpha_1+\alpha_2|/\alpha_0} \\ g_4 &= -g_2/2 \end{aligned}$$

The final choice between the two solutions depends on the specific requirements of the generic insertion. As an example the selection criterion in the LHC application was to minimise the β_x and β_y maximum amplitudes within the triplet.

4 FULL INSERTION FROM DOUBLE WAIST TO FODO LATTICE

Let us now consider the full insertion shown in Figure 1 which is composed of two symmetric triplets (inner and outer) separated by a drift l_5. The inner triplet has the role studied in the previous section, transforming the double waist at the IP into a symmetric $\beta-$ crossing at OTE. The outer triplet matches from this symmetric $\beta-$ crossing to the FODO mid-cell Point. It was proved (Section 3) that there are always two solutions if we do not impose any condition on the phase advances. Thus their parameters are also functions of l_3 and g_2. The whole insertion depends on these two free variables which can be determined by adding two supplementary conditions. In the following development we have chosen to impose the values for the distance d_1 between the inner and the outer triplets and the overall insertion length d_2 (Fig. 1):

$$\begin{aligned} d_1 &= l_{i1} + l_5 + l_{o1} \\ d_2 &= 2(l_{i1} + l_{i3} + l_{o1} + l_{o3}) + l_5 \end{aligned}$$

because they are generally fixed by the geometry (even if more loosely for d_1).

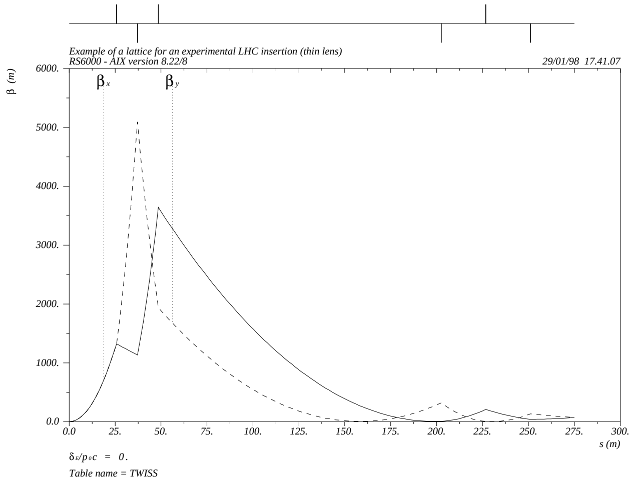

Figure 2: Horizontal and vertical β-functions of an insertion matching the IP to the Mid-cell Point with $\beta = 72$ m and $\alpha = -1$ (thin lens approximation)

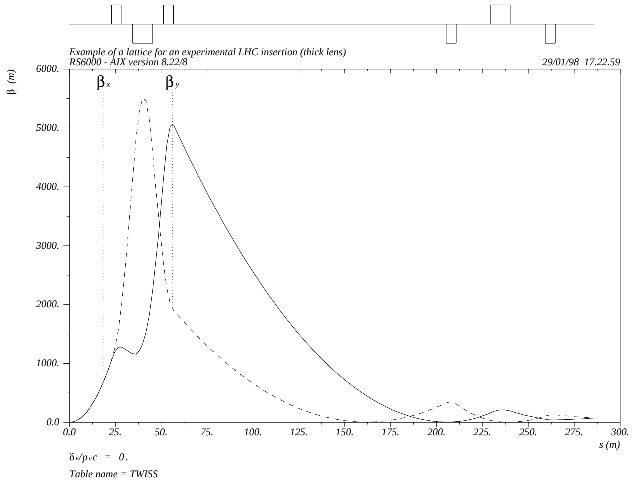

Figure 3: Horizontal and vertical β-functions of an insertion matching the IP to the Mid-cell Point with $\beta = 72$ m and $\alpha = -1$ (thick lens model)

A simple code allows to compute the parameters of the two triplets and the drift length l_5 which satisfy these constraints. A further condition is given by the maximum β value acceptable in the inner triplet, satisfied if the exact value of d_1 is replaced by a range. The figures 2 and 3 show an application to a schematic configuration of one LHC (version 4) low-β insertion [1]. The distance between the IP and the Mid-cell Point of the FODO cell is 275 m while the distance between the last quadrupole of the inner triplet and the first quadrupole of the outer triplet is 154 m in the thin lens approximation. The free distance L_1 is fixed to 23 m and L_3 to 2 m. The quadrupole length l_q is equal to 5.5 m; their gradients should not exceed 225 T/m. The quadrupole separation must be larger than 2 m. The values of the β- and α-function at the IP are 0.5 m and 0 (in both planes, at 7 TeV) and taken equal to 72 m and -1 respectively at the Mid-cell Point (with $\beta_{max} \leq 5500\,m$).

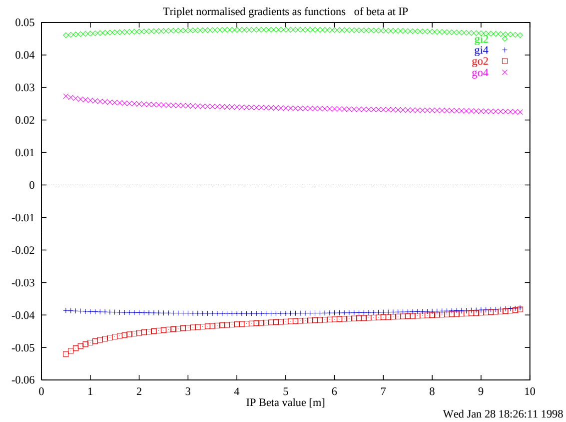

Figure 4: Normalised gradients of the two triplets as function of the β-function value at the IP ($\beta^* = 0.5 - 9.8m$)

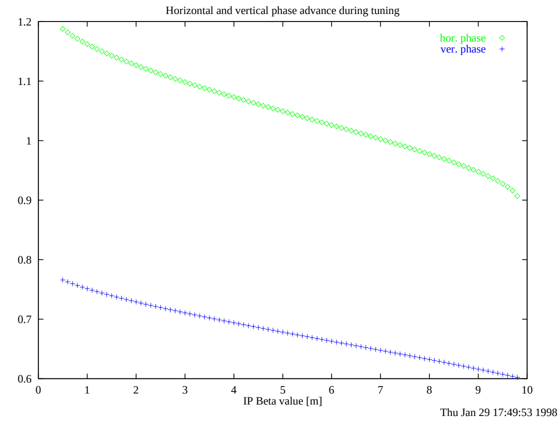

Figure 5: Horizontal and vertical phase advances as function of the β-function value at the IP ($\beta^* = 0.5 - 9.8m$)

Fig. 2 shows the thin lens approximation and Fig. 3 the extension to the thick lens case. A specific program was written to study the behaviour of the quadrupole strengths for an increasing β-amplitude at the IP ("tuning"). Fig. 4 shows that their change is quite smooth for β-values at IP up to near 10 m and Fig. 5 gives the corresponding variation of the horizontal and vertical phase advances. Hence, it is demonstrated that the proposed method allows to determine unequivocally inside the parameter space the existing lattice solutions for a tunable LHC-type experimental insertion, only based on symmetric but not canonical triplets.

5 REFERENCES

[1] LHC Conceptual Design, CERN/AC/95-05 (LHC), 1995.

[2] T.E. d'Amico, 'General Treatment of a matching quadrupole triplet which is symmetric around its median plane', CLIC Note 322, 1997.

[3] T.E. d'Amico, G. Guignard, 'Analysis of generic insertions made of two symmetric triplets', CERN-SL-98-014, 1998.