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Single and Double Universal Seesaw Mechanisms with Universal Strength for Yukawa Couplings

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Abstract

Single and double universal seesaw mechanisms and the hypothesis of *universal strength for Yukawa couplings* are applied to formulate a unified theory of fermion mass spectrum in a model based on an extended Pati-Salam symmetry. Five kinds of Higgs fields are postulated to mediate scalar interactions among electroweak doublets of light fermions and electroweak singlets of heavy exotic fermions with relative Yukawa coupling constants of exponential form. At the first-order seesaw approximation, quasi-democratic mass matrices with equal diagonal elements are derived for all charged fermion sectors and a diagonal mass matrix is obtained for the neutrino sector under an additional ansatz. Assuming the vacuum neutrino oscillation, the problems of solar and atmospheric neutrino anomalies are investigated.

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1 Introduction

The hypothesis of the universal seesaw mechanism (USM) [1–4] was invented to explain the smallness of the charged-fermion masses relative to the electroweak scale by postulating the existence of exotic fermions belonging to electroweak singlets. Although the large observed value of the top-quark mass [5] seems to reduce its original merit as far as the up-quark sector is concerned, it is effective for the down-quark and charged-lepton sectors. Furthermore, the double universal seesaw mechanism [2, 3] is very promising to explain the origin of the tiny masses of neutrinos, for which experimental evidence has been strongly sustained by the recent observation of an atmospheric neutrino anomaly at the Super-Kamiokande [6].

We have developed a unified theory of quark-lepton mass spectrum with the single and double universal seesaw mechanisms (SDUSM) [2, 3] in the model based on the extended Pati-Salam gauge group [7],

$$G \equiv SU(4)_{cL} \times SU(4)_{cR} \times SU(2)_L \times SU(2)_R \times U(1)_X, \tag{1}$$

where a group $U(1)_X$ is generated by a new charge X. In the theory, the $U(1)$ chiral charges [8] were assigned to the fermion and Higgs fields to distinguish generations and to restrict patterns of the Yukawa couplings. Mass matrices of the same extended Fritzsch type [9] were obtained for all four fermion sectors and the problem of solar neutrino deficit was analysed by assuming the Mikheyev-Smirnov-Wolfenstein mechanism [10] and the vacuum neutrino oscillation [11].

The purpose of this article is to formulate a new unified theory of quark-lepton mass spectrum by combining the SDUSM with the hypothesis of *universal strength for Yukawa couplings* (USY) [12,13] in the model based on the same gauge group G in Eq. (1). We obtain quasi-democratic mass matrices [14] of specific forms with equal diagonal elements for the charged-fermion sectors. The hypothesis of the USY is proved to work so favourably on the USM in our formalism that it is applicable also to the up-quark sector [15] . With an additional ansatz, a diagonal mass matrix is obtained for the neutrino sector. We examine the problems of the solar and atmospheric neutrino anomalies by assuming the vacuum neutrino oscillations.

In the SDUSM, two kinds of exotic fermion multiplets belonging to electroweak singlets, i.e. colour quartets $F_{ih}(F = U, D)$ and colour singlets N'_{ih}, are postulated to exist for each generation of colour quartets ψ_{ih} belonging to electroweak doublets, where i ($i = 1, 2, 3$) and h ($h = L, R$) stand for, respectively, the generation and the chirality.

Couplings between the electroweak doublets ψ_{iL} and ψ_{iR} are forbidden by the structure of the fundamental gauge group G. We naturally assume that the purely-neutral fields N'_{ih} have intrinsic bare Dirac masses. Therefore, the *Yukawa couplings* can exist between ψ_{ih} and F_{ih} of colour quartets, between F_{ih} and N'_{ih} of electroweak singlets, and between F_{ih} themselves. Extending the original hypothesis of the USY [12], we require that all three types of these Yukawa couplings depend on generations by their phase factors.

Note that the hypothesis of the USY in the standard model [12] is specified in the gauge-interaction eigenmodes. In our model with the SDUSM, however, there is a large freedom for fermion bases, which allows equivalent eigenmodes for the gauge interaction. For example, arbitrary unitary transformations can be applied to the neutral exotic fields N'_{ih} without changing the gauge interaction. Therefore, in order to state the USY strictly in our model, it is necessary to select a certain preferred base for the gauge interaction eigenmodes (see §2).

2 Model

Fundamental fermions are classified with respect to the basic group G. Dominant components of ordinary quarks and leptons belonging to the i-th generation are described, generically, by the chiral fields ψ_{iL} and ψ_{iR} with the following transformation properties:

$$\psi_{iL} \sim (4,1,2,1;0), \quad \psi_{iR} \sim (1,4,1,2;0). \tag{2}$$

In order to implement the USM, it is necessary to introduce the electroweak singlets of chiral fermion fields

$$\begin{array}{ll} U_{iL} \sim (1,4,1,1;1), & U_{iR} \sim (4,1,1,1;1), \\ D_{iL} \sim (1,4,1,1;-1), & D_{iR} \sim (4,1,1,1;-1) \end{array} \tag{3}$$

as the seesaw partners for each generation. Note here that the colour gauge fields of $SU(4)_L$ and $SU(4)_R$ groups interact with the fermion multiplets $(\psi_{iL}, U_{iR}, D_{iR})$ and $(\psi_{iR}, U_{iL}, D_{iL})$, respectively. With these specific choice of the fermion multiplets, the cancellation of triangular anomalies [16] is ensured even when the exact left-right symmetry in the gauge interaction is broken [17]. For the USM to be induced doubly in the neutral fermion sector, electroweak singlets of colourless fermions described by the chiral fields

$$N'_{iL} \sim (1,1,1,1;0), \quad N'_{iR} \sim (1,1,1,1;0) \tag{4}$$

are postulated to exist per generation.

Symmetry breakings from $SU(4)_{cL} \times SU(4)_{cR}$ down to $SU(3)_c$ through $SU(3)_{cL} \times SU(3)_{cR}$ are built in by colour quartets and a bi-quartet of Higgs fields

$$\chi_{cL} \sim (4,1,1,1;1), \quad \chi_{cR} \sim (1,4,1,1;1), \quad \Phi \sim (4,\overline{4},1,1;0), \tag{5}$$

which develop vacuum expectation values

$$\langle \chi_{cL} \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ v_L \end{pmatrix}, \quad \langle \chi_{cR} \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ v_R \end{pmatrix}, \quad \langle \Phi \rangle = \begin{pmatrix} v & 0 & 0 & 0 \\ 0 & v & 0 & 0 \\ 0 & 0 & v & 0 \\ 0 & 0 & 0 & v' \end{pmatrix}. \tag{6}$$

Electroweak doublets of Higgs fields

$$\chi_L \sim (1,1,2,1;-1), \quad \chi_R \sim (1,1,1,2;-1) \tag{7}$$

with vacuum expectation values

$$\langle \chi_L \rangle = \begin{pmatrix} w_L \\ 0 \end{pmatrix}, \quad \langle \chi_R \rangle = \begin{pmatrix} w_R \\ 0 \end{pmatrix} \tag{8}$$

are necessary to break the chiral symmetry and the Weinberg-Salam symmetry in electroweak interaction. The vacuum expectation values are assumed to be real and to satisfy the hierarchy

$$v_R,\, v_L \gg v,\, v' > w_R > w_L, \quad (v^2,\, v'^2 \gg w_R^2 \gg w_L^2). \tag{9}$$

As pointed out in the introduction, it is necessary to specify the base of gauge interaction eigenmodes of fundamental fermions in order to state the hypothesis of the universal strength for Yukawa couplings in the SDUSM without ambiguity. Here, we take the base in which a *bare-mass* matrix of the colourless neutral fields N'_{ih} is diagonal, and scalar interactions between electroweak singlets F_{iL} and F_{iR} are also diagonal. Our hypothesis on the USY in the SDUSM then is that the coupling constants for the Yukawa interactions have generation dependence described by phase factors. We express the *Yukawa coupling constants* for scalar interactions between ψ_{iL} and F_{jR} by $Y_f e^{i\phi_{ij}^f}$ and those between F_{iL} and N'_{jR} by $Y' e^{i\varphi_{ij}^N}$, where ϕ_{ij}^f and φ_{ij}^N are real, $f = (u,d)$ and $F = (U,D)$. Without loss of generality, the interactions of the bi-quartet Higgs field Φ with the electroweak singlet fields F_{iL} and F_{iR} are set to be described by a coupling constant Y_F in a generation-independent way since phase factors can be eliminated by adjusting phases of the singlet fields.

The most general form of the Lagrangian density $\mathcal{L}_Y$ for the bare masses and scalar interactions of fermions, which satisfies the USY and is invariant under the left-right symmetric gauge group G, is written as

$$\begin{aligned}
\mathcal{L}_Y &= \textstyle\sum_{i,j} \Big\{ Y_u e^{i\phi^u_{ij}} \left(\bar{\psi}_{iL}\chi_L U_{jR} + \bar{\psi}_{iR}\chi_R U_{jL} \right) \\
&\qquad + Y_d\, e^{i\phi^d_{ij}} \left(\bar{\psi}_{iL}\tilde{\chi}_L D_{jR} + \bar{\psi}_{iR}\tilde{\chi}_R D_{jL} \right) \Big\} \\
&\quad + \textstyle\sum_{i,j} Y' e^{i\varphi^N_{ij}} \left(\bar{N}'_{iL}\chi^\dagger_{cL} U_{jR} + \bar{N}'_{iL}\chi^\dagger_{cR} U_{jL} \right) \\
&\quad - \textstyle\sum_i \left(Y_U \bar{U}_{iR}\Phi U_{iL} + Y_D \bar{D}_{iR}\Phi D_{iL} \right) \\
&\quad + \textstyle\sum_i m^N_i \bar{N}'_{iL} N'_{iR} + \text{h.c.},
\end{aligned} \tag{10}$$

where $\tilde{\chi} = i\sigma_2\chi^*$ and $m^N_i (m^N_1 < m^N_2 < m^N_3)$ are the bare Dirac masses of the colourless neutral fields N'_{ih}. Henceforth the strengths of the Yukawa coupling constants Y_f, Y' and Y_F are considered to have nearly the same order of magnitude.

3 Seesaw mass matrices with pure phase submatrices

Spontaneous breakdowns of the underlying symmetry G induce 6×6 seesaw mass matrices for the charged-fermion sectors and a 9×9 seesaw mass matrix for the neutral-fermion sector. For general representations of mass matrices, it is convenient to use the following bases in the generation space as

$$f = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}, \quad F = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix}, \quad F' = \begin{pmatrix} F'_1 \\ F'_2 \\ F'_3 \end{pmatrix}, \tag{11}$$

where the components f_i, F_i and F'_i are the multiplets ψ_{ih}, (U_{ih}, D_{ih}) and N'_{ih}, respectively. The mass matrices are expressed in the seesaw block-matrix forms

$$\left(\bar{f}_L \quad \bar{F}_L \right) \begin{pmatrix} 0 & M^f_L \\ M^f_R & M^F \end{pmatrix} \begin{pmatrix} f_R \\ F_R \end{pmatrix} + \text{h.c.}, \tag{12}$$

for the charged-fermion sectors in the (f, F) base, and

$$\left(\bar{f}_L \quad \bar{F}_L \quad \bar{F}'_L \right) \begin{pmatrix} 0 & M^f_L & 0 \\ M^f_R & M^F & M^{F'}_R \\ 0 & M^{F'}_L & M^{N'} \end{pmatrix} \begin{pmatrix} f_R \\ F_R \\ F'_R \end{pmatrix} + \text{h.c.}, \tag{13}$$

for the neutral-fermion sector in the $(f, F, F') = (\nu, N, N')$ base.

The 3×3 submatrices M^f_L and M^f_R are given by

$$M^f_L = Y_f w_L M^f_\phi, \quad M^f_R = Y_f w_R M^{f\dagger}_\phi \tag{14}$$

in terms of the pure phase matrices

$$M_\phi^f \equiv \begin{pmatrix} e^{i\phi_{11}^f} & e^{i\phi_{12}^f} & e^{i\phi_{13}^f} \\ e^{i\phi_{21}^f} & e^{i\phi_{22}^f} & e^{i\phi_{23}^f} \\ e^{i\phi_{31}^f} & e^{i\phi_{32}^f} & e^{i\phi_{33}^f} \end{pmatrix}. \tag{15}$$

The submatrix M^F has the structure

$$M^F = -Y_F v E, \quad M^F = -Y_F v' E \tag{16}$$

for the quark and lepton sector, respectively, where E is the 3×3 unit matrix. As in Eq. (14), the submatrices $M_L^{F'}$ and $M_R^{F'}$ are expressed as

$$M_L^{F'} = Y' v_L M_\varphi^N, \quad M_R^{F'} = Y' v_R M_\varphi^{N\dagger} \tag{17}$$

in terms of the pure phase matrix

$$M_\varphi^N \equiv \begin{pmatrix} e^{i\varphi_{11}^N} & e^{i\varphi_{12}^N} & e^{i\varphi_{13}^N} \\ e^{i\varphi_{21}^N} & e^{i\varphi_{22}^N} & e^{i\varphi_{23}^N} \\ e^{i\varphi_{31}^N} & e^{i\varphi_{32}^N} & e^{i\varphi_{33}^N} \end{pmatrix}. \tag{18}$$

$M^{N'}$ is the diagonal matrix

$$M^{N'} = \begin{pmatrix} m_1^N & 0 & 0 \\ 0 & m_2^N & 0 \\ 0 & 0 & m_3^N \end{pmatrix}. \tag{19}$$

4 Effective mass matrices

On the assumption that $Y_F^2 v^2, Y_F^2 v'^2 \gg Y_f^2 w_R^2 \gg Y_f^2 w_L^2$, the seesaw mass matrices in Eqs. (12) and (13) are block-diagonalized by the bi-unitary transformations. The single universal seesaw mechanism results in the effective mass matrices

$$M_{\text{eff}}^f = -M_L^f (M^F)^{-1} M_R^f \tag{20}$$

for ordinary low-lying charged fermions. Substituting the submatrices, the effective mass matrices of the charged fermions are written in the form

$$M_{\text{eff}}^f = M_f \tilde{\Omega}_f, \tag{21}$$

where

$$\tilde{\Omega}_f = \frac{1}{3} \begin{pmatrix} 1 & a_3^f\, e^{i\delta_{12}^f} & a_2^f\, e^{-i\delta_{31}^f} \\ a_3^f\, e^{-i\delta_{12}^f} & 1 & a_1^f\, e^{i\delta_{23}^f} \\ a_2^f\, e^{i\delta_{31}^f} & a_1^f\, e^{-i\delta_{23}^f} & 1 \end{pmatrix}. \tag{22}$$

Here, a_i^f and δ_{ij}^f are real parameters, which are expressed in terms of the original phases ϕ_{ij}^f in Eq. (15), as follows:

$$a_i^f\, e^{i\delta_{jk}^f} = \frac{1}{3}\sum_l e^{i(\phi_{jl}^f - \phi_{kl}^f)} \tag{23}$$

where $(i,j,k) = \text{cyclic}(1,2,3)$. The mass scales M_f $(f = u,d,l)$ are fixed by

$$M_u = 9\frac{Y_u^2}{Y_U}\frac{w_L w_R}{v}, \quad M_d = 9\frac{Y_d^2}{Y_D}\frac{w_L w_R}{v}, \quad M_l = 9\frac{Y_d^2}{Y_D}\frac{w_L w_R}{v'}. \tag{24}$$

The masses of ordinary charged fermions m_j^f and the eigenvalues $\tilde{\omega}_j^f$ of the Hermitian matrix $\tilde{\Omega}_f$ are related as follows:

$$m_j^f = M_f \tilde{\omega}_j^f, \qquad \sum_j \tilde{\omega}_j^f = 1. \tag{25}$$

From Eqs. (24) and (25), we find

$$\frac{m_u + m_c + m_t}{m_d + m_s + m_b} = \frac{Y_u^2}{Y_d^2}\frac{Y_D}{Y_U} \tag{26}$$

for the masses of up- and down-quark sectors. Owing to the large symmetry of the group G, the quasi-democratic matrix is common to the down-quark and charged-lepton sectors, i.e. $\tilde{\Omega}_d = \tilde{\Omega}_l$. Therefore, their mass spectra have the same infrastructure. From Eqs. (24) and (25), we obtain the relations

$$\frac{m_e}{m_d} = \frac{m_\mu}{m_s} = \frac{m_\tau}{m_b} = \frac{v}{v'} \tag{27}$$

for the masses of the charged-lepton and down-quark sectors. The quasi-democratic mass matrices M_{eff}^f in Eq. (22) have been derived and analysed in the previous articles [15]. We can inherit here all the results of those analyses.

Provided that $Y'^2 v_L^2, Y'^2 v_R^2 \gg Y_U^2 v'^2, m_i^2 \gg Y_f^2 w_R^2 \gg Y_f^2 w_L^2$, the double universal seesaw mechanism [2] leads to the effective mass matrix

$$\begin{aligned} M_{\text{eff}}^\nu &= -M_L^\nu \left(M^N - M_R^{N'} \left(M^{N'}\right)^{-1} M_L^{N'} \right)^{-1} M_R^\nu \\ &\approx M_L^\nu \left(M_L^{N'}\right)^{-1} M^{N'} \left(M_R^{N'}\right)^{-1} M_R^\nu \\ &\approx \left(\frac{Y_u}{Y'}\right)^2 \frac{w_L w_R}{v_L v_R} [M_\phi^u \left(M_\varphi^N\right)^{-1}] M^{N'} [M_\phi^u \left(M_\varphi^N\right)^{-1}]^\dagger \end{aligned} \tag{28}$$

for the neutrino sector. While the quasi-democratic mass matrix M_{eff}^f for the charged fermion sector is made out of 9 phases ϕ_{ij}^f of the matrix in Eq. (15), the effective mass matrix $M^\nu_{\rm eff}$ for the neutrino sector includes 18 phases ϕ^u_{ij} and φ^N_{ij} of the matrices in Eqs. (15) and (18). Without a skilful reduction of these unknown phases, it is impossible to determine a concrete form of $M^\nu_{\rm eff}$. Here, resorting to the principle of simplicity, we postulate an additional ansatz that the pure phase matrices M^u_ϕ and M^N_φ are identical, i.e. $\phi^u_{ij} = \varphi^N_{ij}$, mod$(2\pi)$. This ansatz enables us to eliminate all the unknown phases and to simplify the effective mass matrix $M^\nu_{\rm eff}$ into the diagonal form

$$M^\nu_{\rm eff} = S_\nu M^{N'}, \quad S_\nu = \left(\frac{Y_u}{Y'}\right)^2 \frac{w_L w_R}{v_L v_R}. \tag{29}$$

Accordingly we obtain the simple mass formula for the neutrions as follows:

$$m^\nu_i = S_\nu m^N_i. \tag{30}$$

5 Vacuum neutrino oscillation

The weak mixing matrix V in the leptonic sector is determined by deriving a diagonalization matrix for the effective mass matrix $M^l_{\rm eff}$ of the charged lepton sector since the neutrino mass matrix is already given in the diagonal form. In order to obtain the diagonalization matrix, which is calculable in terms of the charged-lepton masses exclusively, *CP*-violation effects are neglected and a simplification $a^l_1 = a^l_2$ is made in the matrix $\tilde{\Omega}_l$. Following the method used in the previous paper [15], we introduce the following parametrization as

$$a^l_1 = a^l_2 = \frac{3}{2\sqrt{2}}\rho_l \sin\theta_l, \quad a^l_3 = 3\rho_l\cos\theta_l. \tag{31}$$

Then the eigenvalue problem for the matrix

$$M^l_{\rm eff} = M_l\tilde{\Omega}_l \approx M_l\Omega_l \tag{32}$$

with

$$\Omega_l = \frac{1}{3}\begin{pmatrix} 1 & 3\rho_l\cos\theta_l & \frac{3}{2\sqrt{2}}\rho_l\sin\theta_l \\ 3\rho_l\cos\theta_l & 1 & \frac{3}{2\sqrt{2}}\rho_l\sin\theta_l \\ \frac{3}{2\sqrt{2}}\rho_l\sin\theta_l & \frac{3}{2\sqrt{2}}\rho_l\sin\theta_l & 1 \end{pmatrix} \tag{33}$$

turns out to be solvable in a simple analytical form. With the orthogonal matrix

$$U_l = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 & 0 \\ \sin\frac{1}{2}\theta_l & \sin\frac{1}{2}\theta_l & -\sqrt{2}\cos\frac{1}{2}\theta_l \\ \cos\frac{1}{2}\theta_l & \cos\frac{1}{2}\theta_l & \sqrt{2}\sin\frac{1}{2}\theta_l \end{pmatrix}, \tag{34}$$

the quasi-democratic matrix Ω_l is diagonalized as

$$U_l \Omega_l U_l^\dagger = \mathrm{diag}\left(\omega_1^l,\, \omega_2^l,\, \omega_3^l\right), \tag{35}$$

where

$$\omega_1^l = \frac{1}{3} - \rho_l \cos\theta_l,\ \omega_2^l = \frac{1}{3} + \frac{1}{2}\rho_l(-1+\cos\theta_l),\ \omega_3^l = \frac{1}{3} + \frac{1}{2}\rho_l(1+\cos\theta_l). \tag{36}$$

The parameters ρ_l and $\cos\theta_l$ are expressed in terms of the charged-lepton masses by

$$\rho_l = 1 - \frac{2m_\mu + m_e}{m_\tau + m_\mu + m_e}, \quad \cos\theta_l = \frac{1}{3}\left(1 + 2\frac{m_\mu - m_e}{m_\tau - m_\mu}\right). \tag{37}$$

Since the diagonalization matrix Ω_l depends only on the parameter θ_l, the weak mixing matrix $V = U_l$ is calculable in terms of the charged-lepton masses in this approximation. Note that the matrix Ω_l takes the democratic form and its eigenvalues results in $(\omega_1^l,\, \omega_2^l,\, \omega_3^l) = (0,\, 0,\, 1)$ at the limit $\rho_l = 1$ and $\cos\theta_l = \frac{1}{3}$.

The experimental data of neutrino anomalies are usually presented in terms of two flavour-mixing parameters $\Delta m_{ij}^2 = |m_i^2 - m_j^2|$ and $\sin^2 2\theta_{ij}$. The most natural solution of the atmospheric neutrino anomaly is considered to be given by the $\nu_\mu \leftrightarrow \nu_\tau$ oscillation with the following ranges of parameters [6,18]

$$\Delta m_{\mathrm{atm}}^2 \approx (0.5 - 6) \times 10^{-3}\mathrm{eV}^2, \quad \sin^2 2\theta_{\mathrm{atm}} > 0.82 \tag{38}$$

If we apply the mechanism of neutrino oscillation to interpret also the phenomenon of the solar-neutrino deficit, the most probable candidate is the $\nu_e \leftrightarrow \nu_\mu$ oscillation, with the following ranges of parameters [19]:

$$\Delta m_{\mathrm{sol}}^2 \approx (0.6 - 1.1) \times 10^{-10}\mathrm{eV}^2, \quad \sin^2 2\theta_{\mathrm{sol}} \approx 0.7 - 1. \tag{39}$$

In these interpretations where the parameters Δm_{ij}^2 satisfy $\Delta m_{21}^2 \ll \Delta m_{32}^2 \approx \Delta m_{31}^2$, the angles of two-flavour mixings can be approximated by the matrix elements of the three-flavour mixing matrix $V = U_l$ as

$$\sin^2 2\theta_{\mathrm{atm}} = 4|V_{23}|^2(|V_{21}|^2 + |V_{22}|^2) \tag{40}$$

and

$$\sin^2 2\theta_{\mathrm{sol}} = 4|V_{11}|^2|V_{12}|^2 \tag{41}$$

in good approximation.

Using Eqs. (34) and (37), we obtain

$$\sin^2 2\theta_{\rm atm} = \sin^2\theta_l = \frac{4}{9}\left(1 - \frac{m_\mu - m_e}{m_\tau - m_\mu}\right)\left(2 + \frac{m_\mu - m_e}{m_\tau - m_\mu}\right) \tag{42}$$

and

$$\sin^2 2\theta_{\rm sol} = 1. \tag{43}$$

These values representing the large mixings are consistent with the experimental data in Eqs. (38) and (39). The latter is identical to the result obtained at the democratic limit by Fritzsch and Xing [20] and the former reduces to the relation $\sin^2 2\theta_{\rm atm} = 8/9$ discovered by them at the same limit.

As for the neutrino masses, two interpretations are often adovocated, i.e. (i) the almost degenerate spectrum with $m_1^\nu \approx m_2^\nu \approx m_3^\nu$ and (ii) the hierarchical spectrum with $m_1^\nu \ll m_2^\nu \ll m_3^\nu$. Both interpretations are applicable to the present model. Since $m_i^\nu = S_\nu m_i^N$, these characteristics of the neutrino mass spectrum are translated into those of the exotic neutral fields. In the former case, it is possible to identify the hot component of dark matter in the Universe with the neutrinos that have $m_i^\nu \approx 2.5$ eV [21].

6 Discussion

For the up-quark sector the seesaw approximation is usually considered to be disqualified, since the magnitude of the Yukawa coupling constant has to be much larger than 1 for the top-quark mass m_t to be close to the electroweak scale w_L. In our theory the numerical factor 9, which appears through the product of $M_\phi^f M_\phi^{f\dagger}$ in Eq. (20), acts to improve the situation considerably. Equation (24) and the relation $M_u \approx m_t \approx w_L$ lead to the estimate $Y_U v \approx 9Y_u^2 w_R$. This estimate tells us that the condition for the seesaw approximation $Y_U^2 v^2 \gg Y_u^2 w_R^2$ holds if the criterion $81Y_u^2 \gg 1$ is satisfied. Therefore, for the Yukawa coupling constant Y_u of the order, say, of around $1/2$, the seesaw approximation turns out to be applicable to the up-quark sector.

Departures of the mass scales of the down-quark and charged-lepton sectors from the electroweak scale w_L must be explained, respectively, by the seesaw factors $9Y_d^2 w_R/Y_D v$ and $9Y_d^2 w_R/Y_D v'$. The scale difference of up- and down-quark sectors has to be ascribed to the difference between the strengths of the Yukawa coupling constants Y_f and Y_F. Namely those constants must be tuned so as to satisfy the relation $Y_u^2 Y_D/Y_d^2 Y_U \simeq m_t/m_b$. This tuning seems to be executable without changing the orders of the constants Y_f and Y_F. In the present stage of our model, the down-quark and charged-lepton sectors have similar mass matrices and mass spectra. It is necessary to find a mechanism of $SU(4)_h$ symmetry breakings to generate fine variations in these two sectors. To explain the scale difference of the spectra, the vacuum expectation values of the bi-quartet Φ must satisfy the relation $v/v' \simeq m_\tau/m_b$.

The masses of the neutrinos and the exotic neutral fields are proportional to each other. To explain the strong suppression of the neutrino mass scale, vacuum expectation values of χ_{ch} must be assumed to be sufficiently larger than those of χ_h. Note that, owing to the result of the CERN precision measurement [22], the bare masses of the neutral exotic fields must be larger than the half-value of the Z boson mass, i.e. $m_i^N > \frac{1}{2}m_Z$. Therefore, if we adopt the almost degenerate mass spectrum (i) for the neutrinos and identify the hot dark matter with them, the scale factor $S_\nu \approx w_L w_R/v_L v_R$ is subject to the restriction $S_\nu < 2m_i^\nu/m_Z \approx 5 \times 10^{-11}$. In the case of the hierarchical mass spectrum (ii), the scale factor must satisfy the more stringent relation $S_\nu < 2m_1^\nu/m_Z \ll 2m_2^\nu/m_Z \approx 10^{-16}$. Therefore the breakdowns of the $SU(4)_h$ colour symmetries have to occur at much higher energy scales in the hierarchical case (ii) than in the degenerate case (i). Here it should be mentioned that, in our model with the USM, no Majorana mass is assumed to exist. All masses are of the Dirac type. Therefore the observed bound of the neutrinoless $\beta\beta$-decay [23] does not impose any restriction on our theory.

In this model the hypothesis of the USY is stated in the simplest fermion base, where the neutral exotic fields have the diagonal mass matrix and the electroweak singlets of colour quartets interact in the generation diagonal manner. It is possible to choose other fermion bases. If an off-diagonal mass matrix is adapted for the neutral exotic fields, it affects the weak mixing matrix and accordingly the estimates in Eqs. (42) and (43). The additional ansatz $M_\phi^u = M_\varphi^N$ plays the decisive role to render the simple mass matrix for the neutrinos by totally eliminating the contributions from unknown submatrices in this model. Its physical and geometrical meanings must be investigated.

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