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The impedance of layered vacuum chambers

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Abstract

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THE IMPEDANCE OF LAYERED VACUUM CHAMBERS

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Using an algorithm for the calculation of electro-magnetic fields of oscillating circular cylindrical beams surrounded by any number of annular layers of arbitrary material properties, we calculate the transverse resistive wall impedance of the beam screens for the LHC and FLHC, a larger hadron collider. These consist of tubes of stainless steel with thin inside layers of copper. The results are compared with those of an approximate analytic calculation which gives incorrect results when the skin depth becomes so large that it approaches the wall radius.

1 INTRODUCTION

The transverse resistive wall instability is one of the performance limitations of the Large Hadron Collider LHC [1], in particular when a beam screen or “liner” is installed inside the cold vacuum chamber to reduce the cryogenic heat load due to synchrotron radiation. In order to withstand the mechanical forces during quenches of the surrounding super-conducting magnets, the liners are made of stainless steel. To increase the threshold of the resistive wall instability the liner should be coated on the inside with a layer of copper. Estimates of the impedance and instability growth rate as function of the coating thickness were made using approximate analytic expressions with a method using “wall penetration factors” [2]. Here we apply a more exact technique[3, 4] for computing the impedance of “layered” vacuum chamber walls, consisting of any number of concentric, annular layers with arbitrary material properties surrounding a circular cylindrical beam to the LHC, and FLHC[5].

2 DESCRIPTION OF THE TECHNIQUE

The electro-magnetic field components, excited at circular frequency ω by a transversely oscillating, circular cylindrical particle beam moving with ultra-relativistic velocity on the axis of any number of concentric annular layers of arbitrary material properties, can be written in general as infinite series over products of (modified) Bessel functions for the radial, and exponential functions for the axial direction, containing four sets of arbitrary expansion coefficients in each (intermediate) layer. Matching the four tangential field components at consecutive boundaries between layers, starting from the outside, permits determining these coefficients uniquely in terms of the beam oscillation parameters. The results depend on the choice of the outermost layer (with only two unknown coefficients): here we assume that vacuum extends to a perfect conductor at the radius of the beam pipe. The transverse impedance can then be expressed by the coefficients in the beam region. We assume here that the liner is surrounded by a perfect conductor at the radius of the beam pipe. These calculations were done with the Mathematica package LAWAT available at http://wwwslap.cern.ch/∼keil/Math/Lawat [6].

Table 1: LHC and FLHC parameters

Machine	LHC	FLHC
Average radius R/m	4243	36452
Revolution freq. f_0/Hz	11245	1309
Relativistic factor γ_0	480	5330
Tune Q	63.3	101.3
Average beta fct. $\overline{\beta}$/m	85	431
Lowest mode freq./Hz	3373	393
Bunch population N_b	10^{11}	$9 \cdot 10^9$
Number of bunches k_b	2835	20361

3 LHC BEAM SCREEN

In Tab. 2 we list the properties of all cylindrical layers. The outermost layer is the vacuum chamber proper that we may represent with little error by a perfect conductor. We assume throughout that the relative permeabilities and permittivities are $\varepsilon_i = \mu_i = 1$. The skin depths at the lowest betatron frequency is larger than the thickness of the conducting layers in both cases. The conductivity of the copper layer takes due account of the magneto-resistance effect at the LHC injection field $B = 0.56$ T.

Table 2: Layer number i, material, radius a_i, thickness d_i, conductivity σ_i for the layers of the vacuum chamber

i	Material	a_i	d_i	σ_i
-	-	mm	mm	Sm^{-1}
1	Beam	3	3	0
2	Vacuum	19	16	0
3	Cu layer	19.05	0.05	$5.5 \cdot 10^9$
4	Steel screen	21.05	2.0	$2 \cdot 10^6$
5	Vacuum	24.5	3.45	0
6	Outer chamber	30	5.5	∞

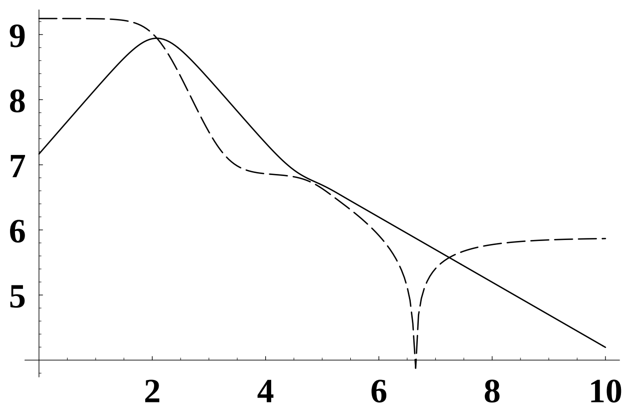

Figure 1: Decimal logarithm of real (full line) and imaginary (dashed line) parts of the transverse impedance in /m of the LHC for the vacuum chamber parameters listed in Tab. 2 on a logarithmic frequency scale from 1 to 10^{10} Hz

Fig. 1 shows logarithmically the real and imaginary parts of the transverse impedance of the LHC vacuum chamber versus frequency. The real part rises linearly at low frequencies, reaches a maximum, and decreases when the skin depth becomes small compared to the thickness of the Cu layer, asymptotically dropping as 1 $\overline{f}$. The absolute value of the imaginary part is nearly constant up to the frequency where reaches the maximum, and then drops essentially as 1 $\overline{f}$. At the frequency where the constant negative reactance of the direct space charge term cancels the decreasing positive inductance of the metal wall, changes sign which appears as a dip in the logarithmic plot of the absolute value (near 10 Hz in the figure). At even higher frequencies the negative space charge reactance dominates, and becomes again independent of frequency. For higher values of the energy factor γ_0, the dip moves towards higher frequencies in proportion to γ_0.

The traditional formula for the resistive-wall growth rate $^{-1}$ of the most unstable mode is [7]:

$$\frac{1}{\;} = \frac{\;}{2\;\;_0}\,\frac{N_b k_b}{\gamma R}\,\overline{\beta} \qquad (1)$$

Here, N_b is the bunch population, k_b is the number of bunches, $\overline{\beta}$ is the average β-function, is the transverse impedance at the most unstable mode, γ is the usual relativistic factor, and and $_0$ are natural constants.

Tab. 3 shows a comparison of the results for the product of the average β-function and the real part of the transverse impedance $\overline{\beta}$ and the growth times at injection energy as a function of the thickness of the Cu layer for $\overline{\beta} =$ m, obtained at frequency $\omega =$ $Q\ \omega_0$ $0.\ \omega_0$ in two different ways:

1. With the standard expression for the resistive wall impedance, modified by "wall penetration factors" [2], taking into account the effect of the two layers (superscript "P");
2. With the LAWAT package [6] (superscript "L").

We show the results in Tab. 3 in terms of $\overline{\beta}$ and in order to facilitate comparison with Tab. 9 in [2]. The agreement between our results and the former ones is fair: the difference is only about one percent for a Cu layer of 50 μm thickness. However, it increases for thinner Cu layers, and reaches 8 % for 10 μm thick Cu layers. The transverse impedances depend only little on the thickness of the stainless steel liner when computed with LAWAT, while these values are approached for thicker layers when calculated with penetration factors. The difference is only +2 % for 10 μm Cu on a 4 mm thick stainless steel liner.

Table 3: Product of average β-function and real part of the transverse **LHC impedance** and growth time at injection energy, computed with wall penetration factors (P) and LAWAT (L) for thickness d of the Cu layer.

d	$\overline{\beta}$	$\overline{\beta}$		
μm	G	G	ms	ms
10	27.17	29.39	6.24	5.77
20	15.31	15.51	11.08	10.94
30	10.49	10.49	16.17	16.17
40	7.96	7.92	20.02	21.41
50	6.42	6.37	26.43	26.64

4 APPLICATION TO FLHC

The overall FLHC parameters are listed in Tab. 1. We assume that the parameters of the beam screen in the FLHC are identical to those of the LHC, shown in Tab. 2. The skin depths are much larger than the thicknesses of the layers.

The variations of the real and imaginary parts of the impedance of the FLHC vacuum chamber with frequency are shown in Fig. 2. They are similar to those for the LHC in Fig. 1, but is higher in the ratio of the radii of the machines. Since the injection energy in the FLHC is higher than in the LHC, the dip of occurs beyond 10^{10} Hz.

Despite the larger circumference of the FLHC, the maximum of the real part of the transverse impedance still occurs at a frequency f not much higher than that of the lowest betatron mode. Again we evaluate the transverse impedance, both with wall penetration factors and with LAWAT. Tab. 4 shows the results for several values of the Cu-layer thickness d and a SS liner with $d\ = 2$ mm. For $d\ = 50\,\mu$m and $^{-1}$ are underestimated by about 20 % with wall penetration factors. For smaller values of d the discrepancy is even larger, almost reaching a factor of 3 at $d\ = 10\ \mu$m. For a thicker liner, the results agree better: for $d\ = 50\ \mu$m and $d\ =$ mm, the difference between the two calculations is only about 10 %. The calculation with wall penetration factors overestimates

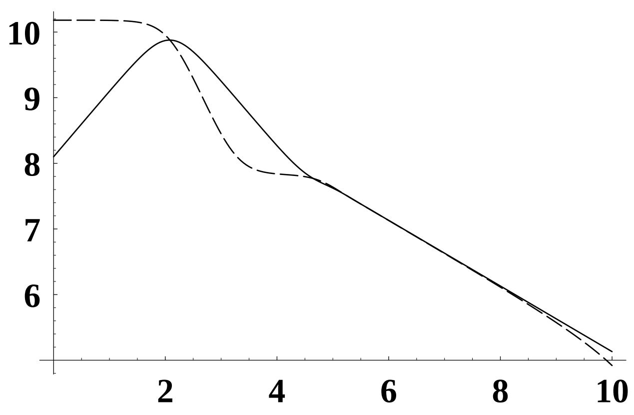

Figure 2: Decimal logarithm of real (full line) and imaginary (dashed line) parts of the transverse impedance in /m of the FLHC for the vacuum chamber parameters listed in Tab. 2 on a logarithmic frequency scale from 1 to 10^{10} Hz

Table 4: Product of average β-function and real part of the transverse **FLHC impedance** and growth time at injection energy, computed with wall penetration factors (P) and LAWAT (L) for thickness d of the Cu layer

d	$\overline{\beta}$	$\overline{\beta}$		
μm	T	T	ms	ms
10	1.088	3.041	60.96	21.81
20	1.632	3.131	40.63	21.18
30	1.741	2.621	38.08	25.30
40	1.656	2.163	40.04	30.06
50	1.514	1.816	43.80	36.51

and hence underestimates . Both and vary by less than a factor of two when d changes from 10 μm to 50 μm.

5 CONCLUSIONS

We calculate the transverse impedance of a circular cylindrical particle beam in a concentric vacuum chamber consisting of a number of layers of different materials by field matching, and thus obtain the transverse impedance of the the LHC and the FLHC beam screens made of stainless steel with a thin inside layer of copper whose thickness is varied. The beam screen is surrounded by a vacuum chamber which has been assumed to be perfectly conducting. The real part of the transverse impedance rises linearly at low frequencies and reaches a maximum below the frequency of the lowest transverse mode. At higher frequencies, where the skin depth is smaller than the thickness of the Cu layer, it behaves asymptotically as $1 \quad \overline{f}$.

Our results for impedances and growth rates are compared with previous calculations using wall penetration factors. For the 2 mm thick liners with a copper layer of 50 μm in the LHC, our results agree with the previous ones within a few percent, while for thinner Cu layers the difference becomes larger. The results of the calculation using wall penetration factors depend on the thickness of the stainless steel layer while our results do not.

In the larger FLHC machine with a similar beam screen, the revolution frequency is so small that the skin depth at the lowest betatron mode is much larger than the thickness of the beam screen. It becomes an appreciable fraction of the chamber radius, and then the method using wall penetration factors is no longer a good approximation. In these circumstances the impedances and growth rates can still be obtained with the LAWAT package.

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