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Pseudo-anomalous $U(1)$ symmetry in the strong coupling limit of the heterotic string

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Abstract

We discuss, in the context of the strongly-coupled $E_8 \times E_8$ heterotic string proposed by Hořava and Witten, the appeareance of anomalous $U(1)_X$ symmetries of a nonperturbative origin, related to the presence, after compactification, of five-branes in the five-dimensional bulk of the theory. We compute the gauge anomalies and the induced Fayet-Iliopoulos terms on each boundary, which we find to be lower than the universal one induced in the weakly coupled case.

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The role played by a pseudo-anomalous abelian symmetry [1] present in many models constructed from the weakly coupled heterotic string theory has been increasingly recognized. It may hold a function as a family symmetry to explain quark and lepton mass hierarchies [2]-[10], as a mediator of supersymmetry breaking [11]-[13], in cosmology [14, 15]. The anomaly cancellation mechanism is a four-dimensional remnant of the Green-Schwarz mechanism of anomaly cancellation which makes use of the coupling of the dilaton-axion to the gauge degrees of freedom. The physics of this anomalous $U(1)$ depends primarily on the scale ξ at which the corresponding symmetry is broken. This scale may be computed from the underlying string theory and lies one or two orders of magnitude below the Planck scale, which may be suitable for family symmetry purposes but probably too high for cosmology [16, 17].

In this article we discuss the possible origin of an anomalous $U(1)$ symmetry in the context of the strong coupling limit of the heterotic string constructed by Hořava and Witten [18]. In this picture, the observable and hidden gauge degrees of freedom live on two distinct 10-dimensional boundary planes. Because of its non-vanishing mixed anomalies, the anomalous $U(1)$ couples to both types of degrees of freedom and must therefore be found in the 11-dimensional bulk. This restricts its possible origin and makes the eleventh orbifold-like dimension between the two boundary planes play a key role in unravelling its structure. This may also help, as we will see, to evaluate the associated scale ξ.

Let us start by recalling some of the properties of the pseudo-anomalous $U(1)_X$ as it appears in the weakly coupled heterotic string. The anomaly cancellation rests on the universal coupling of the dilaton superfield S to the gauge degrees of freedom:

$$\mathcal{L} = -\frac{1}{4}\mathrm{Re}\ S \sum_a k_a F^{a\mu\nu} F^a_{\mu\nu} + \frac{1}{4}\mathrm{Im}\ S \sum_a k_a F^{a\mu\nu} \tilde{F}^a_{\mu\nu}, \qquad (1)$$

where k_a is the Kac-Moody level of the gauge symmetry group G_a. Anomaly cancellation is thus ensured by a Peccei-Quinn transformation of the axion string Im S at the sole condition that the mixed $U(1)_X - G_a - G_a$ anomaly coefficients C_a satisfy:

$$\frac{C_1}{k_1} = \frac{C_2}{k_2} = \cdots = \frac{C_a}{k_a} \equiv \delta_{GS}. \qquad (2)$$

The Green-Schwarz coefficient δ_{GS} is non-vanishing which imposes that the mixed anomaly coefficients for both observable and hidden sector gauge symmetries are non-zero. This in turn implies that fermions of both sectors are charged under the anomalous symmetry which therefore couples observable and hidden sector. This will be a key property to help us identify the origin of similar anomalous symmetries in the context of the strongly coupling limit of the heterotic string.

In the Hořava-Witten construction, this limit is described by eleven-dimensional supergravity compactified on $\mathbf{R}^{10} \times \mathbf{S}^1/Z_2$, coupled with gauge fields which are ten-dimensional vector multiplets that propagate on the boundary of space-time. The corresponding bosonic action reads:

$$\begin{aligned}
\mathcal{S} &= \frac{1}{\kappa_{11}^2}\int d^{11}x\sqrt{g^{(11)}}\left(-\frac{1}{2}\mathcal{R}^{(11)} - \frac{1}{48}G_{IJKL}G^{IJKL}\right) \\
&\quad -\frac{\sqrt{2}}{3456\kappa_{11}^2}\int d^{11}x\epsilon^{I_1\cdots I_{11}}C_{I_1I_2I_3}G_{I_4\cdots I_7}G_{I_8\cdots I_{11}} \\
&\quad -\frac{1}{8\pi(4\pi\kappa_{11}^2)^{2/3}}\int d^{10}x\sqrt{g^{(10)}}\,\mathrm{tr}F_{AB}F^{AB},
\end{aligned} \tag{3}$$

where $I, J, \cdots$ are eleven-dimensional indices, $A, B, \cdots$ ten-dimensional indices and G is the field strength of the three-form C ($G = 6dC + \cdots$). The Z_2 projection corresponds to the reflection on the eleventh coordinate ($x^{11} \to -x^{11}$) and acts as the chirality projector on the gravitino degrees of freedom. The 3-form C is odd under this projection whereas the metric tensor is even.

The bosonic action (3) and the corresponding fermionic one constructed in [18] do not form the complete quantum action but they are rather an effective description including the lowest orders of an expansion in the parameter $\kappa_{11}^{2/3}$. Because of the presence of the boundary, the fermionic action includes divergent terms proportional to $\delta(0)$ (as well as its derivatives) possibly to some power: in a full quantum treatment, the boundary presumably acquires a non-zero thickness of order M^{-1} and the divergent $\delta(0)$ terms are smoothed out into terms of order M, where M is the fundamental mass scale:

$$4\pi\kappa_{11}^2 = (2\pi M^{-1})^9. \tag{4}$$

One may compactify this theory down to 5 dimensions [20]-[26]. With a standard embedding of the $SU(3)$ holonomy group of the 6-dimensional compact manifold, one may consider that the (E_6-type) gauge fields of the observable sector live on one boundary, whereas the (E_8-type) gauge fields of the hidden sector live on the other one. The different scales involved will play an important part in what follows. Let us therefore review them. We will adopt a simplified compactification scheme [19] which includes the most generic properties of more realistic scenarios: we keep only the two moduli which describe respectively the radii of the six-dimensional compact manifold (compactification from 11 to 5 dimensions) and of the orbifoldlike 11th dimension (counted from now on as the fifth dimension). In terms of the eleven-dimensional metric $g_{IJ}^{(11)}$ one may write:

$$\begin{aligned}
g_{ab}^{(11)} &= e^{\sigma}g_{ab}^{(0)}, \qquad \textstyle\int d^6x\sqrt{g^{(0)}} = (2\pi M^{-1})^6 \\
g_{\mu\nu}^{(11)} &= e^{a\sigma}g_{\mu\nu}^{(5)}
\end{aligned} \tag{5}$$

where $a, b \in \{6, \cdots, 11\}$ and $\mu, \nu \in \{1, \cdots, 5\}$, and

$$\begin{aligned}
g_{55}^{(5)} &= e^{2\gamma}\hat{g}^{(0)}, \qquad \textstyle\int dx^5\sqrt{\hat{g}^{(0)}} = \pi M^{-1} \\
g_{mn}^{(5)} &= e^{b\gamma}g_{mn}
\end{aligned} \tag{6}$$

where $m, n \in \{1, \cdots, 4\}$. In these formulas, M is the fundamental mass scale but the rescalings undergone by the four-dimensional metric may change its physical interpretation in 4-dimensional spacetime. In the absence of rescaling ($a = b = 0$) it is simply the fundamental scale of the original eleven-dimensional theory, usually[3] denoted by M_{11}. A look at the Einstein term in the four-dimensional Lagrangian shows that the choice $a = -2, b = -1$ yields the 4-dimensional Planck scale $M = m_{Pl}\sqrt{2\pi}$ (m_{Pl} being the reduced Planck scale). Finally, the choice $a = -2, b = 0$ corresponds to the 5-dimensional Planck scale [22] as well as 5-brane unit mass scale in the 10-dimensional theory [28]; we will denote it by $M = M_5$.

Our simplified compactification scheme amounts to the presence of only the dilaton S and a single Kähler modulus T. Their real parts s and t expressed in general M units simply read:

$$s = e^{3\sigma}, \quad t = e^{\gamma} e^{(a+2)\sigma/2}. \tag{7}$$

The mass scale which corresponds to the inverse radius of the 6-dimensional compact manifold is the scale where the theory becomes 11-dimensional and therefore corresponds to the unification of all couplings; we denote it by M_U. In original units, it simply reads:

$$M_U = \frac{M_{11}}{s^{1/6}}. \tag{8}$$

One can easily obtain from (3) the gauge kinetic terms in four dimensions:

$$\mathcal{L} = -\frac{1}{4} \int d^4x \sqrt{g} \frac{s}{2\pi} F^{mn} F_{mn}, \tag{9}$$

which shows that the gauge coupling at the unification scale $\alpha_U = g_U^2/(4\pi)$ simply reads:

$$\alpha_U = \frac{1}{2s} \ . \tag{10}$$

Notice that this gauge coupling is universal at the tree level, for all the gauge groups living on the boundary, which leads to the universality of the Green-Schwarz mechanism in four dimensions.

In what follows, we will be interested mainly in expressing the scales in 5-dimensional units. One easily obtains the following relations:

$$m_{Pl} = M_5 \left(\frac{t}{2\pi}\right)^{1/2}, \ M_U = M_5 s^{-1/2}, \ R_5^{-1} = M_5 t^{-1}, \tag{11}$$

where R_5 is the radius of the orbifold dimension. One then obtains:

$$M_5 \quad = \quad M_U (2\alpha_U)^{-1/2} \sim 3.4 \ M_U \ ,$$

[3] In fact, our M_{11} is ℓ_{11}^{-1} of ref. [27] and $2\pi M_{11}(4\pi)^{-1/9}$ of ref. [18].

$$
\begin{aligned}
R_5^{-1} &= \frac{1}{2\pi}\frac{M_U^3}{m_{Pl}^2}(2\alpha_U)^{-3/2} \sim 10^{-3}\ M_U\ , \qquad (12)\\
M_{11} &= M_U(2\alpha_U)^{-1/6} \sim 1.5\ M_U\ ,
\end{aligned}
$$

where we have used $M_U = 3\ 10^{16}$ GeV, $\alpha_U^{-1} = 23.3$ and $m_{Pl} = M_{Pl}/\sqrt{8\pi} = 2.4\ 10^{18}$ GeV.

It is important to note that, because the original theory is not fully determined by the action (3), its compactified version is valid only for a certain range of mass scales. In particular, we disregarded the non-zero thickness of the boundary, presumably associated with some non-perturbative effect in quantum M-theory. Had we restored a non-vanishing thickness, the gauge fields of the boundary would propagate in the corresponding layer (of width of order M^{-1}): this would generate in the 4-dimensional theory massive states of mass M. Since we consider on the other hand the Kaluza-Klein states of mass R_5^{-1}, our treatment is not consistent unless we impose the condition (in our 5-dimensional units, $M = M_5$)[4]:

$$
M_5 R_5 = t \gg 1. \qquad (13)
$$

This is obviously verified if we plug in the data (12). Notice that, physically, $M_5 R_5$ is the number of Kaluza-Klein states of mass less than M_5, which contributes to computations involving Kaluza-Klein states running in loops.

Let us now turn to the anomalous $U(1)$ symmetry in this context. Since it necessarily couples the observable and the hidden sector which lie on the two boundaries of 11-dimensional spacetime, the corresponding gauge degrees of freedom necessarily live in the 11-dimensional bulk (or at least, when we consider the compactified theory, in the 5-dimensional bulk). Indeed, consider the limiting case when the two boundaries are far apart, $t \to \infty$ and therefore they do not interact with each other. In this case, the $U(1)_X$ gauge coupling should vanish and therefore the $U(1)_X$ gauge group is intimately related to the presence of the extra dimension. In particular, the heterotic perturbative gauge group, with a gauge coupling given by (10) does not satisfy this constraint and cannot describe, in the M-theory regime, a perturbative physics.

An obvious candidate for our anomalous $U(1)_X$ would be the 5-dimensional gauge field $C_{\mu IJ}$ (among which is found the graviphoton which we denote by C_μ) but it is odd under the Hořava-Witten Z_2 parity and therefore only the 4-dimensional scalar field C_{5IJ} has non-vanishing zero modes on the boundaries. We thus have to assume that an anomalous $U(1)_X$ symmetry has a non-perturbative origin (from the point of view of the weakly coupled heterotic string) and that the corresponding gauge field is even under the Z_2 parity. This

[4]This may also be seen in a more technical way: the delta functions δ which appear are invariant and therefore incorporate a factor $1/\sqrt{g_{5,5}}$ which is $1/t$ in our 5-dimensional units; thus expansion in the number of δ factors amounts in 4 dimensions to an expansion in t^{-1} in our units where the rescaling of the 4-dimensional metric is t-independent ($b = 0$ in (6)). Similarly derivatives such as δ' include a factor $1/g_{5,5}$ and therefore yield higher powers in $1/t$.

is indeed a generic situation after compactification, where the 5d bulk contain nonperturbative gauge fields and charged matter coming from 5-branes. The perturbative gauge fields on the boundary are interpreted in an open string language as coming from 9-branes. There are also mixed, $5-9$ sectors, corresponding, in an effective Horava-Witten type lagrangian, to boundary fields charged under the nonperturbative $U(1)_X$ gauge field [29].

We will illustrate how the anomaly arises on a 5-dimensional toy model, containing both 9-branes and 5-branes. Let us consider a 5-dimensional supergravity theory compactified on $\mathbf{R}^4 \times \mathbf{S}^1/Z_2$. The gauge degrees of freedom consist of a 5-dimensional vector superfield A_μ, *even under the Hořava-Witten Z_2 parity* and a 4-dimensional vector superfields A_m^a which propagate on the boundary of spacetime. The matter fields consist of: (i) bulk hypermultiplets $(\phi_+^a, \phi_-^a, \psi_+^a, \psi_-^a)$, where the subscripts $+$ and $-$ denote the Z_2 parity of the corresponding field, $\phi_\pm$ are complex scalar fields and $\psi_\pm$ are Weyl fermions, (ii) ordinary 4-dimensional matter living on the boundary of spacetime.

As is well-known [31], the supersymmetric Lagrangian describing the interactions of the vector superfields is determined by a function $\mathcal{N}(\xi^C, \xi^A)$, a homogeneous cubic polynomial of the coordinates ξ^C and ξ^A, which are in correspondence with the vector bosons C^μ and A^μ. The scalar component of the $U(1)_X$ vector multiplet parametrizes a one-dimensional hypersurface of equation $\mathcal{N}(\xi^C, \xi^A) = 1$ in the 2-dimensional manifold of coordinates (ξ^C, ξ^A). These variables have an Horava-Witten parity which is the opposite of the parity of the corresponding vector field, that is -1 for ξ^A and $+1$ for ξ^C.[5] Therefore, the function $\mathcal{N}$ compatible with the Horava-Witten parity is simply:

$$\mathcal{N}(\xi^C, \xi^A) = (\xi^C)^3 - \frac{3}{2}\xi^C(\xi^A)^2 \ , \tag{14}$$

where we have chosen the normalisation in such a way that one recovers diagonal and conveniently normalized gauge kinetic terms at the point $\xi^A = 0$ (see below).

The corresponding Lagrangian for the 5-dimensional gauge fields reads:

$$\begin{aligned}\mathcal{L} \;=\; & \frac{1}{\pi}\int d^5x \left[\sqrt{g^{(5)}}\left(-\frac{1}{4}G_{AA}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}G_{AC}F^{\mu\nu}C_{\mu\nu} - \frac{1}{4}G_{CC}C^{\mu\nu}C_{\mu\nu}\right)\right. \\ & \left. +\frac{1}{8}\epsilon^{\mu\nu\rho\sigma\lambda}C_{\mu\nu}C_{\rho\sigma}C_\lambda - \frac{3}{16}\epsilon^{\mu\nu\rho\sigma\lambda}F_{\mu\nu}F_{\rho\sigma}C_\lambda\right] , \end{aligned} \tag{15}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is our $U(1)_X$ field strength, $C_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu$ is the graviphoton field strength and $G_{\Lambda\Sigma} = -(1/2)\partial_\Lambda\partial_\Sigma \ln \mathcal{N}|_{\mathcal{N}=1}$. The topological terms are fixed by the requirements of gauge invariance and supersymmetry in 5 dimensions.

[5] This is more easily seen when compactifying to 4 dimensions where ξ^C (resp. ξ^A) lies in the same supermultiplet as C_5 (resp. A_5).

After compactification to 4 dimensions, one finds two chiral supermultiplets of respective scalar fields:

$$\begin{aligned} T^C &= t\xi^C + iC_5 \ , \\ T^A &= t\xi^A + iA_5 \ . \end{aligned} \tag{16}$$

We work now in Planck mass units where $t = e^\gamma$. The normalisation of the kinetic terms is evaluated at $\mathcal{N}(t\xi^C, t\xi^A) = t^3$ and, since T^A does not survive the Hořava-Witten projection, at $\xi^C = 1$. The Kähler potential for the remaining scalar field T^C is simply [31] $-\ln\mathcal{N} = -3\ln(T^C + \bar{T}^C)$. Taking into account the fact that the gauge field C_m does not survive either the Hořava-Witten projection, the 4-dimensional Lagrangian reads, including the scalar kinetic terms:

$$\begin{aligned} \mathcal{L} &= \int d^4x \left(\sqrt{g}\left[-\frac{3}{8} t F^{mn} F_{mn} + \frac{3}{(T^C + \bar{T}^C)^2} \partial^m T^C \partial_m \bar{T}^C \right] \right. \\ &\quad \left. -\frac{3}{8} \mathrm{Im} T^C F_{mn} \tilde{F}^{mn} \right) \ , \end{aligned} \tag{17}$$

Comparing the gauge kinetic term with (9), we see the $S \leftrightarrow T$ exchange characterizing perturbative-nonperturbative mapping in 4 dimensions [28].

Our model is somewhat reminiscent of a model discussed recently by Mirabelli and Peskin [30] and we follow the method devised by these authors to couple 4-dimensional boundary fields with the fields living in the 5-dimensional bulk.

In the following we need the propagators of the bulk hypermultiplets, which we compute in the limit of interest $R_5 M_5 \gg 1$. The Kaluza-Klein decomposition reads[6]:

$$\begin{aligned} (\phi_+, \psi_+) &= \frac{1}{\sqrt{2\pi}} \sum_{n=0}^{\infty} \cos nx_5 \ (\phi_+^{(n)}, \psi_+^{(n)}), \\ (\phi_-, \psi_-) &= \frac{1}{\sqrt{2\pi}} \sum_{n=1}^{\infty} \sin nx_5 \ (\phi_-^{(n)}, \psi_-^{(n)}) \ . \end{aligned} \tag{18}$$

The Feynman propagators are then computed to be

$$\begin{aligned} < 0|T\phi_+(x, x_5)\phi_+^*(y, y_5)|0 > &= \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \cos nx_5 \cos ny_5 \ \Delta_F^{(n)} \ , \\ < 0|T\phi_-(x, x_5)\phi_-^*(y, y_5)|0 > &= \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \sin nx_5 \sin ny_5 \ \Delta_F^{(n)} \ , \\ < 0|T\phi_+(x, x_5)\phi_-^*(y, y_5)|0 > &= 0 \ , \end{aligned}$$

[6]From now on, we use the dimensionless angular variable Mx_5, where M is the fundamental mass unit $m_{Pl}\sqrt{2\pi}$ introduced earlier, and for simplicity still denote it by x_5 $(-\pi < x_5 < \pi)$.

$$
\begin{aligned}
< 0|T\psi_+(x,x_5)\bar{\psi}_+(y,y_5)|0 > &= \frac{1}{2\pi}\sum_{n=-\infty}^{\infty} \cos nx_5 \cos ny_5 \; S_{F,++}^{(n)} \; , \\
< 0|T\psi_-(x,x_5)\bar{\psi}_-(y,y_5)|0 > &= \frac{1}{2\pi}\sum_{n=-\infty}^{\infty} \sin nx_5 \sin ny_5 \; S_{F,--}^{(n)} \; , \\
< 0|T\psi_+(x,x_5)\psi_-(y,y_5)|0 > &= \frac{1}{2\pi}\sum_{n=-\infty}^{\infty} \cos nx_5 \sin ny_5 \; S_{F,+-}^{(n)} \; , \qquad (19)
\end{aligned}
$$

where for example the fields $\phi_\pm^{(-n)} = \phi_\pm^{(n)}$, $\Delta_F^{(n)}$ is the Feynman propagator for a complex scalar field of mass $m_n^2 = n^2/R_5^2$ and we defined the massive fermion propagators

$$
\begin{aligned}
S_{F,++}^{(n)}(x,y) &= \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} \; \frac{\bar{\sigma}^m p_m}{p^2 - m_n^2 + i\epsilon} \; , \\
S_{F,+-}^{(n)}(x,y) &= \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)} \; \frac{m_n}{p^2 - m_n^2 + i\epsilon} \; , \qquad (20)
\end{aligned}
$$

and $S_{F,--}^{(n)} = S_{F,++}^{*(n)}$. By using a Schwinger proper-time representation, we can write, for example,

$$
< 0|T\phi_+(x,x_5)\phi_+^*(y,y_5)|0 >= \frac{1}{2\pi}\int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip(x-y)}}{p^2 + i\epsilon} J(x_5,y_5,pR_5) \; , \qquad (21)
$$

where

$$
J(x_5,y_5,pR_5) = \int_0^\infty dt e^{-t} \sum_n \cos nx_5 \cos ny_5 e^{-\frac{tn^2}{p^2R^2}} \simeq \pi[\delta(x_5+y_5)+\delta(x_5-y_5)] \; , \qquad (22)
$$

where, in the last step, we have evaluated the function J in the ultraviolet region $p \sim M_5$, that is, in the region of interest (13), $pR_5 \gg 1$. Indeed, in the following, we are interested in computing the triangle gauge anomaly and the induced Fayet-Iliopoulos term, the computation of which involves the ultraviolet behaviour of the propagators. In this case, the sum over Kaluza-Klein modes can be approximated by an integral. The final result is [7]:

$$
\begin{aligned}
< 0|T\phi_+(x,x_5)\phi_+^*(y,y_5)|0 > &= \frac{1}{2}[\delta(x_5-y_5)+\delta(x_5+y_5)]D_F(x-y) \; , \\
< 0|T\phi_-(x,x_5)\phi_-^*(y,y_5)|0 > &= \frac{1}{2}[\delta(x_5-y_5)-\delta(x_5+y_5)]D_F(x-y) \; , \\
< 0|T\phi_+(x,x_5)\phi_-^*(y,y_5)|0 > &= 0 \; , \qquad (23) \\
< 0|T\psi_+(x,x_5)\bar{\psi}_+(y,y_5)|0 > &= \frac{1}{2} \; [\delta(x_5-y_5)+\delta(x_5+y_5)]S_F(x-y) \; ,
\end{aligned}
$$

[7] All the δ functions in our paper are periodic of period 2π, $\delta(x_5) = \delta(x_5 - 2\pi)$.

$$< 0|T\psi_-(x,x_5)\bar{\psi}_-(y,y_5)|0> \;=\; \frac{1}{2}[\delta(x_5-y_5)-\delta(x_5+y_5)]S_F(x-y)\ ,$$
$$< 0|T\psi_+(x,x_5)\psi_-(y,y_5)|0> \;=\; \frac{1}{2R_5}[\delta'(x_5-y_5)-\delta'(x_5+y_5)]D_F(x-y),$$

where $D_F(x-y)$ and $S_F(x-y)$ are the Feynman propagators for a 4-dimensional complex massless boson and massless Weyl fermion, respectively. Consistently with our previous discussion, we neglect δ' type terms and therefore the last correlator in (24) is zero in the limit (13).

Using these results, we first compute the $U(1)_X^3$ triangle anomaly contribution coming from bulk hypermultiplets containing the fermions ψ_+^a, ψ_-^a of charges $\pm X_+^a$, described by the 5-dimensional Dirac fermions $\Psi^a = \begin{pmatrix} \psi_+^a \\ \bar{\psi}_-^a \end{pmatrix}$ (we use the Weyl basis in the following) and from the boundary fermions $\psi_{\varphi,i}$ living on the two boundaries $i=1,2$. More precisely, we compute (see figure 1) $< 0|\partial^m J_m|\gamma^{(0)}\gamma^{(0)}|0>$, where $\gamma^{(0)}$ is the zero-mode of the $U(1)_X$ gauge field and

$$\begin{aligned} J_m(x,x_5) &= \sum_a X_+^a \bar{\Psi}^a(x,x_5)\gamma_m \Psi^a(x,x_5) + \sum_i \delta(x_5-x_{5,i})X_{\varphi,i}\bar{\psi}_{\varphi,i}\bar{\sigma}_m\psi_{\varphi,i} \\ &= \sum_a \left[X_-^a\bar{\psi}_-^a(x,x_5)\bar{\sigma}_m\psi_-^a(x,x_5)+X_+^a\bar{\psi}_+^a(x,x_5)\bar{\sigma}_m\psi_+^a(x,x_5)\right] \\ &\quad + \sum_i \delta(x_5-x_{5,i})X_{\varphi,i}\bar{\psi}_{\varphi,i}\bar{\sigma}_m\psi_{\varphi,i}\ , \end{aligned} \tag{24}$$

where $X_-^a = -X_+^a$.

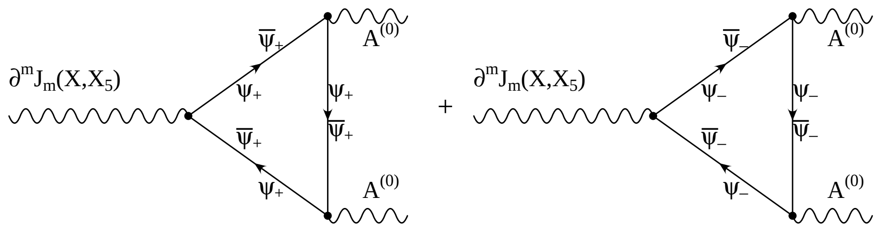

Figure 1: *Triangle diagram for the* $U(1)_X^3$ *anomaly.*

In this computation, we are using the following identity, valid for periodic delta functions:

$$\delta(2x_5) = \frac{1}{2}\left[\delta(x_5)+\delta(x_5-\pi)\right]. \tag{25}$$

By using the above propagators (24), we find

$$\partial^m J_m(x,x_5) = \ A_+ + A_- + \sum_i A_{\varphi,i} = \tag{26}$$

$$\left[(TrX^3_{\varphi,1} + \frac{1}{2}TrX^3_+)\delta(x_5) + (TrX^3_{\varphi,2} + \frac{1}{2}TrX^3_+)\delta(x_5-\pi)\right] \frac{g_4^2}{16\pi^2} F^{mn}\tilde{F}_{mn} \ ,$$

where A_+, A_- and $A_{\varphi,i}$ are the contributions of ψ_+, ψ_- and respectively $\psi_{\varphi,i}$ running in the loop, as in figure 1, and their explicit expression read

$$\begin{aligned} A_+ &= \frac{1}{4}\left[\delta(x_5) + \delta(x_5-\pi) + 2\delta(0)\right] TrX^3_+ \frac{g_4^2}{16\pi^2} F^{mn}\tilde{F}_{mn} \ , \\ A_- &= -\frac{1}{4}\left[\delta(x_5) + \delta(x_5-\pi) - 2\delta(0)\right] TrX^3_- \frac{g_4^2}{16\pi^2} F^{mn}\tilde{F}_{mn} \ , \\ A_{\varphi,i} &= \delta(x_5 - x_{5,i}) TrX^3_{\varphi,i} \frac{g_4^2}{16\pi^2} F^{mn}\tilde{F}_{mn} \ , \end{aligned} \tag{27}$$

Strictly speaking, there are other diagrams contributing to the anomaly, which however, give a result proportional to δ'' functions and are therefore consistently neglected within our hypothesis. A similar computation for $\partial^5 J_5$ gives a similar result involving δ'' . Similar computations can be made for the other, mixed gauge anomalies. The corresponding anomaly coefficients are

$$\begin{aligned} U(1)^3_X &: \ \sum_i (TrX^3_{\varphi,i} + \frac{1}{2}TrX^3_+)\delta(x_5 - x_{5,i}) \ , \\ U(1)_X &: \ \sum_i (TrX_{\varphi,i} + \frac{1}{2}TrX_+)\delta(x_5 - x_{5,i}) \ , \\ U(1)_X G_1^a G_1^b &: \ \delta^{ab} \sum_{R_a} TrX_{\varphi,1} T(R_a)\delta(x_5) \ , \\ U(1)_X G_2^c G_2^d &: \ \delta^{cd} \sum_{R_c} TrX_{\varphi,2} T(R_c)\delta(x_5-\pi) \ , \\ U(1)_X G_1^a G_2^c &: \ 0 \ . \end{aligned} \tag{28}$$

In (28), G_i stand for gauge groups on the boundary i and $T(R)$ for Dynkin index of charged fermions. The last mixed anomaly is automatically zero as no fermion living on the boundaries can be simultaneously charged under G_1 and G_2 gauge groups. The anomaly-free conditions are easily read from (28) which, if violated, signal the presence of a anomalous $U(1)_X$ in the theory.

It is interesting to notice the close analogy between (27) and the modified Bianchi identity [18] (written here in differential form language)

$$dG = -\frac{3\sqrt{2}}{2\pi}\left(\frac{\kappa_{11}}{4\pi}\right)^{\frac{2}{3}} \sum_i \delta(x_5 - x_{5,i})(trF_i^2 - \frac{1}{2}trR^2)dx^5 \ , \tag{29}$$

where G is the field strength of the three form , F_i is the gauge field form on the boundary i and trR^2 is computed from the curvature two form. The interpretation of (27) is, of course, similar to (29), i.e. the anomaly coming from the 5d hypermultiplets is equally distributed on the two boundaries, which was expected physically. If we integrate over x_5 in (27) in order to find the global anomaly term, we find, as usual

$$\partial^m J_m^{(0)} = (TrX_{\varphi,1}^3 + TrX_{\varphi,2}^3 + TrX_+^3)\frac{g_4^2}{16\pi^2}F^{mn}\tilde{F}_{mn} \ . \tag{30}$$

We now come to our main goal, the computation of the one-loop Fayet-Iliopoulos terms induced through the orbifold Z_2 projection. To accomodate the general case, we assume that both the fields living on the boundaries (denoted by φ, i in the following) and the bulk fields contribute to the anomaly and therefore generate Fayet-Iliopoulos terms. These can be simply found by computing the induced mass terms for the charged scalar fields, of either boundary or bulk type, to be obtained from the diagrams shown in figure 2. The results, obtained by using the interaction terms:

$$\begin{aligned} V_D &= \frac{g_5^2}{2}\int d^5x\sqrt{g}\left[X_+\phi_+\frac{\partial K}{\partial\phi_+} + X_-\phi_-\frac{\partial K}{\partial\phi_-}\right. \\ &\quad \left. + \sum_i \delta(x_5 - x_{5,i})X_{\varphi,i}\varphi_i\frac{\partial K}{\partial\varphi_i}\right]^2 \end{aligned} \tag{31}$$

and the propagators (24) are

$$\begin{aligned} m_{\varphi,i}^2 &= g_4^2\delta(0)(TrX_{\varphi,i} + \frac{1}{2}TrX_+)X_{\varphi,i}\frac{1}{192\pi^2}M_5^2 \ , \\ m_{X_+}^2 &= g_4^2(TrX_{\varphi,1} + TrX_{\varphi,2} + TrX_+)X_+\frac{1}{192\pi^2}M_5^2 \ , \end{aligned} \tag{32}$$

where we choose the ultraviolet regulator in figure 2, in complete analogy with the weakly-coupled case, as in the second reference in [1]. More precisely, we cut-off the momentum integration at $p^2 = (1/3)M_5^2$, as the effective field theory description breaks down above M_5^2 and the numerical factor $1/3$ is due to the stringy cut-off [1].

The results can be interpreted as the generation in the effective lagrangian of a Fayet-Iliopoulos term on each boundary ξ_i, such that the $U(1)_X$ D-term in the scalar potential reads

$$\begin{aligned} V_D &= \frac{g_5^2}{2}\int d^5x\sqrt{g}\left[X_+\phi_+\frac{\partial K}{\partial\phi_+} + X_-\phi_-\frac{\partial K}{\partial\phi_-}\right. \\ &\quad \left. + \sum_i \delta(x_5 - x_{5,i})X_{\varphi,i}\varphi_i\frac{\partial K}{\partial\varphi_i} + \sum_i \delta(x_5 - x_{5,i})\xi_i\right]^2 \ , \end{aligned} \tag{33}$$

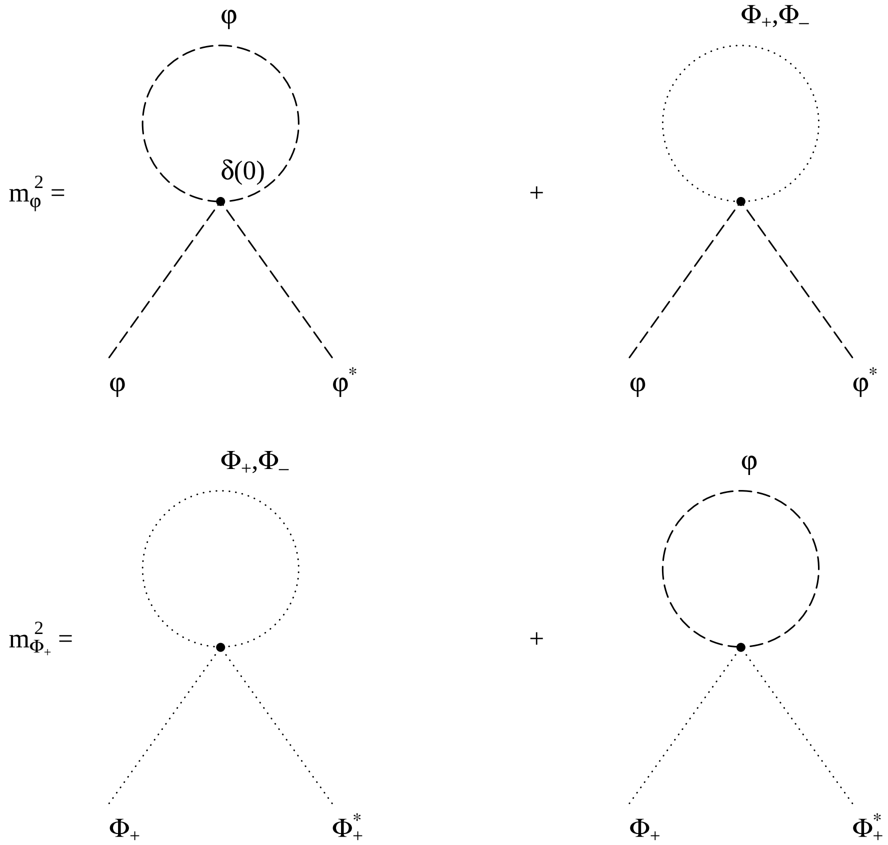

Figure 2: *One loop diagrams involved in the computation of the induced mass terms for the charged scalar fields, on the boundary (φ) or in the bulk (Φ).*

where

$$\xi_1 = (TrX_{\varphi,1} + \frac{1}{2}TrX_+)\frac{1}{192\pi^2}M_5^2 \ , \xi_2 = (TrX_{\varphi,2} + \frac{1}{2}TrX_+)\frac{1}{192\pi^2}M_5^2 \quad (34)$$

and $g_4^2 = 2\pi g_5^2/t$. In analogy with (27), here also the global Fayet-Iliopoulos term, obtained by integrating over x_5 the above densities is the expression

$$\xi = (TrX_{\varphi,1} + TrX_{\varphi,2} + TrX_+)\frac{1}{192\pi^2}M_5^2 \ , \quad (35)$$

very similar to the one obtained in the perturbative heterotic string where in (35) M_5 is replaced by m_{Pl}.

This gives a slightly lower scale of breaking for the anomalous $U(1)_X$, which goes in the right direction for solving the cosmological issues related with such a symmetry [15, 16].

A supersymmetry preserving vacuum in (33) is found if on each boundary there is at least one scalar field φ, i of charge opposite in sign to the corresponding Fayet-Iliopoulos term ξ_i, which takes a compensating v.e.v. [8]

$$\varphi_1 = -\frac{\xi_1}{X_{\varphi,1}} \ , \varphi_2 = -\frac{\xi_2}{X_{\varphi,2}} \ . \tag{36}$$

Supersymmetry can be broken in this context if a gaugino condensate $< \lambda\lambda >$ forms on one boundary (2, for concreteness, with a Fayet-Iliopoulos term ξ_2) and the corresponding F-term condition is incompatible with the $U(1)_X$ D-term condition. As in [11], the resulting soft terms in the observable sector (boundary 1) are

$$\tilde{m} \sim \frac{< \lambda\lambda >}{\xi_2} \ . \tag{37}$$

The usual supergravity-induced soft terms by the gaugino condensation [24] are here much smaller, therefore the anomalous $U(1)_X$ contributions are dominant. Phenomenologically relevant soft terms ask therefore for a condensate scale $< \lambda\lambda > \sim (10^{11} GeV)^3$, lower than the analogous one in the perturbative heterotic case.

The generalization of the above results to the case of more than one perturbative C_μ^α and nonperturbative gauge fields A_μ^i is straightforward. The five-dimensional bosonic spectrum related to gauge fields contains the gravitational multiplet $(g_{\mu\nu}, C_\mu)$, perturbative vector multiplets $(C_\mu^\alpha, \xi_C^\alpha)$, $\alpha = 1 \cdots h_{1,1} - 1$, odd under Z_2 projection and even nonperturbative vector multiplets (A_μ^i, ξ_A^i), where $h_{1,1}$ characterizes the 6-dimensional compact manifold with the intersection numbers $c_{\alpha\beta\gamma}$. The 5-dimensional prepotential describing the vector action is

$$\mathcal{N} = \frac{1}{6} c_{\alpha\beta\gamma} \xi_C^\alpha \xi_C^\beta \xi_C^\gamma - \frac{1}{2} c_{\alpha i i} \xi_C^\alpha (\xi_A^i)^2 \ . \tag{38}$$

The four-dimensional action is found by putting $\mathcal{N} = 1$, $\xi_A^i = 0$ and the 4-dimensional even complex moduli are

$$T_C^\alpha = t \xi_C^\alpha + i C_5^\alpha \ . \tag{39}$$

The 4d gauge kinetic function of the nonperturbative gauge fields then read

$$f_{ij} = -\frac{1}{2} \partial_i \partial_j \ln \mathcal{N}|_{\mathcal{N}=1, \xi_A^i = 0} = c_{\alpha ij} T_C^\alpha \tag{40}$$

and are therefore non-universal, depending on compactification details. The 4-dimensional Green-Schwarz mechanism will generalize accordingly, involving

[8] A v.e.v. for a bulk field of order $\delta(x_5 - x_{5,i}) \xi_i$ would not minimize the kinetic energy density.

more moduli axions able to shift gauge anomalies, in analogy with the six-dimensional case [32].

Note added

After this work was completed, we received the preprint [33], where the question of anomalous $U(1)_X$ in M-theory and open strings was studied. Our arguments seem to indicate that the universal anomalous $U(1)_X$ of [1] considered in [33] cannot give a one-loop Fayet-Iliopoulos term in M-theory and rather describes some unkonwn, nonperturbative physics.

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