

# A No-Lose Theorem for Higgs Searches at a Future Linear Collider

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Assuming perturbativity up to a high energy scale  $\sim 10^{16}$  GeV, we demonstrate that a future  $e^+e^-$  linear collider operating at  $\sqrt{s} = 500$  GeV with  $\int \mathcal{L} = 500$  fb<sup>-1</sup> per year (such as the recently proposed TESLA facility) will detect a Higgs boson signal regardless of the complexity of the Higgs sector and of how the Higgs bosons decay.

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**1.** In this paper, we show that an  $e^+e^-$  collider of sufficient luminosity (which we quantify) will be guaranteed to find a signal for a Higgs boson [1] if the model remains perturbative up to energy scales of order  $10^{16}$  GeV. The maximal luminosity is that needed to detect a broad enhancement in the  $M_X$  spectrum in  $e^+e^- \rightarrow ZX \rightarrow e^+e^-X + \mu^+\mu^-X$  over the perturbatively allowed  $M_X$  mass region.

For simplicity, we will adopt a notation that assumes CP conservation in the Higgs sector; generalization is straightforward. A general neutral Higgs field  $\Phi_\alpha$  in an arbitrary  $(t_\alpha, Y_\alpha)$  representation is written in terms of its real (CP-even) and imaginary (CP-odd) components as  $\Phi_\alpha^0 = \frac{1}{\sqrt{2}}(h_\alpha^{0r} + v_\alpha + ih_\alpha^{0i})$ , where  $v_\alpha$  is the vacuum expectation value acquired by the field as a result of electroweak symmetry breaking.

The crucial ingredient for the no-lose theorem is that there must be Higgs bosons with mass below and in the vicinity of a perturbatively limited mass scale  $m_B$  (of order the Fermi scale) which, in aggregate, have net coupling-squared to  $ZZ$  that is at least as large as the SM value. The proof begins with two crucial inequalities. Defining

$$\phi_Z^0 = \frac{1}{N_Z} \sum_\alpha t_{3\alpha}^2 h_\alpha^{0r} v_\alpha, \quad (1)$$

where  $N_Z$  is a normalization constant, the first inequality is [2,3]

$$\langle \phi_Z^0 | M^2 | \phi_Z^0 \rangle \leq \lambda v^2 \equiv m_B^2, \quad (2)$$

where  $M^2$  is the mass-squared operator for the neutral Higgs bosons,  $\lambda$  is some adimensional

quartic Higgs coupling and  $v^2 = 1/(\sqrt{2}G_F) \sim (246 \text{ GeV})^2$ . The coupling  $\lambda$  is not asymptotically free and can be bounded above by triviality arguments assuming it remains perturbative up to a high-energy scale  $\Lambda$ . The model-dependence of  $m_B$  will be discussed below. The second inequality is

$$g_{ZZ\phi_Z^0}^2 \geq g_{ZZh_{\text{SM}}}^2 = g^2 m_Z^2 / \cos^2 \theta_W. \quad (3)$$

(The equality holds only for  $t_{3\alpha} = 0, \pm 1/2$ .) We now define  $K_i = |\langle \phi_Z^0 | h_i^0 \rangle|^2$ , where  $\sum_i K_i = 1$ , and insert complete sets of  $h_i^0$  states in (2) to obtain

$$\langle \phi_Z^0 | M^2 | \phi_Z^0 \rangle = \sum_i K_i m_{h_i^0}^2 \leq m_B^2. \quad (4)$$

Further, since  $\phi_Z^0$  defines the combination of states that couples to  $ZZ$ , we have  $g_{ZZh_i^0}^2 = K_i g_{ZZ\phi_Z^0}^2$ . Similar results apply in the  $WW$  sector.

**2.** In the SM with  $\Lambda \sim M_{Pl}$ ,  $m_B \sim 190$  GeV [4], and modest luminosity at  $\sqrt{s} = 500$  GeV would be sufficient to detect the unique Higgs boson [5]. In the MSSM, there are two CP-even Higgs bosons, and, for  $\Lambda_{SUSY} \leq 1$  TeV and  $\Lambda = \Lambda_{GUT} \sim 10^{16}$  GeV,  $m_B \lesssim 125$  GeV [6]. Eq. (4) implies that, when  $K_1 \rightarrow 0$ ,  $m_{h_2} \leq m_B$  and  $K_2 \rightarrow 1$ . For  $\sqrt{s}$  well above  $m_B$ , this is sufficient to guarantee Higgs detection in the  $h_{1,2}Z$  channels alone [5]. (At LEP2, the complementary channel  $h_1A$  must be used to cover the  $K_1 \rightarrow 0$  case although large radiative corrections render that channel inoperative in some small region of parameter space [7].) In the NMSSM (the MSSM extended by a

singlet chiral multiplet),  $m_B \sim 140$  GeV [8] for the same  $\Lambda_{GUT}$ , and there are three CP-even Higgs bosons. Using the fact that  $\sigma_{Zh}(m_h)$  is a decreasing function of  $m_h$  (for constant  $g_{ZZh}^2$ ) and using our previous results, the minimum cross section for any one of the Higgs bosons,

$$\sigma_{\min} \equiv \min [\{\max\}_i \sigma_{Zh}(h_i^0)] , \quad (5)$$

[where  $\sigma_{Zh}(h_i^0) = K_i \sigma_{SM}(m_{h_i^0})$ ] must be bigger than [9,8]  $\sigma_{SM}(m_B)/3$ , as achieved for  $m_{h_i^0} = m_B$  and  $K_i = 1/3$  for  $i = 1, 2, 3$ . For  $\sqrt{s} = 300$  GeV (and  $m_B = 140$  GeV),  $\sigma_{\min} \sim 61$  fb. (This improves over Ref. [9] by a different use of the mass inequalities.) For this  $\sigma_{\min}$ , an integrated luminosity of  $L = 10 \text{ fb}^{-1}$  would be more than adequate to detect at least one of the three CP-even scalars. In contrast, there are substantial regions of NMSSM parameter space for which none of the Higgs bosons will be discovered at the LHC [10].

In general SUSY models with arbitrary particle content,  $m_B$  can be as high as 200 GeV for  $\Lambda \sim 10^{17}$  GeV [11]. As fundamental bosons at the electroweak scale are natural only in SUSY, we will take  $m_B \lesssim 200$  GeV for the general case. Experimentally, the success of fits to precision electroweak data using a single light Higgs boson with SM-like couplings and mass  $\lesssim 200$  GeV implies that, in a multi-Higgs model, the Higgs bosons with large ZZ coupling must have average mass  $\lesssim 200$  GeV.

**3.** The minimal cross section (5) gets smaller if more singlets are added to the model [9] (a large number of singlets is predicted in some string models) or if there is CP violation with mixing of scalar and pseudoscalar Higgses. Roughly,  $\sigma_{\min} \sim \sigma_{SM}(m_B)/N$  where  $N$  counts the number of neutral mass eigenstates that can mix with  $\phi_Z^0$ . However, for large  $N$  there is inadequate mass resolution to separate the various Higgs bosons if they all have mass  $\sim m_B$ . One should simply start adding their signals together (in all our discussions, we assume that the widths of the individual Higgs boson eigenstates are sufficiently small that interference effects can be neglected) inside mass bins of size determined by the experimental resolution,  $\Delta m$  (defined as full width at half maximum). Consequently, the worst-case situation in the limit of large  $N$  arises when all of the Higgs bosons that can couple to  $\phi_Z^0$  have mass spread out below and somewhat above  $m_B$ , but are spaced more closely than  $\Delta m$ ,

and have coupling  $K_i$  such that  $K_i \sigma_{SM}(m_{h_i^0})$  closely mimics the background of relevance. In this way, the only observable becomes the excess of overall rate as compared to that expected from background over some mass interval. The resonance peaks of the Higgs bosons will not be observable.

To analyze this situation more quantitatively, we first note that once the spacing is smaller than  $\Delta m$  it becomes irrelevant as to how many Higgs bosons there are. Thus, we will simply convert to an integral notation. Our crucial sum rules become:

$$\int dm_h K(m_h) = 1 \quad (6)$$

$$\int dm_h K(m_h) m_h^2 \leq m_B^2. \quad (7)$$

Since it seems overly perverse to assume that  $K(m_h) \sigma_{SM}(m_h)$  is exactly proportional to the background cross section as a function of  $m_h$  (after cuts and efficiencies), we assume a spectrum of Higgs bosons that is uniform in  $m_h$  up to some maximum value  $m_h^{\max}$  starting from  $m_h^{\min}$ . For  $K(m_h) = K$ , (6) implies  $K = 1/(m_h^{\max} - m_h^{\min})$ , and (7) implies

$$m_B^2 \geq \frac{1}{3} ([m_h^{\max}]^2 + m_h^{\max} m_h^{\min} + [m_h^{\min}]^2). \quad (8)$$

The maximal spread is achieved for  $m_h^{\min} = 0$ , in which case (8) requires  $m_h^{\max} \leq \sqrt{3} m_B$ . (The minimal spread, arising when  $m_h^{\max} = m_h^{\min} \leq m_B$ , is not of interest for our worst-case considerations.) To have the greatest sensitivity to the maximal spread region, we ideally require  $\sqrt{s}$  large enough that  $\sigma_{SM}(m_h)$  is large for all  $m_h \lesssim \sqrt{3} m_B$ . For  $m_B = 200$  GeV,  $\sqrt{3} m_B \sim 340$  GeV and quite large  $\sqrt{s}$  would be required to achieve this ideal. However,  $\sqrt{s} \sim 500$  GeV would not be too bad. For this  $\sqrt{s}$ ,  $\sigma_{SM}(m_h)$  varies from  $\sim 70$  fb at low  $m_h$  falling to  $\sim 15$  fb at  $m_h \sim 340$  GeV. At  $m_h \sim 200$  GeV,  $\sigma_{SM}(m_h)$  has only fallen to  $\sim 42$  fb. For this reason, we will confine ourselves to the  $\leq 200$  GeV window in  $m_h$  in discussing experimental sensitivity.

Another important issue is the channel(s) in which we must detect the Higgs bosons. In the worst case, the Higgs bosons will decay invisibly (*e.g.* to two LSP's in a supersymmetric model) or to a large number of channels. In these cases, identification of the final state would either not be possible or would not be useful because of the large background in

any one channel (respectively). The only robust procedure is to look for  $e^+e^- \rightarrow Zh$  and, possibly,  $e^+e^- \rightarrow e^+e^-h$  ( $ZZ$ -fusion) production in the  $e^+e^- \rightarrow ZX$  (with  $Z \rightarrow e^+e^-$  and  $\mu^+\mu^-$ ) and  $e^+e^- \rightarrow e^+e^-X$  channels, respectively — we employ cuts that separate  $e^+e^- \rightarrow Zh, Z \rightarrow e^+e^-$  from  $e^+e^- \rightarrow e^+e^-h$  via  $ZZ$ -fusion — as an excess in the recoil  $M_X$  distribution due to the continuum of Higgs bosons. Our focus will be on the  $Zh$  channel.

4. We find it most convenient to use the analysis of Ref. [12]. Although the cuts employed there restrict the analysis to the region  $m_h = 70 - 200$  GeV, the results will be very representative of what can be achieved. Roughly, a fraction  $130 \text{ GeV}/340 \text{ GeV} \sim 0.4$  of the signal weight would be in this region for  $m_h^{\min} = 0$  and  $m_h$ -independent  $K$ . Of course, roughly  $1/5$  of this worst-case continuum Higgs spectrum would lie in the  $\lesssim 70$  GeV region to which LEP2 certainly has sensitivity. We believe that more weight in this region can probably already be eliminated using current data. If the LEP limits prove even stronger, then we would have to increase  $m_h^{\min}$  to an appropriate lower limit and our conclusions for a future  $e^+e^-$  machine would be improved. For example, for  $m_h^{\min} = 70$  GeV (*i.e.* very small  $K$  being required by LEP2 data below this), then  $m_h^{\max} \sim 300$  GeV is required for  $m_B = 200$  GeV. and a fraction  $130/230 \sim 0.55$  of the Higgs continuum signal would be in the mass window we shall consider.

The results of Ref. [12] are best summarized in two tables. In Table I, we give the approximate signal and background rates for  $L = 500 \text{ fb}^{-1}$  for the  $Zh$  and  $ZZ$ -fusion contributions to the  $e^+e^- \rightarrow e^+e^-h$  channel, and the corresponding  $S/\sqrt{B}$  values obtained by integrating over two different mass intervals:  $70 - 200$  GeV, which includes the  $Z$  peak in the background; and  $100 - 200$  GeV, which starts above the  $Z$  peak. For both intervals we have assumed a “continuum” of Higgs bosons in the mass interval, the sum of whose coupling-squared to  $ZZ$  is equal to that of the SM Higgs boson. The  $Zh$  with  $Z \rightarrow \mu^+\mu^-$  channel would give  $S$  and  $B$  values that are essentially the same as for  $Zh$  with  $Z \rightarrow e^+e^-$ . Summing the  $e^+e^-$  and  $\mu^+\mu^-$   $Zh$  channels yields  $S \sim 1350$ , and  $S/\sqrt{B} \sim 17$  and  $\sim 26$  for the  $70 - 200$  GeV and  $100 - 200$  GeV windows in  $M_X$ , respectively. Of course, both  $S$  and  $S/\sqrt{B}$  must be reduced by the fraction  $f$  of the Higgs signal

contained in the mass interval. Above, we argued that  $f \sim 0.4 - 0.55$  is an appropriate range to consider. Then, for  $L = 500 \text{ fb}^{-1}$ , we would have a net signal of  $S \sim 1350f$  with a background of either  $B = 6350$  or  $B = 2700$ , depending on which of the two mass intervals are considered. For the  $70 - 200$  GeV ( $100 - 200$  GeV) interval, we have to detect the presence of a broad  $\sim 21\%f$  ( $\sim 50\%f$ ) excess over background, respectively. Understanding the absolute normalization of the backgrounds to the  $20\%f$  level of accuracy is probably feasible at an  $e^+e^-$  collider for  $f \gtrsim 0.4$ ; a  $50\%f$  excess would certainly be observable.

We have included the  $e^+e^- \rightarrow e^+e^-h$   $ZZ$ -fusion channel in Table I to indicate how difficult its use would be in this situation. Even though  $ZZ$ -fusion yields a large nominal  $S/\sqrt{B}$ , the  $S/B$  values are  $0.032f$  and  $0.058f$  for the  $70 - 200$  GeV and  $100 - 200$  GeV intervals, respectively. The first excess would almost certainly be impossible to detect as a broad enhancement. Even the  $\sim 6\%$  ( $f = 1$ ) excess over the latter interval would be difficult to be certain of.

If we decrease  $L$  to  $50 \text{ fb}^{-1}$ , we lose a factor of  $\sqrt{10}$  in our  $S/\sqrt{B}$  values, leaving effects at the  $S/\sqrt{B} = 5.3f$  and  $8.2f$  level in the  $Zh$  (with  $Z \rightarrow e^+e^- + \mu^+\mu^-$ ) channel for the  $70 - 200$  GeV and  $100 - 200$  GeV intervals, respectively, which is borderline for  $f \sim 0.4 - 0.55$ .

5. If such a broad continuum-like signal is observed, the next important step will be to divide the  $M_X$  spectrum up into bins, knowing that in each bin there is presumably an excess of events due to some set of Higgs bosons with some portion of the net SM  $ZZ$  coupling. Just to illustrate what the situation might be, suppose we distribute the roughly 1350 SM signal events (for  $Zh$  with  $Z \rightarrow e^+e^-$  and  $\mu^+\mu^-$ ) equally in the thirteen 10 GeV bins from 70 to 200 GeV. The level of signal and background, and corresponding  $S/\sqrt{B}$  value, in each bin is given in Table II. Note that both  $S$  and  $S/\sqrt{B}$  would more generally be reduced by the fraction  $f$  of the SM signal present in the  $70 - 200$  GeV mass window.

Table II shows that  $L = 500 \text{ fb}^{-1}$  would yield  $S/\sqrt{B} > 3$  only for the  $M_X \gtrsim 120$  GeV bins when  $f \sim 0.5$ . With only  $L = 100 \text{ fb}^{-1}$  (as might be achieved after a few years of running at a lower luminosity design), this bin-by-

bin type of analysis would not be possible for 10 GeV bins if  $f \sim 0.5$ .

It is useful to determine the number of Higgs bosons that would necessitate considering the continuum limit. Suppose there were 13 spread out over the 70 GeV to 200 GeV interval, *i.e.* a Higgs boson every 10 GeV. The resulting signals would be impossible to resolve into separate mass peaks if  $\Delta m \gtrsim 10$  GeV. In fact, this is nearly twice as good as the typical  $M_X$  resolution found in Ref. [12] using ‘super-JLC’ tracking (which, in turn, is substantially better than currently planned for first-round NLC detectors). Thus, for current detector designs it would appear that a model with  $\gtrsim 10$  Higgs bosons that can couple to  $ZZ$  would be in danger of leading to a continuum Higgs signal.

**6.** In summary, we have considered a ‘worst-case’ Higgs scenario with a large number of Higgs bosons which cannot be separated from one another given expected detector resolutions, and which all decay invisibly or to many final states. In this case, the only viable signal is a continuum excess in the  $e^+e^- \rightarrow ZX$  (with  $Z \rightarrow e^+e^-$  and  $\mu^+\mu^-$ ) recoil  $M_X$  spectrum. We have shown that the assumption of perturbativity up to high scales and the bounds/inequalities derived therefrom, guarantee the observability of this signal provided that an integrated luminosity in excess of  $L = 100 \text{ fb}^{-1}$  is accumulated. Bin-by-bin analysis of such a continuum excess would very possibly require  $L \geq 500 \text{ fb}^{-1}$ , implying that this situation provides a rather strong motivation for going to a very high  $L$  collider design.

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TABLE I. Approximate  $S$ ,  $B$  and  $S/\sqrt{B}$  values for the  $Zh$  and  $ZZ$ -fusion contributions to  $e^+e^- \rightarrow e^+e^-h$ , after integrating the  $M_X$  recoil mass spectrum from (a) 70 GeV to 200 GeV and (b) 100 GeV to 200 GeV, assuming that many Higgs bosons are distributed evenly throughout the interval and have  $\sum_h g_{ZZh}^2 = g_{ZZh_{SM}}^2$ . Results are for  $\sqrt{s} = 500 \text{ GeV}$ ,  $L = 500 \text{ fb}^{-1}$ , and assume the cuts and efficiencies of Ref. [12].

$M_X$ Interval	$Zh$			$ZZ$ -fusion		
	$S$	$B$	$S/\sqrt{B}$	$S$	$B$	$S/\sqrt{B}$
70 – 200	675	3170	12.0	2615	80100	9.2
100 – 200	678	1350	18.5	2300	39600	11.5

TABLE II. Approximate  $S$ ,  $B$  and  $S/\sqrt{B}$  values for  $Zh$  (with  $Z \rightarrow e^+e^- + \mu^+\mu^-$ ) in each of the thirteen 10 GeV bins in  $M_X$  from 70 to 200 GeV, assuming that  $S \sim 1350$  events, from Higgs bosons with  $\sum_h g_{ZZh}^2 = g_{ZZh_{SM}}^2$ , are distributed equally among these bins. We assume  $\sqrt{s} = 500 \text{ GeV}$ ,  $L = 500 \text{ fb}^{-1}$ , and use the results of Ref. [12].

Bin No.	1	2	3	4	5	6	7	8–13
$S$	104	104	104	104	104	104	104	104
$B$	1020	1560	1440	734	296	162	125	$\sim 130$
$S/\sqrt{B}$	3.3	2.6	2.7	3.8	6.0	8.2	9.3	9.1