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Exploring the light-cone through semi-inclusive hadronic distributions

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Abstract: Light-cone dominance is established for the energy angular distribution of fast hadrons in e^+e^- annihilation. The analysis presented does not explicitly rely on perturbation theory and is based on the space-time description of the scattering process.

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Contents

1. Introduction

Perturbative QCD provides a successful approximation scheme for the description of hadronic processes involving large momentum transfers. While, originally, quantities reliably computable in perturbation theory were restricted to those related to various forms of operator expansion (light-cone or short-distance), subsequently the observables amenable to perturbative evaluation have been enlarged to encompass the so-called infrared- and collinear-safe ones [1]. Observables related to short-distance or light-cone singularities, however, still maintain a privileged status. In fact, while from one side the use of perturbation theory for their evaluation is justified from first principles, on the other side there is also, at least in principle, the possibility to control pre-leading terms through the use of the operator product expansion.

$e^+e^- \to$ hadrons was among the first reactions treated through operator singularity techniques [2]. Its peculiar interest lies in the fact that, contrary to the other classical light-cone dominated reactions, the deep-inelastic scattering, it does not require the introduction of non-perturbative hadron parameters.

However, as soon as we ask more detailed questions related to the structure of the hadronic final state, the only available theoretical instrument is renormalization improved perturbation theory. Using this technique it has been possible to formulate plausible arguments showing the jet-like distribution of the produced hadrons[3].

In this paper we show, through a non perturbative analysis based on the space-time description of the scattering process, that the energy angular distribution of fast hadrons in e^+e^- annihilation, first discussed in ref. [4], can be interpreted in terms of light-cone singularities of three local operators: two insertions of the electromagnetic current and the energy-momentum tensor. Such connection was, in fact, shadowed by the authors of ref. [4], but not exploited by them.

Similar considerations apply to the complete hierarchy of energy-energy correlation observables considered in ref.[4], but, for simplicity, we will not address to them explicitly.

Approaches somewhat related to the one presented here are discussed in refs. [5, 6]: in particular some of the ingredients of the present paper can be found in ref. [6]. However the main point here, as contrasted to refs. [5, 6], is the use of localized observables which, only, allows the exploration of light-cone singularities.

The use of local observables, however, brings in two basic difficulties:

- the need of localizing the interaction region, in order to be able to define its light-cone;
- the intrinsic non-additivity of local operators.

As for the first point, we will use the space-time approach to scattering, which has been recently shown [7] to be quite successful in the discussion of the properties of unstable states in relativistic quantum field theory and we will show that the connectivity properties of matrix elements provide a natural way to deal with the second point.

In sections 2 and 3 we will discuss the structure of the hadron energy distribution in terms of hadronic intermediate states, both in the massive and in the massless situation; in section 4 we will establish its light-cone dominance. In section 5 we present the conclusions.

2. Hadronic analysis

In order not to obscure the exposition with kinematical details we will treat a schematized problem in which electrons and positrons are scalar particles which interact with hadrons through a contact interaction, with an action:

$$S_I = \int d^4x e^\dagger(x) e(x) J(x) \,, \tag{2.1}$$

where $e(x)$ denotes the (scalar) electron field, $J(x)$ the hadronic current and the coupling constant has been reabsorbed in the definition of the current (for instance $J(x) = g\,\phi^2(x)$).

The starting point is the construction of the initial state. It consists of an e^+ and an e^- with wave functions localized far apart, at large negative times, and overlapping around the origin of coordinates, around time $t = 0$:

$$|in\rangle = \int d^3p_1 d^3p_2 f_{\underline{p}_1} g_{\underline{p}_2} |\underline{p}_1, \underline{p}_2; in\rangle \,. \tag{2.2}$$

Considering the space-time wave functions $f(\underline{x}, t)$ and $g(\underline{y}, t)$ associated to $f_{\underline{p}}$ and $g_{\underline{p}}$, where, e.g.,

$$f(\underline{x}, t) \equiv \int \frac{d^3p}{\sqrt{(2\pi)^3 2\omega_{\underline{p}}}} f_{\underline{p}} e^{-i\omega_{\underline{p}} t + i\underline{p}\cdot\underline{x}} \,, \tag{2.3}$$

the above requirements amount to say that the supports in space of $f(\underline{x}, t)$ and $g(\underline{y}, t)$ are disjoint for negative time t, while overlapping around time $t = 0$.

We will further assume that the momentum spreads of $f_{\underline{p}}$ and $g_{\underline{p}}$ are very narrow, so that $f_{\underline{p}}$ is strongly peaked around some given momentum $\underline{p}_0$ and $g_{\underline{p}}$ around momentum $-\underline{p}_0$ so that the total four-momentum is essentially equal to

$$Q^\star \approx (2\omega_{\underline{p}_0}, \underline{0}) \tag{2.4}$$

and we are considering the limit $|\underline{p}_0| \to \infty$.

The state $|in\rangle$ is a superposition of a freely propagating e^+e^--pair state

$$|out\rangle = \int d^3p_1 d^3p_2 f_{\underline{p}_1} g_{\underline{p}_2} |\underline{p}_1, \underline{p}_2; out\rangle \tag{2.5}$$

and a state

$$|h\rangle \equiv |in\rangle - |out\rangle \tag{2.6}$$

in which the interaction actually takes place, giving rise to hadron production. The norm $\langle h|h\rangle$ of the state defined in eq. (2.6) is the probability that hadron production actually occurs in the scattering and is therefore proportional to the total hadron cross section $\sigma_{e^+e^- \to hadrons}$.

At the lowest order in the interaction described by eq. (2.1), $|h\rangle$ is given by

$$\begin{aligned} |h\rangle &= i \int d^4x d^3p_1 d^3p_2 \frac{f_{\underline{p}_1} e^{-ip_1x}}{\sqrt{(2\pi)^3 2\omega_{\underline{p}_1}}} \frac{g_{\underline{p}_2} e^{-ip_2x}}{\sqrt{(2\pi)^3 2\omega_{\underline{p}_2}}} J(x)|0\rangle = \\ &= i \int d^4x f(x) g(x) J(x)|0\rangle \equiv \\ &\equiv i \int d^4x F(x) J(x)|0\rangle \,, \end{aligned} \tag{2.7}$$

where

$$F(x) \equiv f(x) g(x) \,. \tag{2.8}$$

In view of the assumptions made on $f(x)$ and $g(x)$, $F(x)$ has a support well localized around the origin of space-time and its Fourier transform

$$\tilde{F}(Q) \equiv \int d^4x F(x) e^{iQx} \tag{2.9}$$

is narrowly peaked around the four-vector $Q^\star$ defined in eq. (2.4).

The probability of hadron production can then be written as

$$\begin{aligned} \langle h|h\rangle &= \int d^4x d^4y F^*(y) F(x) \langle 0|J(y)J(x)|0\rangle = \\ &= \int d^4x d^4y F^*(y) F(x) \langle 0|J(0)J(x-y)|0\rangle = \\ &= \int d^4x d^4y F^*(y) F(x+y) \langle 0|J(0)J(x)|0\rangle = \end{aligned}$$

$$
\begin{aligned}
&= \int d^4Q|\tilde{F}(Q)|^2 \int \frac{d^4x}{(2\pi)^4} e^{iQx} \langle 0|J(0)J(x)|0\rangle \approx \\
&\approx \int \frac{d^4x}{(2\pi)^4} e^{iQ^\star x} \langle 0|J(0)J(x)|0\rangle \int d^4Q|\tilde{F}(Q)|^2 \equiv \\
&\equiv \Pi(Q^{\star 2}) \int d^4Q|\tilde{F}(Q)|^2 \,. \qquad (2.10)
\end{aligned}
$$

In eq. (2.10) we used the narrow packet approximation in order to factorize the initial wave function dependence in the factor $\int d^4Q|\tilde{F}(Q)|^2$.

The observable we want to study is related to the expectation value of the hadronic energy-momentum tensor evaluated at a space-time point z:

$$
\langle \theta^{\mu\nu}(z)\rangle \equiv \frac{\langle h|\theta^{\mu\nu}(z)|h\rangle}{\langle h|h\rangle} \equiv \frac{S^{\mu\nu}(z)}{\langle h|h\rangle}\,, \qquad (2.11)
$$

where $S^{\mu\nu}(z)$ is defined as

$$
S^{\mu\nu}(z) \equiv \langle h|\theta^{\mu\nu}(z)|h\rangle = \int d^4x d^4y F^*(y)F(x)\langle 0|J(y)\theta^{\mu\nu}(z)J(x)|0\rangle\,. \qquad (2.12)
$$

The space-time point z is where (and when) the experimental counters are placed. We will take it far from the interaction region, and on its light-cone ($z^2 \approx 0$).

Out of $\langle \theta^{\mu\nu}(z)\rangle$ we will construct an observable which, for large $Q^{\star 2}$, will be dominated by the short distance region $x \approx y$ and by the light-cone regions $(z-x)^2 \approx 0$ and $(z-y)^2 \approx 0$.

We start studying how $S^{\mu\nu}(z)$ can be expressed in terms of hadron quantities, by inserting a double completeness sum over outgoing hadronic states:

$$
\begin{aligned}
&\langle 0|J(y)\theta^{\mu\nu}(z)J(x)|0\rangle = \\
&\quad = \sum_n \sum_m \langle 0|J(y)|n; out\rangle\langle n; out|\theta^{\mu\nu}(z)|m; out\rangle\langle m; out|J(x)|0\rangle\,. \qquad (2.13)
\end{aligned}
$$

As discussed in detail in the following, it is a general feature of the Haag-Ruelle scattering theory[8] that, for asymptotically large z, the semi-disconnected parts of the matrix element $\langle n; out|\theta^{\mu\nu}(z)|m; out\rangle$ will dominate in eqs. (2.12), (2.13). Qualitatively this is due to the fact that, in the very distant future the outgoing hadronic state will look like a bunch of sparse, practically free, particles. We also notice that, due to the convolution with $F^*(y)F(x)$ in eq. (2.12), the total momentum transfer between $|n; out\rangle$ and $|m; out\rangle$ is of the order of the energy-momentum spread in $\tilde{F}(Q)$, so that $\theta^{\mu\nu}(z)$ carries essentially zero four-momentum.

Semi-disconnected contributions are of the form

$$
\begin{aligned}
&\langle p_1', p_2', \ldots, p_n'; out|\theta^{\mu\nu}(z)|p_1, p_2, \ldots, p_n; out\rangle_{(s-d)} = \\
&\quad = \sum_{h=1}^{n} \delta(\underline{p}_1' - \underline{p}_1)\ldots\langle p_h'|\theta^{\mu\nu}(z)|p_h\rangle\ldots\delta(\underline{p}_n' - \underline{p}_n)\,. \qquad (2.14)
\end{aligned}
$$

In view of the very small momentum carried by $\theta^{\mu\nu}(z)$ we can write

$$\langle p'_h;|\theta^{\mu\nu}(z)|p_h;\rangle \approx \frac{{p_h}^\mu {p_h}^\nu}{(2\pi)^3\omega_{\underline{p}_h}} e^{i(p'_h-p_h)z} . \tag{2.15}$$

In eq. (2.15) the variation of the matrix element of the hadronic energy-momentum tensor on the scale of the wave packet momentum spread was neglected, while keeping the complete momentum dependence of the rapidly varying exponential factor.

In the following we will denote by n the ensemble of the particles in the intermediate state with momenta $p_1 \dots p_n$ and by $\tilde{n}_h$ the ensemble obtained by n after taking out the particle h, so that $n = \tilde{n}_h + h$.

From eqs. (2.13), (2.14) and (2.15) we get

$$\begin{aligned} S^{\mu\nu}_{(s-d)}(z) = & \int d^4x d^4y F^*(y) F(x) \sum_{p_1 \dots p_n} |\langle 0|J(0)|n; out\rangle|^2 \times \\ & \times \sum_{h=1}^{n} \sum_{p'_h} e^{-ip_{\tilde{n}_h}(y-x)} \frac{{p_h}^\mu {p_h}^\nu}{(2\pi)^3\omega_{\underline{p}_h}} e^{ip'_h(z-y)} e^{-ip_h(z-x)} . \end{aligned} \tag{2.16}$$

After the insertion of a factor $1 = \int d^4Q \; \delta^{(4)}(Q - p_n)$, eq. (2.16) becomes

$$\begin{aligned} S^{\mu\nu}_{(s-d)}(z) = & \int d^4Q \int d^4x d^4y F^*(y) F(x) \sum_{p_1 \dots p_n} |\langle 0|J(0)|n; out\rangle|^2 \times \\ & \times \sum_{h=1}^{n} \sum_{p'_h} e^{-ip_{\tilde{n}_h}(y-x)} \frac{{p_h}^\mu {p_h}^\nu}{(2\pi)^3\omega_{\underline{p}_h}} e^{ip'_h(z-y)} e^{-ip_h(z-x)} \delta^{(4)}(Q - p_n) = \\ = & \int d^4Q \tilde{F}(Q) \sum_{p_1 \dots p_n} |\langle 0|J(0)|n; out\rangle|^2 \sum_{h=1}^{n} \sum_{p'_h} \tilde{F}^*(Q + p'_h - p_h) \times \\ & \times \frac{{p_h}^\mu {p_h}^\nu}{(2\pi)^3\omega_{\underline{p}_h}} e^{i(p'_h-p_h)z} \delta^{(4)}(Q - p_{\tilde{n}_h} - p_h) . \end{aligned} \tag{2.17}$$

Since we are interested in observations taking place very far from the interaction region, z is large and the integrations over $\underline{p}_h$ and $\underline{p}'_h$ in eq. (2.17) can be performed through the stationary-phase method, as discussed in ref.[8].

Both integrals on $\underline{p}_h$ and $\underline{p}'_h$ have the same stationary point, $\underline{p}^\star{}_h$, determined by

$$\left. \frac{\partial \omega_{\underline{p}_h}}{\partial {p_h}^j} \right|_{\underline{p}^\star{}_h} = \frac{(p^\star{}_h)^j}{\omega_{\underline{p}^\star{}_h}} \equiv (v^\star{}_h)^j = \frac{z^j}{z^0} . \tag{2.18}$$

Equation (2.18) simply says that only states with at least one hadron with the correct velocity $\underline{v}^\star{}_h$ to go from any finite region of space-time up to z, will contribute to the sum in eq. (2.17).

An important point should be remarked: while $\underline{v}^\star{}_h$ is independent on the mass m_h of the hadron h, the corresponding energy, $\omega_{\underline{p}^\star{}_h} = \frac{m_h}{\sqrt{1-(\underline{v}^\star{}_h)^2}}$, and momentum, $\underline{p}^\star{}_h = m_h \frac{\underline{v}^\star{}_h}{\sqrt{1-(\underline{v}^\star{}_h)^2}}$, are indeed strongly h-dependent.

The exponential factors in eq. (2.17) must now be expanded around $\underline{p}^\star{}_h$. We have, for example

$$\underline{p}_h = \underline{p}^\star{}_h + \underline{\eta} \tag{2.19}$$

$$e^{-ip_h z} \approx e^{-i\frac{z^0}{2\omega_{\underline{p}^\star{}_h}}[\underline{\eta}\cdot\underline{\eta}-(\underline{\eta}\cdot\underline{v}^\star{}_h)^2]}\,. \tag{2.20}$$

The quadratic form, $\underline{\eta}\cdot\underline{\eta}-(\underline{\eta}\cdot\underline{v}^\star{}_h)^2$, in the exponent of eq. (2.20), has the eigenvalues

$$\lambda_1 = \lambda_2 = 1\,, \qquad \lambda_3 = 1 - (\underline{v}^\star{}_h)^2\,, \tag{2.21}$$

so that eq. (2.17) becomes

$$\begin{aligned} S^{\mu\nu}(z) &\approx 1/(z^0)^3 \int d^4Q|\tilde{F}(Q)|^2 \sum_{h=1}^{n}\sum_{\tilde{n}_h} |\langle 0|J(0)|\tilde{n}_h + p^\star{}_h; out\rangle|^2 \times \\ &\quad\times \frac{(\omega_{\underline{p}^\star{}_h})^2}{1-(\underline{v}^\star{}_h)^2}(p^\star{}_h)^\mu (p^\star{}_h)^\nu \delta^{(4)}(Q - p_{\tilde{n}_h} - p^\star{}_h) \approx \\ &\approx \frac{\int d^4Q|\tilde{F}(Q)|^2}{(z^0)^3} \sum_{h=1}^{n}\sum_{\tilde{n}_h} |\langle 0|J(0)|\tilde{n}_h + p^\star{}_h; out\rangle|^2 \times \\ &\quad\times \frac{(\omega_{\underline{p}^\star{}_h})^2}{1-(\underline{v}^\star{}_h)^2}(p^\star)^\mu (p^\star)^\nu \delta^{(4)}(Q^\star - p_{\tilde{n}_h} - p^\star{}_h)\,, \end{aligned} \tag{2.22}$$

where we have used the narrowness of the initial wave packets and dropped the subscript $(s-d)$ which reminded us we are considering semi-disconnected contributions. In fact it should by now be clear why we can rigorously restrict our considerations to the semi-disconnected contributions, eq. (2.14): all other (more-connected) contributions will be depressed by powers of $1/(z^0)^3$ with respect to the semi-disconnected ones.

Through eq. (2.22) we can now determine physical observables. We can, for example, compute the energy flux through a given portion Σ of the spherical surface of radius $|\underline{z}|$, as

$$\Phi_\Sigma(z^0) \equiv \int_\Sigma \langle \theta^{0i}(z)\rangle n^i d^2\Sigma\,, \tag{2.23}$$

where

$$d^2\Sigma = |\underline{z}|^2 d^2\Omega_h \tag{2.24}$$

is the (spherical) surface element of Σ around $\underline{z}$ and $d^2\Omega_h$ is the solid angle element around $\underline{z}$ and therefore, by eq. (2.18), around $\underline{v}^\star{}_h$. For $z^0 \approx |\underline{z}|$, i.e. on the light-cone of the interaction region, we have

$$\begin{aligned} \Phi_\Sigma(z^0) &\approx \frac{1}{\Pi(Q^{\star 2})z^0} \sum_{h=1}^{n}\sum_{\tilde{n}_h} \int_\Sigma d^2\Omega_h |\langle 0|J(0)|\tilde{n}_h + p^\star{}_h; out\rangle|^2 \times \\ &\quad\times \frac{\omega^4_{\underline{p}^\star{}_h}}{1-(\underline{v}^\star{}_h)^2}\delta^{(4)}(Q^\star - p_{\tilde{n}_h} - p^\star{}_h)\,, \end{aligned} \tag{2.25}$$

where we are considering the ultra-relativistic limit $(\underline{v}^{\star}{}_h)^2 \approx 1$, corresponding to our choice of position and switching on of the measuring apparatus. In this situation we do not distinguish between $\omega_{\underline{p}^{\star}{}_h}$ and $|\underline{p}^{\star}{}_h|$.

Equation (2.25) can also be written as

$$\Phi_\Sigma(z^0) = \frac{1}{\Pi(Q^{\star 2}) z^0} \sum_{h=1}^{n} \sum_{\tilde{n}_h} \int_\Sigma d^2\Omega_h |\langle 0|J(0)|\tilde{n}_h + p^{\star}{}_h; out\rangle|^2 \times \\ \times \frac{\omega^6_{\underline{p}^{\star}{}_h}}{m_h{}^2} \delta^{(4)}(Q^{\star} - p_{\tilde{n}_h} - p^{\star}{}_h)\,, \tag{2.26}$$

which shows that $\Phi_\Sigma(z^0)$ is not a nice inclusive quantity. In particular it does not have a smooth massless limit and is not an I.R. safe quantity. From a physical point of view it is in fact much more sensible to integrate the energy flux over some interval of time, in order to get the total energy going through Σ during the corresponding time interval. More generally, we can integrate the energy flux over some function $\Lambda(t)$ representing the response of the physical apparatus:

$$\Psi_\Sigma(\Lambda) \equiv \int dz^0 \Phi_\Sigma(z^0) \Lambda(z^0 - T)\,, \tag{2.27}$$

where $\Lambda(t)$ is well localized around zero and

$$T \equiv |\underline{z}| \tag{2.28}$$

so that the measuring region will still be localized around the light-cone of the interaction region. A "perfect" counter corresponds to $\Lambda(t) = 1$ for $-\epsilon < t < \epsilon$ and 0 otherwise. The z^0 integration in eq. (2.27) is equivalent to an integration over the speed $v^{\star}{}_h$ of the detected hadron:

$$\begin{aligned} v^{\star}{}_h &= |\underline{z}|/z^0 \\ dz^0/z^0 &= v^{\star}{}_h d(1/v^{\star}{}_h) \approx -dv^{\star}{}_h \end{aligned} \tag{2.29}$$

always in the ultra-relativistic approximation ($v^{\star}{}_h \approx 1$).

Equation (2.27) then becomes

$$\Psi_\Sigma(\Lambda) \approx \frac{1}{\Pi(Q^{\star 2})} \sum_{h=1}^{n} \sum_{\tilde{n}_h} \int_0^1 dv_h \int_\Sigma d^2\Omega_h |\langle 0|J(0)|\tilde{n}_h + p_h; out\rangle|^2 \times \\ \times \frac{\omega^4_{\underline{p}_h}}{1 - (\underline{v}_h)^2} \delta^{(4)}(Q^{\star} - p_{\tilde{n}_h} - p_h) \Lambda[|\underline{z}|(1/v_h - 1)]\,, \tag{2.30}$$

where we dropped the $\star$ superscript on the momentum and velocity of the h-th hadron, because they are from now on dummy integration variables. In eq. (2.30) we used

$$\Lambda(z^0 - T) = \Lambda[|\underline{z}|(1/v_h - 1)]\,. \tag{2.31}$$

Due to the limited support of $\Lambda(t)$, we see that, for large $|\underline{z}|$, the integral over v_h in eq. (2.30) is restricted to a very small region just below 1, which justifies our ultra-relativistic approximations.

We also observe that

$$\frac{dv_h}{1-(\underline{v}_h)^2} \approx \frac{d\omega_{\underline{p}_h}}{\omega_{\underline{p}_h}}\,, \tag{2.32}$$

where $\omega_h \equiv \omega_{\underline{p}_h}$, so that eq. (2.30) can be rewritten as

$$\Psi_\Sigma(\Lambda) \approx \frac{1}{\Pi(Q^{\star 2})} \sum_{h=1}^{n} \sum_{\tilde{n}_h} \int {\omega_h}^3 d\omega_h \int_\Sigma d^2\Omega_h |\langle 0|J(0)|\tilde{n}_h + p_h; out\rangle|^2 \times \\ \times \delta^{(4)}(Q^\star - p_{\tilde{n}_h} - p_h)\Lambda[|\underline{z}|(1/v_h - 1)]\,. \tag{2.33}$$

In eq. (2.33) we can now make the identification

$${\omega_h}^2 d\omega_h d^2\Omega_h = d^3p_h\,, \tag{2.34}$$

where d^3p_h is the integration measure of the detected hadron, so that

$$\begin{aligned}\Psi_\Sigma(\Lambda) &\approx \frac{1}{\Pi(Q^{\star 2})} \sum_{h=1}^{n} \sum_{\tilde{n}_h} \int_\Sigma d^3p_h \omega_h |\langle 0|J(0)|\tilde{n}_h + p_h; out\rangle|^2 \times \\ &\quad \times \delta^{(4)}(Q^\star - p_{\tilde{n}_h} - p_h)\Lambda[|\underline{z}|(1/v_h - 1)] = \\ &= \frac{1}{\Pi(Q^{\star 2})} \sum_n |\langle 0|J(0)|n; out\rangle|^2 \delta^{(4)}(Q^\star - p_n)\,\cdot \\ &\quad \times \sum_{h=1}^{n} \omega_h \Lambda[|\underline{z}|(1/v_h - 1)]\theta(\underline{p}_h \Rightarrow \Sigma)\,,\end{aligned} \tag{2.35}$$

where $\theta(\underline{p}_h \Rightarrow \Sigma)$ is 1, if $\underline{p}_h$ crosses Σ and 0 otherwise, and reminds us that the sum runs over intermediate states containing at least a hadron with momentum direction contained within Σ; besides, in eq. (2.35) the $\underline{p}_h$ integration region is also limited to hadrons whose velocity is close to 1, through the presence of $\Lambda[|\underline{z}|(1/v_h - 1)]$. The quantity $\Psi_\Sigma(\Lambda)$ is therefore a measure of the energy transported through Σ by the fastest hadrons.

3. The massless limit

We discuss in this section the massless limit of $\Psi_\Sigma(\Lambda)$. This is important for two reasons:

- in the massless case the space-time behaviour of correlation functions is rather different from the one found in section 2 and this discussion will convince us of the infrared safety of $\Psi_\Sigma(\Lambda)$;

- in section 4 $\Psi_\Sigma(\Lambda)$ will be shown to be light-cone dominated, so that its leading contribution can be reliably computed in massless perturbation theory. We must therefore find out to which perturbative quantity, expressed through massless quarks and gluons, it corresponds.

The main difference with the treatment of the massive hadron case, discussed in section 2, is that in the massless case the stationary phase condition eq. (2.18) *requires*, for consistency, $z^2 = 0$ *and* does not fix $(p^\star)^\mu$ completely, but only its direction. In fact the solution of eq. (2.18), in the massless case, is

$$(p^\star)^\mu = \lambda z^\mu \tag{3.1}$$

with an arbitrary $\lambda \geq 0$. The existence of a continuous line of stationary phase points results in a zero eigenvalue of the quadratic part of the exponential factor, eq. (2.20), as in fact confirmed by eq. (2.21). The correct strategy to adopt in this case is to integrate exactly along the "valley", eq. (3.1), and apply the stationary phase approximation in the transverse directions. In order to carry on this procedure we choose to parametrize $(p^\star)^\mu$ as follows:

$$(p^\star)^\mu = \omega \frac{z^\mu}{|\underline{z}|} \tag{3.2}$$

This means that the integrations over $\underline{p}_h$ and $\underline{p}'_h$ will be replaced by two one-dimensional integrations over the corresponding energies, ω_h and $\omega_h{}'$. We have, therefore,

$$\begin{aligned} S_{m.l.}{}^{\mu\nu}(z) \approx \frac{1}{(z^0)^2} \int d^4Q \tilde{F}(Q) \sum_{h=1}^{n} \sum_{\tilde{n}_h} \int d\omega_h d\omega'_h \tilde{F}^*[Q + p^{\star\prime}{}_h - p^\star{}_h] \times \\ \times |\langle 0|J(0)|\tilde{n}_h + p^\star{}_h; out\rangle_{m.l.}|^2 \, \omega_h \frac{(p^\star{}_h)^\mu (p^\star{}_h)^\nu}{2\pi} e^{i(p^\star{}_h{}' - p^\star{}_h)z} \times \\ \times \delta^{(4)}(Q - p_{\tilde{n}_h} - p^\star{}_h)\,, \end{aligned} \tag{3.3}$$

where the subscript $m.l.$ reminds us that we are in the massless limit. It should be noticed, that, due to the zero eigenvalue in eq. (2.21) ($(\underline{v}^\star{}_h)^2 = 1$), in eq. (3.3) the overall $(1/z^0)^2$ replaces the $(1/z^0)^3$ behaviour of the massive case of eq. (2.22).

Another important difference with respect to eq. (2.22) is that, in eq. (3.3), the integrals over ω_h and ω'_h, along the valley direction, are regulated by the presence of $\tilde{F}^*(Q + p^\star{}_h{}' - p^\star{}_h)$, so that the dependence from the initial wave packets cannot be factorized. Therefore $S_{m.l.}{}^{\mu\nu}(z)$ will depend on the details of the preparation of the initial e^+e^- state. This fact has a physical interpretation: a sharp space-time observation at z, would reveal a light front structure reflecting the detailed characteristics of the initial beam wave packet.

As discussed in section 2, a physically more realistic observable is $\Psi_\Sigma(\Lambda)$, defined in eq. (2.27), which, in the present case, reads

$$\Pi(Q^{\star 2})\Big|_{m.l.} \int d^4Q |\tilde{F}(Q)|^2 \, \Psi_\Sigma(\Lambda)|_{m.l.} =$$

$$
\begin{aligned}
&= \int d^4Q\tilde{F}(Q) \int_\Sigma d^2\Sigma \sum_{h=1}^{n} \sum_{\tilde{n}_h} \int d\omega_h d\omega'_h \tilde{F}^*(Q + {p^\star}_h{}' - {p^\star}_h) \times \\
&\quad \times |\langle 0|J(0)|\tilde{n}_h + {p^\star}_h; out\rangle_{m.l.}|^2 \,\frac{\omega_h}{(2\pi)^3}\delta^{(4)}(Q - p_{\tilde{n}_h} - {p^\star}_h) \times \\
&\quad \times\tilde{\Lambda}(\omega'_h - \omega_h) \exp +i[(\omega'_h - \omega_h)T - ({\underline{p}^\star}_h{}' - {\underline{p}^\star}_h) \times \underline{z}] = \\
&= \frac{|\underline{z}|^2}{T^2} \int d^4Q\tilde{F}(Q) \sum_{h=1}^{n} \sum_{\tilde{n}_h} \int_\Sigma d^2\Omega_h \int d\omega_h d\omega'_h \tilde{F}^*[Q + p_h{}' - p_h] \times \\
&\quad \times |\langle 0|J(0)|\tilde{n}_h + p_h; out\rangle_{m.l.}|^2 \,\frac{(\omega_h)^3}{2\pi}\delta^{(4)}(Q - p_{\tilde{n}_h} - p_h)\tilde{\Lambda}(\omega_h{}' - \omega_h)\,,
\end{aligned}
\tag{3.4}
$$

where we dropped again the $\star$ superscript in the last step and we put the exponential to 1 because of eq. (2.28).

If the support of the smearing function $\Lambda(t)$ is larger than the overlap time of the initial wave packets, the corresponding support of its Fourier transform, $\tilde{\Lambda}(\omega)$, will be smaller than that of $\tilde{F}$, thus allowing to neglect $p_h{}' - p_h$ in $\tilde{F}^*[Q + p_h{}' - p_h]$. Therefore we have

$$
\begin{aligned}
\Psi_\Sigma(\Lambda)|_{m.l.} &= \frac{1}{\Pi(Q^{\star 2})|_{m.l.}} \sum_{h=1}^{n} \sum_{\tilde{n}_h} \int_\Sigma d^2\Omega_h \int d\omega_h d\omega_h{}' \,|\langle 0|J(0)|\tilde{n}_h + p_h; out\rangle_{m.l.}|^2 \times \\
&\quad \times \frac{(\omega_h)^3}{2\pi}\delta^{(4)}(Q^\star - p_{\tilde{n}_h} - p_h)\tilde{\Lambda}(\omega_h{}' - \omega_h) = \\
&= \frac{\Lambda(0)}{\Pi(Q^{\star 2})|_{m.l.}} \sum_{h=1}^{n} \sum_{\tilde{n}_h} \int_\Sigma d^2\Omega_h \int d\omega_h \,|\langle 0|J(0)|\tilde{n}_h + p_h; out\rangle_{m.l.}|^2 \times \\
&\quad \times(\omega_h)^3\delta^{(4)}(Q^\star - p_{\tilde{n}_h} - p_h)\,.
\end{aligned}
\tag{3.5}
$$

Recalling eq. (2.34), we get

$$
\begin{aligned}
\Psi_\Sigma(\Lambda)|_{m.l.} &= \frac{\Lambda(0)}{\Pi(Q^{\star 2})|_{m.l.}} \sum_n |\langle 0|J(0)|n; out\rangle_{m.l.}|^2 \,\delta^{(4)}(Q^\star - p_n) \sum_{h=1}^{n} \omega_h \theta(\underline{p}_h \Rightarrow \Sigma) \equiv \\
&\equiv \Lambda(0)E(\Sigma)\,,
\end{aligned}
\tag{3.6}
$$

where

$$
E(\Sigma) = 1/\left.\Pi(Q^{\star 2})\right|_{m.l.} \sum_n |\langle 0|J(0)|n; out\rangle_{m.l.}|^2 \,\delta^{(4)}(Q^\star - p_n) \sum_{h=1}^{n} \omega_h \theta(\underline{p}_h \Rightarrow \Sigma) \tag{3.7}
$$

is precisely the first of a hierarchy of observables studied in ref. [4].

The factorization of $\Lambda(0)$ in eq. (3.6) shows that, in the massless case, the hadron shock wave produced from e^+e^- annihilation reaches the experimental apparatus quite sharply, at the time $T = |\underline{z}|$. In the case of a "perfect" counter $\Lambda(0) = 1$ and $\Psi_\Sigma(\Lambda)|_{m.l.} = E(\Sigma)$.

The result of a perturbative computation will have the form given in eq. (3.6), with intermediate states composed of massless quarks and gluons. Light-cone dominance, discussed in the next section, implies that, for large $Q^{\star 2}$, perturbation theory should reliably reproduce $\Psi_\Sigma(\Lambda)$. Thus, taken together, eqs.(2.35) and (3.6) imply that, as $Q^{\star 2} \to \infty$, the hadron velocity distribution will become more and more peaked around 1, so that $\Psi_\Sigma(\Lambda)$ becomes proportional to $\Lambda(0)$, as its perturbative counterpart. Besides, the average value of the energy of the fastest hadrons crossing Σ, i.e. $\Psi_\Sigma(\Lambda)/\Lambda(0)$, should be well approximated by $E(\Sigma)$. This means, in particular, that the energy angular distribution of fast hadrons asymptotically coincides with that of the corresponding perturbative massless quark-gluon matter.

4. Light cone dominance

In order to establish the light-cone dominance of the observable $\Psi_\Sigma(\Lambda)$ defined in eq. (2.27), we use translational invariance and a change of variables to write:

$$\begin{aligned}
&\sum_i \int d\mu_{(\Sigma,\Lambda)}{}^i(z)\ S^{0i}(z) = \\
&\quad = \sum_i \int d\mu_{(\Sigma,\Lambda)}{}^i(z)\ d^4x d^4y F^*(y) F(x) \langle 0|J(y)\theta^{0i}(z)J(x)|0\rangle = \\
&\quad = \sum_i \int d\mu_{(\Sigma,\Lambda)}{}^i(z)\ d^4x d^4y F^*(y) F(x) \langle 0|J(0)\theta^{0i}(z-y)J(x-y)|0\rangle = \\
&\quad = \sum_i \int d\mu_{(\Sigma,\Lambda)}{}^i(z+y)\ d^4x d^4y F^*(y) F(x+y) \langle 0|J(0)\theta^{0i}(z)J(x)|0\rangle\,, \qquad (4.1)
\end{aligned}$$

where the integration measure $d\mu_{(\Sigma,\Lambda)}{}^i(z)$ summarizes the angular and temporal averages described in section 2.

Since z is very large we can safely approximate, in eq. (4.1), $d\mu_{(\Sigma,\Lambda)}{}^i(z+y) \approx d\mu_{(\Sigma,\Lambda)}{}^i(z)$, so that we can factorize the wave function dependence and get:

$$\Psi_\Sigma(\Lambda) \approx \frac{1}{\Pi(Q^{\star 2})} \sum_i \int d\mu_{(\Sigma,\Lambda)}{}^i(z) \int \frac{d^4x}{(2\pi)^4} e^{-iQ^\star x} \langle 0|J(0)\theta^{0i}(z)J(x)|0\rangle \qquad (4.2)$$

where, once again, the narrowness of the initial wave-packets has been exploited.

Equation (4.2) explicitly shows the light-cone nature of $\Psi_\Sigma(\Lambda)$. In fact, since the dependence from $Q^\star$ is expressed in the form of a Fourier transform, the large $Q^\star$ limit will be dominated by the most singular regions of the integrand, i.e. its simultaneous short-distance and light-cone singularities.

5. Conclusions

We have shown that the energy angular distribution of fast hadrons in e^+e^- annihilation can be related to light-cone and short-distance singularities of products of local operators. Similar considerations apply also to the whole set of "energy-energy correlators" considered in ref.[4]. The operator formulation allows to deduce, with a certain rigor, the energy angular distribution of fast hadrons from first principles, without explicitly invoking quark-hadron duality. For example, on the basis of these arguments, we are able to exclude the possibility that the typical high $Q^\star$ hadronic event consists of a large number of slowly moving hadrons. The present approach could also allow a systematic study of pre-asymptotic contributions coming from pre-leading singularities in the short-distance, light-cone operator product expansion.

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