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On-shell two-loop three-gluon vertex

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Abstract

The two-loop three-gluon vertex is calculated in an arbitrary covariant gauge, in the limit when two of the gluons are on the mass shell. The corresponding two-loop results for the ghost-gluon vertex are also obtained. It is shown that the results are consistent with the Ward–Slavnov–Taylor identities.

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1 Introduction

Calculation of radiative corrections to certain jet processes is becoming extremely important (see, for instance, the reviews [1, 2] and references therein). In particular, the two-loop three-gluon vertex with two external gluons on shell (i.e. when two external momenta squared vanish) is a part of the next-to-next-to-leading order (NNLO) contributions to $gg \to gg$ and $q\overline{q} \to gg$ processes (see, for instance, Fig. 1). The next-to-leading order (NLO) contributions to these processes were considered in ref. [3][1] (see also in refs. [4]). For a complete calculation of all relevant NNLO contributions, one also needs to calculate the two-loop four-point functions (see, for instance, in [5, 6]).

Studying the three-gluon vertex (and other QCD vertices) in this on-shell limit is also important, in order to understand the structure of infrared (on-shell) singularities, to illustrate how the corresponding Ward–Slavnov–Taylor (WST) identities work in this divergent case and to develop calculational techniques that can then be generalized to deal with more complicated (e.g. off-shell) configurations. Performing the calculations in an arbitrary covariant gauge is useful, in order to exploit the consequences of gauge invariance (the WST identities) at all stages. In the calculation of physical quantities, the independence of the gauge parameter usually serves as an important check.

The one-loop QCD vertices have been known for quite some time. The one-loop result for the three-gluon vertex, for off-shell gluons (with $p_1^2 = p_2^2 = p_3^3$) and in an arbitrary covariant gauge, was presented in [7, 8]. The general off-shell case, but restricted to the Feynman gauge, was considered in [9]. Various on-shell results have also been given: in [10], restricted to the infrared-singular parts only (in an arbitrary covariant gauge), and in [11], with the finite parts for the case of two gluons being on-shell (in the Feynman gauge). The most general results, valid for arbitrary values of the space-time dimension and the covariant-gauge parameter, have been presented in an earlier paper [12], where we have also collected the results for all on-shell limits of interest. Some results for the one-loop quark–gluon vertex (or its Abelian part, which is related to the QED vertex) can be found in [13].

At the two-loop level, the QCD vertices were mainly studied in the zero-momentum limit [14, 15], i.e. when one of the external momenta is zero. This limit is useful for studying the renormalization properties of QCD, since (for the considered vertices) it does not bring in any infrared (on-shell) singularities. In [14], the renormalized results in the Feynman gauge were presented. In [15], the unrenormalized and renormalized results for the three-gluon and ghost-gluon vertices have been obtained in an arbitrary covariant gauge, and the corresponding differential WST identity in QCD was analysed in detail. The relevant techniques for on-shell two-loop calculations have been studied in refs. [16, 17, 18]. In ref. [16], the two-loop electromagnetic quark form factor in massless QCD was considered. Corrected results for this form factor were later presented in [19, 20]. In ref. [17], the two-loop scalar form factor was calculated. We also note that the structure and factorization properties of infrared singularities of two-loop order QCD amplitudes were recently discussed in ref. [21].

In the present paper, we discuss an algorithm to calculate two-loop three-point diagrams with two external legs on shell. Then, we present the on-shell results for the

[1] For a complete list of NLO contributions, see Figs. 6 and 8 of [3].

three-gluon vertex and the ghost-gluon vertex, in an arbitrary covariant gauge, keeping the finite parts of the expansion in the dimensional regularization [22] parameter ε. We consider the relevant WST identities in the on-shell limit and confirm that the results obtained are consistent with these identities.

2 Preliminaries and WST identities

The three-gluon vertex is defined as (see Fig. 2)

$$\Gamma^{a_1a_2a_3}_{\mu_1\mu_2\mu_3}(p_1,p_2,p_3) \equiv -\mathrm{i}\; g\; f^{a_1a_2a_3}\; \Gamma_{\mu_1\mu_2\mu_3}(p_1,p_2,p_3). \tag{2.1}$$

Here, the $f^{a_1a_2a_3}$ are the totally antisymmetric colour structures corresponding to the adjoint representation of the gauge group (for example, SU(N) or any other semi-simple gauge group). Other colour structures do not appear in the perturbative calculation of QCD three-point vertices, at least at the one- and two-loop levels. To regulate the ultraviolet and infrared (on-shell) divergences occurring at the one- and two-loop levels, we shall use dimensional regularization [22], with the space-time dimension $n = 4 - 2\varepsilon$.

Since the $f^{a_1a_2a_3}$ are antisymmetric, $\Gamma_{\mu_1\mu_2\mu_3}(p_1,p_2,p_3)$ must also be *antisymmetric* under any interchange of a pair of gluon momenta and the corresponding Lorentz indices. Therefore, in the limit of interest ($p_1^2 = p_2^2 = 0,\; p_3^2 \equiv p^2$) it can be presented as

$$\begin{aligned}
\Gamma_{\mu_1\mu_2\mu_3}(p_1,p_2,p_3)\big|_{p_1^2=p_2^2=0,\; p_3^2\equiv p^2} &= g_{\mu_1\mu_2}\;(p_1-p_2)_{\mu_3}\; U_1(p^2) \\
&+ [g_{\mu_1\mu_3}p_{1\mu_2} - g_{\mu_2\mu_3}p_{2\mu_1}]\; U_2(p^2) + [g_{\mu_1\mu_3}p_{2\mu_2} - g_{\mu_2\mu_3}p_{1\mu_1}]\; U_3(p^2) \\
&+ p_{1\mu_1}p_{2\mu_2}(p_1-p_2)_{\mu_3}\; U_4(p^2) + p_{2\mu_1}p_{1\mu_2}(p_1-p_2)_{\mu_3}\; U_5(p^2) \\
&+ [p_{1\mu_1}p_{1\mu_2}p_{1\mu_3} - p_{2\mu_1}p_{2\mu_2}p_{2\mu_3}]\; U_6(p^2) + [p_{1\mu_1}p_{1\mu_2}p_{2\mu_3} - p_{2\mu_1}p_{2\mu_2}p_{1\mu_3}]\; U_7(p^2).
\end{aligned} \tag{2.2}$$

This decomposition is similar to eq. (29) of ref. [10][2]. All terms are explicitly antisymmetric with respect to $(p_1,\mu_1) \leftrightarrow (p_2,\mu_2)$. At the lowest, "zero-loop" order,

$$U_1^{(0)} = 1, \quad U_2^{(0)} = -2, \quad U_3^{(0)} = -1, \quad U_4^{(0)} = U_5^{(0)} = U_6^{(0)} = U_7^{(0)} = 0, \tag{2.3}$$

so we get the well-known tensor structure

$$g_{\mu_1\mu_2}(p_1-p_2)_{\mu_3} + g_{\mu_2\mu_3}(p_2-p_3)_{\mu_1} + g_{\mu_3\mu_1}(p_3-p_1)_{\mu_2}. \tag{2.4}$$

The lowest-order gluon propagator is[3]

$$\delta^{a_1a_2}\;\frac{1}{p^2}\left(g_{\mu_1\mu_2} - \xi\;\frac{p_{\mu_1}p_{\mu_2}}{p^2}\right), \tag{2.5}$$

where $\xi \equiv 1-\alpha$ is the gauge parameter corresponding to a general covariant gauge, defined in such a way that $\xi = 0$ ($\alpha = 1$) is the Feynman gauge. The gluon polarization operator is defined as

$$\Pi^{a_1a_2}_{\mu_1\mu_2}(p) \equiv -\delta^{a_1a_2}\left(p^2 g_{\mu_1\mu_2} - p_{\mu_1}p_{\mu_2}\right) J(p^2), \tag{2.6}$$

[2]For details, see section 4C and appendix F of ref. [12].

[3]Here and henceforth, a causal prescription is understood, $1/p^2 \to 1/(p^2+\mathrm{i}0)$.

while the ghost self-energy is

$$\widetilde{\Pi}^{a_1a_2}(p^2) = \delta^{a_1a_2}\ p^2\ \left[G(p^2)\right]^{-1}. \tag{2.7}$$

In the lowest-order approximation $J^{(0)} = G^{(0)} = 1$.

The ghost-gluon vertex can be represented as

$$\widetilde{\Gamma}^{a_1a_2a_3}_{\mu_3}(p_1,p_2;p_3) \equiv -\mathrm{i}g\ f^{a_1a_2a_3}\ {p_1}^{\mu}\ \widetilde{\Gamma}_{\mu\mu_3}(p_1,p_2;p_3), \tag{2.8}$$

where p_1 is the out-ghost momentum, p_2 is the in-ghost momentum, p_3 and μ_3 are the momentum and the Lorentz index of the gluon (all momenta are ingoing). For $\widetilde{\Gamma}_{\mu\mu_3}$, we use following decomposition (see [9][4]):

$$\begin{aligned}\widetilde{\Gamma}_{\mu\mu_3}(p_1,p_2;p_3) = g_{\mu\mu_3}a(p_3^2,p_2^2,p_1^2) - p_{3\mu}p_{2\mu_3}b(p_3^2,p_2^2,p_1^2) + p_{1\mu}p_{3\mu_3}c(p_3^2,p_2^2,p_1^2)\\ +p_{3\mu}p_{1\mu_3}d(p_3^2,p_2^2,p_1^2) + p_{1\mu}p_{1\mu_3}e(p_3^2,p_2^2,p_1^2).\end{aligned} \tag{2.9}$$

At the "zero-loop" level, all the scalar functions involved in (2.9) vanish at this order, except one, $a^{(0)} = 1$.

Whenever possible, we adopt the notation used in our previous papers [12, 15]. In particular, for a quantity X (e.g. any of the scalar functions contributing to the propagators or the vertices), we denote the zero-loop order contribution as $X^{(0)}$ (cf. eq. (2.3)), the one-loop order contribution as $X^{(1)}$, and the two-loop order contribution as $X^{(2)}$. In this paper, as a rule,

$$X^{(L)} = X^{(L,\xi)} + X^{(L,q)}, \tag{2.10}$$

where $X^{(L,\xi)}$ denotes the contribution of gluon and ghost loops in a general covariant gauge (2.5) (in particular, $X^{(L,0)}$ corresponds to the Feynman gauge, $\xi = 0$), while $X^{(L,q)}$ represents the contribution of the quark loops.

In general, the WST identity [23] for the three-gluon vertex reads (see e.g. in [24]):

$$\begin{aligned}p_3^{\mu_3}\ \Gamma_{\mu_1\mu_2\mu_3}(p_1,p_2,p_3) = -J(p_1^2)\ G(p_3^2)\ \left[{g_{\mu_1}}^{\mu_3}p_1^2 - p_{1\mu_1}p_1^{\mu_3}\right]\ \widetilde{\Gamma}_{\mu_3\mu_2}(p_1,p_3;p_2)\\ +J(p_2^2)\ G(p_3^2)\ \left[{g_{\mu_2}}^{\mu_3}p_2^2 - p_{2\mu_2}p_2^{\mu_3}\right]\ \widetilde{\Gamma}_{\mu_3\mu_1}(p_2,p_3;p_1).\end{aligned} \tag{2.11}$$

It is easy to see that the c and e functions from the ghost-gluon vertex (2.9) do not contribute to the WST identity (2.11). In the on-shell case, some of the momenta squared vanish. Note that (in the case of massless quarks) $J(0) = G(0) = 1$. In eq. (2.11) we can also consider permutations of the indices 1, 2, 3 corresponding to the contractions of the three-gluon vertex with $p_1^{\mu_1}$ or $p_2^{\mu_2}$. To get all relations between the scalar functions in the case of interest ($p_1^2 = p_2^2 = 0,\ p_3^2 \equiv p^2$), it is sufficient to consider the contractions with $p_3^{\mu_3}$ and $p_1^{\mu_1}$.

The WST identity (2.11) (contraction with $p_3^{\mu_3}$) yields just one condition on the scalar functions:

$$\begin{aligned}U_2(p^2) - U_3(p^2) + \tfrac{1}{2}p^2U_6(p^2) + \tfrac{1}{2}p^2U_7(p^2)\\ = -J(0)\ G(p^2)\left[a(0,p^2,0) + \tfrac{1}{2}p^2b(0,p^2,0) + \tfrac{1}{2}p^2d(0,p^2,0)\right].\end{aligned} \tag{2.12}$$

[4]Adopting the notation used in ref. [9], we have written in previous papers [12, 15] the arguments of these ghost-gluon scalar functions a, b, c, d and e as momenta (vectors). In the present paper, it is more convenient to write the arguments as momenta *squared*. The *order* (3,2,1) of the arguments on the r.h.s. of eq. (2.9) corresponds to ref. [9] and has not been changed.

Considering contraction with $p_1^{\mu_1}$, we get four relations:

$$U_1(p^2) + \tfrac{1}{2}p^2 U_5(p^2) = G(0)\ J(p^2)\left[a(0,0,p^2) + \tfrac{1}{2}p^2 b(0,0,p^2) + \tfrac{1}{2}p^2 d(0,0,p^2)\right], \qquad (2.13)$$

$$U_2(p^2) = -2\ G(0)\ J(p^2)\ a(0,0,p^2), \qquad (2.14)$$

$$U_3(p^2) - \tfrac{1}{2}p^2 U_7(p^2) = G(0)J(0)\ \tfrac{1}{2}p^2 b(p^2,0,0) - G(0)J(p^2)\left[a(0,0,p^2) - \tfrac{1}{2}p^2 d(0,0,p^2)\right], \qquad (2.15)$$

$$\begin{aligned}\tfrac{1}{2}p^2 U_6(p^2) = &-G(0)\ J(0)\ \left[a(p^2,0,0) - \tfrac{1}{2}p^2 d(p^2,0,0)\right] \\ &+G(0)\ J(p^2)\left[a(0,0,p^2) + \tfrac{1}{2}p^2 d(0,0,p^2)\right]. \end{aligned} \qquad (2.16)$$

Note that the l.h.s. of eq. (2.12) can be constructed from the l.h.s.'s of eqs. (2.14)–(2.16). This gives a condition on the scalar functions from the ghost-gluon vertex:

$$\begin{aligned}G(p^2)\ &\left[a(0,p^2,0) + \tfrac{1}{2}p^2 b(0,p^2,0) + \tfrac{1}{2}p^2 d(0,p^2,0)\right] \\ &= G(0)\ \left[a(p^2,0,0) + \tfrac{1}{2}p^2 b(p^2,0,0) - \tfrac{1}{2}p^2 d(p^2,0,0)\right]. \end{aligned} \qquad (2.17)$$

Therefore, the conditions (2.13)–(2.17) may be considered as a set of independent corollaries of the WST identities (2.11).

At the one-loop order, the diagrams contributing to the three-gluon vertex, two-point functions and the ghost-gluon vertex are shown in Figs. 1, 2 and 3 of ref. [12]. The one-loop expressions for the U_i functions are presented in Appendix F of ref. [12][5], for arbitrary values of the space-time dimension and of the covariant-gauge parameter ξ. In ref. [12], they were obtained as a limiting case of the general off-shell expressions. Of course, they may also be obtained by direct calculation, using results for the on-shell one-loop integrals collected in Appendix A of the present paper. The corresponding one-loop expressions for the ghost-gluon scalar functions are, for the on-shell limits of interest, collected in Appendix B. They can be obtained from the expressions for general momenta presented in Appendix D of ref. [12], or by direct calculation. The one-loop expressions for the two-point functions can be found, for instance, in ref. [25] (see also in [12] and Appendix C of the present paper). Collecting all the mentioned one-loop results, we have checked that they satisfy the conditions (2.13)–(2.17), in any space-time dimension n and for any ξ.

In the one-loop expressions, the following notation is used[6]:

$$\kappa(p^2) \equiv -\frac{2}{(n-3)(n-4)}\ (-p^2)^{(n-4)/2} = \frac{1}{\varepsilon(1-2\varepsilon)}\ (-p^2)^{-\varepsilon}, \qquad (2.18)$$

$$\begin{aligned}\eta\ &\equiv\ \frac{\Gamma^2(\frac{n}{2}-1)}{\Gamma(n-3)}\,\Gamma(3-\tfrac{n}{2}) = \frac{\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\,\Gamma(1+\varepsilon) \\ &=\ e^{-\gamma\varepsilon}\left[1 - \frac{1}{12}\pi^2\varepsilon^2 - \frac{7}{3}\zeta_3\varepsilon^3 - \frac{47}{1440}\pi^4\varepsilon^4 + \mathcal{O}(\varepsilon^5)\right], \end{aligned} \qquad (2.19)$$

[5]The divergent parts were presented earlier in ref. [10], whereas the Feynman-gauge results, including the finite terms in ε, are available in [11].

[6]Below, we shall also use the factor η in two-loop results (they are proportional to η^2). Since the two-loop contributions involve poles up to $1/\varepsilon^4$, we need the expansion of η up to the ε^4 term.

where $\gamma \simeq 0.57721566...$ is the Euler constant, whilst $\zeta_3 \equiv \zeta(3) = \sum_{j=1}^{\infty} j^{-3} \simeq 1.2020569...$ is the value of Riemann's zeta function.

We also use the standard notations C_A for the eigenvalue of the quadratic Casimir operator in the adjoint representation,

$$f^{acd} f^{bcd} = C_A\, \delta^{ab} \quad (C_A = N \text{ for the SU}(N) \text{ group}), \tag{2.20}$$

and C_F for the eigenvalue of the quadratic Casimir operator in the fundamental representation. For the SU(N) group, $C_F = (N^2-1)/(2N)$. Furthermore,

$$T \equiv N_f T_R, \qquad T_R = \tfrac{1}{8}\,\mathrm{Tr}(I) = \tfrac{1}{2}\,, \tag{2.21}$$

where N_f is the number of quark flavours, and I is the "unity" in the space of Dirac matrices (we assume that $\mathrm{Tr}(I) = 4$).

3 Planar two-loop three-point integrals

To calculate two-loop contributions to the three-gluon vertex (shown in Fig. 1 of ref. [15]), we consider contractions of eq. (2.2) with all possible tensor structures carrying three Lorentz indices μ_1, μ_2 and μ_3. Note that non-planar graphs do not contribute to the two-loop vertex, since their over-all colour factors vanish, due to the Jacobi identity (see Fig. 6 of ref. [26], where this is explained).

Technically, the problem is therefore reduced to the calculation of scalar integrals corresponding to the planar two-loop vertex graph shown in Fig. 3a (and similar graphs with cyclic permutation of external momenta p_1, p_2 and p_3). However, as a result of contracting the tensor structures, we get in the numerator some polynomials in scalar products of external and loop momenta. The complete basis for expanding these polynomials (see ref. [27]) includes (i) three external momentum invariants (e.g. p_1^2, p_2^2 and p_3^2), (ii) six squared momenta corresponding to the six denominators shown in Fig. 3a, and (iii) one additional invariant, which can be chosen as q^2. Diagrammatically, the latter member of the basis can be associated with the seventh line of an auxiliary "forward-scattering" four-point diagram shown in Fig. 3b.

Since q^2 is missing in the original set of denominators (Fig. 3a), it always remains as a numerator, which cannot be cancelled against any of the denominators involved. Therefore, it is referred to as an *irreducible* numerator (see, for instance, ref. [27]). In general, integrals with irreducible numerators require a special consideration [27, 28]. However, as we shall see below, this problem is not so serious in the on-shell case as in the general off-shell case, since the relevant "boundary" integrals can be calculated for any (integer) powers of the numerator q^2.

In terms of an algorithm, it is convenient to consider q^2 as an extra denominator, remembering that its power ν_7 is usually non-positive. Thus, let us consider integrals corresponding to the auxiliary diagram in Fig. 3b:

$$K_3(n;\nu_1,\nu_2,\nu_3,\nu_4,\nu_5,\nu_6,\nu_7) \equiv \int\!\!\int \frac{\mathrm{d}^n q\; \mathrm{d}^n r}{[(p_1+r)^2]^{\nu_1}[(p_1+q)^2]^{\nu_2}[(p_2-r)^2]^{\nu_3}[(p_2-q)^2]^{\nu_4}(r^2)^{\nu_5}[(q-r)^2]^{\nu_6}(q^2)^{\nu_7}}, \tag{3.1}$$

where $n = 4-2\varepsilon$ is the space-time dimension. We shall also need diagrams corresponding to the permutations of the external momenta in the diagram (3.1). They are:

$$K_2(n;\nu_1,\nu_2,\nu_3,\nu_4,\nu_5,\nu_6,\nu_7) \equiv \text{ Eq. (3.1) with } (p_1,p_2,p_3)\to(p_3,p_1,p_2), \tag{3.2}$$

$$K_1(n;\nu_1,\nu_2,\nu_3,\nu_4,\nu_5,\nu_6,\nu_7) \equiv \text{ Eq. (3.1) with } (p_1,p_2,p_3)\to(p_2,p_3,p_1). \tag{3.3}$$

The corresponding diagrams are the same as given in Fig. 3; the only thing to do is to permute the external momenta p_i. Note that for the integrals K_2 and K_1 not all external lines of the four-point function are on shell, since some of them carry p_3 with $p_3^2 \equiv p^2 \neq 0$. Instead, one of the corresponding Mandelstam variables vanishes.

To construct a procedure of calculating the integrals K_i with different integer ν_i, the integration-by-parts procedure [29] is useful[7]. If we introduce the notation $\mathbf{j}^+$ to denote an *increase* of ν_j by one unit (and similarly let $\mathbf{j}^-$ denote a *decrease* of ν_j by one unit), then the set of independent integration-by-parts relations for the functions (3.1) can (for arbitrary momenta) be written as follows[8]:

$$\Big[p_3^2\nu_3\mathbf{3}^+ + p_1^2\nu_5\mathbf{5}^+ - \nu_3\mathbf{1}^-\mathbf{3}^+ - \nu_5\mathbf{1}^-\mathbf{5}^+ + \nu_6\mathbf{6}^+\left(\mathbf{2}^- - \mathbf{1}^-\right) + (n-2\nu_1-\nu_3-\nu_5-\nu_6)\Big]\, K_3 = 0, \tag{3.4}$$

$$\Big[p_3^2\nu_1\mathbf{1}^+ + p_2^2\nu_5\mathbf{5}^+ - \nu_1\mathbf{1}^+\mathbf{3}^- - \nu_5\mathbf{3}^-\mathbf{5}^+ + \nu_6\mathbf{6}^+\left(\mathbf{4}^- - \mathbf{3}^-\right) + (n-\nu_1-2\nu_3-\nu_5-\nu_6)\Big]\, K_3 = 0, \tag{3.5}$$

$$\Big[p_1^2\nu_1\mathbf{1}^+ + p_2^2\nu_3\mathbf{3}^+ - \nu_1\mathbf{1}^+\mathbf{5}^- - \nu_3\mathbf{3}^+\mathbf{5}^- + \nu_6\mathbf{6}^+\left(\mathbf{7}^- - \mathbf{5}^-\right) + (n-\nu_1-\nu_3-2\nu_5-\nu_6)\Big]\, K_3 = 0, \tag{3.6}$$

$$\Big[\nu_1\mathbf{1}^+\left(\mathbf{2}^- - \mathbf{6}^-\right) + \nu_3\mathbf{3}^+\left(\mathbf{4}^- - \mathbf{6}^-\right) + \nu_5\mathbf{5}^+\left(\mathbf{7}^- - \mathbf{6}^-\right) + (n-\nu_1-\nu_3-\nu_5-2\nu_6)\Big]\, K_3 = 0, \tag{3.7}$$

$$\Big[p_3^2\nu_4\mathbf{4}^+ + p_1^2\nu_7\mathbf{7}^+ - \nu_4\mathbf{2}^-\mathbf{4}^+ + \nu_6\mathbf{6}^+\left(\mathbf{1}^- - \mathbf{2}^-\right) - \nu_7\mathbf{2}^-\mathbf{7}^+ + (n-2\nu_2-\nu_4-\nu_6-\nu_7)\Big]\, K_3 = 0, \tag{3.8}$$

$$\Big[p_3^2\nu_2\mathbf{2}^+ + p_2^2\nu_7\mathbf{7}^+ - \nu_2\mathbf{2}^+\mathbf{4}^- + \nu_6\mathbf{6}^+\left(\mathbf{3}^- - \mathbf{4}^-\right) - \nu_7\mathbf{4}^-\mathbf{7}^+ + (n-\nu_2-2\nu_4-\nu_6-\nu_7)\Big]\, K_3 = 0, \tag{3.9}$$

$$\Big[\nu_2\mathbf{2}^+\left(\mathbf{1}^- - \mathbf{6}^-\right) + \nu_4\mathbf{4}^+\left(\mathbf{3}^- - \mathbf{6}^-\right) + \nu_7\mathbf{7}^+\left(\mathbf{5}^- - \mathbf{6}^-\right) + (n-\nu_2-\nu_4-2\nu_6-\nu_7)\Big]\, K_3 = 0, \tag{3.10}$$

$$\Big[p_1^2\nu_2\mathbf{2}^+ + p_2^2\nu_4\mathbf{4}^+ - \nu_2\mathbf{2}^+\mathbf{7}^- - \nu_4\mathbf{4}^+\mathbf{7}^- + \nu_6\mathbf{6}^+\left(\mathbf{5}^- - \mathbf{7}^-\right) + (n-\nu_2-\nu_4-\nu_6-2\nu_7)\Big]\, K_3 = 0. \tag{3.11}$$

Analogous relations for K_2 and K_1 can be obtained by permuting the subscripts of p_i^2, according to eqs. (3.2) and (3.3).

Now, if we recall that we are dealing with the on-shell case, $p_1^2 = p_2^2 = 0$, we can see that some terms on the r.h.s. of eqs. (3.4)–(3.11) (and in the analogous relations for the K_2 and K_1 integrals) vanish. In this case, the following symmetry property is valid:

$$K_1(n;\nu_1,\nu_2,\nu_3,\nu_4,\nu_5,\nu_6,\nu_7) = K_2(n;\nu_3,\nu_4,\nu_1,\nu_2,\nu_5,\nu_6,\nu_7) \qquad (\text{when } p_1^2 = p_2^2). \tag{3.12}$$

Therefore, it is sufficient to consider the K_3 and K_2 integrals.

[7] The application of this procedure to the calculation of the integral $K_3(n;1,1,1,1,1,1,0)$ was presented in [18] (see also eq. (3.13) below). In a more general context, it was also discussed in [30].

[8] The set of recurrence relations for calculating one-loop three-point functions was considered in ref. [31].

When certain powers of propagators ν_i vanish[9], the corresponding integrals K_i can be calculated in terms of Γ functions or finite sums over terms involving Γ functions. A collection of relevant results for such "boundary" integrals is presented in Appendix D. Using the relations (3.4)–(3.11), the integrals K_i with integer powers of propagators ν_i can be reduced to a set of such "boundary" integrals, or to analogous integrals where some ν's are negative, i.e. the corresponding denominators are in the numerator. The latter integrals can also be reduced to boundary integrals (with the corresponding ν's equal to zero), by using the tensor decomposition in appropriate self-energy-type sub-loops, i.e. formulae similar to those collected in Appendix A of ref. [18] (see also [29, 32]).

It should be noted that, starting from integrals with non-positive ν_7 and using relations (3.4)–(3.11), we never get integrals with positive ν_7, since $\mathbf{7}^+$ is always accompanied by ν_7 and one cannot "overcome" $\nu_7 = 0$. Nevertheless, in some cases integrals with positive ν_7 may appear, due to the above-mentioned tensor decomposition in the sub-loops. Since the relevant boundary integrals can be calculated for an arbitrary ν_7 (see Appendix D), this does not create extra problems, even if all seven ν's are positive (see eq. (D.6)).

Let us illustrate this procedure by some important examples. One of them is the integral $K_3(1,1,1,1,1,1,0)$ (Fig. 3a), which was calculated in [16, 17, 18]:

$$\begin{aligned} &K_3(n;1,1,1,1,1,1,0) \\ &\quad = \frac{1}{2\varepsilon^2}\left[K_3(n;0,2,1,0,1,2,0) - 2K_3(n;0,1,0,2,2,1,0) + 2K_3(n;1,1,2,2,0,0,0)\right] \\ &\qquad\qquad = -\pi^{4-2\varepsilon}(-p^2)^{-2-2\varepsilon}\eta^2\left[\frac{1}{4\varepsilon^4} + \frac{1}{4\varepsilon^2}\pi^2 + \frac{6}{\varepsilon}\zeta_3 + \frac{3}{20}\pi^4 + \mathcal{O}(\varepsilon)\right]. \end{aligned} \quad (3.13)$$

To reduce the number of terms, some obvious symmetry properties of the boundary integrals have been used. Another example is the K_2 integral with the same ν's:

$$\begin{aligned} &K_2(n;1,1,1,1,1,1,0) \\ &= -\frac{1}{2\varepsilon(1+2\varepsilon)}\left[K_2(n;1,0,0,2,1,2,0) + K_2(n;0,1,0,1,2,2,0) + 2K_2(n;1,0,0,3,1,1,0)\right] \\ &\qquad = \pi^{4-2\varepsilon}(-p^2)^{-2-2\varepsilon}\eta^2\left[-\frac{3}{4\varepsilon^3} + \frac{3}{2\varepsilon^2} - \frac{3}{\varepsilon} + \frac{1}{4\varepsilon}\pi^2 + 6 - \frac{1}{2}\pi^2 + 6\zeta_3 + \mathcal{O}(\varepsilon)\right]. \end{aligned} \quad (3.14)$$

It corresponds to Fig. 3a, with the left lower momentum off shell. Expanding the η factor (2.19), one can see that our results (3.13)–(3.14) coincide with those presented in [16, 17, 18]. Our eq. (3.13) corresponds to the result for diagram 6A (with numerator = 1) presented in the Appendix of ref. [16], to the first line of Table 4 of ref. [17] ("fig. 2"), and to eq. (12) of ref. [18]. Our eq. (3.14) corresponds to the second line of Table 4 of ref. [17] ("fig. 4"), and to eq. (27) of ref. [18][10]. We have also checked the results for other scalar integrals listed in refs. [16, 17, 18], namely: all results for the diagrams from 6A to 3A in the Appendix of ref. [16] (including those with numerators[11], but excluding

[9] Diagrammatically, this can be understood as shrinking the corresponding line to a point.

[10] In eq. (27) of ref. [18], $9\zeta_4$ should read $9\zeta_2$. We also note another obvious misprint: in the last diagram on the r.h.s. of eq. (9) of [18], the "upper" line should contain a dot, i.e. the power of the propagator is equal to 2, which is clear from their eqs. (8) and (10).

[11] We note a misprint in the result for the diagram 4C with numerator $(l \cdot p)$, $15/(16\omega)$ should read $5/(16\omega)$ (in our notation, $\omega \leftrightarrow -\varepsilon$). In addition, l^2 is forgotten in the product of denominators in diagram 5F, and the denominator $(l-r)^2$ should read $(l+r)^2$ in diagram 4D.

the non-planar diagram 6B); the three remaining lines in Table 4 of ref. [17], as well as the results listed in Appendix C of [17][12] (excluding the non-planar stuff, eqs. (C.6) and (C.7)); eqs. (26) and (28) of ref. [18].

The algorithm discussed in this section makes it possible to evaluate all relevant integrals (corresponding to planar two-loop diagrams) for an *arbitrary* value of the space-time dimension n. However, since in physical applications one usually needs the expansion in $\varepsilon = (4-n)/2$, we present below the results for the two-loop vertices in an expanded form, up to ε^0, i.e. keeping the poles and the finite terms.

4 Two-loop results for the three-gluon vertex

Below, we list the unrenormalized two-loop contributions to the functions $U_i(p^2)$ occurring in the three-gluon vertex. They were calculated using a set of REDUCE [33] programs based on the algorithm described in the previous section. The two-loop diagrams contributing to the three-gluon vertex are shown in Fig. 1 of ref. [15]. The results are expanded in ε up to the finite terms. The factor η is defined (and its expansion in ε is given) in eq. (2.19). The colour factors C_A, T and C_F are defined at the end of section 2.

The contributions of the diagrams without quark loops, in an arbitrary covariant gauge (for the definition of the gauge parameter ξ, see eq. (2.5)) are

$$\begin{aligned}
U_1^{(2,\xi)}(p^2) &= C_A^2\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon^4}\left(\frac{21}{64}+\frac{9}{128}\xi\right)+\frac{1}{\varepsilon^3}\left(\frac{11}{12}+\frac{335}{384}\xi+\frac{25}{256}\xi^2\right)\right.\\
&+\frac{1}{\varepsilon^2}\left(-\frac{37}{72}+\frac{19}{192}\pi^2+\frac{623}{576}\xi+\frac{3}{128}\pi^2\xi+\frac{103}{128}\xi^2+\frac{1}{32}\xi^3\right)\\
&+\frac{1}{\varepsilon}\left(-\frac{239}{54}+\frac{1}{72}\pi^2+\frac{11}{4}\zeta_3+\frac{3343}{1728}\xi+\frac{37}{384}\pi^2\xi\right.\\
&\left.+\frac{21}{32}\zeta_3\xi+\frac{137}{384}\xi^2-\frac{1}{256}\pi^2\xi^2+\frac{9}{16}\xi^3\right)\\
&-\frac{6653}{324}-\frac{119}{432}\pi^2+\frac{139}{12}\zeta_3+\frac{21}{320}\pi^4-\frac{1333}{5184}\xi+\frac{37}{192}\pi^2\xi+\frac{191}{32}\zeta_3\xi\\
&\left.+\frac{1}{64}\pi^4\xi+\frac{1993}{1152}\xi^2-\frac{3}{128}\pi^2\xi^2-\frac{11}{64}\zeta_3\xi^2+\frac{19}{16}\xi^3+\frac{1}{16}\xi^4\right\}+\mathcal{O}(\varepsilon), \qquad (4.1)
\end{aligned}$$

$$\begin{aligned}
U_2^{(2,\xi)}(p^2) &= C_A^2\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{-\frac{5}{16\varepsilon^4}-\frac{1}{\varepsilon^3}\left(\frac{19}{16}+\frac{5}{8}\xi\right)\right.\\
&+\frac{1}{\varepsilon^2}\left(\frac{15}{8}-\frac{5}{48}\pi^2-\frac{1}{12}\xi-\frac{1}{192}\pi^2\xi-\frac{61}{64}\xi^2\right)\\
&+\frac{1}{\varepsilon}\left(\frac{593}{48}-\frac{1}{36}\pi^2-\frac{7}{4}\zeta_3-\frac{67}{288}\xi-\frac{1}{12}\pi^2\xi-\frac{1}{32}\zeta_3\xi+\frac{31}{48}\xi^2-\frac{13}{16}\xi^3\right)\\
&+\frac{17939}{288}+\frac{19}{108}\pi^2-\frac{107}{12}\zeta_3-\frac{1}{20}\pi^4+\frac{8381}{1728}\xi-\frac{31}{96}\pi^2\xi-5\zeta_3\xi\\
&\left.-\frac{1}{640}\pi^4\xi-\frac{41}{288}\xi^2-\frac{1}{192}\pi^2\xi^2+\frac{1}{16}\zeta_3\xi^2-\frac{11}{8}\xi^3-\frac{1}{8}\xi^4\right\}+\mathcal{O}(\varepsilon), \qquad (4.2)
\end{aligned}$$

[12] In eq. (C.5) of ref. [17], in the second term in the square brackets (the term containing D) the numerator $2+\varepsilon^2$ should read $2+\frac{1}{2}\varepsilon^2$ (note that the ε used in [17] corresponds to our -2ε).

$$
\begin{aligned}
U_3^{(2,\xi)}(p^2) &= C_A^2 \, \frac{g^4 \, \eta^2}{(4\pi)^n} (-p^2)^{-2\varepsilon} \left\{ \frac{1}{\varepsilon^4} \left(\frac{1}{16} - \frac{1}{8}\xi - \frac{1}{32}\xi^2 \right) \right. \\
&+ \frac{1}{\varepsilon^3} \left(\frac{79}{96} - \frac{65}{192}\xi - \frac{103}{192}\xi^2 - \frac{27}{256}\xi^3 \right) \\
&+ \frac{1}{\varepsilon^2} \left(\frac{95}{72} + \frac{17}{48}\pi^2 + \frac{157}{288}\xi - \frac{13}{128}\pi^2\xi - \frac{751}{1152}\xi^2 - \frac{1}{192}\pi^2\xi^2 - \frac{81}{256}\xi^3 - \frac{1}{32}\xi^4 \right) \\
&+ \frac{1}{\varepsilon} \left(-\frac{6125}{864} + \frac{155}{72}\pi^2 + \frac{13}{4}\zeta_3 + \frac{97}{1728}\xi - \frac{1}{48}\pi^2\xi - \frac{135}{64}\zeta_3\xi \right. \\
&- \frac{1433}{864}\xi^2 + \frac{3}{128}\pi^2\xi^2 - \frac{1}{32}\zeta_3\xi^2 - \frac{29}{32}\xi^3 - \frac{1}{768}\pi^2\xi^3 - \frac{1}{8}\xi^4 \Big) \\
&- \frac{265205}{5184} + \frac{206}{27}\pi^2 + \frac{227}{12}\zeta_3 + \frac{1}{8}\pi^4 + \frac{140815}{10368}\xi - \frac{319}{288}\pi^2\xi - \frac{239}{32}\zeta_3\xi \\
&- \frac{71}{1280}\pi^4\xi - \frac{23687}{5184}\xi^2 + \frac{1}{6}\pi^2\xi^2 - \frac{9}{64}\zeta_3\xi^2 - \frac{1}{640}\pi^4\xi^2 - \frac{145}{64}\xi^3 \\
&\left. - \frac{5}{768}\pi^2\xi^3 + \frac{5}{32}\zeta_3\xi^3 - \frac{13}{32}\xi^4 \right\} + \mathcal{O}(\varepsilon), \qquad (4.3)
\end{aligned}
$$

$$
\begin{aligned}
p^2 U_4^{(2,\xi)}(p^2) &= -C_A^2 \, \frac{g^4 \, \eta^2}{(4\pi)^n} (-p^2)^{-2\varepsilon} \left\{ \frac{1}{\varepsilon^4} \left(\frac{11}{32} + \xi + \frac{55}{128}\xi^2 + \frac{5}{128}\xi^3 \right) \right. \\
&+ \frac{1}{\varepsilon^3} \left(\frac{11}{16} + \frac{301}{96}\xi + \frac{539}{192}\xi^2 + \frac{65}{48}\xi^3 + \frac{25}{128}\xi^4 \right) \\
&+ \frac{1}{\varepsilon^2} \left(\frac{9}{16} + \frac{11}{96}\pi^2 - \frac{389}{144}\xi + \frac{9}{16}\pi^2\xi + \frac{1541}{1152}\xi^2 + \frac{49}{192}\pi^2\xi^2 \right. \\
&\left. + \frac{4267}{2304}\xi^3 + \frac{1}{32}\pi^2\xi^3 + \frac{3}{16}\xi^4 + \frac{1}{16}\xi^5 \right) \\
&+ \frac{1}{\varepsilon} \left(\frac{2195}{96} - \frac{271}{144}\pi^2 + \frac{19}{8}\zeta_3 - \frac{34549}{1728}\xi + \frac{7}{9}\pi^2\xi + \frac{15}{2}\zeta_3\xi + \frac{7031}{1728}\xi^2 - \frac{5}{576}\pi^2\xi^2 \right. \\
&\left. + \frac{221}{64}\zeta_3\xi^2 + \frac{20881}{3456}\xi^3 - \frac{1}{16}\pi^2\xi^3 + \frac{21}{64}\zeta_3\xi^3 + \frac{293}{256}\xi^4 + \frac{1}{384}\pi^2\xi^4 + \frac{3}{32}\xi^5 \right) \\
&+ \frac{84683}{576} - \frac{2473}{216}\pi^2 + \frac{89}{24}\zeta_3 + \frac{1}{16}\pi^4 - \frac{1212661}{10368}\xi + \frac{685}{216}\pi^2\xi \\
&+ \frac{1661}{48}\zeta_3\xi + \frac{19}{80}\pi^4\xi - \frac{39857}{5184}\xi^2 + \frac{1327}{3456}\pi^2\xi^2 + \frac{109}{6}\zeta_3\xi^2 \\
&+ \frac{139}{1280}\pi^4\xi^2 + \frac{87085}{5184}\xi^3 + \frac{73}{768}\pi^2\xi^3 - \frac{21}{32}\zeta_3\xi^3 + \frac{3}{256}\pi^4\xi^3 \\
&\left. + \frac{89}{32}\xi^4 + \frac{1}{192}\pi^2\xi^4 - \frac{7}{32}\zeta_3\xi^4 + \frac{7}{32}\xi^5 \right\} + \mathcal{O}(\varepsilon), \qquad (4.4)
\end{aligned}
$$

$$
\begin{aligned}
p^2 U_5^{(2,\xi)}(p^2) &= C_A^2 \, \frac{g^4 \, \eta^2}{(4\pi)^n} (-p^2)^{-2\varepsilon} \left\{ -\frac{1}{\varepsilon^4} \left(\frac{1}{8} + \frac{9}{64}\xi \right) + \frac{1}{\varepsilon^3} \left(\frac{1}{24} - \frac{155}{192}\xi - \frac{25}{128}\xi^2 \right) \right. \\
&+ \frac{1}{\varepsilon^2} \left(\frac{10}{9} - \frac{1}{16}\pi^2 - \frac{653}{288}\xi - \frac{7}{192}\pi^2\xi - \frac{67}{128}\xi^2 - \frac{1}{16}\xi^3 \right) \\
&+ \frac{1}{\varepsilon} \left(\frac{385}{216} - \frac{11}{72}\pi^2 - \frac{21}{8}\zeta_3 - \frac{4303}{864}\xi - \frac{5}{192}\pi^2\xi - \frac{5}{4}\zeta_3\xi - \frac{63}{64}\xi^2 + \frac{1}{128}\pi^2\xi^2 - \frac{5}{16}\xi^3 \right) \\
&- \frac{9557}{1296} - \frac{41}{108}\pi^2 - \frac{113}{12}\zeta_3 - \frac{9}{160}\pi^4 - \frac{12889}{1296}\xi + \frac{9}{32}\pi^2\xi - \frac{103}{16}\zeta_3\xi - \frac{9}{320}\pi^4\xi
\end{aligned}
$$

$$
-\frac{457}{192}\xi^2+\frac{7}{128}\pi^2\xi^2+\frac{9}{32}\zeta_3\xi^2-\xi^3\Big\}+\mathcal{O}(\varepsilon), \tag{4.5}
$$

$$
\begin{aligned}
p^2U_6^{(2,\xi)}(p^2) &= -C_A^2\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{3}{32\varepsilon^4}\xi+\frac{1}{\varepsilon^3}\left(-\frac{1}{3}+\frac{41}{192}\xi+\frac{41}{128}\xi^2\right)\right.\\
&+\frac{1}{\varepsilon^2}\left(\frac{157}{144}-\frac{1}{48}\pi^2-\frac{53}{144}\xi+\frac{9}{64}\pi^2\xi-\frac{155}{128}\xi^2+\frac{13}{384}\pi^2\xi^2+\frac{29}{256}\xi^3\right)\\
&+\frac{1}{\varepsilon}\left(\frac{2365}{864}-\frac{1}{36}\pi^2-\frac{1}{8}\zeta_3-\frac{3565}{1728}\xi+\frac{5}{64}\pi^2\xi+\frac{33}{32}\zeta_3\xi\right.\\
&\left.-\frac{73}{192}\xi^2+\frac{11}{128}\pi^2\xi^2+\frac{13}{64}\zeta_3\xi^2-\frac{27}{64}\xi^3\right)\\
&+\frac{40429}{5184}-\frac{47}{432}\pi^2+\frac{31}{12}\zeta_3-\frac{1}{160}\pi^4-\frac{150367}{10368}\xi+\frac{269}{288}\pi^2\xi-\frac{3}{2}\zeta_3\xi+\frac{29}{640}\pi^4\xi\\
&\left.-\frac{29}{9}\xi^2+\frac{113}{384}\pi^2\xi^2+\frac{1}{2}\zeta_3\xi^2+\frac{13}{1280}\pi^4\xi^2-\frac{3}{4}\xi^3-\frac{1}{768}\pi^2\xi^3-\frac{3}{32}\xi^4\right\}+\mathcal{O}(\varepsilon),
\end{aligned} \tag{4.6}
$$

$$
\begin{aligned}
p^2U_7^{(2,\xi)}(p^2) &= C_A^2\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon^4}\left(\frac{21}{32}-\frac{5}{32}\xi-\frac{1}{16}\xi^2\right)\right.\\
&+\frac{1}{\varepsilon^3}\left(\frac{193}{48}+\frac{139}{192}\xi-\frac{289}{384}\xi^2-\frac{27}{128}\xi^3\right)\\
&+\frac{1}{\varepsilon^2}\left(\frac{125}{144}+\frac{83}{96}\pi^2+\frac{119}{144}\xi-\frac{1}{24}\pi^2\xi-\frac{355}{576}\xi^2+\frac{7}{384}\pi^2\xi^2-\frac{133}{256}\xi^3-\frac{1}{16}\xi^4\right)\\
&+\frac{1}{\varepsilon}\left(-\frac{29137}{864}+\frac{311}{72}\pi^2+\frac{19}{2}\zeta_3-\frac{2027}{1728}\xi+\frac{43}{192}\pi^2\xi-\frac{49}{16}\zeta_3\xi\right.\\
&\left.-\frac{8621}{1728}\xi^2+\frac{43}{384}\pi^2\xi^2+\frac{7}{64}\zeta_3\xi^2-\frac{39}{64}\xi^3-\frac{1}{384}\pi^2\xi^3-\frac{1}{4}\xi^4\right)\\
&-\frac{1100617}{5184}+\frac{6377}{432}\pi^2+\frac{337}{6}\zeta_3+\frac{53}{160}\pi^4+\frac{50779}{10368}\xi-\frac{19}{32}\pi^2\xi\\
&-\frac{107}{16}\zeta_3\xi-\frac{19}{320}\pi^4\xi-\frac{15529}{1296}\xi^2+\frac{55}{96}\pi^2\xi^2-\frac{1}{32}\zeta_3\xi^2+\frac{7}{1280}\pi^4\xi^2\\
&\left.-\frac{81}{32}\xi^3-\frac{11}{768}\pi^2\xi^3+\frac{5}{16}\zeta_3\xi^3-\frac{21}{32}\xi^4\right\}+\mathcal{O}(\varepsilon).
\end{aligned} \tag{4.7}
$$

The unrenormalized two-loop contributions of the diagrams involving quark loops are

$$
\begin{aligned}
U_1^{(2,q)}(p^2) &= C_AT\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{-\frac{1}{\varepsilon^3}\left(\frac{17}{24}+\frac{1}{6}\xi\right)+\frac{1}{\varepsilon^2}\left(\frac{17}{18}-\frac{29}{18}\xi\right)\right.\\
&\left.+\frac{1}{\varepsilon}\left(\frac{383}{108}+\frac{1}{72}\pi^2-\frac{85}{54}\xi-\frac{2}{3}\xi^2\right)-\frac{449}{324}+\frac{43}{108}\pi^2+\frac{19}{3}\zeta_3-\frac{197}{162}\xi-\frac{41}{18}\xi^2\right\}\\
&+C_FT\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{2}{\varepsilon}+\frac{61}{3}-16\zeta_3\right\}+\mathcal{O}(\varepsilon),
\end{aligned} \tag{4.8}
$$

$$
\begin{aligned}
U_2^{(2,q)}(p^2) &= C_AT\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon^3}+\frac{1}{\varepsilon^2}\left(-\frac{3}{2}+\frac{5}{3}\xi\right)\right.\\
&\left.-\frac{1}{\varepsilon}\left(\frac{91}{12}+\frac{1}{36}\pi^2+\frac{5}{9}\xi-\frac{4}{3}\xi^2\right)-\frac{2113}{72}-\frac{23}{108}\pi^2-\frac{85}{6}\zeta_3-\frac{190}{27}\xi+\frac{44}{9}\xi^2\right\}\\
&+C_FT\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{-\frac{4}{\varepsilon}-\frac{110}{3}+32\zeta_3\right\}+\mathcal{O}(\varepsilon),
\end{aligned} \tag{4.9}
$$

$$
\begin{aligned}
U_3^{(2,q)}(p^2) &= C_A T\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon^3}\left(-\frac{7}{12}+\frac{1}{3}\xi+\frac{1}{6}\xi^2\right)\right.\\
&+\frac{1}{\varepsilon^2}\left(-\frac{35}{36}-\frac{1}{6}\pi^2+\frac{1}{18}\xi+\frac{11}{18}\xi^2\right)+\frac{1}{\varepsilon}\left(\frac{4405}{216}-\frac{151}{72}\pi^2-\zeta_3-\frac{23}{108}\xi+\frac{47}{27}\xi^2\right)\\
&\left.+\frac{152755}{1296}-\frac{1661}{216}\pi^2-\frac{133}{12}\zeta_3-\frac{1}{20}\pi^4-\frac{235}{324}\xi+\frac{1}{36}\pi^2\xi+2\zeta_3\xi+\frac{337}{81}\xi^2\right\}\\
&+C_F T\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{-\frac{1}{3\varepsilon}\pi^2+\frac{26}{3}-2\pi^2-10\zeta_3\right\}+\mathcal{O}(\varepsilon), \qquad (4.10)
\end{aligned}
$$

$$
\begin{aligned}
p^2U_4^{(2,q)}(p^2) &= C_A T\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon^3}\left(\frac{5}{4}+\frac{29}{24}\xi+\frac{19}{12}\xi^2+\frac{1}{3}\xi^3\right)\right.\\
&+\frac{1}{\varepsilon^2}\left(\frac{103}{4}-\frac{1}{3}\pi^2-\frac{203}{72}\xi+\frac{73}{18}\xi^2+\frac{7}{18}\xi^3\right)\\
&+\frac{1}{\varepsilon}\left(\frac{5125}{24}-\frac{53}{9}\pi^2-2\zeta_3-\frac{457}{54}\xi-\frac{17}{72}\pi^2\xi+\frac{1111}{108}\xi^2-\frac{1}{36}\pi^2\xi^2+\frac{25}{27}\xi^3\right)\\
&+\frac{153277}{144}-\frac{3307}{108}\pi^2-\frac{113}{6}\zeta_3-\frac{1}{10}\pi^4-\frac{3346}{81}\xi-\frac{31}{216}\pi^2\xi+\frac{28}{3}\zeta_3\xi\\
&\left.+\frac{1919}{81}\xi^2+\frac{1}{27}\pi^2\xi^2+\frac{7}{3}\zeta_3\xi^2+\frac{131}{81}\xi^3\right\}\\
&-C_F T\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{8}{\varepsilon^2}+\frac{1}{\varepsilon}\left(52+\frac{2}{3}\pi^2\right)+214+\frac{4}{3}\pi^2+52\zeta_3\right\}+\mathcal{O}(\varepsilon) \qquad (4.11)
\end{aligned}
$$

$$
\begin{aligned}
p^2U_5^{(2,q)}(p^2) &= C_A T\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon^3}\left(-\frac{1}{12}+\frac{1}{3}\xi\right)+\frac{1}{\varepsilon^2}\left(-\frac{14}{9}+\frac{14}{9}\xi\right)\right.\\
&\left.+\frac{1}{\varepsilon}\left(-\frac{64}{27}-\frac{1}{36}\pi^2+\frac{100}{27}\xi\right)+\frac{8101}{324}-\frac{37}{54}\pi^2+\frac{1}{3}\zeta_3+\frac{767}{81}\xi-\frac{1}{3}\xi^2\right\}\\
&-4C_F T\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}+\mathcal{O}(\varepsilon), \qquad (4.12)
\end{aligned}
$$

$$
\begin{aligned}
p^2U_6^{(2,q)}(p^2) &= C_A T\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon^3}\left(-\frac{2}{3}+\frac{11}{24}\xi\right)+\frac{1}{\varepsilon^2}\left(\frac{59}{36}-\frac{65}{72}\xi\right)\right.\\
&+\frac{1}{\varepsilon}\left(\frac{2387}{216}-\frac{13}{18}\pi^2+\frac{49}{27}\xi-\frac{1}{24}\pi^2\xi-\frac{7}{6}\xi^2\right)\\
&\left.+\frac{42515}{1296}-\frac{223}{108}\pi^2+\frac{35}{3}\zeta_3+\frac{1519}{162}\xi-\frac{11}{72}\pi^2\xi+\zeta_3\xi-\frac{77}{18}\xi^2\right\}\\
&+C_F T\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{4}{\varepsilon}+\frac{110}{3}-32\zeta_3\right\}+\mathcal{O}(\varepsilon), \qquad (4.13)
\end{aligned}
$$

$$
\begin{aligned}
p^2U_7^{(2,q)}(p^2) &= C_A T\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon^3}\left(-\frac{8}{3}+\frac{5}{24}\xi+\frac{1}{3}\xi^2\right)\right.\\
&\left.+\frac{1}{\varepsilon^2}\left(-\frac{37}{36}-\frac{1}{3}\pi^2-\frac{167}{72}\xi+\frac{11}{9}\xi^2\right)\right.
\end{aligned}
$$

$$+\frac{1}{\varepsilon}\left(\frac{9419}{216}-\frac{125}{36}\pi^2-2\zeta_3-\frac{61}{54}\xi+\frac{1}{24}\pi^2\xi+\frac{107}{54}\xi^2\right)$$
$$+\frac{334055}{1296}-\frac{352}{27}\pi^2-\frac{29}{6}\zeta_3-\frac{1}{10}\pi^4+\frac{263}{81}\xi+\frac{5}{24}\pi^2\xi+3\zeta_3\xi+\frac{457}{162}\xi^2\Big\}$$
$$+C_F T\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon}\left(4-\frac{2}{3}\pi^2\right)+54-4\pi^2-52\zeta_3\right\}+\mathcal{O}(\varepsilon). \quad (4.14)$$

We have also obtained the two-loop results for the ghost-gluon scalar functions in all on-shell limits of interest. They are given in Appendix E, whereas the relevant two-loop contributions to the two-point functions $J(p^2)$ and $G(p^2)$ are collected in Appendix C. Using all these expressions, together with the one-loop contributions, we have checked that all the results obtained satisfy the WST identities (2.13)–(2.17), as they should.

The renormalization of the results for the three-gluon vertex (and other three- and two-point functions involved) was discussed in detail in section 8 of ref. [15]. The corresponding renormalization factors (Z_1, $\widetilde{Z}_1$, Z_3 and $\widetilde{Z}_3$) in the MS (or $\overline{\rm MS}$) scheme have been presented in refs. [34, 35]. For a detailed discussion of these results, together with a list of misprints, see Appendix B of ref. [15]. To construct renormalized expressions for the three-gluon vertex functions U_i at the two-loop level, we need
(i) to take the sum of (unrenormalized) zero-, one- and two-loop contributions[13], considering the coupling constant and the gauge parameter as "bare" quantities, $g \to g_{\rm B}$ and $\xi \to \xi_{\rm B}$;
(ii) to substitute $g_{\rm B}$ and $\xi_{\rm B}$ in terms of the renormalized g and ξ, multiplied by the appropriate Z-factors (see eqs. (8.8) and (8.9) of ref. [15]);
(iii) to multiply the resulting expression by the corresponding Z-factor[14], namely Z_1 (see eq. (B.1) of ref. [15]).

Since the resulting renormalized expressions are as cumbersome as the unrenormalized ones (and can easily be obtained from the latter ones), we do not present them here. They also contain infrared (on-shell) poles in ε up to $1/\varepsilon^4$.

5 Conclusion

In the limit when two external gluons are on shell, we have calculated the two-loop contributions to the three-gluon vertex, in an arbitrary covariant gauge, keeping finite terms of the expansion in $\varepsilon = (4-n)/2$. In this limit, the three-gluon vertex is described by seven scalar functions $U_i(p^2)$ associated with different tensor structures, see eq. (2.2). The results (listed in section 4) contain on-shell singularities up to $1/\varepsilon^4$. The ultraviolet singularities are at most $1/\varepsilon^2$ and should be removed by the renormalization. In a realistic physical calculation of squared amplitudes, the infrared (on-shell) singularities should be cancelled by the contributions of one-loop diagrams with soft emission from the external legs, etc., according to the Kinoshita–Lee–Nauenberg mechanism [36]. In this way, our result will be useful as a "block" in the calculation of NNLO corrections to physical amplitudes.

[13]Note that the one-loop expressions should be expanded up to ε^2 terms, since they may be multiplied by other one-loop contributions involving $1/\varepsilon^2$ poles.

[14]For the ghost-gluon vertex (see Appendix E), the renormalization factor $\widetilde{Z}_1$ is required (see, for instance, eq. (B.2) in ref. [15]).

We have also calculated the ghost-gluon vertex (2.9) in all on-shell limits of interest; the results are collected in Appendix E. We have confirmed that the obtained results obey the corresponding WST identities (2.13)–(2.17).

We note that in the on-shell case considered, the problem of irreducible numerators in three-point two-loop integrals already shows up, but it can be overcome in a relatively simple way, since the relevant boundary integrals can be calculated for any integer powers of this numerator (see Appendix D). In the zero-momentum calculation [15], there was no such problem at all. In the general off-shell calculation, though, the problem of irreducible numerators is much more severe [27].

Our results can be considered as a further step, in addition to [14, 15], towards calculating the two-loop QCD vertices in more complicated cases, such as the on-shell limit with just one gluon on shell, or the general off-shell case[15].

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[15] In principle, the techniques for calculating the relevant off-shell scalar integrals are already available [37, 27].

Appendix A: Some useful one-loop formulae

The "triangle" integral with the external momenta p_1, p_2 and $p_3 = -p_1 - p_2$ is defined as

$$J(n;\nu_1,\nu_2,\nu_3) \equiv \int \frac{\mathrm{d}^n q}{[(p_2-q)^2]^{\nu_1}[(p_1+q)^2]^{\nu_2}(q^2)^{\nu_3}}, \tag{A.1}$$

where $n = 4-2\varepsilon$ is the space-time dimension. For general values of n and ν_i, the result for the integral (A.1) can be expressed in terms of hypergeometric functions of two variables [38, 31].

When two external legs are on shell, $p_1^2 = p_2^2 = 0$ ($p_3^2 \equiv p^2$), the following simple formula can be easily obtained:

$$J(n;\nu_1,\nu_2,\nu_3)\Big|_{\substack{p_1^2=p_2^2=0\\ p_3^2\equiv p^2}} = \mathrm{i}^{1-n}\pi^{n/2}(p^2)^{n/2-\Sigma\nu_i}\,\frac{\Gamma\left(\frac{n}{2}-\nu_1-\nu_3\right)\Gamma\left(\frac{n}{2}-\nu_2-\nu_3\right)\Gamma\left(\sum\nu_i-\frac{n}{2}\right)}{\Gamma(\nu_1)\Gamma(\nu_2)\Gamma\left(n-\sum\nu_i\right)}. \tag{A.2}$$

In particular, we shall need this formula for the case when one of the indices is a negative integer, $\nu_3 = -s$. This means that $(q^2)^s$ is in the numerator. When $\nu_3 = 0$, the r.h.s. of (A.2) gives the well-known result for the one-loop two-point function. When ν_1 or ν_2 are non-positive integers, we get zero.

We also need the result for the triangle integral with one leg on shell. Assuming that $p_1^2 \neq 0$, $p_2^2 \neq 0$ and $p_3^2 = 0$, we get

$$\begin{aligned} J(n;\nu_1,\nu_2,\nu_3)\Big|_{p_3^2=0} &= \mathrm{i}^{1-n}\pi^{n/2}(p_1^2)^{n/2-\Sigma\nu_i}\,\frac{\Gamma\left(\frac{n}{2}-\nu_1-\nu_2\right)}{\Gamma(\nu_3)\Gamma\left(n-\sum\nu_i\right)} \\ &\times\left\{\frac{\Gamma\left(\frac{n}{2}-\nu_1-\nu_3\right)\Gamma\left(\sum\nu_i-\frac{n}{2}\right)}{\Gamma(\nu_2)}\,{}_2F_1\left(\begin{array}{c}\nu_1,\ \Sigma\nu_i-\frac{n}{2}\\ \nu_1+\nu_3-\frac{n}{2}+1\end{array}\,\middle|\,\frac{p_2^2}{p_1^2}\right)\right. \\ &\left.+\frac{\Gamma\left(\nu_1+\nu_3-\frac{n}{2}\right)\Gamma\left(\frac{n}{2}-\nu_3\right)}{\Gamma(\nu_1)}\,\left(\frac{p_2^2}{p_1^2}\right)^{n/2-\nu_1-\nu_3}\,{}_2F_1\left(\begin{array}{c}\nu_2,\ \frac{n}{2}-\nu_3\\ \frac{n}{2}-\nu_1-\nu_3+1\end{array}\,\middle|\,\frac{p_2^2}{p_1^2}\right)\right\}, \end{aligned} \tag{A.3}$$

where ${}_2F_1$ is the Gauss hypergeometric function. If we use the well-known formula of analytic continuation of ${}_2F_1$ function from the argument z to $1-z$ (with $z = p_2^2/p_1^2$), we reproduce the result presented in Appendix A of ref. [39] (namely, their K-integral at $N = 0$). When $\nu_1 = -s$ (where s is a non-negative integer), the second term in the braces of (A.3) vanishes, and we get

$$\begin{aligned} &J(n;-s,\nu_2,\nu_3)\Big|_{p_3^2=0} = \mathrm{i}^{1-n}\pi^{n/2}(p_1^2)^{n/2+s-\nu_2-\nu_3} \\ &\times\frac{\Gamma\left(\frac{n}{2}+s-\nu_2\right)\Gamma\left(\frac{n}{2}+s-\nu_3\right)\Gamma\left(\nu_2+\nu_3-s-\frac{n}{2}\right)}{\Gamma(\nu_2)\Gamma(\nu_3)\Gamma\left(n+s-\nu_2-\nu_3\right)}\,{}_2F_1\left(\begin{array}{c}-s,\ \nu_2+\nu_3-s-\frac{n}{2}\\ \nu_3-s-\frac{n}{2}+1\end{array}\,\middle|\,\frac{p_2^2}{p_1^2}\right), \end{aligned} \tag{A.4}$$

with a terminating ${}_2F_1$ series containing $(s+1)$ terms.

Appendix B: One-loop results for the ghost-gluon vertex

At the zero-loop level, we have

$$a^{(0)}(p^2,0,0) = a^{(0)}(0,p^2,0) = a^{(0)}(0,0,p^2) = 1, \tag{B.1}$$

whereas all other ghost-gluon functions are equal to zero at this order.

The diagrams contributing to the ghost-gluon vertex at the one-loop level are shown in Fig. 3 of ref. [12]. The general expressions listed in Appendix D of [12] give the following results in the on-shell limits of interest:

$$\begin{aligned}
a^{(1)}(p^2,0,0) &= \frac{g^2\,\eta}{(4\pi)^{n/2}}\,\frac{C_A}{16(n-4)}\,\kappa(p^2) \\
&\quad\times\Big[4(n-4)-2\xi(2n^2-9n+8)+\xi^2(n-2)(n-4)\Big], && \text{(B.2)}\\
a^{(1)}(0,p^2,0) &= -\frac{g^2\,\eta}{(4\pi)^{n/2}}\,\frac{C_A}{4(n-4)}\,\kappa(p^2)\,\Big[2+\xi(n^2-6n+7)\Big], && \text{(B.3)}\\
a^{(1)}(0,0,p^2) &= \frac{g^2\,\eta}{(4\pi)^{n/2}}\,\frac{C_A}{8(n-4)}\,\kappa(p^2)\,\left[2(3n-8)-\xi(n-4)\right], && \text{(B.4)}\\
b^{(1)}(p^2,0,0) &= \frac{g^2\,\eta}{(4\pi)^{n/2}}\,\frac{C_A}{16(n-4)p^2}\,\kappa(p^2) \\
&\quad\times\Big[16(n-4)+4\xi(2n^2-15n+30)-\xi^2(n^2-8n+20)\Big], && \text{(B.5)}\\
b^{(1)}(0,p^2,0) &= \frac{g^2\,\eta}{(4\pi)^{n/2}}\,\frac{C_A}{8(n-4)p^2}\,\kappa(p^2) \\
&\quad\times\Big[8(n-4)+4\xi(n^2-7n+13)-\xi^2(n^2-8n+14)\Big], && \text{(B.6)}\\
b^{(1)}(0,0,p^2) &= \frac{g^2\,\eta}{(4\pi)^{n/2}}\,\frac{C_A}{4(n-4)p^2}\,\kappa(p^2)\,\Big[8+2\xi(2n-5)-\xi^2\Big], && \text{(B.7)}\\
c^{(1)}(p^2,0,0) &= -\frac{g^2\,\eta}{(4\pi)^{n/2}}\,\frac{C_A}{4(n-4)p^2}\,\kappa(p^2)\,\xi\,(n-6+2\xi), && \text{(B.8)}\\
c^{(1)}(0,p^2,0) &= \frac{g^2\,\eta}{(4\pi)^{n/2}}\,\frac{C_A}{8(n-4)p^2}\,\kappa(p^2) \\
&\quad\times\Big[4(n-6)+2\xi(n-2)(n-6)+\xi^2(n^2-10n+20)\Big], && \text{(B.9)}\\
c^{(1)}(0,0,p^2) &= -\frac{g^2\,\eta}{(4\pi)^{n/2}}\,\frac{C_A}{16(n-4)p^2}\,\kappa(p^2)\,(n-6)\,\Big[8+4\xi(n-2)+\xi^2(n-4)\Big], && \text{(B.10)}\\
d^{(1)}(p^2,0,0) &= \frac{g^2\,\eta}{(4\pi)^{n/2}}\,\frac{C_A}{16(n-4)p^2}\kappa(p^2)\,\Big[8(n-6)-4\xi(5n-18)+\xi^2(n^2-4n-4)\Big], && \text{(B.11)}\\
d^{(1)}(0,p^2,0) &= -\frac{g^2\,\eta}{(4\pi)^{n/2}}\,\frac{C_A}{8(n-4)p^2}\,\kappa(p^2) \\
&\quad\times\Big[8(n-6)+2\xi(2n^2-15n+32)-\xi^2(n^2-8n+14)\Big], && \text{(B.12)}\\
d^{(1)}(0,0,p^2) &= \frac{g^2\,\eta}{(4\pi)^{n/2}}\,\frac{C_A}{4(n-4)p^2}\,\kappa(p^2)\,\Big[2(n-6)-2\xi(2n-5)+\xi^2\Big], && \text{(B.13)}
\end{aligned}$$

$$e^{(1)}(p^2,0,0) \;=\; -\frac{g^2\,\eta}{(4\pi)^{n/2}}\,\frac{C_A}{2(n-4)p^2}\,\kappa(p^2)\,\left[2+\xi(2n-9)+2\xi^2\right], \tag{B.14}$$

$$e^{(1)}(0,p^2,0) \;=\; e^{(1)}(0,0,p^2) \;=\; -\frac{g^2\,\eta}{(4\pi)^{n/2}}\,\frac{C_A}{4p^2}\,\kappa(p^2)\,\left[2+\xi(n-3)\right], \tag{B.15}$$

where η and $\kappa(p^2)$ are defined by eqs. (2.19) and (2.18), respectively. These results are valid for arbitrary values of n and ξ. Note that there are no quark-loop contributions at the one-loop level.

Appendix C: Two-point functions

For arbitrary values of n and ξ, the results for the one-loop two-point functions (see eqs. (2.6) and (2.7)) are available elsewhere (see [35, 25, 12]). For completeness, we also present them here:

$$J^{(1)}(p^2) = \frac{g^2\,\eta}{(4\pi)^{n/2}}\,\kappa(p^2)\left\{-\frac{C_A}{8}\left[\frac{4(3n-2)}{n-1}+4(2n-7)\xi-(n-4)\xi^2\right]+2T\frac{n-2}{n-1}\right\}, \tag{C.1}$$

$$G^{(1)}(p^2) = \frac{g^2\,\eta}{(4\pi)^{n/2}}\,\frac{C_A}{4}\,\kappa(p^2)\,\left[2+(n-3)\xi\right], \tag{C.2}$$

where $\kappa(p^2)$ and η are defined in eqs. (2.18) and (2.19), respectively. The corresponding diagrams are shown in Fig. 2 of ref. [12].

Two-loop diagrams contributing to the gluon polarization operator are shown in Fig. 3 of ref. [15]. Calculating their sum, we get the following unrenormalized results [15]:

$$J^{(2,\xi)}(p^2) = C_A^2\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon^2}\left(-\frac{25}{12}+\frac{5}{24}\xi+\frac{1}{4}\xi^2\right)+\frac{1}{\varepsilon}\left(-\frac{583}{72}+\frac{113}{144}\xi-\frac{19}{24}\xi^2+\frac{3}{8}\xi^3\right)\right.$$
$$\left.-\frac{14311}{432}+\zeta_3+\frac{425}{864}\xi+2\xi\zeta_3-\frac{71}{72}\xi^2+\frac{9}{16}\xi^3+\frac{1}{16}\xi^4\right\}+\mathcal{O}(\varepsilon), \tag{C.3}$$

$$J^{(2,q)}(p^2) = C_A T\frac{g^4\,\eta^2}{(4\pi)^n}\,(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon^2}\left(\frac{5}{3}-\frac{2}{3}\xi\right)+\frac{1}{\varepsilon}\left(\frac{101}{18}+\frac{8}{9}\xi-\frac{2}{3}\xi^2\right)\right.$$
$$\left.+\frac{1961}{108}+8\zeta_3+\frac{142}{27}\xi-\frac{22}{9}\xi^2\right\}$$
$$+C_F T\frac{g^4\,\eta^2}{(4\pi)^n}\,(-p^2)^{-2\varepsilon}\left\{\frac{2}{\varepsilon}+\frac{55}{3}-16\zeta_3\right\}+\mathcal{O}(\varepsilon), \tag{C.4}$$

where C_A, T and C_F are defined at the end of section 2.

The two-loop ghost self-energy diagrams are shown in Fig. 4 of ref. [15]. Their sum yields the following unrenormalized results:

$$G^{(2,\xi)}(p^2) = C_A^2\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon^2}\left(\frac{5}{4}+\frac{7}{16}\xi-\frac{1}{32}\xi^2\right)+\frac{1}{\varepsilon}\left(\frac{83}{16}+\frac{7}{32}\xi\right)\right.$$
$$\left.+\frac{599}{32}-\frac{3}{4}\zeta_3-\frac{9}{64}\xi+\frac{3}{8}\xi^2-\frac{3}{16}\xi^2\zeta_3\right\}+\mathcal{O}(\varepsilon). \tag{C.5}$$

$$G^{(2,q)}(p^2) = C_A T \frac{g^4\,\eta^2}{(4\pi)^n}\,(-p^2)^{-2\varepsilon}\left\{-\frac{1}{2\varepsilon^2}-\frac{7}{4\varepsilon}-\frac{53}{8}\right\}+\mathcal{O}(\varepsilon). \qquad \text{(C.6)}$$

The discussion of renormalization, as well as the renormalized results for J and G can be found in section 8 of ref. [15] (see also [14], where these results in the Feynman gauge are presented).

Appendix D: Boundary integrals

Below the case $p_1^2 = p_2^2 = 0,\ p_3^2 \equiv p^2$ is understood. Using the integration-by-parts relations (3.4)–(3.11), we can reduce the integrals with six denominators to integrals where some of the lines are shrunk, i.e. some of the ν's vanish. Here we list explicit results for such "boundary" integrals of interest. Apart from the one-loop formulae listed in Appendix A, the following well-known (see, for example, in [18, 5]) trick is useful, to combine two internal lines attached to a triple vertex with the remaining external line on shell:

$$\left.\frac{1}{[(p-q)^2]^{\nu}\ (q^2)^{\nu'}}\right|_{p^2=0} = \frac{\Gamma(\nu+\nu')}{\Gamma(\nu)\,\Gamma(\nu')}\int\limits_0^1 \frac{\alpha^{\nu-1}(1-\alpha)^{\nu'-1}\,\mathrm{d}\alpha}{[(q-\alpha p)^2]^{\nu+\nu'}}. \qquad \text{(D.1)}$$

In fact, this is nothing but the Feynman parametrization of a product of two propagators, where it is taken into account that the difference of their momenta is light-like.

The diagrams formally corresponding to the boundary cases of the integrals K_3 and K_2 (listed below) are drawn in Fig. 4 and Fig. 5, respectively. To distinguish an off-shell external line (corresponding to p_3) from the on-shell ones (corresponding to p_1 and p_2), the former is drawn as a double line, which can be associated with the sum of p_1 and p_2. Note that when using the tensor decomposition in the sub-loops we may get some integrals with positive ν_7. This is not a problem, because the relevant integrals can be calculated for any ν_7 (see below). When the result is valid for an arbitrary ν_7, the corresponding line is solid; when it is valid only for non-positive integer ν_7, this is indicated by a dashed line, as in Fig. 3b.

The results (D.2), (D.4) and (D.9) can be obtained by repeated use of the one-loop formulae (see Appendix A). To get other results, the trick (D.1) has been used. For the integrals (D.3), (D.7), (D.8) and (D.10), this trick was used for two pairs of propagators. The notation $\Sigma\nu_i$ means $\sum_{i=1}^7 \nu_i$, i.e. the sum over all ν's involved (excluding those equal to zero). In particular, if $\nu_7 = -s$ then $\Sigma\nu_i = \sum_{i=1}^6 \nu_i - s$.

D.1: K_3 integrals

The following boundary integrals can be expressed in terms of one-term products of Γ functions:

$$K_3(n;\nu_1,\nu_2,\nu_3,\nu_4,\nu_5,0,\nu_7) = \mathrm{i}^{2-2n}\pi^n (p^2)^{n-\Sigma\nu_i}\,\Gamma\Big(\nu_1+\nu_3+\nu_5-\tfrac{n}{2}\Big)\,\Gamma\Big(\nu_2+\nu_4+\nu_7-\tfrac{n}{2}\Big)$$
$$\times\frac{\Gamma\Big(\frac{n}{2}-\nu_1-\nu_5\Big)\,\Gamma\Big(\frac{n}{2}-\nu_3-\nu_5\Big)\,\Gamma\Big(\frac{n}{2}-\nu_2-\nu_7\Big)\,\Gamma\Big(\frac{n}{2}-\nu_4-\nu_7\Big)}{\Gamma(\nu_1)\Gamma(\nu_2)\Gamma(\nu_3)\Gamma(\nu_4)\Gamma\left(n-\nu_1-\nu_3-\nu_5\right)\Gamma\left(n-\nu_2-\nu_4-\nu_7\right)}, \qquad \text{(D.2)}$$

$$K_3(n;0,\nu_2,\nu_3,0,\nu_5,\nu_6,\nu_7) = \mathrm{i}^{2-2n}\pi^n (p^2)^{n-\Sigma\nu_i}\,\Gamma(\Sigma\nu_i - n)$$
$$\times\frac{\Gamma\left(\frac{n}{2}-\nu_2-\nu_7\right)\Gamma\left(\frac{n}{2}-\nu_3-\nu_5\right)\Gamma\left(\frac{n}{2}-\nu_6\right)\Gamma(n-\nu_2-\nu_5-\nu_6-\nu_7)\,\Gamma(n-\nu_3-\nu_5-\nu_6-\nu_7)}{\Gamma(\nu_2)\Gamma(\nu_3)\Gamma(\nu_6)\Gamma\left(n-\nu_2-\nu_6-\nu_7\right)\Gamma\left(n-\nu_3-\nu_5-\nu_6\right)\Gamma\left(\frac{3n}{2}-\Sigma\nu_i\right)}, \tag{D.3}$$

$$K_3(n;0,\nu_2,0,\nu_4,\nu_5,\nu_6,\nu_7) = \mathrm{i}^{2-2n}\pi^n (p^2)^{n-\Sigma\nu_i}\,\Gamma(\Sigma\nu_i - n)$$
$$\times\frac{\Gamma\left(\frac{n}{2}-\nu_5\right)\Gamma\left(\frac{n}{2}-\nu_6\right)\Gamma\left(\nu_5+\nu_6-\frac{n}{2}\right)\Gamma(n-\nu_2-\nu_5-\nu_6-\nu_7)\,\Gamma(n-\nu_4-\nu_5-\nu_6-\nu_7)}{\Gamma(\nu_2)\Gamma(\nu_4)\Gamma(\nu_5)\Gamma(\nu_6)\Gamma\left(n-\nu_5-\nu_6\right)\Gamma\left(\frac{3n}{2}-\Sigma\nu_i\right)}. \tag{D.4}$$

In the following formula we assume that $\nu_7 = -s$ is a non-positive integer:

$$K_3(n;\nu_1,\nu_2,\nu_3,0,0,\nu_6,-s) = \mathrm{i}^{2-2n}\pi^n (p^2)^{n-\Sigma\nu_i}\,\Gamma(\Sigma\nu_i - n)$$
$$\times\frac{\Gamma\left(\frac{n}{2}+s-\nu_2\right)\Gamma\left(\frac{n}{2}-\nu_3\right)\Gamma\left(\frac{n}{2}+s-\nu_6\right)\Gamma(n+s-\nu_1-\nu_2-\nu_6)}{\Gamma(\nu_2)\Gamma(\nu_3)\Gamma(\nu_6)\Gamma\left(\frac{3n}{2}-\Sigma\nu_i\right)\Gamma\left(n+s-\nu_2-\nu_6\right)}$$
$$\times\frac{\Gamma\left(\nu_2+\nu_6-s-\frac{n}{2}\right)}{\Gamma\left(\nu_1+\nu_2+\nu_6-s-\frac{n}{2}\right)}\,{}_3F_2\left(\begin{matrix}-s,\ \frac{n}{2}-\nu_3,\ \nu_2+\nu_6-s-\frac{n}{2}\\ \nu_6-s-\frac{n}{2}+1,\ \nu_1+\nu_2+\nu_6-s-\frac{n}{2}\end{matrix}\,\middle|\,1\right). \tag{D.5}$$

Here, ${}_3F_2$ denotes a generalized hypergeometric series. In fact, we have a terminating ${}_3F_2$ series of unit argument, since one of the upper parameters is equal to $-s$. This may be considered just as a compact representation of a finite sum containing $(s+1)$ terms, each term being a product of Γ functions.

Using the results for the boundary integrals with $\nu_7 > 0$, one can also calculate the planar forward-scattering double-box diagram (see Fig. 3b),

$$K_3(n;1,1,1,1,1,1,1) = -\pi^{4-2\varepsilon}(-p^2)^{-3-2\varepsilon}\eta^2\left[\frac{1}{\varepsilon^3}-\frac{5}{\varepsilon^2}+\frac{16}{\varepsilon}+\frac{\pi^2}{\varepsilon}-44-\pi^2+24\zeta_3+\mathcal{O}(\varepsilon)\right], \tag{D.6}$$

and similar integrals with higher powers of the propagators.

D.2: K_2 integrals

In the following two formulae, we also assume that $\nu_7 = -s$ is a non-positive integer:

$$K_2(n;0,\nu_2,\nu_3,\nu_4,\nu_5,\nu_6,-s) = \mathrm{i}^{2-2n}\pi^n (p^2)^{n-\Sigma\nu_i}\,\Gamma(\Sigma\nu_i - n)$$
$$\times\frac{\Gamma\left(\frac{n}{2}-\nu_2-\nu_4\right)\Gamma\left(\frac{n}{2}-\nu_3-\nu_5\right)\Gamma\left(\frac{n}{2}-\nu_6\right)\Gamma(n-\nu_2-\nu_3-\nu_4-\nu_6)\,\Gamma(n+s-\nu_3-\nu_4-\nu_5-\nu_6)}{\Gamma(\nu_2)\Gamma(\nu_5)\Gamma(\nu_6)\Gamma\left(\frac{3n}{2}-\Sigma\nu_i\right)\Gamma\left(n-\nu_2-\nu_4-\nu_6\right)\Gamma\left(n-\nu_3-\nu_5-\nu_6\right)}$$
$$\times\frac{\Gamma\left(\nu_3+\nu_5+\nu_6-\frac{n}{2}\right)}{\Gamma\left(\nu_3+\nu_5+\nu_6-s-\frac{n}{2}\right)}\,{}_3F_2\left(\begin{matrix}-s,\ \nu_3,\ \frac{n}{2}-\nu_2-\nu_4\\ n-\nu_2-\nu_4-\nu_6,\ \nu_3+\nu_5+\nu_6-s-\frac{n}{2}\end{matrix}\,\middle|\,1\right), \tag{D.7}$$

$$K_2(n;\nu_1,0,\nu_3,\nu_4,\nu_5,\nu_6,-s) = \mathrm{i}^{2-2n}\pi^n (p^2)^{n-\Sigma\nu_i}\,\Gamma\left(\Sigma\nu_i - n\right)$$
$$\times\frac{\Gamma\left(\frac{n}{2}-\nu_4+s\right)\Gamma\left(\frac{n}{2}-\nu_6+s\right)\Gamma(n-\nu_1-\nu_3-\nu_4-\nu_6+s)\,\Gamma(n-\nu_3-\nu_4-\nu_5-\nu_6+s)}{\Gamma(\nu_1)\Gamma(\nu_4)\Gamma(\nu_5)\Gamma(\nu_6)\Gamma\left(n-\nu_4-\nu_6+s\right)\Gamma\left(\frac{3n}{2}-\nu_1-\nu_3-\nu_4-\nu_5-\nu_6+s\right)}$$
$$\times\Gamma\left(\nu_4+\nu_6-s-\tfrac{n}{2}\right)\ {}_3F_2\left(\begin{matrix}-s,\ 1-\nu_5,\ \nu_4+\nu_6-s-\frac{n}{2}\\ \nu_6-s-\frac{n}{2}+1,\ \nu_1+\nu_3+\nu_4+\nu_6-s-n+1\end{matrix}\,\middle|\,1\right). \tag{D.8}$$

Here, we also have terminating ${}_3F_2$ series of unit argument, containing $(s+1)$ terms. However, in some cases they may contain less terms. For example, for an integer $\nu_5 > 0$ the number of terms in ${}_3F_2$ from (D.8) is $\min(s+1, \nu_5)$. In particular, when $s = 0$ or $\nu_5 = 1$ we get just one term.

When $\nu_3 = 0$, eq. (D.7) also contains just one term, since one of the upper parameters in ${}_3F_2$ vanishes, so that ${}_3F_2 = 1$. Moreover, the corresponding result can be extended to an arbitrary value of ν_7:

$$K_2(n;0,\nu_2,0,\nu_4,\nu_5,\nu_6,\nu_7) = \mathrm{i}^{2-2n}\pi^n (p^2)^{n-\Sigma\nu_i}\,\Gamma\left(\Sigma\nu_i - n\right) \times \frac{\Gamma\left(\frac{n}{2}-\nu_5\right)\Gamma\left(\frac{n}{2}-\nu_6\right)\Gamma\left(\frac{n}{2}-\nu_2-\nu_4\right)\Gamma\left(\nu_5+\nu_6-\frac{n}{2}\right)\Gamma(n-\nu_4-\nu_5-\nu_6-\nu_7)}{\Gamma(\nu_2)\Gamma(\nu_5)\Gamma(\nu_6)\Gamma\left(\nu_5+\nu_6+\nu_7-\frac{n}{2}\right)\Gamma\left(n-\nu_5-\nu_6\right)\Gamma\left(\frac{3n}{2}-\Sigma\nu_i\right)}. \quad \text{(D.9)}$$

An important special case of eq. (D.7) is $\nu_7 = 0$ ($s = 0$). In this case, the ${}_3F_2$ function is equal to 1 (since one of the upper parameters is zero) and we get[16]

$$K_2(n;0,\nu_2,\nu_3,\nu_4,\nu_5,\nu_6,0) = \mathrm{i}^{2-2n}\pi^n (p^2)^{n-\Sigma\nu_i}\,\Gamma\left(\Sigma\nu_i - n\right) \times \frac{\Gamma\left(\frac{n}{2}-\nu_2-\nu_4\right)\Gamma\left(\frac{n}{2}-\nu_3-\nu_5\right)\Gamma\left(\frac{n}{2}-\nu_6\right)\Gamma(n-\nu_2-\nu_3-\nu_4-\nu_6)\,\Gamma(n-\nu_3-\nu_4-\nu_5-\nu_6)}{\Gamma(\nu_2)\Gamma(\nu_5)\Gamma(\nu_6)\Gamma\left(n-\nu_2-\nu_4-\nu_6\right)\Gamma\left(n-\nu_3-\nu_5-\nu_6\right)\Gamma\left(\frac{3n}{2}-\Sigma\nu_i\right)}. \quad \text{(D.10)}$$

This result corresponds to the last diagram shown in Fig. 5.

Appendix E: Two-loop results for the ghost-gluon vertex

Here we present the unrenormalized expressions for two-loop contributions to the scalar functions occurring in the ghost-gluon vertex (2.9), in all on-shell limits of interest. To calculate these functions, the same algorithms (and the same REDUCE program) as for the three-gluon vertex have been employed. The two-loop diagrams contributing to the ghost-gluon vertex are shown in Fig. 2 of ref. [15]. Their renormalization is similar to that of the three-gluon vertex (see section 8 and Appendix B of ref. [15]); the renormalization factor $\widetilde{Z}_1$ should be used.

E.1: Non-zero gluon momentum squared

$$a^{(2,\xi)}(p^2,0,0) \;=\; C_A^2\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{\xi}{\varepsilon^4}\left(-\frac{1}{32}+\frac{1}{128}\xi\right)+\frac{1}{\varepsilon^3}\left(\frac{5}{96}+\frac{17}{192}\xi-\frac{21}{128}\xi^2+\frac{1}{128}\xi^3\right)\right.$$

[16] This diagram has been considered in ref. [40], using the method of negative-dimensional integration. We note that in their result (11) (or in the definition (5)) the parameters m and n (corresponding, up to a sign, to ν_4 and ν_3 in our eq. (D.10)) should be interchanged. Also, $(-\pi)^D$ should read π^D (they use Euclidean metric). We are grateful to the authors of [40] for confirming these misprints. The other diagrams considered in [40] (see also [41]) correspond to integrals that can be obtained by repeated use of one-loop formulae. The results are given in eqs. (18), (21) and (23) of [40], and they correspond to the special cases of our eqs. (D.8) (at $s = 0$), (D.2) (at $\nu_7 = 0$) and (D.4) (at $\nu_7 = 0$), respectively.

$$
\begin{aligned}
&+\frac{1}{\varepsilon^2}\left(\frac{19}{72}-\frac{5}{96}\pi^2-\frac{25}{288}\xi-\frac{3}{128}\pi^2\xi+\frac{73}{384}\xi^2+\frac{1}{384}\pi^2\xi^2-\frac{19}{128}\xi^3+\frac{1}{768}\pi^2\xi^3\right)\\
&+\frac{1}{\varepsilon}\left(\frac{2141}{864}-\frac{101}{288}\pi^2-\frac{5}{16}\zeta_3+\frac{125}{1728}\xi-\frac{45}{64}\zeta_3\xi-\frac{1}{72}\xi^2-\frac{5}{384}\pi^2\xi^2\right.\\
&\left.-\frac{1}{32}\zeta_3\xi^2-\frac{1}{8}\xi^3+\frac{7}{768}\pi^2\xi^3+\frac{1}{128}\zeta_3\xi^3-\frac{1}{32}\xi^4\right)\\
&+\frac{57245}{5184}-\frac{895}{864}\pi^2-\frac{179}{48}\zeta_3-\frac{1}{64}\pi^4-\frac{9139}{10368}\xi+\frac{3}{32}\pi^2\xi-\frac{81}{32}\zeta_3\xi\\
&-\frac{21}{1280}\pi^4\xi+\frac{587}{1728}\xi^2-\frac{1}{384}\pi^2\xi^2-\frac{7}{16}\zeta_3\xi^2-\frac{23}{64}\xi^3+\frac{19}{768}\pi^2\xi^3\\
&+\frac{7}{128}\zeta_3\xi^3+\frac{1}{2560}\pi^4\xi^3-\frac{3}{32}\xi^4\Big\}+\mathcal{O}(\varepsilon), \qquad \text{(E.1)}
\end{aligned}
$$

$$
\begin{aligned}
p^2b^{(2,\xi)}(p^2,0,0) \;=\; & C_A^2\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{\xi}{\varepsilon^4}\left(\frac{3}{64}-\frac{3}{128}\xi\right)+\frac{1}{\varepsilon^3}\left(-\frac{1}{3}+\frac{19}{96}\xi+\frac{1}{24}\xi^2-\frac{13}{128}\xi^3\right)\right.\\
&+\frac{1}{\varepsilon^2}\left(\frac{17}{72}+\frac{1}{24}\pi^2+\frac{17}{18}\xi+\frac{1}{192}\pi^2\xi-\frac{83}{576}\xi^2-\frac{7}{384}\pi^2\xi^2-\frac{5}{32}\xi^3\right.\\
&\left.+\frac{1}{384}\pi^2\xi^3-\frac{1}{32}\xi^4\right)\\
&+\frac{1}{\varepsilon}\left(\frac{779}{432}+\frac{31}{144}\pi^2+\frac{1}{4}\zeta_3+\frac{1093}{864}\xi+\frac{1}{96}\pi^2\xi+\frac{1}{2}\zeta_3\xi+\frac{331}{864}\xi^2-\frac{1}{64}\pi^2\xi^2\right.\\
&\left.-\frac{11}{32}\zeta_3\xi^2-\frac{67}{128}\xi^3+\frac{1}{128}\pi^2\xi^3+\frac{1}{64}\zeta_3\xi^3-\frac{1}{8}\xi^4\right)\\
&+\frac{24959}{2592}+\frac{293}{432}\pi^2+\frac{25}{24}\zeta_3+\frac{1}{80}\pi^4+\frac{5371}{5184}\xi-\frac{11}{96}\pi^2\xi+\frac{35}{16}\zeta_3\xi+\frac{3}{320}\pi^4\xi\\
&+\frac{5549}{2592}\xi^2+\frac{7}{192}\pi^2\xi^2-\frac{7}{4}\zeta_3\xi^2-\frac{3}{320}\pi^4\xi^2-\frac{11}{8}\xi^3+\frac{1}{48}\pi^2\xi^3+\frac{3}{16}\zeta_3\xi^3\\
&\left.+\frac{1}{1280}\pi^4\xi^3-\frac{13}{32}\xi^4\right\}+\mathcal{O}(\varepsilon), \qquad \text{(E.2)}
\end{aligned}
$$

$$
\begin{aligned}
p^2c^{(2,\xi)}(p^2,0,0) \;=\; & C_A^2\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{\xi}{\varepsilon^4}\left(\frac{1}{64}-\frac{3}{128}\xi+\frac{1}{256}\xi^2\right)\right.\\
&+\frac{1}{\varepsilon^3}\left(-\frac{25}{48}+\frac{7}{32}\xi-\frac{7}{384}\xi^2-\frac{45}{256}\xi^3\right)\\
&+\frac{1}{\varepsilon^2}\left(-\frac{163}{72}-\frac{1}{24}\pi^2+\frac{5}{4}\xi-\frac{1}{32}\pi^2\xi-\frac{5}{72}\xi^2-\frac{1}{128}\pi^2\xi^2-\frac{85}{256}\xi^3\right.\\
&\left.+\frac{3}{256}\pi^2\xi^3-\frac{1}{16}\xi^4\right)\\
&+\frac{1}{\varepsilon}\left(-\frac{2407}{432}-\frac{3}{8}\pi^2-\frac{1}{4}\zeta_3+\frac{409}{96}\xi-\frac{1}{8}\pi^2\xi-\frac{9}{32}\zeta_3\xi-\frac{1331}{1728}\xi^2\right.\\
&\left.+\frac{9}{128}\pi^2\xi^2-\frac{15}{32}\zeta_3\xi^2-\frac{77}{128}\xi^3+\frac{7}{256}\pi^2\xi^3+\frac{3}{64}\zeta_3\xi^3-\frac{1}{4}\xi^4\right)\\
&-\frac{26449}{2592}-\frac{29}{16}\pi^2-\frac{1}{4}\zeta_3-\frac{1}{80}\pi^4+\frac{4651}{576}\xi-\frac{11}{48}\pi^2\xi-\frac{5}{16}\zeta_3\xi-\frac{7}{640}\pi^4\xi\\
&-\frac{4471}{5184}\xi^2+\frac{73}{192}\pi^2\xi^2-\frac{87}{32}\zeta_3\xi^2-\frac{3}{320}\pi^4\xi^2-\frac{17}{8}\xi^3+\frac{79}{768}\pi^2\xi^3+\frac{3}{8}\zeta_3\xi^3
\end{aligned}
$$

$$+\frac{1}{320}\pi^4\xi^3-\frac{3}{4}\xi^4\Big\}+\mathcal{O}(\varepsilon), \tag{E.3}$$

$$\begin{aligned}
p^2 d^{(2,\xi)}(p^2,0,0) \;=\; & C_A^2\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{-\frac{1}{\varepsilon^4}\left(\frac{3}{32}+\frac{1}{64}\xi+\frac{1}{128}\xi^2\right)\right.\\
&+\frac{1}{\varepsilon^3}\left(\frac{5}{48}+\frac{5}{16}\xi-\frac{55}{192}\xi^2-\frac{11}{128}\xi^3\right)\\
&+\frac{1}{\varepsilon^2}\left(\frac{119}{72}-\frac{3}{32}\pi^2+\frac{17}{24}\xi-\frac{1}{32}\pi^2\xi+\frac{263}{1152}\xi^2-\frac{7}{384}\pi^2\xi^2-\frac{29}{64}\xi^3\right.\\
&\left.+\frac{1}{192}\pi^2\xi^3-\frac{1}{32}\xi^4\right)\\
&+\frac{1}{\varepsilon}\left(\frac{248}{27}-\frac{1}{2}\pi^2-\frac{3}{4}\zeta_3+\frac{31}{18}\xi+\frac{1}{32}\pi^2\xi-\frac{27}{32}\zeta_3\xi+\frac{307}{864}\xi^2-\frac{1}{16}\pi^2\xi^2\right.\\
&\left.-\frac{7}{16}\zeta_3\xi^2-\frac{99}{128}\xi^3+\frac{5}{192}\pi^2\xi^3+\frac{1}{32}\zeta_3\xi^3-\frac{3}{16}\xi^4\right)\\
&+\frac{24947}{648}-\frac{103}{72}\pi^2-\frac{17}{2}\zeta_3-\frac{1}{32}\pi^4+\frac{523}{432}\xi+\frac{11}{96}\pi^2\xi-\frac{25}{8}\zeta_3\xi-\frac{13}{640}\pi^4\xi\\
&+\frac{7553}{2592}\xi^2-\frac{13}{384}\pi^2\xi^2-\frac{11}{4}\zeta_3\xi^2-\frac{7}{640}\pi^4\xi^2-\frac{67}{32}\xi^3+\frac{9}{128}\pi^2\xi^3+\frac{19}{64}\zeta_3\xi^3\\
&\left.+\frac{1}{640}\pi^4\xi^3-\frac{19}{32}\xi^4\right\}+\mathcal{O}(\varepsilon),
\end{aligned} \tag{E.4}$$

$$\begin{aligned}
p^2 e^{(2,\xi)}(p^2,0,0) \;=\; & C_A^2\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon^4}\left(-\frac{3}{32}+\frac{1}{64}\xi-\frac{3}{64}\xi^2+\frac{1}{128}\xi^3\right)\right.\\
&+\frac{1}{\varepsilon^3}\left(-\frac{11}{16}+\frac{23}{32}\xi-\frac{11}{96}\xi^2-\frac{45}{128}\xi^3\right)\\
&+\frac{1}{\varepsilon^2}\left(-\frac{19}{12}-\frac{7}{48}\pi^2+\frac{93}{32}\xi-\frac{1}{16}\pi^2\xi-\frac{29}{144}\xi^2-\frac{1}{32}\pi^2\xi^2-\frac{85}{128}\xi^3\right.\\
&\left.+\frac{3}{128}\pi^2\xi^3-\frac{1}{8}\xi^4\right)\\
&+\frac{1}{\varepsilon}\left(\frac{35}{288}-\frac{67}{72}\pi^2-\frac{17}{16}\zeta_3+\frac{1681}{192}\xi-\frac{17}{96}\pi^2\xi-\frac{27}{32}\zeta_3\xi-\frac{1439}{864}\xi^2\right.\\
&+\frac{7}{64}\pi^2\xi^2-\frac{33}{32}\zeta_3\xi^2-\frac{77}{64}\xi^3+\frac{7}{128}\pi^2\xi^3+\frac{3}{32}\zeta_3\xi^3-\frac{1}{2}\xi^4\Big)\\
&+\frac{33431}{1728}-\frac{1795}{432}\pi^2-\frac{125}{24}\zeta_3-\frac{3}{64}\pi^4+\frac{19117}{1152}\xi-\frac{5}{16}\pi^2\xi-\frac{9}{4}\zeta_3\xi\\
&-\frac{17}{640}\pi^4\xi-\frac{409}{648}\xi^2+\frac{55}{96}\pi^2\xi^2-\frac{171}{32}\zeta_3\xi^2-\frac{3}{128}\pi^4\xi^2-\frac{17}{4}\xi^3+\frac{79}{384}\pi^2\xi^3\\
&\left.+\frac{3}{4}\zeta_3\xi^3+\frac{1}{160}\pi^4\xi^3-\frac{3}{2}\xi^4\right\}+\mathcal{O}(\varepsilon),
\end{aligned} \tag{E.5}$$

$$\begin{aligned}
a^{(2,q)}(p^2,0,0) \;=\; & C_A T\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon^3}\left(-\frac{1}{6}+\frac{1}{6}\xi\right)+\frac{1}{\varepsilon^2}\left(-\frac{7}{36}+\frac{1}{36}\xi+\frac{1}{6}\xi^2\right)\right.\\
&+\frac{1}{\varepsilon}\left(-\frac{235}{216}+\frac{5}{36}\pi^2+\frac{11}{108}\xi+\frac{4}{9}\xi^2\right)\\
&\left.-\frac{4963}{1296}+\frac{77}{216}\pi^2+\frac{11}{6}\zeta_3+\frac{157}{324}\xi+\zeta_3\xi+\frac{61}{54}\xi^2\right\}+\mathcal{O}(\varepsilon),
\end{aligned} \tag{E.6}$$

$$p^2 b^{(2,q)}(p^2,0,0) = C_A T \frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon^3}\left(\frac{1}{6}-\frac{1}{3}\xi+\frac{1}{6}\xi^2\right)+\frac{1}{\varepsilon^2}\left(-\frac{1}{18}-\frac{11}{9}\xi+\frac{11}{18}\xi^2\right)\right.$$
$$+\frac{1}{\varepsilon}\left(\frac{23}{108}-\frac{2}{9}\pi^2-\frac{103}{27}\xi+\frac{103}{54}\xi^2\right)$$
$$\left.-\frac{313}{648}-\frac{67}{108}\pi^2-\frac{7}{3}\zeta_3-\frac{827}{81}\xi+\frac{827}{162}\xi^2\right\}+\mathcal{O}(\varepsilon), \tag{E.7}$$

$$p^2 c^{(2,q)}(p^2,0,0) = C_A T \frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon^3}\left(\frac{1}{6}-\frac{1}{2}\xi+\frac{1}{3}\xi^2\right)+\frac{1}{\varepsilon^2}\left(\frac{17}{18}-2\xi+\frac{11}{9}\xi^2\right)\right.$$
$$+\frac{1}{\varepsilon}\left(\frac{239}{108}+\frac{1}{6}\pi^2-\frac{16}{3}\xi+\frac{94}{27}\xi^2\right)$$
$$\left.+\frac{2387}{648}+\frac{11}{12}\pi^2-\frac{116}{9}\xi+\zeta_3\xi+\frac{728}{81}\xi^2\right\}+\mathcal{O}(\varepsilon), \tag{E.8}$$

$$p^2 d^{(2,q)}(p^2,0,0) = -C_A T \frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon^3}\left(\frac{1}{3}-\frac{1}{6}\xi^2\right)+\frac{1}{\varepsilon^2}\left(\frac{8}{9}+\frac{7}{6}\xi-\frac{17}{18}\xi^2\right)\right.$$
$$+\frac{1}{\varepsilon}\left(\frac{88}{27}+\frac{65}{18}\xi-\frac{151}{54}\xi^2\right)$$
$$\left.+\frac{1945}{162}+\frac{1}{18}\pi^2-2\zeta_3+\frac{499}{54}\xi-2\zeta_3\xi-\frac{1193}{162}\xi^2\right\}+\mathcal{O}(\varepsilon), \tag{E.9}$$

$$p^2 e^{(2,q)}(p^2,0,0) = C_A T \frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{\xi}{\varepsilon^3}\left(-1+\frac{2}{3}\xi\right)+\frac{1}{\varepsilon^2}\left(\frac{5}{12}-4\xi+\frac{22}{9}\xi^2\right)\right.$$
$$+\frac{1}{\varepsilon}\left(-\frac{73}{72}+\frac{5}{18}\pi^2-\frac{32}{3}\xi+\frac{188}{27}\xi^2\right)$$
$$\left.-\frac{4801}{432}+\frac{44}{27}\pi^2+\frac{5}{3}\zeta_3-\frac{232}{9}\xi+2\zeta_3\xi+\frac{1456}{81}\xi^2\right\}+\mathcal{O}(\varepsilon). \tag{E.10}$$

E.2: Non-zero in-ghost momentum squared

$$a^{(2,\xi)}(0,p^2,0) = C_A^2 \frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon^4}\left(-\frac{3}{64}+\frac{3}{128}\xi\right)+\frac{1}{\varepsilon^3}\left(\frac{5}{96}-\frac{21}{128}\xi+\frac{19}{256}\xi^2\right)\right.$$
$$+\frac{1}{\varepsilon^2}\left(\frac{31}{72}-\frac{13}{192}\pi^2-\frac{93}{64}\xi+\frac{7}{384}\pi^2\xi+\frac{67}{128}\xi^2-\frac{1}{384}\pi^2\xi^2\right)$$
$$+\frac{1}{\varepsilon}\left(\frac{355}{108}-\frac{11}{36}\pi^2-\frac{1}{2}\zeta_3-\frac{305}{64}\xi+\frac{11}{128}\pi^2\xi+\frac{5}{32}\zeta_3\xi+\frac{187}{128}\xi^2\right.$$
$$\left.-\frac{11}{768}\pi^2\xi^2-\frac{1}{64}\zeta_3\xi^2\right)$$
$$+\frac{37027}{2592}-\frac{791}{864}\pi^2-\frac{175}{48}\zeta_3-\frac{7}{320}\pi^4-\frac{263}{16}\xi+\frac{23}{96}\pi^2\xi+\frac{33}{16}\zeta_3\xi+\frac{1}{160}\pi^4\xi$$
$$\left.+\frac{541}{128}\xi^2-\frac{3}{64}\pi^2\xi^2-\frac{1}{16}\zeta_3\xi^2-\frac{1}{1280}\pi^4\xi^2\right\}+\mathcal{O}(\varepsilon), \tag{E.11}$$

$$
\begin{aligned}
p^2 b^{(2,\xi)}(0,p^2,0) \;=\; & C_A^2\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{\xi}{\varepsilon^4}\left(-\frac{1}{64}+\frac{1}{128}\xi+\frac{1}{128}\xi^2\right)\right.\\
&+\frac{1}{\varepsilon^3}\left(-\frac{1}{6}-\frac{73}{192}\xi+\frac{11}{128}\xi^2+\frac{7}{64}\xi^3\right)\\
&+\frac{1}{\varepsilon^2}\left(\frac{407}{144}-\frac{5}{48}\pi^2-\frac{323}{144}\xi+\frac{1}{32}\pi^2\xi-\frac{1}{128}\xi^2+\frac{1}{96}\pi^2\xi^2+\frac{85}{256}\xi^3+\frac{1}{384}\pi^2\xi^3\right)\\
&+\frac{1}{\varepsilon}\left(\frac{13391}{864}-\frac{31}{48}\pi^2-\frac{5}{8}\zeta_3-\frac{14383}{1728}\xi-\frac{1}{64}\pi^2\xi+\frac{9}{32}\zeta_3\xi+\frac{75}{64}\xi^2\right.\\
&\left.-\frac{11}{384}\pi^2\xi^2+\frac{25}{64}\zeta_3\xi^2+\frac{101}{128}\xi^3+\frac{1}{96}\pi^2\xi^3-\frac{1}{32}\zeta_3\xi^3\right)\\
&+\frac{315815}{5184}-\frac{17}{9}\pi^2-\frac{47}{8}\zeta_3-\frac{1}{32}\pi^4-\frac{376861}{10368}\xi+\frac{31}{48}\pi^2\xi+\frac{35}{16}\zeta_3\xi+\frac{7}{640}\pi^4\xi\\
&\left.+\frac{131}{32}\xi^2-\frac{25}{384}\pi^2\xi^2+\frac{13}{16}\zeta_3\xi^2+\frac{11}{1280}\pi^4\xi^2+\frac{17}{8}\xi^3+\frac{23}{768}\pi^2\xi^3-\frac{7}{32}\zeta_3\xi^3\right\}\\
&+\mathcal{O}(\varepsilon),
\end{aligned}
\tag{E.12}
$$

$$
\begin{aligned}
p^2 c^{(2,\xi)}(0,p^2,0) \;=\; & C_A^2\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon^4}\left(\frac{5}{32}-\frac{3}{32}\xi\right)+\frac{1}{\varepsilon^3}\left(\frac{109}{48}+\frac{31}{96}\xi-\frac{3}{128}\xi^2-\frac{45}{256}\xi^3\right)\right.\\
&+\frac{1}{\varepsilon^2}\left(\frac{775}{72}+\frac{1}{32}\pi^2+\frac{1231}{576}\xi-\frac{5}{96}\pi^2\xi+\frac{11}{32}\xi^2-\frac{1}{192}\pi^2\xi^2-\frac{1}{4}\xi^3-\frac{1}{192}\pi^2\xi^3\right)\\
&+\frac{1}{\varepsilon}\left(\frac{1931}{54}+\frac{19}{48}\pi^2-\frac{3}{2}\zeta_3+\frac{185}{27}\xi-\frac{11}{72}\pi^2\xi-\frac{1}{2}\zeta_3\xi+\frac{5}{8}\xi^2+\frac{7}{128}\pi^2\xi^2\right.\\
&\left.-\frac{13}{32}\zeta_3\xi^2-\frac{29}{32}\xi^3+\frac{1}{256}\pi^2\xi^3-\frac{1}{32}\zeta_3\xi^3\right)\\
&+\frac{17443}{162}+\frac{107}{48}\pi^2-15\zeta_3-\frac{3}{160}\pi^4+\frac{70727}{2592}\xi-\frac{755}{1728}\pi^2\xi-\frac{317}{48}\zeta_3\xi\\
&-\frac{3}{160}\pi^4\xi+\frac{45}{16}\xi^2+\frac{7}{32}\pi^2\xi^2-\frac{57}{32}\zeta_3\xi^2-\frac{1}{128}\pi^4\xi^2-\frac{115}{64}\xi^3-\frac{7}{128}\pi^2\xi^3\\
&\left.+\frac{27}{64}\zeta_3\xi^3-\frac{1}{640}\pi^4\xi^3\right\}+\mathcal{O}(\varepsilon),
\end{aligned}
\tag{E.13}
$$

$$
\begin{aligned}
p^2 d^{(2,\xi)}(0,p^2,0) \;=\; & C_A^2\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon^4}\left(\frac{3}{16}-\frac{1}{32}\xi-\frac{1}{128}\xi^2-\frac{1}{128}\xi^3\right)\right.\\
&+\frac{1}{\varepsilon^3}\left(-\frac{1}{48}+\frac{43}{48}\xi-\frac{15}{64}\xi^2-\frac{7}{64}\xi^3\right)\\
&+\frac{1}{\varepsilon^2}\left(-\frac{875}{144}+\frac{13}{48}\pi^2+\frac{1429}{288}\xi-\frac{5}{64}\pi^2\xi-\frac{25}{32}\xi^2-\frac{85}{256}\xi^3-\frac{1}{384}\pi^2\xi^3\right)\\
&+\frac{1}{\varepsilon}\left(-\frac{27539}{864}+\frac{61}{48}\pi^2+2\zeta_3+\frac{32149}{1728}\xi-\frac{17}{96}\pi^2\xi-\frac{21}{32}\zeta_3\xi-\frac{127}{32}\xi^2\right.\\
&\left.+\frac{5}{64}\pi^2\xi^2-\frac{21}{64}\zeta_3\xi^2-\frac{101}{128}\xi^3-\frac{1}{96}\pi^2\xi^3+\frac{1}{32}\zeta_3\xi^3\right)\\
&-\frac{651695}{5184}+\frac{541}{144}\pi^2+\frac{67}{4}\zeta_3+\frac{7}{80}\pi^4+\frac{736825}{10368}\xi-\frac{7}{6}\pi^2\xi-\frac{97}{16}\zeta_3\xi\\
&-\frac{17}{640}\pi^4\xi-\frac{841}{64}\xi^2+\frac{43}{192}\pi^2\xi^2-\frac{3}{16}\zeta_3\xi^2-\frac{7}{1280}\pi^4\xi^2-\frac{17}{8}\xi^3
\end{aligned}
$$

$$-\frac{23}{768}\pi^2\xi^3+\frac{7}{32}\zeta_3\xi^3\Big\}+\mathcal{O}(\varepsilon), \tag{E.14}$$

$$\begin{aligned}
p^2e^{(2,\xi)}(0,p^2,0) \;=\; & -C_A^2\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon^3}\left(\frac{1}{16}+\frac{1}{32}\xi\right)\right.\\
& +\frac{1}{\varepsilon^2}\left(\frac{23}{24}+\frac{1}{96}\pi^2+\frac{3}{16}\xi-\frac{1}{192}\pi^2\xi-\frac{11}{32}\xi^2+\frac{1}{48}\pi^2\xi^2\right)\\
& +\frac{1}{\varepsilon}\left(\frac{1201}{288}+\frac{1}{48}\pi^2+\frac{1}{16}\zeta_3-\frac{15}{64}\xi-\frac{1}{32}\pi^2\xi-\frac{1}{32}\zeta_3\xi-\frac{15}{32}\xi^2\right.\\
& \left.+\frac{1}{48}\pi^2\xi^2+\frac{1}{8}\zeta_3\xi^2\right)\\
& +\frac{22921}{1728}+\frac{35}{144}\pi^2-\zeta_3+\frac{1}{320}\pi^4-\frac{237}{128}\xi-\frac{5}{96}\pi^2\xi-\frac{3}{8}\zeta_3\xi\\
& \left.-\frac{1}{640}\pi^4\xi-2\xi^2+\frac{13}{96}\pi^2\xi^2-\frac{1}{16}\zeta_3\xi^2+\frac{1}{160}\pi^4\xi^2\right\}+\mathcal{O}(\varepsilon),
\end{aligned} \tag{E.15}$$

$$\begin{aligned}
a^{(2,q)}(0,p^2,0) \;=\; & C_AT\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{-\frac{1}{24\varepsilon^3}-\frac{1}{36\varepsilon^2}+\frac{1}{\varepsilon}\left(-\frac{23}{27}+\frac{1}{36}\pi^2\right)\right.\\
& \left.-\frac{2339}{648}+\frac{5}{108}\pi^2+\frac{5}{12}\zeta_3\right\}+\mathcal{O}(\varepsilon),
\end{aligned} \tag{E.16}$$

$$\begin{aligned}
p^2b^{(2,q)}(0,p^2,0) \;=\; & C_AT\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon^3}\left(-\frac{1}{6}+\frac{1}{24}\xi\right)-\frac{1}{\varepsilon^2}\left(\frac{25}{36}+\frac{7}{72}\xi\right)\right.\\
& +\frac{1}{\varepsilon}\left(-\frac{1081}{216}+\frac{1}{6}\pi^2-\frac{13}{27}\xi+\frac{1}{24}\pi^2\xi\right)\\
& \left.-\frac{17257}{1296}-\frac{1}{18}\pi^2+2\zeta_3-\frac{241}{162}\xi+\frac{1}{24}\pi^2\xi\right\}+\mathcal{O}(\varepsilon),
\end{aligned} \tag{E.17}$$

$$\begin{aligned}
p^2c^{(2,q)}(0,p^2,0) \;=\; & -C_AT\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon^3}\left(\frac{5}{12}+\frac{1}{12}\xi\right)+\frac{1}{\varepsilon^2}\left(\frac{35}{18}+\frac{5}{36}\xi\right)\right.\\
& +\frac{1}{\varepsilon}\left(\frac{295}{54}-\frac{17}{108}\xi+\frac{1}{36}\pi^2\xi\right)\\
& \left.+\frac{1240}{81}-\frac{5}{2}\zeta_3-\frac{103}{81}\xi+\frac{11}{108}\pi^2\xi-\frac{1}{3}\zeta_3\xi\right\}+\mathcal{O}(\varepsilon),
\end{aligned} \tag{E.18}$$

$$\begin{aligned}
p^2d^{(2,q)}(0,p^2,0) \;=\; & C_AT\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon^3}\left(\frac{5}{12}-\frac{1}{24}\xi\right)+\frac{1}{\varepsilon^2}\left(\frac{79}{36}+\frac{7}{72}\xi\right)\right.\\
& +\frac{1}{\varepsilon}\left(\frac{2485}{216}-\frac{1}{6}\pi^2+\frac{13}{27}\xi-\frac{1}{24}\pi^2\xi\right)\\
& \left.+\frac{48793}{1296}+\frac{1}{9}\pi^2-\frac{7}{2}\zeta_3+\frac{241}{162}\xi-\frac{1}{24}\pi^2\xi\right\}+\mathcal{O}(\varepsilon),
\end{aligned} \tag{E.19}$$

$$p^2e^{(2,q)}(0,p^2,0)=C_AT\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{5}{12\varepsilon^2}+\frac{95}{72\varepsilon}+\frac{2135}{432}-\frac{1}{18}\pi^2\right\}+\mathcal{O}(\varepsilon). \tag{E.20}$$

E.3: Non-zero out-ghost momentum squared

$$
\begin{aligned}
a^{(2,\xi)}(0,0,p^2) \;=\; & C_A^2\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{5}{32\varepsilon^4}+\frac{1}{\varepsilon^3}\left(-\frac{23}{96}+\frac{1}{16}\xi\right)\right.\\
&+\frac{1}{\varepsilon^2}\left(-\frac{143}{144}+\frac{5}{96}\pi^2+\frac{1}{384}\pi^2\xi+\frac{5}{128}\xi^2\right)\\
&+\frac{1}{\varepsilon}\left(-\frac{2813}{864}+\frac{1}{72}\pi^2+\frac{7}{8}\zeta_3-\frac{33}{64}\xi+\frac{1}{24}\pi^2\xi+\frac{1}{64}\zeta_3\xi+\frac{5}{32}\xi^2\right)\\
&-\frac{52535}{5184}-\frac{19}{216}\pi^2+\frac{83}{24}\zeta_3+\frac{1}{40}\pi^4-\frac{1145}{384}\xi+\frac{31}{192}\pi^2\xi+\frac{1}{2}\zeta_3\xi\\
&\left.+\frac{1}{1280}\pi^4\xi+\frac{107}{192}\xi^2+\frac{1}{384}\pi^2\xi^2-\frac{1}{32}\zeta_3\xi^2\right\}+\mathcal{O}(\varepsilon), \qquad \text{(E.21)}
\end{aligned}
$$

$$
\begin{aligned}
p^2b^{(2,\xi)}(0,0,p^2) \;=\; & C_A^2\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon^4}\left(\frac{7}{16}+\frac{9}{64}\xi-\frac{3}{128}\xi^2\right)\right.\\
&+\frac{1}{\varepsilon^3}\left(-\frac{1}{8}-\frac{89}{192}\xi+\frac{25}{128}\xi^2-\frac{5}{128}\xi^3\right)\\
&+\frac{1}{\varepsilon^2}\left(-\frac{215}{48}+\frac{1}{8}\pi^2-\frac{433}{144}\xi+\frac{1}{6}\pi^2\xi+\frac{105}{128}\xi^2+\frac{1}{96}\pi^2\xi^2-\frac{59}{256}\xi^3+\frac{1}{384}\pi^2\xi^3\right)\\
&+\frac{1}{\varepsilon}\left(-\frac{5939}{288}+\frac{7}{144}\pi^2+\frac{21}{8}\zeta_3-\frac{14183}{1728}\xi+\frac{53}{192}\pi^2\xi+\frac{53}{32}\zeta_3\xi+\frac{179}{64}\xi^2\right.\\
&\left.+\frac{19}{384}\pi^2\xi^2-\frac{11}{64}\zeta_3\xi^2-\frac{113}{128}\xi^3+\frac{1}{128}\pi^2\xi^3+\frac{1}{64}\zeta_3\xi^3\right)\\
&-\frac{136931}{1728}-\frac{43}{108}\pi^2+\frac{367}{24}\zeta_3+\frac{11}{160}\pi^4-\frac{360077}{10368}\xi+\frac{55}{36}\pi^2\xi+\frac{95}{16}\zeta_3\xi\\
&+\frac{39}{640}\pi^4\xi+\frac{255}{32}\xi^2+\frac{35}{128}\pi^2\xi^2-\frac{23}{16}\zeta_3\xi^2-\frac{1}{1280}\pi^4\xi^2-3\xi^3+\frac{5}{256}\pi^2\xi^3\\
&\left.+\frac{3}{16}\zeta_3\xi^3+\frac{1}{1280}\pi^4\xi^3\right\}+\mathcal{O}(\varepsilon), \qquad \text{(E.22)}
\end{aligned}
$$

$$
\begin{aligned}
p^2c^{(2,\xi)}(0,0,p^2) \;=\; & C_A^2\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon^4}\left(\frac{7}{32}+\frac{1}{8}\xi-\frac{1}{128}\xi^2-\frac{1}{256}\xi^3\right)\right.\\
&+\frac{1}{\varepsilon^3}\left(\frac{1}{16}-\frac{11}{48}\xi+\frac{11}{128}\xi^2-\frac{1}{256}\xi^3\right)\\
&+\frac{1}{\varepsilon^2}\left(-\frac{41}{12}+\frac{11}{96}\pi^2-\frac{167}{72}\xi+\frac{17}{192}\pi^2\xi+\frac{83}{128}\xi^2+\frac{1}{192}\pi^2\xi^2-\frac{7}{64}\xi^3\right)\\
&+\frac{1}{\varepsilon}\left(-\frac{2467}{144}+\frac{25}{72}\pi^2+\frac{13}{8}\zeta_3-\frac{6961}{864}\xi+\frac{31}{144}\pi^2\xi+\frac{29}{32}\zeta_3\xi+\frac{37}{16}\xi^2\right.\\
&\left.+\frac{11}{384}\pi^2\xi^2-\frac{7}{64}\zeta_3\xi^2-\frac{73}{128}\xi^3+\frac{1}{256}\pi^2\xi^3+\frac{3}{128}\zeta_3\xi^3\right)\\
&-\frac{57757}{864}+\frac{161}{216}\pi^2+\frac{31}{3}\zeta_3+\frac{1}{20}\pi^4-\frac{147967}{5184}\xi+\frac{505}{864}\pi^2\xi+\frac{127}{24}\zeta_3\xi\\
&+\frac{21}{640}\pi^4\xi+\frac{231}{32}\xi^2+\frac{17}{128}\pi^2\xi^2-\frac{7}{16}\zeta_3\xi^2-\frac{1}{1280}\pi^4\xi^2-\frac{133}{64}\xi^3+\frac{1}{96}\pi^2\xi^3\\
&\left.+\frac{3}{16}\zeta_3\xi^3+\frac{1}{2560}\pi^4\xi^3\right\}+\mathcal{O}(\varepsilon), \qquad \text{(E.23)}
\end{aligned}
$$

$$
\begin{aligned}
p^2 d^{(2,\xi)}(0,0,p^2) \;=\; & C_A^2 \,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon^4}\left(-\frac{7}{32}-\frac{9}{64}\xi+\frac{3}{128}\xi^2\right)\right. \\
& +\frac{1}{\varepsilon^3}\left(-\frac{1}{48}+\frac{101}{192}\xi-\frac{25}{128}\xi^2+\frac{5}{128}\xi^3\right) \\
& +\frac{1}{\varepsilon^2}\left(\frac{559}{144}-\frac{3}{32}\pi^2+\frac{221}{72}\xi-\frac{31}{192}\pi^2\xi-\frac{13}{16}\xi^2-\frac{1}{96}\pi^2\xi^2+\frac{59}{256}\xi^3-\frac{1}{384}\pi^2\xi^3\right) \\
& +\frac{1}{\varepsilon}\left(\frac{16447}{864}-\frac{29}{144}\pi^2-\frac{3}{2}\zeta_3+\frac{13589}{1728}\xi-\frac{37}{192}\pi^2\xi-\frac{13}{8}\zeta_3\xi-\frac{179}{64}\xi^2\right. \\
& \left.-\frac{19}{384}\pi^2\xi^2+\frac{11}{64}\zeta_3\xi^2+\frac{113}{128}\xi^3-\frac{1}{128}\pi^2\xi^3-\frac{1}{64}\zeta_3\xi^3\right) \\
& +\frac{393163}{5184}-\frac{77}{216}\pi^2-\frac{251}{24}\zeta_3-\frac{7}{160}\pi^4+\frac{333023}{10368}\xi-\frac{341}{288}\pi^2\xi-\frac{87}{16}\zeta_3\xi \\
& -\frac{19}{320}\pi^4\xi-\frac{257}{32}\xi^2-\frac{13}{48}\pi^2\xi^2+\frac{23}{16}\zeta_3\xi^2+\frac{1}{1280}\pi^4\xi^2+3\xi^3-\frac{5}{256}\pi^2\xi^3 \\
& \left.-\frac{3}{16}\zeta_3\xi^3-\frac{1}{1280}\pi^4\xi^3\right\}+\mathcal{O}(\varepsilon), \qquad \text{(E.24)}
\end{aligned}
$$

$$
\begin{aligned}
p^2 e^{(2,\xi)}(0,0,p^2) \;=\; & C_A^2 \,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon^3}\left(\frac{1}{8}+\frac{1}{16}\xi\right)\right. \\
& -\frac{1}{\varepsilon^2}\left(\frac{1}{2}+\frac{1}{48}\pi^2+\frac{1}{16}\xi+\frac{1}{192}\pi^2\xi-\frac{3}{32}\xi^2-\frac{1}{384}\pi^2\xi^2\right) \\
& -\frac{1}{\varepsilon}\left(\frac{39}{16}+\frac{1}{12}\pi^2+\frac{1}{8}\zeta_3-\frac{23}{32}\xi+\frac{1}{96}\pi^2\xi+\frac{1}{32}\zeta_3\xi+\frac{3}{32}\xi^2-\frac{1}{96}\pi^2\xi^2-\frac{1}{64}\zeta_3\xi^2\right) \\
& -\frac{929}{96}-\frac{13}{48}\pi^2+\frac{3}{4}\zeta_3-\frac{1}{160}\pi^4+\frac{499}{192}\xi-\frac{1}{96}\pi^2\xi-\frac{1}{8}\zeta_3\xi-\frac{1}{640}\pi^4\xi \\
& \left.-\frac{85}{96}\xi^2+\frac{1}{32}\pi^2\xi^2+\frac{9}{32}\zeta_3\xi^2+\frac{1}{1280}\pi^4\xi^2\right\}+\mathcal{O}(\varepsilon), \qquad \text{(E.25)}
\end{aligned}
$$

$$
\begin{aligned}
a^{(2,q)}(0,0,p^2) \;=\; & C_A T \,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{6\varepsilon^3}+\frac{19}{36\varepsilon^2}+\frac{1}{\varepsilon}\left(\frac{319}{216}+\frac{1}{72}\pi^2\right)\right. \\
& \left.+\frac{5245}{1296}+\frac{23}{216}\pi^2-\frac{11}{12}\zeta_3\right\}+\mathcal{O}(\varepsilon), \qquad \text{(E.26)}
\end{aligned}
$$

$$
\begin{aligned}
p^2 b^{(2,q)}(0,0,p^2) \;=\; & C_A T \,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon^3}\left(\frac{1}{2}+\frac{5}{24}\xi\right)+\frac{1}{\varepsilon^2}\left(\frac{37}{12}+\frac{25}{72}\xi\right)\right. \\
& +\frac{1}{\varepsilon}\left(\frac{469}{72}+\frac{4}{9}\pi^2+\frac{13}{27}\xi+\frac{1}{24}\pi^2\xi\right) \\
& \left.+\frac{9085}{432}+\frac{38}{27}\pi^2-\frac{1}{3}\zeta_3+\frac{79}{162}\xi+\frac{11}{72}\pi^2\xi-\zeta_3\xi\right\}+\mathcal{O}(\varepsilon), \qquad \text{(E.27)}
\end{aligned}
$$

$$
\begin{aligned}
p^2 c^{(2,q)}(0,0,p^2) \;=\; & C_A T \,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon^3}\left(\frac{1}{2}+\frac{1}{12}\xi\right)+\frac{1}{\varepsilon^2}\left(\frac{17}{6}+\frac{2}{9}\xi\right)\right. \\
& +\frac{1}{\varepsilon}\left(\frac{341}{36}+\frac{5}{36}\pi^2+\frac{4}{27}\xi+\frac{1}{36}\pi^2\xi\right) \\
& \left.+\frac{6503}{216}+\frac{79}{108}\pi^2-\frac{13}{6}\zeta_3-\frac{19}{81}\xi+\frac{7}{54}\pi^2\xi-\frac{1}{3}\zeta_3\xi\right\}+\mathcal{O}(\varepsilon), \qquad \text{(E.28)}
\end{aligned}
$$

$$
\begin{aligned}
p^2 d^{(2,q)}(0,0,p^2) \;=\; & -C_A T\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{\varepsilon^3}\left(\frac{1}{3}+\frac{5}{24}\xi\right)+\frac{1}{\varepsilon^2}\left(\frac{89}{36}+\frac{25}{72}\xi\right)\right. \\
& +\frac{1}{\varepsilon}\left(\frac{1049}{216}+\frac{17}{36}\pi^2+\frac{13}{27}\xi+\frac{1}{24}\pi^2\xi\right) \\
& \left.+\frac{22205}{1296}+\frac{163}{108}\pi^2+\frac{5}{6}\zeta_3+\frac{79}{162}\xi+\frac{11}{72}\pi^2\xi-\zeta_3\xi\right\}+\mathcal{O}(\varepsilon), \qquad \text{(E.29)}
\end{aligned}
$$

$$
p^2 e^{(2,q)}(0,0,p^2) = C_A T\,\frac{g^4\,\eta^2}{(4\pi)^n}(-p^2)^{-2\varepsilon}\left\{\frac{1}{2\varepsilon^2}+\frac{7}{4\varepsilon}+\frac{155}{24}-\frac{1}{12}\pi^2\right\}+\mathcal{O}(\varepsilon). \qquad \text{(E.30)}
$$

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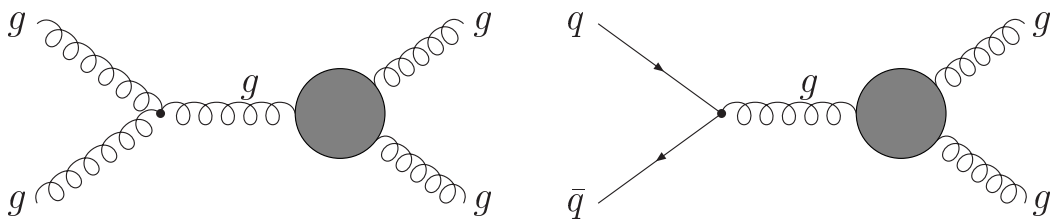


Figure 1: Contributions to the $gg \to gg$ and $q\bar{q} \to gg$ processes which involve the three-gluon vertex

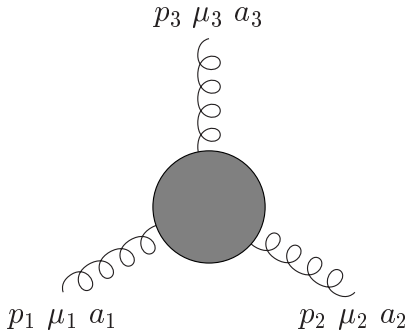


Figure 2: Notation used for gluon momenta ($p_1 + p_2 + p_3 = 0$), Lorentz indices, μ_i, and colour indices, a_i

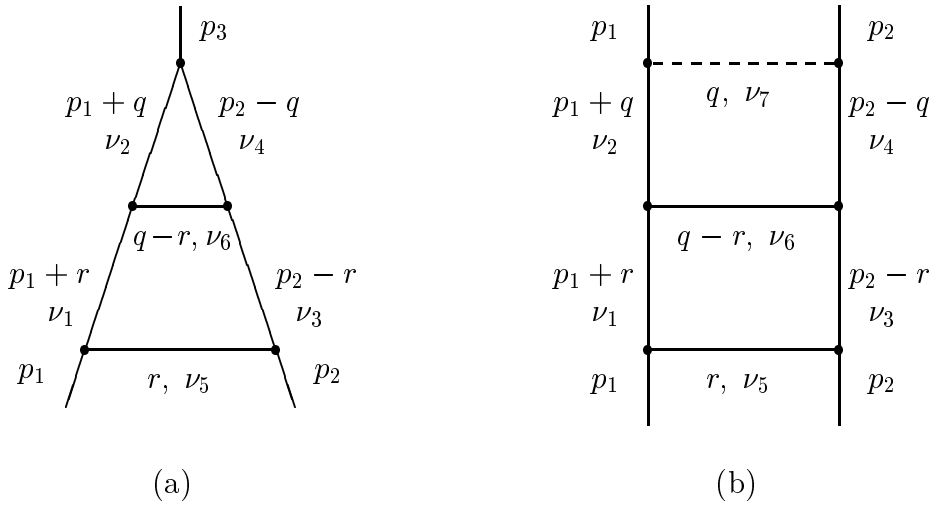
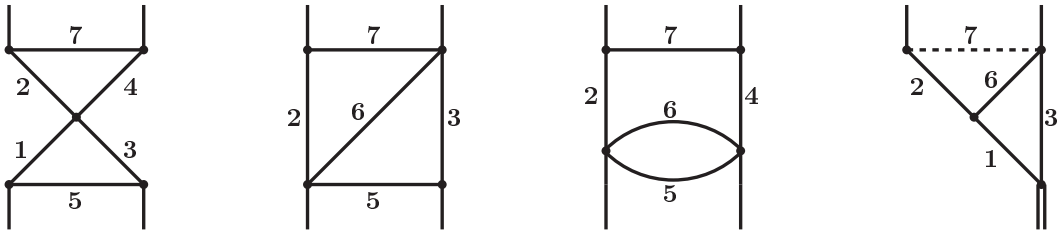
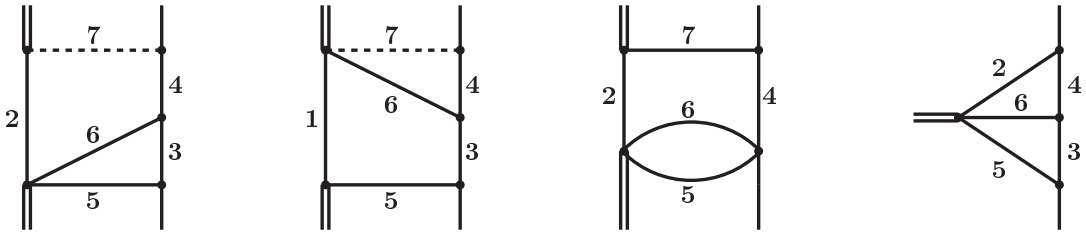


Figure 3: The planar two-loop three-point diagram (a) and an auxiliary four-point function (b)



(D2) (D3) (D4) (D5)

Figure 4: Boundary integrals K_3



(D7) (D8) (D9) (D10)

Figure 5: Boundary integrals K_2