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## F-term Hybrid Inflation in Effective Supergravity Theories

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### Abstract

We show that a particular class of effective low energy supergravity theories motivated by string theory can provide a promising framework for models of hybrid inflation in which the potential energy which drives inflation originates from the F-term of the effective supergravity theory. In the class of models considered the inflaton is protected from receiving mass during inflation by a generalisation of the Heisenberg symmetry present in no-scale supergravity models. The potential during inflation takes the positive definite form  $V \sim |F_S|^2 + |F_T|^2 - 3$ , which allows the possibility that  $V \ll m_{3/2}^2 M_P^2$  through the cancellation of the positive dilaton and moduli contribution against the negative term. We discuss a toy example where this is realised, then describe the application of this result to realistic models focusing on a particular example in which the  $\mu$  problem and the strong CP-problem are addressed.

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# 1 Introduction

Due to its intrinsic elegance, inflation [1] has become the almost universally accepted dogma for accounting for the flatness and homogeneity of the universe. There are various classes of inflation that have been proposed, but possibly the most successful, and certainly one of the most popular versions these days is hybrid inflation [2, 3]. In hybrid inflation, there are (at least) two fields at work: the slowly rolling inflaton field  $\phi$ , and a second field which we shall call  $N$  whose value is held at zero during inflation and whose role is to end inflation by developing a non-zero vacuum expectation value (VEV) when  $\phi$  passes a certain critical value  $\phi_c$  during its slow roll. With  $N = 0$ , the potential along the  $\phi$  direction is approximately flat, with the flatness lifted by a  $\phi$  mass which must be small enough to satisfy the slow-roll conditions for inflation.

The natural framework for hybrid inflation is supersymmetry. SUSY can naturally provide flat directions along which the inflaton can roll, and additionally ensures that the scalar inflaton mass does not have quadratic divergences. The natural size of the inflaton mass in SUSY is of order the SUSY breaking scale, and the slow roll conditions and COBE constraints [4] then determine the height of the potential during inflation  $V(0)^{1/4}$  to be some intermediate scale below the grand unification (GUT) scale. The precise value of  $V(0)$  is model-dependent, however the lower the height of the potential, the flatter the inflaton potential has to be and smaller the inflaton mass.

If one accepts the above framework, one is driven almost inevitably to supergravity (SUGRA). The origin of the vacuum energy  $V(0)$  which drives inflation can only be properly understood within a framework which allows the possibility for the potential energy to settle to zero at the global minimum, and hence lead to an acceptable cosmological constant, and this implies SUGRA. In this way we are led to consider SUGRA models in which the fields are displaced from their global minimum values

during inflation. The potential in SUGRA is given by:

$$V = e^G \left[ G_i (G_j^i)^{-1} G^{\bar{j}} - 3 \right] + |D|^2, \quad (1)$$

in the usual notation, i.e. natural units, with subscripts  $i$  ( $\bar{i}$ ) referring to partial differentiation with respect to the generic field  $\phi_i$  ( $\phi_i^*$ ). We have written the D-terms very schematically as  $|D|^2$ . The Kahler function  $G$  is:

$$G = K + \ln |W|^2, \quad (2)$$

where the Kahler potential  $K$  is a real function of generic fields  $\phi, \phi^*$  and the superpotential  $W$  is an analytic (holomorphic) function of  $\phi$  only. The Kahler metric is  $G_i^{\bar{j}}$  and its inverse satisfies  $(G_j^i)^{-1} G_k^{\bar{j}} = \delta_k^i$ . The SUGRA F-terms are:

$$F^i = e^{G/2} (G_j^i)^{-1} G^{\bar{j}}, \quad (3)$$

whose non-zero value signals SUSY breaking, with a gravitino mass

$$m_{3/2}^2 = e^G = e^K |W|^2. \quad (4)$$

Using the preceding results we can schematically write the potential in SUGRA as:

$$V = |F|^2 + |D|^2 - 3m_{3/2}^2 \tilde{M}_P^2, \quad (5)$$

where we have put back the reduced Planck mass  $\tilde{M}_P$ . Eq. (5) shows that there are two possible sources for the positive vacuum energy  $V(0)$  which drives inflation: the F-term or the D-term. The negative term also allows for eventual cancellation of the potential energy, and is the main motivation for considering SUGRA. Assuming that the D-terms are zero and the F-terms of the same order of magnitude during inflation as they are at the end of inflation, we have<sup>1</sup>

$$V(0) = |F|^2 - 3m_{3/2}^2 \tilde{M}_P^2 \sim m_{3/2}^2 \tilde{M}_P^2 \sim (10^{11} \text{ GeV})^4. \quad (6)$$

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<sup>1</sup>After inflation, since  $V = 0$  we must have  $m_{3/2} = \frac{F}{\sqrt{3}\tilde{M}_P}$  at the global minimum, which is the Deser-Zumino relation. It is hard to see how this relation could be so badly violated during inflation so that typically one expects that during inflation  $m_{3/2} \sim \frac{F}{\tilde{M}_P}$ .

Here the gravitino mass is assumed to be of order of 1 TeV. Now if we make the further assumption that the inflaton  $\phi$  mass is of order  $m_{3/2}$ , this line of reasoning leads to a violation of the slow roll condition,

$$|\eta| = \tilde{M}_P^2 \left( \frac{V''}{V} \right) \ll 1. \quad (7)$$

In fact from Eq. (6) we predict  $|\eta| \sim 1$ . This is the so-called  $\eta$  problem [5]. To overcome it, we must relax one or more of the assumptions in the chain of logic that led to it. For example, one can imagine that both during and after inflation the F-term and the -3 term in Eq. (5) exactly cancel, and that the energy which drives inflation originates from the D-term [6]. Then  $|D|^2$  is allowed to take a higher value than that in Eq. (6), providing it cancels to zero at the end of inflation; indeed the problem in D-term inflation is rather one of keeping the potential small enough compared to the string scale<sup>2</sup> [7]. We will not address this problem in this paper, and from here on we will assume that the D-terms vanish during inflation.

One interesting loop-hole in the above argument which we would like to pursue in the present paper is the possibility that the inflaton mass is in fact much smaller than  $m_{3/2}$  during inflation. For example it is known that in no-scale SUGRA theories, the soft scalar masses are zero (up to radiative corrections) even in the presence of a non-zero gravitino mass [9]. These no-scale results apply for all values of the fields, including during inflation when the fields are away from the global minimum. In fact the general conditions under which the inflaton maintains a zero mass during inflation have been formalised in terms of a Heisenberg symmetry of the Kahler potential [10]. However as shall become clear later the approach that we follow is more general than the Heisenberg symmetry. In fact the basic requirements of a successful F-term theory of inflation will be seen to be a no-scale assumption for the inflaton, together with other conditions which guarantee that it remains massless at tree-level during

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<sup>2</sup>D-term inflation may be more complicated than originally thought due to the unavoidable contribution of the dilaton in the F-term sector to SUSY breaking masses [8].

inflation, plus the requirement that the inflaton couples sufficiently weakly to other sectors so that the radiative corrections to its mass are very small. The requirement of small inflaton mass, combined with the COBE normalisation, imply that the height of the potential during hybrid inflation must be lower than the usual value  $10^{11}$  GeV, and we show how the no-scale structure allows this possibility. In fact we shall give an explicit example of our approach where the SUSY breaking sector, the height of the potential and the inflation sector are all specified. The example involves an ultra-light inflaton whose mass after radiative corrections is in the eV range. For such an ultra-light inflaton, the COBE normalisation demands that the potential during inflation be very low, of order  $10^8$  GeV.

## 2 Effective No Scale Supergravity Models From String Theory.

The simplest example of a no-scale SUGRA model consists of a Kahler potential

$$K = -3 \ln(T + T^* - \phi^* \phi), \quad (8)$$

and a superpotential which is independent of the field  $T$ ,

$$W = W(\phi). \quad (9)$$

When the potential is constructed using Eqs. (1), (2) one finds firstly that certain cross-terms involving derivatives of the superpotential like  $W_\phi$  cancel with each other, and secondly that terms involving derivatives of the Kahler potential cancel with the -3. The only term which survives is:

$$V = e^G \frac{\rho}{3|W|^2} |W_\phi|^2 = \frac{1}{3\rho^2} |W_\phi|^2, \quad (10)$$

where we have defined for convenience:

$$\rho \equiv T + T^* - \phi^* \phi. \quad (11)$$

The potential is minimised along the flat direction  $W_\phi = 0$ , and in fact is identically zero for all values of  $\rho$ . Supersymmetry is broken by the non-zero value of the F-term  $F_T = -\rho e^{G/2}$  but neither it nor the gravitino mass  $m_{3/2} = e^{G/2}$  is determined since  $\rho$  is not fixed at tree-level. However, assuming  $\rho$  is fixed by radiative/non-perturbative corrections, we conclude that supersymmetry is broken, the gravitino has a mass, but the fields  $\phi$  remain massless. The masslessness of the inflaton may be associated with the so called Heisenberg symmetry [10] which is more or less equivalent to the observation that the fields  $T, T^*$  only ever appear in the Kahler function in the combination  $\rho$ , and so the degrees of freedom may be regarded as  $\rho = \rho^*$  and  $\phi$ , and the masslessness of  $\phi$  is guaranteed for any Kahler potential of the form  $K_0(\rho)$ . On the other hand, we emphasise that if one allows a superpotential with a non-trivial  $T$  dependence then extra terms appear in the potential proportional to  $W_T$  which invalidate these results.

Although the no-scale SUGRA mechanism looks quite specialised, it turns out that it fits in quite well within 4d effective string theories. For example, in orbifold constructions [12] the tree-level Kahler potential is given by,

$$K = -\ln(S + S^*) - 3\ln(T + T^*) + \sum_a (T + T^*)^{n_a} C_a^* C_a, \quad (12)$$

where in the usual notation,  $S$  is the dilaton,  $T$  is an over-all modulus and  $C_a$  observable (matter) superfields with modular weights  $n_a$ . Untwisted sector superfields have  $n_a = -1$ ; fields belonging to the twisted sector without oscillators have usually  $n_a = -2$ , and those with oscillators usually have  $n_a \leq -3$ , but in general any integer  $n_a \leq 0$  is possible for twisted fields. The assumption of an over-all modulus  $T$  is a simplified one and generically there are at least three  $T_i$  moduli fields, the over-all modulus case corresponding to the limit  $T_1 = T_2 = T_3$ . Nevertheless, one would expect the values of the moduli fields not to be very different, and approach the simpler over-all modulus situation which we will consider in this paper. The differences that

may appear in the multimoduli case are not relevant for the discussion below.

In dealing with inflation we shall assume that the inflaton  $\phi$  is an untwisted field and distinguish it from all the other matter fields by writing explicitly  $\phi \equiv C_0$ . Then the Kahler potential for the modulus and the inflaton is virtually identical to the non-scale SUGRA Kahler potential in Eq. (8), and we can rewrite  $K$  as,

$$K = -\ln(S + S^*) - 3\ln(T + T^* - \phi^*\phi) + \sum_{a>0} (T + T^*)^{n_a} C_a^* C_a, \quad (13)$$

and define  $\rho = T + T^* - \phi^*\phi$  as done before. The superpotential (which by assumption is independent of  $T$ ) may be written as:

$$W = \hat{W}(S) + \tilde{W}(\phi, C_a), \quad (14)$$

where for simplicity we have assumed that  $\tilde{W}(\phi, C_a)$  has no  $S$  dependence, so that  $\hat{W}$  is the superpotential in the “hidden” sector, and  $\tilde{W}$  is the superpotential in the “observable” sector.

To proceed further with the discussion we need to specify the superpotential  $W = \hat{W} + \tilde{W}$ . This is highly model dependent, and in particular the dilaton superpotential  $\hat{W}(S)$ , which may be responsible for SUSY breaking, is not predicted by string theory. In general both  $K$  and  $W$  may receive corrections related to the problem of dilaton stabilisation [15, 16], and different models have been proposed in the recent literature. For example, a superpotential  $\hat{W}(S)$  can be generated in such a way that the dilaton is stabilised but  $F_S = 0$ , i.e., with zero potential, both during and after inflation, as recently discussed by Riotto and one of us [17]. Here we are most interested in a situation with a non-zero  $F_S$ , and for that purpose we take a model of gaugino condensation in which the dilaton is stabilised by non perturbative corrections to its Kahler potential [16]. Following Ref. [8] we take the particular ansatz:

$$K(S) = -\ln(S + S^*) + \hat{K}_{np}(S), \quad (15)$$

$$\hat{K}_{np}(S) = -\frac{2s_0}{S + S^*} + \frac{b + 4s_0^2}{6(S + S^*)^2}, \quad (16)$$

where  $b$  and  $s_0$  are non-negative constants; we also consider the dilaton superpotential:

$$\hat{W}(S) = \Lambda^3 e^{-S/b_0}, \quad (17)$$

with  $\Lambda$  an unspecified scale for the time being. This effective scale will be later fixed by the requirement of having  $m_{3/2} \approx 1 \text{ TeV}$  after inflation ends. The potential in the  $S$  direction has a minimum at some value  $\text{Re}S < s_0$ , and an exponentially decaying tail for  $\text{Re}S > s_0$ . The constant  $b$  in the non-perturbative Kahler potential controls the height of the barrier between the minima and the exponentially decaying part, being the barrier as larger as smaller is  $b$ .

Including the dilaton contribution in the potential in Eq. (10), the potential in the  $\rho$  direction goes simply as  $\propto 1/\rho^3$ . This would imply that  $\rho \rightarrow \infty$ , unless the potential vanishes, and no period of inflation be allowed. Therefore we must appeal to some additional non-perturbative corrections to the no-scale Kahler potential in order to stabilise the value of  $\rho$ . As a specific example to model the non-perturbative contribution we take:

$$K_{np}(\rho) = \frac{\beta}{\rho^3}, \quad (18)$$

with  $\beta$  a positive constant.

Joining together all the different pieces, Eq. (13) is replaced by:

$$K = -3 \ln(\rho) + \frac{\beta}{\rho^3} - \ln(S + S^*) + \hat{K}_{np}(S) + \sum_{a>0} (T + T^*)^{n_a} C_a^* C_a. \quad (19)$$

Note however that the theory in Eqs. (14), (17) and (19) does not have the Heisenberg symmetry since the fields  $T, T^*$  do not exclusively appear in the combination of Eq. (11). However, since we are only interested in preserving the masslessness of the inflaton *during inflation*, all we require is that the remaining matter fields with non-zero modular weight are switched off during inflation, or at least do not contribute to the potential during inflation. This can be seen as a constraint on the superpotential for such matter fields rather than on the Kahler potential. This means that, apart



from the inflaton, the matter fields with non-zero modular weight have to satisfy the conditions  $C_a = W_a = 0$  in the false vacuum of the theory where inflation is taking place, even if they do not couple to the inflaton. Notice that for the field  $N$  in hybrid inflation, with a typical hybrid superpotential

$$\tilde{W}(\phi, N) = -k\phi N^2, \quad (20)$$

these conditions are immediately fulfilled and thus its Kahler potential may still be arbitrary. We shall assume later that the singlet  $N$  also belongs to the untwisted sector, entering in the definition of  $\rho = T + T^* - \phi^*\phi - N^*N$ . As far as inflation is concerned, this will not make any difference. Note that  $\tilde{W}$  is zero during inflation and therefore the superpotential in Eq. (14) is given by the dilaton superpotential  $\hat{W}$ .

The conditions that we propose to protect the flatness of the inflaton potential, Heisenberg symmetry and stabilising  $\rho$  through non perturbative contributions, should be compared to other approaches. In [13] the requirement is that either the field  $C_a$  or its derivative  $W_a$  to be zero, with the additional constraint that the superpotential during inflation must vanish. This condition ensures that  $dV/d\rho$  vanishes identically, and  $\rho$  is a flat direction. In such a situation the false vacuum energy would be dominated by some non-zero  $W_a$ . By contrast in our present scenario we can relax the assumption of having a zero superpotential by the specific choice of the inflaton Kahler potential we consider in order to stabilise  $\rho$  at some fixed value. In [3] a multimoduli Kahler potential is considered with *all* matter fields having modular weights -1 with respect to one of the moduli  $T_i$ . With such a restriction they are led to conclude that the over-all modulus case is not viable for inflation, because in this case there would be no possible source to generate the potential energy without giving a large mass to the inflaton. In our approach this problem is circumvented by the inclusion of the dilaton contribution. The multimoduli Kahler potential in the

context of inflation has also been recently studied in [14].

### 3 The Height of the SUGRA Inflation Potential.

We now turn to the question of how to achieve a height of the potential below the typical SUGRA value during inflation, when the only relevant fields are  $\rho$  and  $S$ . The potential obtained from Eqs. (1) and (2) is given by,

$$V(\rho, S) = e^K |W|^2 \left( \frac{K'^2}{K''} - 3 + \frac{1}{K_{SS}} \left| K_S + \frac{W_S}{W} \right|^2 \right), \quad (21)$$

where a subscript  $S$  denotes derivative with respect to the dilaton, primes represent derivation with respect to  $\rho$ ; the Kähler potential is given in Eq. (19),  $W$  is given in Eq. (14) with  $\tilde{W} = 0$  and then  $W_S/W = -1/b_0$ . Both fields,  $\rho$  and  $S$ , will have a minimum at some value  $\rho_0$  and  $S_0$  respectively, and they will contribute with a positive term to the potential. Now, the size of the cosmological constant during inflation will depend on the degree of fine-tuning we want to impose on the cancellation of this contribution against the negative term  $-3m_{3/2}^2 \tilde{M}_P^2$ . As discussed in the introduction, we are interested in the situation where they almost cancel, i.e.,

$$V(\rho_0, S_0) = \epsilon m_{3/2}^2 \tilde{M}_P^2, \quad \epsilon \ll 1. \quad (22)$$

The procedure is straightforward: we just minimise the potential with the additional constraint Eq. (22). We find the three set of equations:

$$\frac{K'^2}{K''} - 3 + \frac{1}{K_{SS}} \left| K_S - \frac{1}{b_0} \right|^2 = \epsilon, \quad (23)$$

$$\frac{K' K'''}{K''^2} - 2 = \epsilon, \quad (24)$$

$$\frac{K_{SSS}}{K_{SS}^2} \left( K_S - \frac{1}{b_0} \right) - 2 = \epsilon, \quad (25)$$

which can be solved for  $b_0$ ,  $S$  and  $\rho$ , for given values of  $\beta$ ,  $b$  and  $s_0$ . The solution for  $\rho$  can be found analytically as a series expansion in the parameter  $\epsilon$ , and is given by:

$$\rho_0 = (2\beta)^{1/3} (1 + \epsilon + \epsilon^2 + \dots), \quad (26)$$

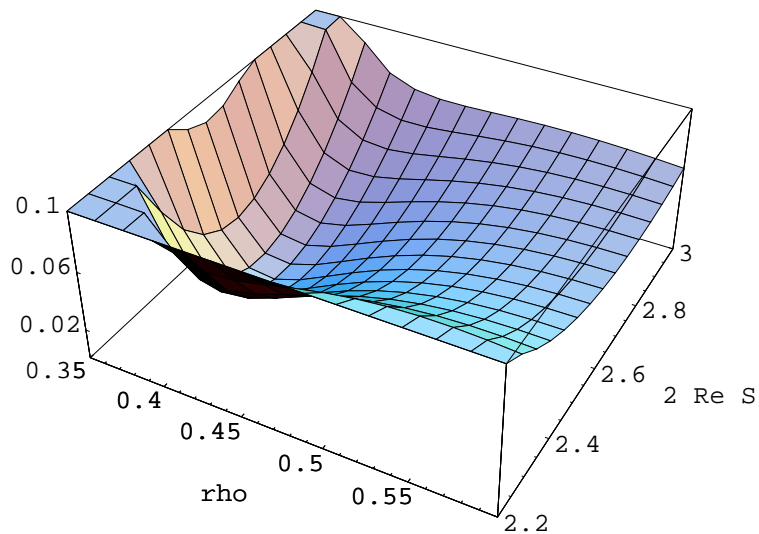


Figure 1: The potential  $V(S, \rho)$  in units of  $m_{3/2}^2 \tilde{M}_P^2$ .  $b = 1$ ,  $s_0 = 4$ ,  $\beta = 1/32$ .

$$\frac{K'^2}{K''} - 3 = -\frac{3}{4}(1 - 3\epsilon^2 + \dots). \quad (27)$$

The values of  $b_0$  and  $S_0$  (value at the minima) are searched numerically. We notice that neither  $b_0$  nor  $S_0$  depend on the parameter  $\beta$ . As an example, in Fig. (1) we have plotted the potential  $V(\rho, S)$ , which clearly shows the presence of a minimum, for the choice  $b = 1$ ,  $\beta = 1/32$  and  $s_0 = 4$ . In Fig. (2) we have plot the projections  $V(\rho_0, S)$  and  $V(\rho, S_0)$  respectively, and in Fig. (3) the computed values of  $b_0$  versus the parameter  $s_0$ . The minimum of the potential is not exactly at zero, but at  $\epsilon \simeq 10^{-10}$ . This implies a potential scale of order  $10^8 \text{ GeV}$  during inflation if the gravitino mass is order  $1 \text{ TeV}$ . Such a low value for  $\epsilon$  can be achieved only by fine-tuning  $b_0$ .

Once we have found the value  $\rho_0$  for which the condition  $dV/d\rho = 0$  is fulfilled, it is not difficult to check that the inflaton will remain massless. The mass matrix for

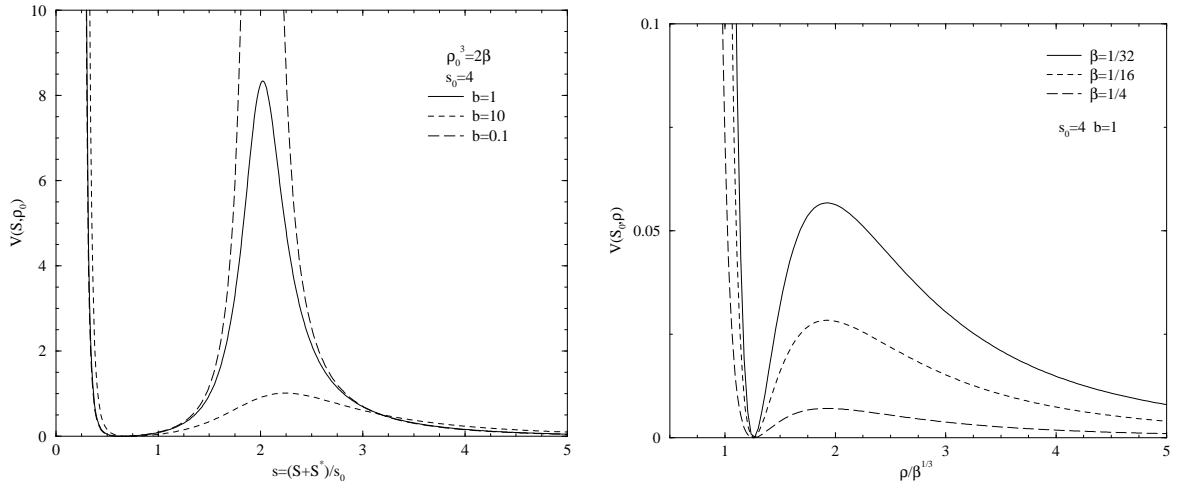


Figure 2: (a) The potential  $V(S, \rho_0)$ , for  $s_0 = 4$ , and different values of  $b$ . (b) The potential  $V(S_0, \rho)$ , for  $s_0 = 4$ ,  $b = 1$  and different values of  $\beta$ . The potential is given in units of  $m_{3/2}^2 M_P^2$ . Note that here we have used scaled variables, whereas in Fig. (1) we used unscaled ones.

the fields  $(T, \phi)$  during inflation is given by:

$$\mathcal{M}^2(T, \phi) = \begin{pmatrix} V'' & -\phi V'' \\ -\phi^* V'' & |\phi|^2 V'' \end{pmatrix}, \quad (28)$$

with  $V'' = d^2V/d\rho^2$  at  $\rho_0$ . It is clear that this matrix has a zero eigenvalue which corresponds to the massless inflaton, that for simplicity we will call  $\phi$ . In other words as the field  $\phi$  rolls the moduli  $T$  will adjust itself in such a way that the overall potential remains flat.

## 4 A Next-to-Minimal Supergravity Model of Hybrid Inflation.

Finally we give an example of a model of hybrid inflation where these results may apply, namely the next-to-minimal supersymmetric model (NMSSM) of hybrid inflation which was recently proposed [18]. This model was proposed in the context of global SUSY, where the origin of the (small) vacuum energy which drives inflation was not

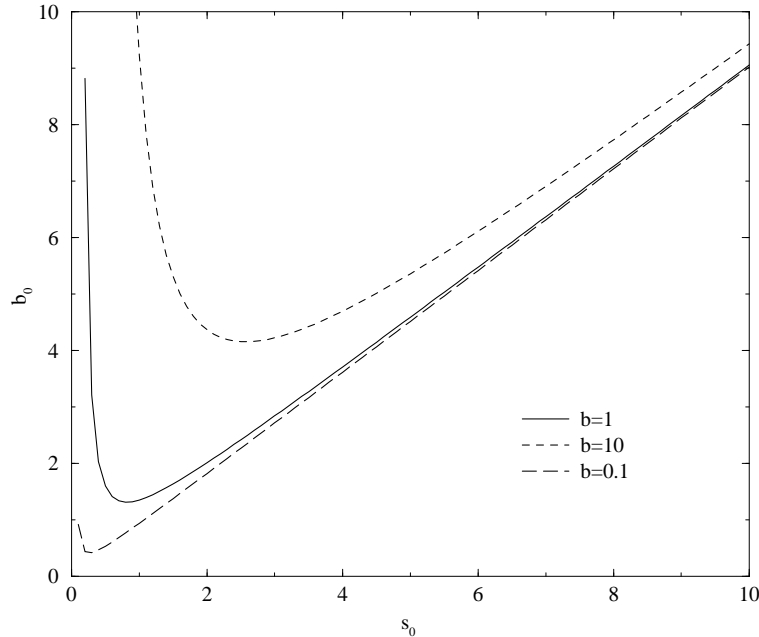


Figure 3: Computed values of  $b_0$ , with the condition  $\epsilon \ll 1$ .

explained, the mechanism of supersymmetry breaking was not discussed, the  $\eta$  problem was not resolved, and the smallness of the inflaton mass was simply assumed to be due to some unspecified no-scale SUGRA model. We are now in a position to address all these issues in the light of the preceding discussion.

We begin with a brief resume of the global SUSY model. The model is based on the superpotential:

$$\tilde{W} = \lambda N H_1 H_2 - k \phi N^2, \quad (29)$$

with the fields  $\phi, N$  being gauge singlets, and the first term coupling the singlet  $N$  to the MSSM Higgs doublets  $H_1, H_2$ . Since the VEV of  $N$  generates the effective  $\mu$  mass term coupling the two Higgs doublets, we require that  $\lambda \langle N \rangle \sim 1$  TeV as in the well known particle physics NMSSM model. The Higgs doublets develop electroweak VEVs, much smaller than  $\langle N \rangle$ , and they may be ignored in the analysis. Therefore,

the potential for the real components of the complex singlets reads,

$$V(\phi, N) = k^2 N^4 + (m_N^2 - 2kA_k\phi + 4k^2\phi^2)N^2 + m_\phi^2\phi^2, \quad (30)$$

where the soft parameters above occur in the soft SUSY breaking potential  $V_{soft} = m_N N^2 - A_k k \phi N^2$ , but  $m_\phi^2$  owes its origin to radiative corrections to the potential controlled by the small coupling  $k$ . We have fixed our convention of signs and phases such that  $kA_k > 0$  but appears with a negative sign in the potential. This negative contribution is necessary not only for the purpose of inflation but in order to get electroweak symmetry breaking afterwards.

For large values of the field  $\phi$  the effective  $N$  mass is positive and during inflation the field  $N = 0$ ;  $\phi$  slowly rolls until it reaches a critical value:

$$\phi_c^\pm = \frac{A_k}{4k} \left( 1 \pm \sqrt{1 - 4 \frac{m_N^2}{A_k^2}} \right). \quad (31)$$

Depending on the sign of the mass squared  $m_\phi^2$ ,  $\phi$  can roll towards  $\phi^+$  from above ( $m_\phi^2 > 0$ , hybrid inflation) or towards  $\phi^-$  from below ( $m_\phi^2 < 0$ , inverted hybrid inflation [19]). Either way, when the critical value is reached, inflation ends and the following global minimum is achieved:

$$\langle \phi \rangle = \frac{A_k}{4k}, \quad (32)$$

$$\langle N \rangle = \frac{A_k}{2\sqrt{2}k} \sqrt{1 - 4 \frac{m_N^2}{A_k^2}} = \sqrt{2} |\phi_c^\pm - \langle \phi \rangle|, \quad (33)$$

During inflation the potential energy of the vacuum takes the positive value  $V(0)$  (which will be shortly explained in the context of a SUGRA model) which drives inflation. After inflation ends  $V(0)$  is assumed to remain unchanged, but be cancelled by a negative contribution from the remaining part of the potential at the global minimum:

$$V(\langle \phi \rangle, \langle N \rangle) = -k^2 \langle N \rangle^4 = -4k^2 (\phi_c^\pm - \langle \phi \rangle)^4. \quad (34)$$

The cancellation mechanism is beyond the scope of this paper (this is the problem of the cosmological constant).

We now wish to elevate this model to an SUGRA theory, using the results of earlier sections, and in particular the Kahler potential in Eq. (19). The inflaton  $\phi$  is in the untwisted sector, and appears with the overall modulus  $T$  in the combination  $\rho$ . As before  $T$  is assumed not to enter the superpotential. The singlet  $N$  and the remaining MSSM superfields are in the twisted sector with canonical Kahler potentials (zero modular weights) to begin with. The vacuum energy driven inflation  $V(\rho_0, S_0)$  is achieved by the partial cancellation between the  $\rho$  and the dilaton contributions. The superpotential  $W$  is from Eq. (14),

$$W = -kN^2\phi + \hat{W}(S), \quad (35)$$

where  $\hat{W}(S)$  was given in Eq. (17). In the present SUGRA framework we can compute the soft parameters  $m_N$  and  $A_k$ , and the related critical value  $\phi_c^\pm$  and vevs of the fields, in term of the gravitino mass. We are now interested in what happens after inflation to the fields  $\phi$  and  $N$ , so first we have to reincluded them in the SUGRA potential. The potential is then that of Eq. (21) plus the contribution of  $\phi$  and  $N$ :

$$V = V(\rho, S) + e^K \left[ -\frac{|W_\phi|^2}{K'} + |N^*W + W_N|^2 \right]. \quad (36)$$

When inflation ends, we can expect the value of  $\rho$  and  $S$  to change, as all the fields will now adjust their values in the global minimum of the theory. Nevertheless, as far as the contribution to the superpotential due to fields  $N$  and  $\phi$  is suppressed respect to the dilaton term, the minima for the fields  $S$  or  $\rho$  (and now by extension also  $T$ ) will be only slightly shifted, and they will safely remain where they were previously. The fact that the minima for  $\rho$  and  $S$  is almost unchanged after inflation also relies on the condition  $\epsilon \ll 1$ . This together with  $\tilde{W}/\hat{W} \sim O(\epsilon)$  makes the minimization conditions for  $\rho$  and  $S$  to look the same than those given in Eqs. (24) and (25), and

at most the values of  $\rho_0$  and  $S_0$  will be shifted only by a factor  $O(\epsilon)$ . Therefore the gravitino mass will also take the same value during and after inflation, i.e.,

$$m_{3/2}^2 = \langle e^G \rangle \simeq e^K \frac{|\hat{W}(S)|^2}{\tilde{M}_P^4} \simeq \frac{\Lambda^6}{\tilde{M}_P^4}. \quad (37)$$

In order to keep a gravitino mass of order  $1 \text{ TeV}$  we require  $\Lambda \approx 10^{13} \text{ GeV}$ .

In the limit of large moduli, we can now expand the potential in powers of  $\phi\phi^*/(T+T^*)$ . The remaining explicit dependence on the modulus  $T$  will be absorbed in the proper normalisation of fields and Yukawas to get the physical degrees of freedom and couplings, and also in the gravitino mass. Therefore we can write our potential as,

$$V = V(\rho_0, S_0) + |W_\phi|^2 + |W_N|^2 + m_{3/2}^2 |N|^2 + m_{3/2} (NW_N + h.c.) - \delta(\tilde{W}(\phi, N) + h.c.) + \dots, \quad (38)$$

where the dots stand for non-renormalisable terms, and from Eq. (22)  $V(\rho_0, S_0) \simeq \epsilon m_{3/2}^2 \tilde{M}_P^2$ . According to Eq. (38) the soft parameters are defined as,

$$m_N^2 \equiv m_{3/2}^2, \quad (39)$$

$$A_k \equiv (2 - \delta)m_{3/2} < 2m_{3/2}, \quad (40)$$

$$\delta = 3 - \frac{K'^2}{K''} + \frac{K_S}{b_0 K_{SS}}. \quad (41)$$

It is clear from Eqs. (33), (39), (40) that in this particular case<sup>3</sup> no minima exists different than  $\langle N \rangle = 0$ , and thus  $V(\langle \phi \rangle, \langle N \rangle) = 0$ .

In order to avoid this problem we allow the field  $N$  to have a non-zero modular weight  $n_N$ . An interesting possibility is to have  $N$  in the untwisted sector, on the same foot in the Kahler potential than the field  $\phi$ . That is,  $N$  will enter into the combination  $\rho$ , will be massless, but still  $N = 0$  during inflation. The Kahler potential and the potential in this case becomes:

$$K = -3 \ln \rho + \frac{\beta}{\rho^3} - \ln(S + S^*) + \hat{K}_{np}(S), \quad (42)$$

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<sup>3</sup>We will see later that in this model the coupling between  $\phi$  and  $N$  is too small, and therefore radiative corrections are not enough to break the relation  $A_k^2 < 4m_N^2$  to the extend needed.



$$\rho = T + T^* - \phi^* \phi - N^* N, \quad (43)$$

$$\begin{aligned} V &= e^K \left[ \left( \frac{K'^2}{K''} - 3 \right) |W|^2 + \frac{1}{K_{SS}} |K_S W + W_S|^2 - \frac{|W_\phi|^2}{K'} - \frac{|W_N|^2}{K'} \right] \\ &= V(\rho_0, S_0) + |W_\phi|^2 + |W_N|^2 - \delta m_{3/2} (\tilde{W}(\phi, N) + h.c.) + \dots, \end{aligned} \quad (44)$$

with the soft parameters given now by<sup>4</sup>,

$$m_N^2 = 0, \quad (45)$$

$$A_k = \delta m_{3/2}, \quad (46)$$

and the condition  $A_k^2 > m_N^2$  trivially satisfied.

The main point to note is that the soft parameters are of order  $m_{3/2} \sim 1$  TeV both during and after inflation, but the inflaton during inflation will have a zero tree-level mass due to the no-scale mechanism. After inflation ends the value of  $V(\rho_0, S_0)$  is assumed to be unchanged, but there is in general a negative contribution from the remaining part of the potential in Eq. (34) whose value depends on the coupling  $k$ , which in this model is fixed to be very small, as we now discuss.

One of the additional features of the NMSSM of hybrid inflation is that it can solve the strong CP problem due to the presence of an  $U(1)_{PQ}$  symmetry in the superpotential given in Eq. (29), broken by the vevs of  $\phi$  and  $N$ . This implies that there is an axion field, and to satisfy the cosmological constraints on its abundance it is required  $\langle N \rangle \sim \langle \phi \rangle \sim 10^{13} \text{ GeV}$ . Looking at Eqs. (32-33) we end with the condition  $k \sim 10^{-10}$ , a quite small coupling constant, but that will render the value of the potential in Eq. (34) of order,

$$V(\langle \phi \rangle, \langle N \rangle) \sim -(10^8 \text{ GeV})^4, \quad (47)$$

This is in fact the same order as the potential  $V(\rho_0, S_0) \simeq \epsilon m_{3/2}^2 \tilde{M}_P^2$  obtained in the previous section when  $\epsilon \simeq 10^{-10}$ . Therefore we may propose that at the global

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<sup>4</sup>We implicitly assume that the sign of the  $k\phi N^2$  coupling in the superpotential may be changed in order to match Eq.(44) with Eq. (30).

minimum these two potentials of opposite sign and the same magnitude accurately cancel to yield an acceptably small cosmological constant. With the extremely small coupling  $k$  radiative corrections yield an (ultralight) inflaton mass during inflation in the eV range [18], assuming that the no-scale mechanism developed in this paper sets its tree level mass during inflation to zero. As discussed in detail elsewhere [18] an inflaton mass in the eV range together with a potential during inflation of height  $V(0)^{1/4} \sim 10^8 \text{ GeV}$  gives an appropriate COBE normalisation.

We end this section by noting that the smallness of  $k$  seems to indicate that in fact the original coupling has a non-renormalisable origin, as previously discussed [18], with  $k$  effectively given as a ratio of two scales, namely  $k \sim (\Lambda/\tilde{M}_P)^2$ , where  $\Lambda$  was identified as the VEV of some new fields. In the present framework we might try to identify the scale  $\Lambda$  with the effective scale in the dilaton superpotential, since these two scales are naturally the same order of magnitude in this model. However we shall not pursue this possibility further here.

## 5 Final Comments

We should comment on the effect of strongly coupled heterotic string theories on models such as this which rely on no-scale SUGRA. In general one might expect that the overall modulus field  $T$  will appear in combination with the dilaton  $S$  in the combination  $S + \alpha T$  in the dilaton superpotential, thereby invalidating the no-scale assumption that  $T$  does not enter the superpotential, and so invalidating our basic assumption that the inflaton field  $\phi$  is massless at the string scale. However the situation is in fact not so clear since some authors claim that the parameter  $\alpha$  may in fact be extremely small even in strongly coupled heterotic string (M) theory. The result may depend on the precise mechanism for SUSY breaking in the hidden sector (e.g. gaugino condensation vs. Scherk-Schwartz breaking) [20]. In any case

the discussion of such issues, along with the dilaton potential, lies beyond the scope of the present paper.

In this paper we have shown that effective SUGRA theories of the no-scale form which may arise naturally from string theory may lead to viable models of F-term hybrid inflation. Whilst it is true that perturbative string theory cannot explain dilaton stabilisation, it is generally accepted that non-perturbative string theory could provide a resolution to this problem in the future, and it has become commonplace to introduce *ad hoc* Kahler potentials to model such an unknown non-perturbative behaviour of string theory. In this spirit we introduced the Kahler potential in Eqs. (16), (18) and the superpotential in Eq. (17) to model the unknown non-perturbative effects of string theory.

The main point of this paper is to construct a general framework for pursuing  $F$  term inflation in effective SUGRA theories which may typically arise in string models. As we have discussed, such theories are typically of the no-scale form, and we have proposed that it is natural to place the inflaton in the untwisted sector along with the moduli fields in order to obtain a massless inflaton and so solve the  $\eta$  or slow roll problem. To summarise, we have assumed the following conditions to hold during inflation:

(a) The superpotential is independent of the modulus  $T$ .

(b) The Kahler potential depends on the combination  $\rho = T + T^* - \sum_i \phi_i^* \phi_i$ , with  $\phi_i$  any untwisted field present in the theory. Here we have only consider those relevant for hybrid inflation, i.e,  $\phi$  and  $N$ . In addition, there are non-perturbative contributions for both the field  $\rho$  and the dilaton  $S$  which help to stabilise them at a fixed value during inflation.

(c) The SUGRA potential during inflation is of the form:

$$V = |F_T|^2 + |F_S|^2 - 3e^G, \quad (48)$$

such that the positive contribution can be fine-tuned to cancel the negative term to the extent needed to generate a positive potential energy suitable for inflation.

The first two conditions ensure the masslessness of the inflaton. The dilaton role is twofold: it works as a source for SUSY breaking, and its positive contribution to the potential helps to keep the potential scale during inflation below the typical SUSY breaking scale.

We emphasise that it is not necessary for the theory to possess a Heisenberg symmetry, but only that the other twisted fields with modular weights  $n_i < -1$  be switched off during inflation. In such theories the partial cancellation of the negative term in the SUGRA potential against the dilaton and moduli contribution allows the possibility that the height of the potential during inflation is much smaller than usually assumed, with  $V \sim \epsilon m_{3/2}^2 \tilde{M}_P^2$ , the size of  $\epsilon$  being model dependent. Smaller potentials are welcome in this kind of approach because one must not only ensure that the inflaton is massless during inflation at tree level, but also explain why radiative corrections do not lead to the inflaton acquiring a mass which would be again too large to avoid the  $\eta$  problem. In order to control the radiative corrections we have simply assumed that they are controlled by very small couplings. The presence of such small couplings is then generally associated with small height potentials which are therefore a generic consequence of our approach. Another good feature of having small couplings in the hybrid superpotential is that their contribution when inflation ends will be highly suppressed respect to the dominant dilaton contribution, and as a result the minima for the moduli and the dilaton are mainly the same during and after inflation. That is, the generic cosmological moduli problem [21] will not be present in our scenario with a small height potential.

To illustrate these features we have revisited an NMSSM model of inflation corresponding to the superpotential in Eq. (29) in which the radiative corrections are

controlled by an extremely small Yukawa coupling  $k \sim 10^{-10}$  which leads to an ultralight inflation in the eV range. Such a light inflaton, combined with the COBE normalisation requires a potential during inflation of  $10^8$  GeV which corresponds to Eq. (22) with  $\epsilon^{1/4} \sim 10^{-3}$ . Of course such small couplings may be regarded a rather extreme example of how to control the radiative corrections, but it is worth recalling that the strength of the couplings are determined by the requirements of satisfying the COBE normalisation solving the strong CP problem, and the  $\mu$  problem at the same time [18]. It is also worth recalling that the origin of such small couplings may involve an additional sector which obeys a discrete  $Z_3 \times Z_5$  symmetry from which the Peccei-Quinn symmetry emerges as an approximation. We have not performed an explicit string compactification to generate the desired symmetry, nor have we derived the superpotential from first principles (which would involve standard techniques of taking vacuum expectations of vertex operators). However explicit string constructions do exist in the literature which involve similar discrete symmetries to those assumed here, and which forbid the lowest dimension operators but allow operators at higher order.

Finally although it might seem that the present model is quite complicated, it is worth emphasising that any realistic supergravity model of inflation will necessarily involve a similar number of free parameters. The literature is replete with toy models or incomplete theories which look simpler than our model precisely because they are not complete. Our model, although not derived from first principles using string theory, nevertheless represents a complete working model of no-scale supergravity inflation. It contains explicit mechanisms for dilaton and moduli stabilisation and supersymmetry breaking, and it successfully resolves the  $\eta$  problem of supergravity hybrid inflation. It looks more complicated precisely because it goes further towards being fully realistic than most other supergravity models. We have already emphasised that the model solves the strong CP problem and the  $\mu$  problem of su-

persymmetry. The model clearly has many virtues, and represents the most realistic attempt at supergravity hybrid inflation (as far as we are aware) available in the current literature. It remains to be seen if its central prediction, a spectral index  $n$  precisely equal to unity [18], will be verified by the forthcoming Map and Planck explorer experiments.

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