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COMPLETENESS OF BOUNDARY CONDITIONS FOR THE CRITICAL THREE-STATE POTTS MODEL

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Abstract

We show that the conformally invariant boundary conditions for the three-state Potts model are exhausted by the eight known solutions. Their structure is seen to be similar to the one in a free field theory that leads to the existence of D-branes in string theory. Specifically, the fixed and mixed boundary conditions correspond to Neumann conditions, while the free boundary condition and the new one recently found by Affleck et al. [1] have a natural interpretation as Dirichlet conditions for a higher-spin current. The latter two conditions are governed by the Lee–Yang fusion rules. These results can be generalized to an infinite series of non-diagonal minimal models, and beyond.

The simplest boundary conditions for a conformal field theory are those which preserve not only the conformal symmetry, but also all other symmetries of the model. In other words, not only do they leave the Virasoro algebra \mathfrak{Vir} invariant, but even the whole chiral algebra \mathfrak{A} . As already argued long ago by Cardy [2], for models in which the torus partition function is given by charge conjugation, such boundary conditions are governed by the fusion rule algebra \mathfrak{N} of the model. In particular, the possible boundary conditions are in one-to-one correspondence to the primary fields (with respect to \mathfrak{A}).

When the chiral algebra \mathfrak{A} is larger than \mathfrak{Vir} , then in general there also exist conformally invariant boundary conditions that do *not* preserve all of \mathfrak{A} . An obvious task is then to identify structures that govern the boundary conditions in this more general situation. This problem can be attacked by identifying suitable compatibility requirements which enforce that the boundary conditions for the fields in the extended algebra cannot be chosen arbitrarily. For instance, for free bosons and for WZW theories such constraints arise from the fact that the energy-momentum tensor is a quadratic expression in the (spin-1) currents. Here we discuss a class of models which are lacking such a direct relation, but whose boundary conditions can nevertheless be classified completely. The simplest example of this class is provided by the (critical) three-state Potts model, for which the chiral algebra \mathfrak{A} is the so-called \mathcal{W}_3 -algebra. Potts models play a prominent role in the study of order-disorder transitions and of high/low temperature duality, they are used for studying critical percolation and linear resistance networks [3, 4], and they are of experimental interest e.g. for describing the adsorption of monolayers on substrates [5]. Therefore (as well as for the sake of definiteness), we first focus our attention to this model, deferring a discussion of other theories to the end of this letter.

As has been established in [6], the classification of the conformally invariant boundary conditions for an arbitrary conformal field theory naturally proceeds in three steps. First one lists all automorphisms of the fusion rules \mathfrak{N} which preserve the conformal weights Δ_λ (not only modulo integers, as would e.g. be required in the case of the torus partition function). The second step consists in implementing such automorphisms on the spaces of chiral blocks. And finally one has to find all solutions for certain scalar factors (which possess an interpretation as reflection coefficients for bulk fields on the disk) that are compatible with the factorization constraints. In string theory terms, this last step amounts to identifying the possible types of Chan–Paton charges. (In string theory, each such Chan–Paton charge comes with its own multiplicity. E.g. in the uncompactified ten-dimensional type I superstring theory, there is a single Chan–Paton charge with multiplicity 32.)

Concerning the first step, we recall that – labelling the primary fields by λ and assigning the special label Ω to the vacuum field – a fusion rule automorphism ω satisfies $N_{\omega(\lambda),\omega(\mu)}^{\omega(\nu)} = N_{\lambda,\mu}^\nu$ and $\omega(\Omega) = \Omega$. Using the Verlinde relation between the fusion coefficients $N_{\lambda,\mu}^\nu$ and the modular S-matrix S of the theory, ω must in particular preserve the quantum dimensions $d_\lambda = S_{\lambda,\Omega}/S_{\Omega,\Omega}$. In the three-state Potts model we employ a special notation for the primaries, which together

with the quantum dimensions and conformal weights is presented in the following table.

λ	Kac labels	d_λ	Δ_λ
Ω	$(11) \oplus (41)$	1	0
ψ	(43)	1	2/3
ψ^+	(43)	1	2/3
ε	$(21) \oplus (31)$	$\frac{1}{2}(1+\sqrt{5})$	2/5
σ	(33)	$\frac{1}{2}(1+\sqrt{5})$	1/15
σ^+	(33)	$\frac{1}{2}(1+\sqrt{5})$	1/15

(1)

Thus from the quantum dimensions d_λ we already learn that ω has to permute the elements of $\{\psi, \psi^+\}$ and of $\{\varepsilon, \sigma, \sigma^+\}$. (As is also already apparent from the values of d_λ , the full modular S-matrix is just the tensor product of the S-matrix for the \mathbb{Z}_3 fusion rules and the one for the Lee–Yang fusion rules [2].) Inspection of the specific fusion rules

$$\psi \star \psi = \psi^+, \quad \psi \star \psi^+ = \Omega, \quad \psi \star \varepsilon = \sigma, \quad \psi \star \sigma = \sigma^+ \quad (2)$$

and

$$\varepsilon \star \varepsilon = \Omega + \varepsilon, \quad \sigma \star \sigma = \psi^+ + \sigma^+, \quad \sigma^+ \star \sigma^+ = \psi + \sigma \quad (3)$$

then tells us that \mathfrak{N} has precisely two automorphisms: either $\omega = \text{id}$ is the identity, or else $\omega = C$ acts as

$$C: \quad \Omega \mapsto \Omega, \quad \psi \leftrightarrow \psi^+, \quad \varepsilon \mapsto \varepsilon, \quad \sigma \leftrightarrow \sigma^+, \quad (4)$$

which is just charge conjugation. Both automorphisms indeed preserve the conformal weights, $\Delta_{\omega(\lambda)} = \Delta_\lambda$.

The next task is to implement the fusion rule automorphism ω on all chiral blocks V of the conformal field theory on arbitrary surfaces. This means that for arbitrary choices of $\lambda_1, \dots, \lambda_m$ and for every value of the insertion points and of the moduli of the surface we must provide a family of associated isomorphisms $\Theta_\omega^{(\vec{\lambda})}$ between the vector bundles $V_{\lambda_1 \dots \lambda_m}$ and $V_{\omega(\lambda_1) \dots \omega(\lambda_m)}$ of blocks. Up to the restrictions originating from factorization, for the specification of such maps it is sufficient to construct an implementation of ω on all the modules (representation spaces) \mathcal{H}_λ of the chiral algebra \mathfrak{A} . That is, we only need to prescribe a family of maps $\omega_\lambda: \mathcal{H}_\lambda \rightarrow \mathcal{H}_{\omega(\lambda)}$ that is consistent with the chiral block structure. (Via state-field correspondence, this also induces a map on the chiral vertex operators, compare also [7].)

We first consider the vacuum sector \mathcal{H}_Ω ; as a Virasoro module it decomposes into irreducible submodules as $\mathcal{H}_\Omega = \mathcal{H}_{(11)} \oplus \mathcal{H}_{(41)}$, where $\mathcal{H}_{(11)} \equiv \mathcal{H}_{\Omega^{(\Lambda)}}$ is the vacuum sector of the tetracritical Ising model – i.e., in terms of the A-D-E classification of minimal models, of the A-type model at the same value $c = 4/5$ of the conformal central charge – while $\mathcal{H}_{(41)}$ is the **Vir**-family that provides the spin-3 current of \mathcal{W}_3 . Preservation of the Virasoro algebra means that ω_Ω acts as the identity on $\mathcal{H}_{(11)}$, and then the \mathcal{W}_3 commutation relations (or equivalently, the fusion rule

(41) \star (41) = (11) of the tetracritical Ising model) imply that ω_Ω acts as $\pm \text{id}$ when restricted to $\mathcal{H}_{(41)}$. Moreover, inspecting the action of \mathfrak{A} on the non-selfconjugate sectors one sees that the two sign choices correspond precisely to $\omega = \text{id}$ and $\omega = C$, respectively. We conclude that each of the two fusion rule automorphisms possesses a unique implementation ω_Ω on \mathcal{H}_Ω .

Next we show that the same result applies to the implementations ω_λ on all other sectors \mathcal{H}_λ as well. We first determine the implementation only up to a non-zero over-all scalar factor, which takes into account the fact that the highest weight state of \mathcal{H}_λ is unique only up to a scalar multiple. Now the \mathfrak{A} -modules \mathcal{H}_λ other than \mathcal{H}_Ω and \mathcal{H}_ε all consist of a single Virasoro module. From the invariance of the Virasoro algebra it therefore follows that in these cases ω_λ is already defined by its action on the highest weight state of \mathcal{H}_λ . Furthermore, the image of the highest weight state of \mathcal{H}_λ under ω_λ must be the highest weight state of $\mathcal{H}_{\omega(\lambda)}$, and hence ω_λ is uniquely determined (up to the freedom of changing the highest weight states by non-zero scalar factors). In short, for $\lambda \in \{\psi, \psi^+, \sigma, \sigma^+\}$ these maps ω_λ just ‘exchange’ the whole modules \mathcal{H}_λ and $\mathcal{H}_{\omega(\lambda)}$. As for the remaining sector \mathcal{H}_ε which decomposes into Virasoro modules as $\mathcal{H}_\varepsilon = \mathcal{H}_{(21)} \oplus \mathcal{H}_{(31)}$, we note that the highest weight vectors of the two components have distinct conformal weights ($2/5$ and $7/5$, respectively). Invariance of the Virasoro algebra therefore implies that ω_ε acts as a multiple of the identity when restricted to $\mathcal{H}_{(21)}$ or $\mathcal{H}_{(31)}$. Up to an over-all normalization, we can set $\omega_\varepsilon = \text{id}$ on $\mathcal{H}_{(21)}$; using also the transformation property of the rest of the chiral algebra, we then see that we must have $\omega_\varepsilon = \pm \text{id}$ when restricted to $\mathcal{H}_{(31)}$, and that the choice of sign again precisely corresponds to the alternative of having $\omega = \text{id}$ and $\omega = C$, respectively.

This finishes the determination of the ‘automorphism type’ [6] of boundary conditions. To complete the classification, we finally need to determine, for each of the two automorphism types separately, the possible values of scalar factors, i.e. of Chan–Paton charges. These charges may be regarded as reflection coefficients $R_{\lambda, \omega(\lambda); \Omega}^A$ for bulk fields on the disk with respect to the vacuum boundary field [8, 9]; they must respect the factorization constraints that implement [10, 8] the compatibility of a conformal field theory on surfaces of different topology. Let us from now on take the torus partition function in the bulk to be given by charge conjugation (when one deals instead with the diagonal torus amplitude, the results for $\omega = \text{id}$ and $\omega = C$ are interchanged). Then the boundary conditions with $\omega = C$ are governed [2] by the fusion rule algebra \mathfrak{N} . In particular, they may be labelled as $A = (C; \lambda)$ by the primary fields, so there are six different boundary conditions, and the reflection coefficients are given by the one-dimensional irreducible representations of \mathfrak{N} , i.e. by the quotients

$$R_{\mu, \mu^+; \Omega}^{(C; \lambda)} = S_{\lambda, \mu} / S_{\lambda, \Omega} \quad (5)$$

of S-matrix elements. For $\omega = \text{id}$ the situation is less familiar. But again the boundary conditions are governed by some finite-dimensional commutative algebra \mathfrak{D} [6]. To construct \mathfrak{D} , we first observe that the fusion rules of the three-state Potts model can be obtained from those of the tetracritical Ising model by combining fields that are related by fusion with the field $\phi_{(41)}$. More precisely,

$$N_{\lambda, \mu}^\nu = {}^{(\text{Is } 4)}N_{\lambda, \dot{\mu}}^{\dot{\nu}} + {}^{(\text{Is } 4)}N_{\lambda, \dot{\mu}}^{(41)\star \dot{\nu}} \quad (6)$$

for $(41) \star \dot{\nu} \neq \dot{\nu}$, while in the case of $(41) \star \dot{\nu} = \dot{\nu}$, i.e. $\nu \in \{\psi, \psi^+, \sigma, \sigma^+\}$, the prescription is more involved (see [11]); here on the right hand side $\dot{\lambda} = \lambda$ when \mathcal{H}_λ is **Vir**-irreducible, while otherwise $\dot{\lambda}$ stands for the label of any of the two **Vir**-irreducible subspaces of \mathcal{H}_λ . Now when expressed in terms of the fields of the tetracritical Ising model, the action of the implementing map C_Ω amounts in particular to mapping $\phi_{(41)}$ to $-\phi_{(41)}$. It follows that the structure constants of the algebra \mathfrak{D} are given by a formula analogous to (6):

$$({}^{\mathfrak{D}})N_{\lambda,\mu}^\nu = ({}^{\text{Is}4})N_{\dot{\lambda},\dot{\mu}}^{\dot{\nu}} - ({}^{\text{Is}4})N_{\dot{\lambda},\dot{\mu}}^{(41)\star\dot{\nu}}. \quad (7)$$

Inspecting the fusion rules $({}^{\text{Is}4})N_{\dot{\lambda},\dot{\mu}}^{\dot{\nu}}$, this relation can be seen to imply that \mathfrak{D} is isomorphic to the fusion rule algebra of the Lee–Yang non-unitary minimal model. In particular, \mathfrak{D} is two-dimensional; its natural basis corresponds to the fields ϕ_Ω and ϕ_ε , while its two one-dimensional irreducible representations are naturally labelled by the \mathbb{Z}_2 -orbits $\{(22), (32)\}$ and $\{(44), (42)\}$ – to which we will refer as ξ and η , respectively – of sectors of the tetracritical Ising model that do not appear in the three-state Potts model. (Note that according to (7) the sign of $({}^{\mathfrak{D}})N_{\lambda,\mu}^\nu$ depends on the choice of representatives $\dot{\lambda}$ of λ ; this does not constitute, however, any physical ambiguity, because these choices must be matched by analogous sign choices for the relevant Ishibashi [2] states, in such a way that the annulus partition function is non-negative.) It follows in particular that

$$R_{\mu,\mu;\Omega}^{(\text{id};\lambda)} = S_{\dot{\lambda},\dot{\mu}}^{(\text{Is}4)} / S_{\dot{\lambda},(11)}^{(\text{Is}4)} \quad (8)$$

with $\lambda \in \{\xi, \eta\}$ and $\mu \in \{\Omega, \varepsilon\}$. Explicitly, we have

$$\begin{aligned} R_{\Omega,\Omega;\Omega}^{(\text{id};\xi)} &= 1 = R_{\Omega,\Omega;\Omega}^{(\text{id};\eta)}, \\ R_{\varepsilon,\varepsilon;\Omega}^{(\text{id};\eta)} &= \frac{1}{2}(1 + \sqrt{5}) = -1/R_{\varepsilon,\varepsilon;\Omega}^{(\text{id};\xi)}. \end{aligned} \quad (9)$$

We can summarize: For the three-state Potts model there are eight distinct conformally invariant boundary conditions $A = (\omega; \lambda)$, six of them with $\omega = C$ and two with $\omega = \text{id}$. It is then straightforward to compute the partition function on an annulus of modular parameter t with boundary conditions A and B and expand it as

$$Z_{AB}(t) = \sum_{\lambda} Z_{AB}^{\lambda} \chi_{\lambda}(it/2) \quad (10)$$

with respect to the characters χ_{μ} . Inspection then allows for the following identification [2, 1] with boundary conditions in the spin chain formulation [12]. For $\omega = C$ the conditions labelled by elements of the \mathbb{Z}_3 -orbit $\{\Omega, \psi, \psi^+\}$ are the three possible *fixed* boundary conditions, while the other \mathbb{Z}_3 -orbit $\{\varepsilon, \sigma, \sigma^+\}$ corresponds to *mixed* boundary conditions where two out of the three possible values of the spin variable are allowed; $\omega = \text{id}$ with $\lambda = \eta$ yields *free* boundary conditions. Finally, the boundary condition $\omega = \text{id}$ with $\lambda = \xi$, which was recently discovered in [1], does not possess an obvious interpretation in terms of the spin variable of the lattice model.

It is not difficult to verify that the boundary conditions obtained above obey the usual consistency conditions. First, as already observed in [2, 1], the numbers Z_{AB}^μ defined by (10) are non-negative integers. Furthermore, they obey the associativity relation

$$\sum_{\mu} Z_{AB}^{\mu} Z_{CD}^{\mu+} = \sum_{\mu} Z_{AC^+}^{\mu} Z_{B^+D}^{\mu+}. \quad (11)$$

And finally, as matrices in the labels A, B they satisfy

$$A^\mu A^\nu = \sum_{\lambda} M^{\mu,\nu}_{\lambda} A^\lambda; \quad (12)$$

the number $M^{\mu,\nu}_{\lambda}$ equals the fusion coefficient of the three-state Potts model when all the labels μ, ν, λ refer to sectors that are present in the model, while it is a linear combination of fusion coefficients of the tetracritical Ising model when precisely two out of these labels refer to the \mathbb{Z}_2 -orbits ξ or η , and is zero otherwise.

Let us now look at these results for the Potts model from the perspective of general conformal field theory. As a matter of fact, the situation encountered above is but a special case of the following general setup. One deals with a ‘D-type’ model which is obtained from an associated ‘A-type’ model – another conformal field theory with the same value of c – by extending the chiral algebra by a primary field ϕ_J that has the simple (A-model) fusion rules $J \star J = \Omega^{(A)}$. (Such a field is called [11] an integer spin simple current; in the Potts case above it is the spin-3 current of \mathcal{W}_3 .) Thus the vacuum sector of the D-model decomposes as $\mathcal{H}_\Omega = \mathcal{H}_{\Omega^{(A)}} \oplus \mathcal{H}_J$ into irreducible modules of the non-extended chiral algebra. Moreover, one can distinguish two types of sectors \mathcal{H}_λ : those which are reducible as modules over the chiral algebra of the A-model and those which are irreducible; we refer to them as A-reducible and A-irreducible sectors, respectively. For A-reducible sectors one has in fact $\mathcal{H}_\lambda = \mathcal{H}_{\dot{\lambda}} \oplus \mathcal{H}_{J \star \dot{\lambda}}$ with $J \star \dot{\lambda} \neq \dot{\lambda}$, while A-irreducible sectors appear only for $J \star \dot{\lambda} = \dot{\lambda}$ and always come in pairs, so we denote them by \mathcal{H}_λ^\pm (the sectors \mathcal{H}_λ^+ and \mathcal{H}_λ^- are isomorphic as representation spaces of the chiral algebra for the A-model, but not of the one for the D-model).

It is also known [13] that in such D-models there is always a non-trivial fusion rule automorphism ω° preserving conformal weights. On A-reducible sectors, in particular on the vacuum sector, this map ω° can be implemented uniquely (up to over-all scalar factors) as

$$\omega_\lambda^\circ = \text{id}_{\mathcal{H}_\lambda} \oplus -\text{id}_{\mathcal{H}_{J \star \lambda}}, \quad (13)$$

while the implementations on A-irreducible sectors read

$$\omega_\lambda^{\circ\pm} : \mathcal{H}_\lambda^\pm \rightarrow \mathcal{H}_\lambda^\mp, \quad (14)$$

with highest weight states mapped to highest weight states. Often ω° will be the only non-trivial implementable fusion rule automorphism (and thus coincide with charge conjugation when C is non-trivial). In the sequel we restrict our attention to models where this is the case.

We denote the fusion rule automorphism that describes the torus partition function by π . As in any conformal field theory, for $\omega = \pi$ the boundary conditions are governed by the fusion rule algebra \mathfrak{N} of the D-model. We find that for $\omega = \pi \circ \omega^\circ$ the role of \mathfrak{N} is taken over by a commutative algebra \mathfrak{D} with the following properties. The dimension of \mathfrak{D} equals the number $N_{A \rightarrow D, \text{red.}}$ of A-reducible sectors, and there is a basis of \mathfrak{D} labelled by those sectors; in this basis the structure constants read

$${}^{(\mathfrak{D})}N_{\lambda, \mu}{}^\nu = {}^{(A)}N_{\lambda, \dot{\mu}}{}^{\dot{\nu}} - {}^{(A)}N_{\lambda, \dot{\mu}}{}^{J \star \dot{\nu}}. \quad (15)$$

Finally, the inequivalent one-dimensional irreducible representations of \mathfrak{D} are labelled by the ($N_{A \neq D}$ many) \mathbb{Z}_2 -orbits $[\lambda] = \{\dot{\lambda}, J \star \dot{\lambda}\}$ of the A-model that do not appear in the spectrum of the D-model; they are given by

$$R_{\mu, \mu; \Omega}^{(\pi \circ \omega^\circ; \lambda)} = S_{\lambda, \dot{\mu}}^{(A)} / S_{\lambda, \dot{\Omega}}^{(A)}. \quad (16)$$

We can also show that \mathfrak{D} is associative and (with an appropriate correlated choice of the signs in (15) for conjugate sectors) has a conjugation such that ${}^{(\mathfrak{D})}N_{\lambda, \mu}{}^\Omega = \delta_{\lambda, \mu+}$. This implies that \mathfrak{D} is semisimple and hence all its irreducible representations are one-dimensional. It follows e.g. that the number of inequivalent irreducible representations of \mathfrak{D} equals its dimension, i.e.

$$N_{A \neq D} = N_{A \rightarrow D, \text{red.}}. \quad (17)$$

Using identities among S-matrix elements such as those employed in [9], by direct computation one can again check that these boundary conditions satisfy the usual consistency relations. That is, the annulus coefficients Z_{AB}^μ defined as in (10) are non-negative integers and they satisfy the relations (11) and (12), with the coefficients $M_{\lambda}^{\mu, \nu}$ in (12) of a form completely analogous to the case of the three-state Potts model.

Besides the three-state Potts model, other examples of this structure are given by the D_{even} -type $\mathfrak{su}(2)$ WZW theories and by free bosons; in the latter case the A-model is the corresponding \mathbb{Z}_2 -orbifold. As an illustration, we note that the distinction between $\omega = C$ and $\omega = \text{id}$ is a direct generalization of the situation in the conformal field theory of a free boson X . In that case the role of $\mathcal{H}_{(41)}$ is taken over by the $\mathfrak{u}(1)$ -current $j = i\partial X$, so that in particular $\omega = C$ which changes the sign of j corresponds to Neumann, while $\omega = \text{id}$ corresponds to Dirichlet boundary conditions. (This applies for the case of the charge conjugation torus partition function $\pi = C$, while for the diagonal torus partition function $\pi = \text{id}$ it is the other way round.)

Similar constructions for the case of several free bosons X^i classify the boundary conditions that correspond to D-branes in string theory. It is natural to use the same nomenclature, i.e. ‘Neumann automorphism type’ for $\omega = \pi$ and ‘Dirichlet automorphism type’ for $\omega = \pi \circ \omega^\circ$, also for any other model of the type described above. Thus in particular in the three-state Potts model the fixed and mixed boundary conditions are analogues of Neumann conditions, while the free boundary condition and the new one of [1] are analogues of Dirichlet conditions. Note that T -duality amounts to exchanging the meaning of Neumann and Dirichlet conditions. We see that T -duality does not act one-to-one between boundary conditions, but rather between suitable orbits of them; e.g. in the three-state Potts model it maps fixed to free conditions and

mixed conditions to the new one of [1], and vice versa. We expect that these orbits come from Galois transformations [14] of S-matrix elements.

When the free boson X is compactified on a circle of rational radius squared, then the Dirichlet-type algebra \mathfrak{D} is two-dimensional and is isomorphic to the \mathbb{Z}_2 fusion rules, while for the non-diagonal $\mathfrak{su}(2)$ WZW theory at level 4ℓ the algebra \mathfrak{D} has dimension ℓ and turns out to be isomorphic to the fusion rule algebra of the non-unitary minimal model of type $(2\ell+1, 2)$.

Another class of such D-models is given by the unitary minimal models of conformal central charge $c = 1 - 6/(m+1)(m+2)$ with $m = 4\ell$ for some $\ell \in \mathbb{Z}_{>0}$, with the chiral algebra extended by the field $\phi_J \equiv \phi_{(m1)}$ of conformal weight $\Delta_J = m(m-1)/4$. (The similar series with $m = 4\ell+1$ can be treated analogously.) These have a total of $2\ell(\ell+2)$ sectors, among them $2\ell^2$ A-reducible modules and 2ℓ pairs of A-irreducible ones. Inspecting the fusion rules of these models, we find that just like in the three-state Potts model (which is obtained for $\ell = 1$), for any ℓ there are only two possible fusion rule automorphisms, namely the identity and charge conjugation. Both of them preserve conformal weights, and up to scalar factors they can be implemented uniquely on the modules \mathcal{H}_λ , in the way described in (13) and (14).

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