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Neutrino transport: no asymmetry in equilibrium

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Abstract

A small asymmetry in the flux of neutrinos emitted by a hot newly-born neutron star could explain the observed motions of pulsars. However, even in the presence of parity-violating processes with anisotropic scattering amplitudes, no asymmetry is generated in thermal equilibrium. We explain why this no-go theorem stymies some of the proposed explanations for the pulsar "kick" velocities.

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The observed rapid motions [1] of magnetized, rotating neutron stars, pulsars, can have a natural explanation if the neutrino emission from a cooling newly-born neutron star exhibits a small, ≈1% asymmetry. One necessary condition for such an asymmetry – a preferred direction – is, clearly, satisfied. A strong magnetic field of the neutron star breaks the spherical symmetry and can, at least in principle, be responsible for some anisotropy in neutrino emission.

One might think that, since the production and scattering of neutrinos in dense, hot nuclear matter is affected by the magnetic field $\mathbf{B}$, some parity-violating processes could produce an asymmetric flux of neutrinos in thermal equilibrium. This conclusion, however, is erroneous.

In fact, no asymmetry can build up in thermal equilibrium even if the scattering probabilities are anisotropic. This was pointed out by one of the authors in Ref. [2]. The argument is similar to that of Weinberg [3] with respect to the baryon asymmetry of the Universe. Our purpose here is to reiterate the discussion of Ref. [2] and to elaborate on this general argument in application to the neutrino emission by a cooling neutron star. This is particularly timely because several recent papers [4, 5] have reached an erroneous conclusion with respect to the size of the recoil velocity the pulsars can receive from neutrino emission.

The Boltzman equation for neutrinos ν scattering off neutrons n (it is straightforward to include the electrons, which we omit for simplicity) can be written as [6]

$$\begin{aligned} q_0 \frac{\partial}{\partial t} f^{(\nu)}(\mathbf{q},t) &= \sum_{\mathbf{s},\mathbf{s}'} \int \frac{d^3\mathbf{p}}{p_0} \frac{d^3\mathbf{p}'}{p_0'} \frac{d^3\mathbf{q}'}{q_0'} \, [f^{(\nu)}(\mathbf{q}',t)\, f^{(n)}_{\mathbf{s}'}(\mathbf{p}',t)\, W(\mathbf{p}',\mathbf{s}',\mathbf{q}'|\mathbf{p},\mathbf{s},\mathbf{q}) \\ & \quad - f^{(\nu)}(\mathbf{q},t)\, f^{(n)}_{\mathbf{s}}(\mathbf{p},t)\, W(\mathbf{p},\mathbf{s},\mathbf{q}|\mathbf{p}',\mathbf{s}',\mathbf{q}')]. \end{aligned} \tag{1}$$

Here $f^{(\nu)}(\mathbf{q})$ and $f^{(n)}(\mathbf{p})$ are the neutrino and neutron distribution functions, respectively; $\mathbf{s}$ denotes a neutron spin; and $W(\mathbf{p}',\mathbf{s}',\mathbf{q}'|\mathbf{p},\mathbf{s},\mathbf{q})$ is the probability of scattering $|n(\mathbf{p},\mathbf{s})\nu(\mathbf{q})\rangle \to |n(\mathbf{p}',\mathbf{s}')\nu(\mathbf{q}')\rangle$ per unit time per unit phase-space volume. The states are normalized for the invariant phase-space volume $d^3\mathbf{p}/p_0$ that appears in the integral. Here we neglected the effects of fermion degeneracy, the inclusion of which is straightforward [3].

We will assume an arbitrary form for the scattering probability W, which may, in particular, be anisotropic. The essential property of W is unitarity,

$$\sum_{\mathbf{s}'} \int \frac{d^3\mathbf{p}'}{p_0'} \frac{d^3\mathbf{q}'}{q_0'} \, W(\mathbf{p}, \mathbf{s}, \mathbf{q}|\mathbf{p}', \mathbf{s}', \mathbf{q}') = \sum_{\mathbf{s}'} \int \frac{d^3\mathbf{p}'}{p_0'} \frac{d^3\mathbf{q}'}{q_0'} \, W(\mathbf{p}', \mathbf{s}', \mathbf{q}'|\mathbf{p}, \mathbf{s}, \mathbf{q}) = 1, \qquad (2)$$

which is merely a requirement that the probability is conserved: with probability one, every initial state scatters into one of the final states, and vice versa.

The unitarity of the scattering matrix, as expressed by equation (2), is sufficient to show that the isotropic time-independent equilibrium distributions, which depend only on energy, satisfy the Boltzman equation (1), regardless of any asymmetries in W. In statistical equilibrium, $f^{(\nu)}(\mathbf{q}) \propto \exp\{-q_0/T\}$ and $f_{\mathbf{s}}^{(n)}(\mathbf{p}) \propto \exp\{-E_{\mathbf{s}}(\mathbf{p})\}$, where $E_{\mathbf{s}}(\mathbf{p}) = \mathbf{p}^2/2m + g_n \mu_{{}_B} \mathbf{s}\cdot\mathbf{B}$. From energy conservation, $q_0 + E_{\mathbf{s}}(\mathbf{p}) = q_0' + E_{\mathbf{s}'}(\mathbf{p}')$, so that

$$f^{(\nu)}(q) f_{\mathbf{s}}^{(n)}(p) = f^{(\nu)}(q') f_{\mathbf{s}'}^{(n)}(p'). \qquad (3)$$

Relation (3) is quite general and can be regarded as an expression of detailed balance in statistical equilibrium [6].

One can now prove that a time-independent spherically-symmetric distribution function, which satisfies the condition of statistical equilibrium (3), is a solution of the Boltzman equation (1) for any form of W. Indeed, if $f^{(\nu)}$ is independent of time, the left-hand side of equation (1) vanishes. The right-hand side,

$$\sum_{\mathbf{s},\mathbf{s}'} \int \frac{d^3\mathbf{p}}{p_0} \frac{d^3\mathbf{p}'}{p_0'} \frac{d^3\mathbf{q}'}{q_0'} \, f^{(\nu)}(\mathbf{q}) \, f_{\mathbf{s}}^{(n)}(\mathbf{p}) \, [W(\mathbf{p}', \mathbf{s}', \mathbf{q}'|\mathbf{p}, \mathbf{s}, \mathbf{q}) - W(\mathbf{p}, \mathbf{s}, \mathbf{q}|\mathbf{p}', \mathbf{s}', \mathbf{q}')], \qquad (4)$$

also vanishes because

$$\sum_{\mathbf{s}'} \int \frac{d^3\mathbf{p}'}{p_0'} \frac{d^3\mathbf{q}'}{q_0'} \, [W(\mathbf{p}', \mathbf{s}', \mathbf{q}'|\mathbf{p}, \mathbf{s}, \mathbf{q}) - W(\mathbf{p}, \mathbf{s}, \mathbf{q}|\mathbf{p}', \mathbf{s}', \mathbf{q}')] = 0 \qquad (5)$$

by virtue of equation (2). So, if the neutrinos are thermally produced in equilibrium, no asymmetry is generated by an anisotropic scattering probability.

It is clear why the analysis of Ref. [4] is in contradiction with the no-go theorem we have proven. The expression for the scattering cross-section in equation (3) of Ref. [4]

explicitly violates unitarity because it depends asymmetrically on the scattering angle of the outgoing neutrino and has no dependency on the initial momentum of the incident neutrino. In our notation, this corresponds to $W \propto (q_0 + \langle \mathbf{s} \rangle \cdot \mathbf{q}')$, where $\langle \mathbf{s} \rangle$ is the average neutron polarization. Since this expression does not satisfy the constraints of unitarity, it can lead to a neutrino asymmetry and hence a pulsar kick. A similar mistake was made in Ref. [5], where it was also claimed that the asymmetry in the distribution of outgoing neutrinos is proportional to the optical depth of the neutron star, that is to the size of the region where neutrinos are in thermal equilibrium with nuclear matter. It should be clear from our discussion, as well as from the general principles of thermodynamics, that the size of a system in equilibrium does not affect the distribution functions.

Of course, a hot neutron star that emerges from a supernova explosion is not fully in equilibrium, and this causes some asymmetry in the flux of outgoing neutrinos [2]. The departure from thermal equilibrium is due to the variation of the macroscopic parameters, such as $\mathbf{B}$, temperature, and the matter density, which occur on some length scale L. Therefore, the asymmetry resulting from the non-equilibrium behavior is proportional to λ/L, where λ is the neutrino mean free path.

In addition, an asymmetry in neutrino emission can arise from some other interactions of neutrinos at or near the neutrinosphere, as suggested in Refs. [7]. Since at that point the neutrinos are free-streaming, they are already out of equilibrium and the above no-go theorem doesn't apply.

To summarize, any attempt to explain the pulsar kick velocities by a "cumulative" amplification of a small asymmetry in the course of numerous collisions [4, 5] is doomed to failure. Multiple collisions imply that the thermally-produced neutrinos continue to be in statistical equilibrium. Therefore, no asymmetry can build up even if the scattering probabilities are anisotropic.

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