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PM-98/1**LOWER BOUNDS ON CHARGED HIGGS BOSONS FROM LEP AND
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ABSTRACT

We point out that charged Higgs bosons can decay into final states different than $\tau^+\nu_\tau$ and $c\bar{s}$, even when they are light enough to be produced at LEP or at the Tevatron, through a t -quark decay. These additional decay modes are overlooked in ongoing searches and alter existing lower bounds on the mass of charged Higgs bosons present in supersymmetric and two Higgs doublets models.

The discovery of a charged Higgs boson would be an unambiguous signal of an extended Higgs sector and possibly of supersymmetry. In supersymmetric models, at least two Higgs doublets are needed to give mass to all fermions: one is coupled only to down-type quarks and leptons; the other, only to up-type quarks. A Two Higgs Doublet Model (2HDM) is said of Type II if the doublets are coupled as in supersymmetric models with minimal particle content. It is said of Type I if one Higgs doublet does not couple to fermions at all and the other couples as the Standard Model (SM) doublet.

After electroweak symmetry breaking, five physical states remain: two CP-even Higgs bosons h and H (with $m_h < m_H$), a CP-odd Higgs boson A , and two charged states H^\pm . The charged Higgs-fermions interaction, can then be comprehensively expressed as:

$$\mathcal{L} = \frac{g}{\sqrt{2}} \left\{ \left(\frac{m_{di}}{M_W} \right) X \bar{u}_{Lj} V_{ji} d_{Ri} + \left(\frac{m_{ui}}{M_W} \right) Y \bar{u}_{Ri} V_{ij} d_{Lj} + \left(\frac{m_{li}}{M_W} \right) Z \bar{\nu}_{Li} e_{Ri} \right\} H^+ + \text{h.c.}, \quad (1)$$

where V is the CKM matrix. The equality $X = Z = 1/Y = \tan\beta$, with $\tan\beta$ the ratio of the two vacuum expectation values, identifies Type II and supersymmetric models; $Y = -X = -Z = \cot\beta$, identifies Type I models.

Besides the mass of h , H , A , and H^\pm , two additional parameters are needed to describe the Higgs sector in 2HDMs of Type I and II: $\tan\beta$ and the mixing angle α . In supersymmetric models, the Higgs sector is more constrained and only two free parameters are needed at the tree-level, m_A and $\tan\beta$. Supersymmetry induces a relation between $\tan 2\beta$ and $\tan 2\alpha$ and the well known tree-level sum rule $m_{H^\pm}^2 = m_W^2 + m_A^2$, which is only mildly altered by one-loop corrections [1]. Together with the experimental lower bound on m_A , $m_A > 76.1$ GeV, for $\tan\beta > 1$ [2], this sum rule makes the supersymmetric charged Higgs a possible candidate for discovery at the Tevatron, but not at LEP II.

Strong constraints on charged Higgs bosons come from searches of processes where H^\pm is exchanged as a virtual particle. Among them, the measurement of the inclusive decay $\bar{B} \rightarrow X_s \gamma$ [3], excludes charged Higgs bosons in a 2HDM of Type II up to ~ 260 GeV [4, 5], but it is, in general, inconclusive for supersymmetric models [6] and 2HDMs of Type I [4, 5]. Other indirect bounds on the ratio $m_{H^\pm}/\tan\beta$ come from inclusive semileptonic b -quark decays $B \rightarrow D \tau \nu_\tau$, $m_{H^\pm} \gtrsim 2.2 \tan\beta$ GeV [7] and from τ -lepton decays, $m_{H^\pm} \gtrsim 1.5 \tan\beta$ GeV [8]. They apply to charged Higgs bosons of Type II in 2HDMs and supersymmetric models. In the former, however, they are non-competitive with the stronger lower bound due to the measurement of $\bar{B} \rightarrow X_s \gamma$; in the latter they are already saturated by the above sum rule and the lower bound on m_A . Constraints on the low $\tan\beta$ region and light H^\pm in Type I models come from the measurement of $Z \rightarrow b\bar{b}$ and $B^0-\bar{B}^0$ mixing (see discussion in [4]).

It is possible that the 2HDMs described above, are only “effective” models, i.e. the low-energy remnant of Multi-Higgs-Doublets models with the same number of degrees

of physical states non-decoupled at the electroweak scale. In this case, more freedom remains in the possible values that X , Y , and Z can acquire. For $X = -1/Y = -a$, with $a \geq 2$, for example, a charged Higgs with $m_{H^\pm} = 100$ GeV can escape the $\bar{B} \rightarrow X_s \gamma$ constraint [5] while having widths for decays into light fermions substantially coinciding with those obtained in a 2HDM of Type II. Moreover, lepton and quark couplings in (1) may be unrelated, thus rendering the indirect bounds from b -quark and τ -lepton decays independent of that due to $\bar{B} \rightarrow X_s \gamma$. Indirect and direct bounds are, therefore, all equally necessary in providing the complementarity that allows to exclude certain ranges of m_{H^\pm} in supersymmetric models, in Type I and Type II 2HDMs, and in those models which may counterfeit them in one specific search.

Charged Higgs bosons are searched at LEP II, above the LEP I limit, in the range $45 \lesssim m_{H^\pm} \lesssim 100$ GeV and at the Tevatron in the range $m_{H^\pm} < m_t - m_b$, i.e. when produced by a decaying t -quark. Searches at LEP II rely on the assumption that no H^+ decay mode, other than $c\bar{s}$ and $\tau^+\nu_\tau$, is kinematically significant and give a limit $m_{H^\pm} \gtrsim 55$ GeV [9] which applies to 2HDMs of Type II and of Type I. Indeed, within the assumption $BR(H^+ \rightarrow c\bar{s}, \tau^+\nu_\tau) \simeq 100\%$, in Type I models the two branching ratios are $\tan\beta$ -independent and approximately equal to those obtained in Type II models with $\tan\beta = 1$. At the Tevatron, searches of an excess of $t\bar{t}$ events in the τ channel provide a $\tan\beta$ - m_{H^\pm} exclusion contour that constrains the very large $\tan\beta$ region in supersymmetric models and 2HDMs of Type II [10], for which the rate of $t \rightarrow H^+ b$ is large. Similarly large is this rate in the region of low $\tan\beta$ ($\tan\beta \lesssim 1$), for Type II yukawa couplings. Searches of H^+ apply in this region to the non-supersymmetric case. They are carried out, specifically for this type of couplings, looking for *i*) a deficit in the e, μ channels, due to $H^+ \rightarrow c\bar{s}$, for $m_{H^\pm} \lesssim 130$ GeV, *ii*) a larger number of taggable b -quarks due to $H^+ \rightarrow t^* b \rightarrow \bar{b} b W$ for $m_{H^\pm} \gtrsim 130$ GeV [11, 12]. Given the limited luminosity at present available at the Tevatron (~ 1 fb $^{-1}$), there is no sensitivity to the intermediate range of $\tan\beta$ where the rate $t \rightarrow H^+ b$ becomes low. This region, partially accessible at the upgraded Tevatron, will be fully covered at the LHC [13]

Aim of this Letter is to show that there exist additional decay modes which are overlooked in ongoing searches of H^\pm within 2HDMs and supersymmetric models, and which alter the existing lower bounds on m_{H^\pm} . In the following, the type of weak scale supersymmetry considered has minimal particle content and R-parity conservation. No specific assumption is made on the superpartner spectrum and on the scale/type of messengers for supersymmetry breaking. All branching ratios presented for supersymmetric models are calculated using HDECAY [14].

In 2HDMs, these modes are $H^+ \rightarrow AW^+$ and/or hW^+ (HW^+). They produce mainly the same final state $\bar{b}bW^+$, than the above mentioned $\bar{b}t^*$ mode, and to a lesser extent the state $\tau^+\tau^-W^+$. Our statement is based on the fact that there is no lower bound on m_A and/or m_h coming from LEP [15]. Indeed, since the mixing angle α is, in this case, a free parameter, one can think of a scenario in which the coupling ZhA vanishes. Being this proportional to $\cos(\beta - \alpha)$, the required direction is $\alpha = \beta \pm \pi/2$.

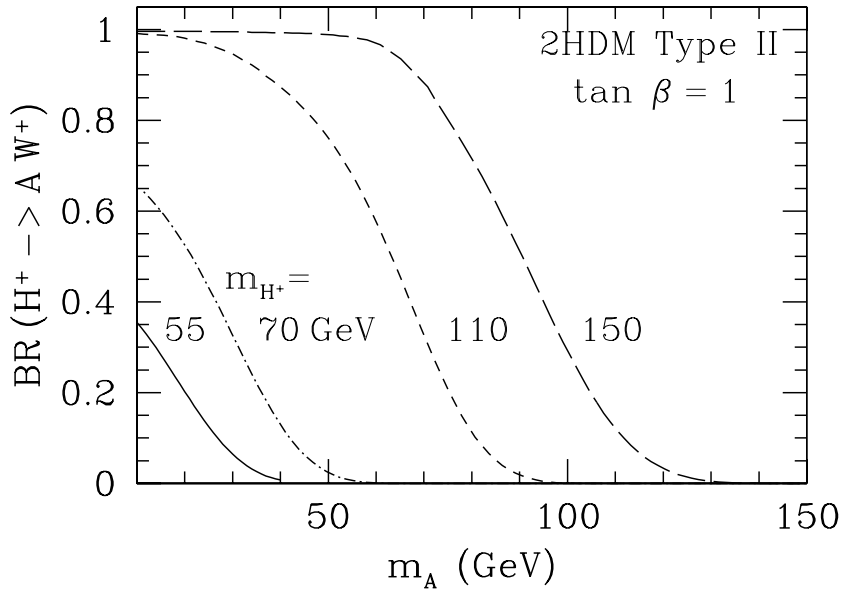


Figure 1: Branching fractions for the decay $H^\pm \rightarrow AW^*$ as a function of m_A for three values of $m_{H^\pm} = 70, 110$ and 150 GeV and $\tan\beta = 1$.

In this case, the process $Z^* \rightarrow hA$ does not occur and the LEP II bound $m_A > 75$ GeV obtained for supersymmetric models does not hold. Nevertheless, the cross section for the process $e^+e^- \rightarrow Z^* \rightarrow hZ$, proportional to $\sin^2(\beta - \alpha)$, is not suppressed compared to that for the corresponding production mechanism of the SM Higgs boson, and the LEP II bound $m_h > 87.9$ GeV [2] applies to our case. Full strength has also the coupling ZHA , still proportional to $\sin(\beta - \alpha)$, whereas HZZ vanishes. The process $Z^* \rightarrow HA$ could in principle provide a bound on m_A depending on m_H and $\tan\beta$. For large m_H , however, no real lower bound can be imposed on m_A . Conversely, even without making specific choices on the angle α , one can assume h to be heavy enough to render impossible any significant lower bound on m_A . The other two production mechanisms possible at LEP I (they require larger numbers of events than LEP II can provide) are the decay $Z \rightarrow A\gamma$ and the radiation out of $b\bar{b}$ and $\tau^+\tau^-$ pairs [16]. The first is mediated only by fermion loops, unlike the decay $Z \rightarrow h\gamma$ which has additional contributions from W -boson loops. The corresponding rate is about two orders of magnitude smaller than that for $Z \rightarrow h\gamma$ and therefore too small to allow for a visible signal [17]. The second process allows for sizable rates only for very large values of $\tan\beta$. No bound can be obtained for non-extreme values of $\tan\beta$ and for 2HDMs of Type I. In general, therefore, one remains with the rather modest bound from the decay $\Upsilon \rightarrow A\gamma$ which has been searched for by the Crystal Ball Collaboration [18], $m_A > 5$ GeV.

If one recalls that the interaction term H^+W^-A is weighted by a gauge coupling, unsuppressed by any projection factor, it is clear that the decay $H^+ \rightarrow AW^+$ can be

rather important for Type I models, or for models of Type II with small $\tan\beta$. This remains true even for an off-shell W -boson, in spite of the additional propagator and weak coupling which are then required. For a 2HDM of Type II with $\tan\beta = 1$, the branching ratio $BR(H^+ \rightarrow AW^+)$ is shown in Fig. 1 as a function of m_A for different values of m_{H^\pm} . Already for $m_{H^\pm} = 55$ GeV, roughly the lower bound obtained at LEP II when $BR(H^+ \rightarrow c\bar{s}, \tau^+\nu_\tau) \simeq 100\%$ is assumed, the branching ratio is 20%–30% for $m_A = 20$ –10 GeV. More strikingly, for heavier H^\pm , when the W -boson is not too far from being on-shell, this decay mode becomes the dominant one. Since the two modes hW^+ and HW^+ are respectively forbidden by our choice of α and the requirement of a very heavy H , the other competing channels are $\tau^+\nu_\tau$, $c\bar{s}$ for m_{H^\pm} in the LEP II range, and $\tau^+\nu_\tau$, $c\bar{s}$, and $\bar{b}t^*$ in the Tevatron' searches. In Fig. 2, the final branching ratio $BR(H^+ \rightarrow \bar{b}bW^+)$ is shown as a function of m_{H^\pm} in a 2HDM of Type II, with our choice of α , for different values of $\tan\beta$ and of m_A . For the larger m_A , the mode AW^+ is forbidden. Indeed, above $m_{H^\pm} = 130$ GeV the mode $c\bar{s}$ is quickly taken over by $\bar{b}t^*$, with the same $\tan\beta$ dependence, but much larger Yukawa couplings which can compensate the virtuality of the t -quark. The deviations from this pattern become striking when the mode AW^+ starts being allowed.

The situation described here corresponds to a particular direction of parameter space. One could have similarly allowed decays into hW^+ and HW^+ . A search strategy based on tagging three b -quarks for each produced t -quark, would then sum over all these decays. The corresponding theoretical branching ratio, however, becomes a function of m_A , m_h , m_H and α , in addition to m_{H^\pm} and $\tan\beta$. Searches at LEP II and the Tevatron aimed at constraining 2HDMs of Type II in the low $\tan\beta$ regime and/or 2HDMs of Type I will have to be modified accordingly. Constraints in the region of very large $\tan\beta$ for Type II couplings, when only the mode $\tau^+\nu_\tau$ survives, remain unchanged.

In supersymmetric models, since m_A cannot be much smaller than m_{H^\pm} and the angle α is not an independent parameter, a non-trivial role is played only by hW^{+*} , once the lower bound $m_h > 72.1$ GeV is implemented [2] and all superpartners are too heavy to open new channels. As before, this type of decay has the largest value of branching fractions for low $\tan\beta$, *i.e.* $\tan\beta \gtrsim 1$. For $m_{H^\pm} = 150$ GeV and $\tan\beta = 2$, the branching ratios for $\tau^+\nu_\tau$, $\bar{b}bW^+$, and hW^{+*} are 32%, 14%, and 52%, respectively. For the same value of $\tan\beta$, when more phase space becomes available due to the increase of m_{H^\pm} , both modes $\bar{b}bW^+$ and hW^{+*} are enhanced against $\tau^+\nu_\tau$. Of the two, hW^{+*} has a more rapid take over, due to the slow growth of m_h . As in 2HDMs of Type II, also in this case $\tau^+\nu_\tau$ remains the dominant mode for very large $\tan\beta$.

In general, however, decays into the lightest chargino χ_1^+ and neutralino χ_1^0 as well as decays into sleptons are still allowed by present experimental data, and they dominate when they occur. (The importance of the channel $\chi_1^+\chi_1^0$ for a constrained minimal supersymmetric model was already discussed in [19].)

The latest lower bounds on χ_1^+ from LEP II, ~ 91 GeV, rely on the assumption of

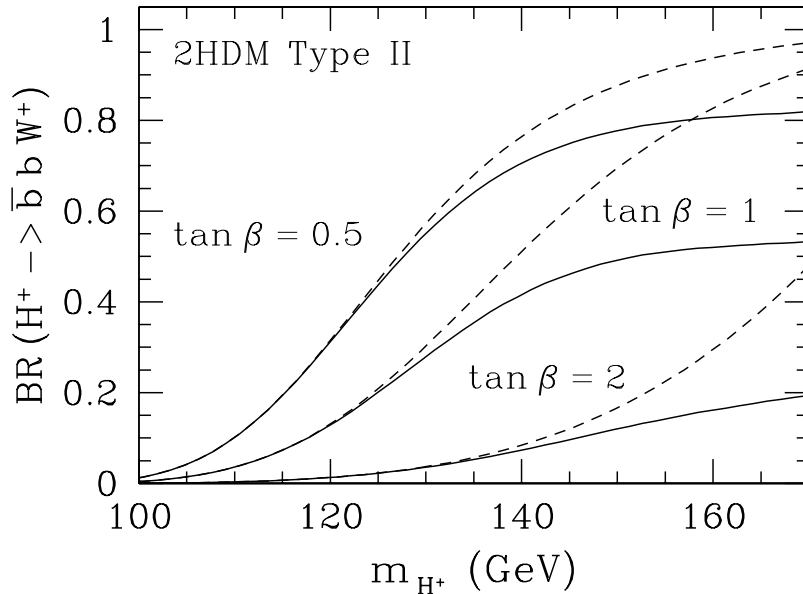


Figure 2: Branching fractions for the decay $H^\pm \rightarrow \bar{b}bW^+$ as a function of m_{H^\pm} for $m_A = 100$ (solid lines) and 200 GeV (dashed lines) and three different values of $\tan\beta$.

very heavy sleptons [2]. For large values of the Higgs–higgsino mass parameter μ , χ_1^+ and χ_1^0 are respectively wino– and bino–like with masses $\sim M_2$ and $\sim M_1$. In this case, even assuming gaugino mass universality at the messenger scale: $M_1 = \frac{5}{3}\text{tg}^2\theta_W M_2 \sim \frac{1}{2}M_2$, the decay channel $H^+ \rightarrow \chi_1^+\chi_1^0$ is possible for $m_{H^\pm} > 150$ GeV. It gives rise to jets or leptons and missing energy and to τ 's and missing energy. The branching ratio $BR(H^+ \rightarrow \chi_1^+\chi_1^0)$ is shown in Fig. 3 as a function of m_{H^+} , for $\tan\beta = 2$, $M_2 = 110$ GeV, $\mu = 500$ GeV, and all remaining sfermion masses at ~ 500 GeV (solid line). For these values of parameters, χ_1^+ and χ_1^0 have respectively masses of 96.5 and $\simeq 50$ GeV. The LEP II limits on χ_1^+ and χ_1^0 become weaker if the assumption on very heavy slepton masses and/or gaugino mass universality is relaxed. In both cases, the channel $\chi_1^+\chi_1^0$ becomes kinematically allowed for lighter H^\pm 's. As an example, we shown in Fig. 3 the branching ratio in a direction of supersymmetric parameter space with M_1 disentangled from M_2 (dashed line). While keeping all other parameters fixed to the previous values, M_1 is set to 30 GeV, which induces a mass for χ_1^0 of $\simeq 26$ GeV. The mode $\chi_1^+\chi_1^0$ opens now already at ~ 120 GeV. Fig. 3 shows clearly that, in the region of small $\tan\beta$, if no other decay of H^+ into superpartners is possible, the mode $\chi_1^+\chi_1^0$ dominates whenever kinematically allowed. For $m_{H^\pm} = 150$ GeV and $\tan\beta = 2$, the contribution of $\tau^+\nu_\tau$, hW^{+*} , and $\chi_1^+\chi_1^0$ to the H^\pm 's width, indeed, is respectively 21%, 14%, 60% for $M_1 = 55$ GeV and 10%, 6%, 80% for $M_1 = 30$ GeV. The $\bar{b}bW^+$ mode has, in both cases, a branching ratio below 5%. An increase of $\tan\beta$ reduces the branching ratio $BR(H^+ \rightarrow \chi_1^+\chi_1^0)$. For $\tan\beta = 10$, this ratio is $\sim 10\%$ for $m_{H^\pm} = 160$ GeV in the scenario with $M_2 = 55$ GeV, $\sim 20\%$ for

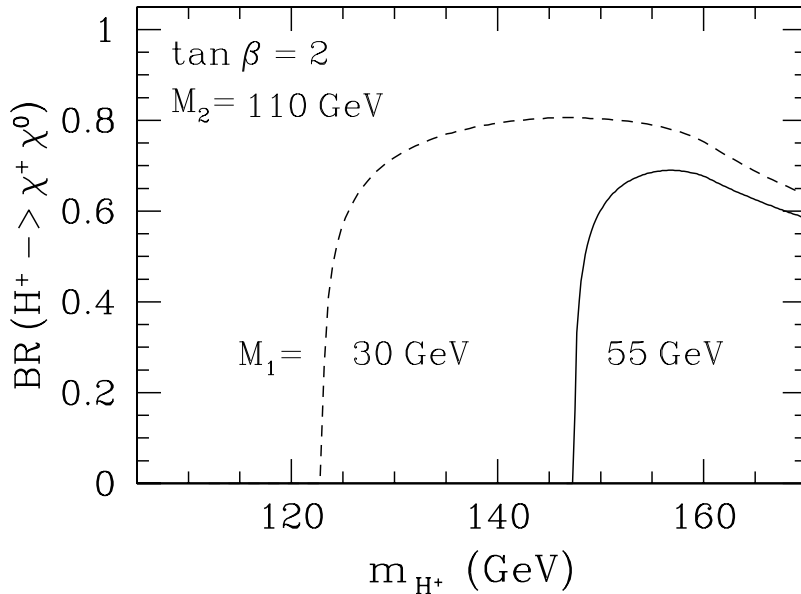


Figure 3: Branching fractions for the decay $H^+ \rightarrow \chi_1^+ \chi_1^0$ as a function of m_{H^+} , for $\tan\beta = 2$, $M_2 = 110$ GeV and two different values of M_1 : $M_1 = 55, 30$ GeV. All other supersymmetric decay modes are kinematically forbidden.

$m_{H^\pm} = 150$ GeV in the case of non-universal gaugino masses, $M_1 = 30$ GeV.

The existing lower bounds on slepton masses from LEP II, are respectively 81, 71, and 65 GeV for \tilde{e} , $\tilde{\mu}$, $\tilde{\tau}$ and $\gtrsim 45$ GeV for sneutrino masses. Hence, the decay $H^+ \rightarrow \tilde{\tau}^+ \tilde{\nu}_\tau$ is therefore kinematically allowed and produces a final τ^+ + missing energy, but with a softer τ^+ than that coming from the direct decay $H^+ \rightarrow \tau^+ \nu_\tau$. We show in Fig. 4 the relative branching ratio for $\tan\beta = 2$, $M_2 = 120$ GeV, $M_1 \sim M_2/2$ and two choices of parameters in the slepton mass matrices: a) $m_{\tilde{l}_L} = m_{\tilde{l}_R} = m_{\tilde{l}} = 75$ GeV, $\mu = 500$ GeV, and $A_\tau = 0$; b) $m_{\tilde{l}} = 90$ GeV, $\mu = -500$ GeV, and $A_\tau = 2$ TeV. (The trilinear soft terms are here assumed to have couplings proportional to the corresponding Yukawa couplings and, therefore, the left-right entries in the slepton mass matrix is still very tiny.) The slepton spectrum is as follows: $m_{\tilde{\nu}} = 56$ GeV, $m_{\tilde{e}} \sim m_{\tilde{\mu}} = 83$ GeV and the two $\tilde{\tau}$ masses are 10 GeV below and above this value, in the first case; $m_{\tilde{\nu}} = 75$ GeV, $m_{\tilde{e}} \sim m_{\tilde{\mu}} = 97$ GeV and $m_{\tilde{\tau}_1} = 63$ GeV, and $m_{\tilde{\tau}_2} = 121$ GeV, in the second. Below thresholds, hW^+ and $\tau^+ \nu_\tau$ account respectively for 15–20% and $\sim 75\%$ of the total width of H^\pm ; the remaining few % are due to AW^+ and $c\bar{s}$. The prominence of $\tilde{\tau}^+ \tilde{\nu}_\tau$ observed above threshold is explained by the coupling of the lagrangian term $H^+ \tilde{\nu}_L^* \tilde{l}_L$, $-(g/\sqrt{2})M_W \sin 2\beta$, very large when compared to the Yukawa coupling $-(g/\sqrt{2})(m_\tau/M_W)$. Due to the $\sin 2\beta$ dependence, this term quickly dies off for increasing $\tan\beta$. In this case, however, there exists other directions of parameter space where this decay mode has still a branching ratio $\sim 100\%$. When $A_\tau \sim \mu \tan\beta$, in fact, the left-right mixing in the slepton mass matrix tends to

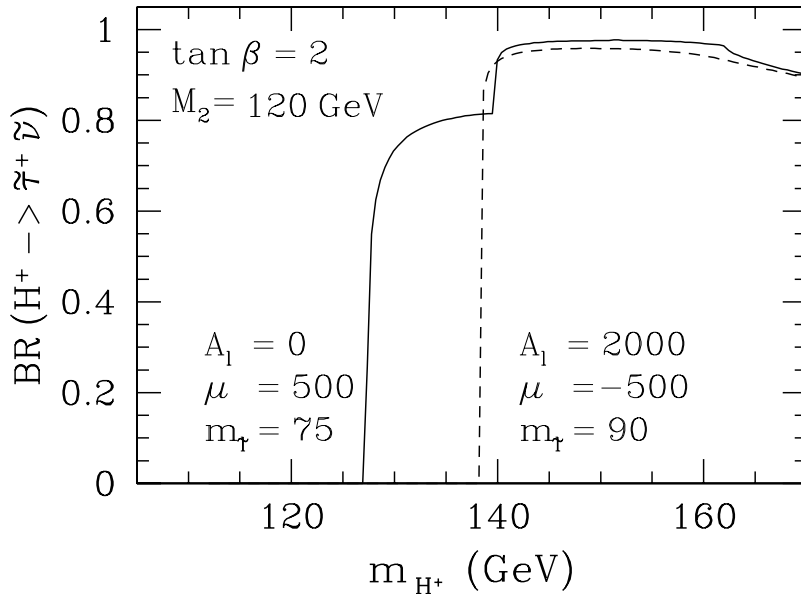


Figure 4: Branching fractions for the decay $H^+ \rightarrow \tilde{\tau}^+ \tilde{\nu}_\tau$ as a function of m_{H^+} , for $\tan\beta = 2$, $M_2 = 120$ GeV, $M_1 = 60$ GeV and two different sets of supersymmetric masses.

vanish, but the coupling of the lagrangian term $H^+ \tilde{\nu}_L^* \tilde{\tau}_R$: $-(g/\sqrt{2})(m_\tau/M_W)(\mu + A_\tau \tan\beta)$ acquires a $1/\cos^2\beta$ dependence, which increases with increasing $\tan\beta$. For $\tan\beta = 20$, for example, the parameters $A_\tau = 2$ TeV, $\mu \sim 200$ GeV, and $m_{\tilde{t}} \sim 90$ GeV, give three sneutrinos with mass ~ 63 GeV, three charged sleptons practically left handed with mass ~ 102 GeV and three mainly right handed at 99.8 GeV. The branching ratio for the channel $\tilde{\tau}^+ \tilde{\nu}_\tau$ is in this case already above 60% for $m_{H^\pm} = 163$ GeV and increases very rapidly for heavier H^\pm . It should be noticed that the spectrum produced by this choice of parameters will survive negative searches at LEP II with a center of mass energy of 200 GeV.

Summarizing, at very large $\tan\beta$, possible excess of τ 's softer than those predicted by a 2HDM of Type II may signal the presence of a heavier H^\pm decaying into $\tilde{\tau}^+ \tilde{\nu}_\tau$. Searches in the region of $\tan\beta \gtrsim 1$, should already consider multi- b signals coming from hW^{+*} , $\bar{b}bW^+$ as well as τ -signals with a wide momentum distribution coming from $\chi_1^+ \chi_1^0$, $\tilde{\tau}^+ \tilde{\nu}_\tau$, and $\tau^+ \nu_\tau$ and jets/leptons +missing energy signals from $\chi_1^+ \chi_1^0$. It is needless to say that all these modes will play an important role in future searches not blind to the intermediate range of $\tan\beta$.

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