Transition Radiation of the Neutrino Toroid Dipole Moment

Elena N. Bukina[a], Vladimir M. Dubovik[a] and Valentin E. Kuznetsov[a,b]
[a]*Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia*
[b]*CERN, CH-1211, Geneva 23, Switzerland*

Abstract

We discuss the transition radiation of a neutrino induced by its toroid dipole moment (τ) when crossing the interface between two different media with different refraction indices. A neutrino of 1 MeV energy emits approximately 10^{-40} keV at $\tau = e\sqrt{2}G_F/\pi^2$. This effect depends very slightly on the neutrino mass and has a finite value in the massless limit.

I. INTRODUCTION

The electromagnetic properties of a spin-1/2 charged particle are described by four independent dipole moments while the neutrino properties by three moments: magnetic (μ), electric (d) and toroid (τ) dipole moments [1,2]. In the Standard Model (SM) they are induced by radiative corrections and have the following theoretical predictions for the electron neutrino [3–5] *

$$\begin{aligned}
\mu_{\nu_e} &= \frac{3eG_F m_{\nu_e}}{8\sqrt{2}\pi^2} = 3\times 10^{-19}\left(\frac{m_{\nu_e}}{1\ \text{eV}}\right)\mu_B,\\
d_{\nu_e} &= 0, \quad E(0) \propto \Delta m,\\
\tau_{\nu_e} &\approx e\frac{\sqrt{2}G_F}{\pi^2} = e\cdot 6.5\times 10^{-34}\ \text{cm}^2 = 8.5\times 10^{-13}\mu_B \lambda_e,
\end{aligned} \tag{1}$$

where G_F, E, μ_B and λ_e are the Fermi constant, the electric form factor of the neutrino, the Bohr magneton and the Compton wavelength of the electron, respectively. At the same time, the experimental bounds on these moments, which can be extracted in diverse ways [6], are really poor [7].

The magnetic and electric dipole moments of neutrinos are well known, but the third electromagnetic characteristic of a neutrino, the toroid dipole moment (TDM), is still under discussion in the literature, see for example [4,5,8] and the references therein. We know that the TDM is the electromagnetic characteristic which the Dirac and Majorana neutrinos posses in both the massive and massless limits. In the non-relativistic limit the interaction energy $\mathcal{H} = -\boldsymbol{\tau}\cdot\mathbf{J} = -\tau\varphi^\dagger\boldsymbol{\sigma}\varphi\left(\text{curl}\,\mathbf{B} - \dot{\mathbf{E}}\right)$ represents a T-invariant electromagnetic

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*The numerical value of τ_{ν_e} corresponds to toroid dipole moment of Majorana neutrino and is obtained summing over all contributions in eq. (16) of [5].

interaction of the particle induced by its TDM which does not conserve P- and C-parity individually. It is useful to remark also that in the massless limit, the electromagnetic properties of Dirac neutrinos are represented by the TDM and the neutrino charge radius which coincide numerically [9]. According to [4,5], the spatial size of the toroid dipole moment (TDM) is formed by the mass of the weak intermidiate boson M_W and does not depend on the inert mass of the particle under consideration. So in the physical hierarchy the TDM is closer to the electric charge than to the magnetic moment.

The permitted forms of coupling for the electromagnetic current $J_\mu^{\rm EM}$ are

$$\begin{aligned} J_\mu^{\rm EM}(q)_{\rm Dirac} &= [\overline{u}_f(\mathbf{p}')\Gamma_\mu(q)u_i(\mathbf{p})]\,, \\ J_\mu^{\rm EM}(q)_{\rm Majorana} &= J_\mu^{\rm EM}(q)_{\rm Dirac} + [\overline{v}_i(\mathbf{p})\Gamma_\mu(q)v_f(\mathbf{p}')]\,, \end{aligned} \tag{2}$$

where the matrix elements are taken between the Dirac or Majorana neutrino states with different masses. A Lorentz-covariant structure of the dressed vertex operator $\Gamma_\mu(q)$ in the toroid parametrization [1,8] is given by

$$\Gamma_\mu(q) = F(q^2)\gamma_\mu + M(q^2)\sigma_{\mu\nu}q^\nu + E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 + i\mathcal{T}(q^2)\epsilon_{\mu\nu\lambda\rho}P_\nu q_\lambda \gamma_\rho, \tag{3}$$

where F, M, E and $\mathcal{T}$ are the normal, anomalous magnetic, electric and toroid dipole form factors respectively, $P_\nu = p_\nu + p'_\nu$ and $\epsilon_{\mu\nu\lambda\rho}$ is the antisymmetric tensor. In the anapole parametrization [10] the vertex operator reads

$$\Gamma_\mu(q) = F(q^2)\gamma_\mu + M(q^2)\sigma_{\mu\nu}q^\nu + E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 + A(q^2)[q^2\gamma_\mu - \widehat{q}q_\mu]\gamma_5 \tag{4}$$

where $A(q^2)$ is the anapole form factor. Using the following identity

$$\begin{aligned} \overline{u}_f(\mathbf{p}')\Big\{\Delta m\sigma_{\mu\nu}q^\nu + \Big(q^2\gamma_\mu - \widehat{q}q_\mu\Big) \\ - i\varepsilon_{\mu\nu\lambda\rho}P^\nu q^\lambda\gamma^\rho\gamma_5\Big\}\gamma_5 u_i(\mathbf{p}) = 0, \end{aligned} \tag{5}$$

we see that the TDM and anapole coincide in the static limit when the initial and final masses of neutrinos are equal to each other [1,8].

It is easy to check, using CPT invariance of Γ_μ and C-, P- and T-properties of each contribution in eqs. (3, 4), that for the Majorana current only the toroid dipole form factor survives [2] and the value of the toroid dipole moment of the Dirac neutrino is just half of the Majorana one. For the above reasons we have not specified the nature of the neutrino and as TDM is a more simple (multipolar) characteristic than anapole, which has the composite structure as it follows from (5), we shall subsequently only use the term TDM. In addition, in the forthcoming calculations the numerical value of TDM from eq. (1) will be used [4,5].

If the toroid dipole moment is observable, what physical consequences does it lead to? Among the several possibilities are the Vavilov-Cherenkov and transition radiations (TR) of particles induced by their dipole moments. This problem for the Dirac neutrino with non-zero magnetic moment was considered in [11,12]. In 1985, Ginzburg and Tsytovich [13], using a classical approach, showed that the macroscopic toroid dipole moment moving in a medium generates Vavilov-Cherenkov and TR radiations as well. Here we present the first quantitative discussion of the transition radiation of a neutrino having non-zero TDM in the framework of quantum theory along the lines of [12].

II. CALCULATION OF TRANSITION RADIATION INTENSITY

Let us consider a neutrino with non-zero toroid dipole moment crossing the interface between two media, see Fig. 1, with refraction indices n_1 and n_2 ($n_1 \neq n_2$). The electromagnetic interactions of neutrinos is described by the Hamiltonian:

$$\mathcal{H}_{\rm int} = ie\mathcal{T}(q^2)\overline{\psi}(x)\varepsilon_{\mu\nu\lambda\rho}P^{\nu}q^{\lambda}\gamma^{\rho}\psi(x)\mathcal{A}^{\mu}(x),$$

using the identity $\varepsilon_{\mu\nu\lambda\rho}\gamma_{\rho} = \frac{i}{2}\left(\gamma_{\mu}\gamma_{\nu}\gamma_{\lambda} - \gamma_{\lambda}\gamma_{\nu}\gamma_{\mu}\right)\gamma_5$, we we obtain

$$\begin{aligned}\mathcal{H}_{\rm int} &\Rightarrow e\mathcal{T}(q^2)\overline{\psi}(x)\gamma_{\mu}\gamma_5\psi(x)\frac{\partial F^{\mu\nu}(x)}{\partial x^{\nu}} \\ &= e\mathcal{T}(q^2)\overline{\psi}(x)\gamma_{\mu}\gamma_5\psi(x)J^{\mu}_{\rm ext}. \end{aligned} \tag{6}$$

Here ψ, $\mathcal{T}(q^2)$, $J^{\mu}_{\rm ext}$, $\mathcal{A}^{\mu}$ and $F^{\mu\nu}$ are the neutrino wave function, neutrino toroid form factor, electromagnetic current, 4−potential and tensor of the electromagnetic field, respectively (the Hamiltonian (6) was also obtained by Zel'dovich [10] using the anapole parametrization).

The transition $\nu(p_1) \to \nu(p_2) + \gamma(k)$ becomes possible due to the TDM of the neutrino †. In a medium with refraction index n, the four-momentum vector of a photon is given by $k^{\mu} = (\omega, \mathbf{k})$, $|\mathbf{k}| = n\omega$ (ω is the energy of a photon), and the transition probability reads $\Gamma = |\mathcal{S}_{fi}|^2 \frac{Vd^3\mathbf{p}_2}{(2\pi)^3}\frac{Vd^3\mathbf{k}}{(2\pi)^3}$, where the transition matrix element is expressed as

$$\begin{aligned}|\mathcal{S}_{fi}|^2 &= (2\pi)^3\ell^2 t\frac{m_{\nu}}{E_1V}\frac{m_{\nu}}{E_2V}\frac{(1-n^2)^2\omega^4}{2\omega n^2V} \\ &\times \delta(p_{1x,y} - p_{2x,y} - k_{x,y})\delta(E_1 - E_2 - \omega) \\ &\times \left|\int_{-\ell/2}^{\ell/2} dz \exp[i(p_{1z} - p_{2z} - k_z)z]\mathcal{M}_{fi}\right|^2. \end{aligned} \tag{7}$$

Here $\mathcal{M}_{fi} = e\mathcal{T}(0)\overline{u}_2\hat{\varepsilon}\gamma_5 u_1$ is the amplitude, and t, ℓ and $V = \ell^3$ denote time, length and volume of the transition region, respectively, and $\ell = \beta t$, where $\beta = p/E$ is the velocity of the neutrino. The phase of the integrand in (7) defines the formation-zone length of the medium as

$$Z(n) = (p_{1z} - p_{2z} - k_z)^{-1} = (p_{1z} - p_{2z} - n\omega\cos\theta)^{-1},$$

where θ is the angle between the photon and the direction of the incident neutrino. The details of further calculations are the same as in [12], and here we present only final results for the energy intensity S per interface

$$\frac{d^2S}{d\theta d\omega} = \frac{\mathcal{T}^2(0)\omega^6\sin\theta}{8\pi^2}(R_1^2 - R_2^2)\left\{2\sin^2\theta\left(1 + \frac{n\omega\cos\theta}{p_{2z}}\right) + \frac{E_{\nu}E_2}{pp_{2z}} - 1 + \frac{m_{\nu}^2}{pp_{2z}}\right\}, \tag{8}$$

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†Recall that both the Vavilov-Cherenkov and transition radiations are not depend on the nature of a moving source [13]

where

$$R_i = \frac{1-n_i^2}{n_i} \frac{1}{p - p_{2z} - n_i \omega \cos\theta}, \quad S = \int_0^{E_\nu - m_\nu} d\omega \int_0^{\theta_{\max}} \frac{d^2 S}{d\theta d\omega}, \tag{9}$$

and

$$p_1^\mu = (E_\nu, 0, 0, p), \quad p_{2z} = \sqrt{E_2^2 - m_\nu^2 - n^2\omega^2 \sin^2\theta}, \quad E_2 = E_\nu - \omega.$$

Using the numerical value (1), $\tau_{\nu_e} = e\mathcal{T}(0)$ with $\mathcal{T}(0) = \sqrt{2}G_F/\pi^2$, and assuming that the refractive index can be expressed as $n_i(\omega) = 1 - \omega_i^2/2\omega^2$ for $\omega \gg \omega_i$ (ω_i is the plasma frequency) for a medium-vacuum transition ($\omega_2 = 0$, $R_2 = 0$), we present the energy spectrum and angular distribution in Figs. 2 and 3. The total energy loss of the neutrino has been computed numerically and is shown as a function of the neutrino mass for $E_\nu = 1$ MeV in Fig. 4. For $m_\nu < 10$ eV the TR energy is approximately constant and equals $S \simeq 2 \times 10^{-40}$ keV. Because of the finite value of the TDM for massless neutrinos [4,5], the TR does not vanish in this limit and has the value $S\Big|_{m_\nu=0} = 2.26 \times 10^{-40}$ keV.

In order to estimate the magnitude of this effect, let us consider a transition radiation detector (TRD) which can be used to measure experimentally such transition radiation of neutrinos. The TRD consists of sets (N_1) of "radiator" and xenon-gas chambers, where one radiator typically comprises of a few hundred layers (N_2) of a polypropylene film ($\omega_p = 20$ eV) and a gas ($\omega_p \ll 1$ eV). The total energy deposition (W) in the TRD is given by

$$W = S \cdot F_\nu \cdot A^2 \cdot T \cdot N_1 \cdot N_2,$$

where F_ν is the neutrino flux, A is the TRD area and T is the time of measurement. For example, we have taken the neutrino flux coming from a nuclear reactor, $F_\nu \sim 10^{13}\overline{\nu}_e/\text{cm}^2$ sec, with the energy of $E_\nu = 1$ MeV and TRD parameters as: $A = 10$ m^2 $N_1 = 10$ sets and $N_2 = 10^4$ layers. The total energy deposition for the TDM $\tau_{\nu_e} = e\sqrt{2}G_F/\pi^2$ with TR energy $S = 10^{-40}$ keV is

$$W = 3 \times 10^{-10} \left(\frac{T}{1 \text{ year}}\right).$$

Unfortunately, this value is extremely small and cannot be extracted from the background of an experimental setup. But this small radiation which always exist for both massive and massless neutrinos, may have interesting consequencies in astrophysics.

III. CONCLUSION

In summary, we have calculated the transition radiation of a neutrino induced by its toroid dipole moment in the framework of quantum theory. Since the TDM of a neutrino is nonzero in the massless limit [4,5], the corresponding TR energy is also nonzero and equals $S \simeq 2.26 \times 10^{-40}$ keV for $\tau_{\nu_e} = e\sqrt{2}G_F/\pi^2$. [‡] In addition the TDM is the weak-electromagnetic characteristic which both Dirac and Majorana neutrinos posses, therefore

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[‡]The transition radiation induced by the neutrino magnetic moment disappeares in the massless limit since $\mu_\nu \sim m_\nu$.

the transition radiation induced by TDM exist indepedently on the nature of the neutrino and its mass.

It is highly plausible that we will stand face-to-face with the dramatic circumstances if either the Dirac neutrinos possess negligible masses and their magnetic moments are also small, or if all neutrinos have a Majorana nature. Then the unique electromagnetic characteristic of such neutrinos will be the toroid moment and we will be forced to seek for some exotic effects generated by it. For instance, if the neutrino is a massless particle then measurement of the transition radiation can be used as a tool to distinguish the nature of the neutrino (since the Dirac TDM is half of the Majorana one and as the energy intensity is proportional to the square of the TDM (8), the TR of Dirac neutrino is 1/4 of the Majorana one).

It is interesting to note that TR energy of the order of 10^{-40} keV for a neutrino with TDM corresponds to the TR energy of a neutrino with anomalous magnetic moment $\mu_\nu \sim 10^{-15}\mu_B$ for $m_\nu = 1$ eV [12]. Such TR of neutrinos induced by their TDMs may have interesting implications for astrophysics as well as the early Universe. However, the conclusions about the magnitude of these effects requires further investigation.

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FIGURES

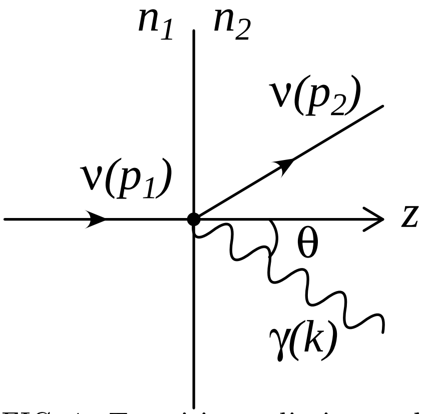

FIG. 1. Transition radiation at the interface of two media: $\nu(p_1) \to \nu(p_2) + \gamma(k)$. The refractive index changes from n_1 to n_2 at $z = 0$.

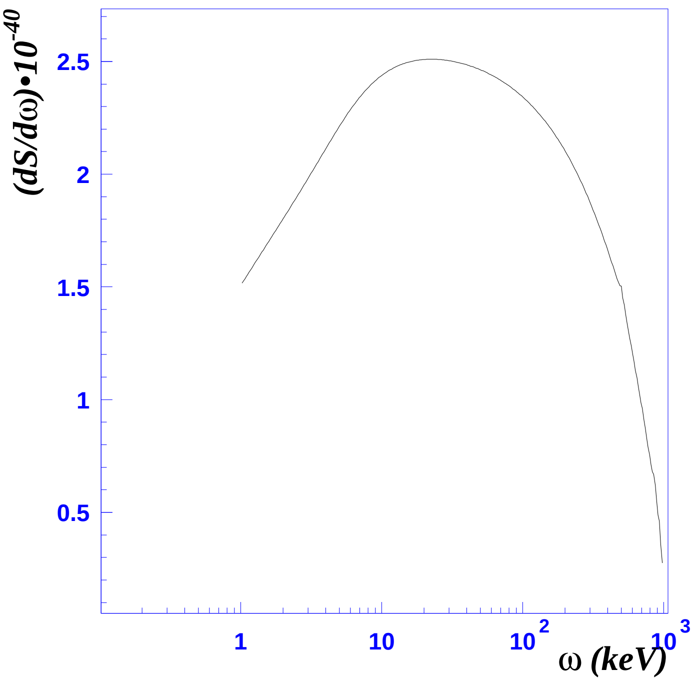

FIG. 2. Energy intensity distribution of the transition radiation of a toroid dipole moment of neutrino as a function of photon energy.

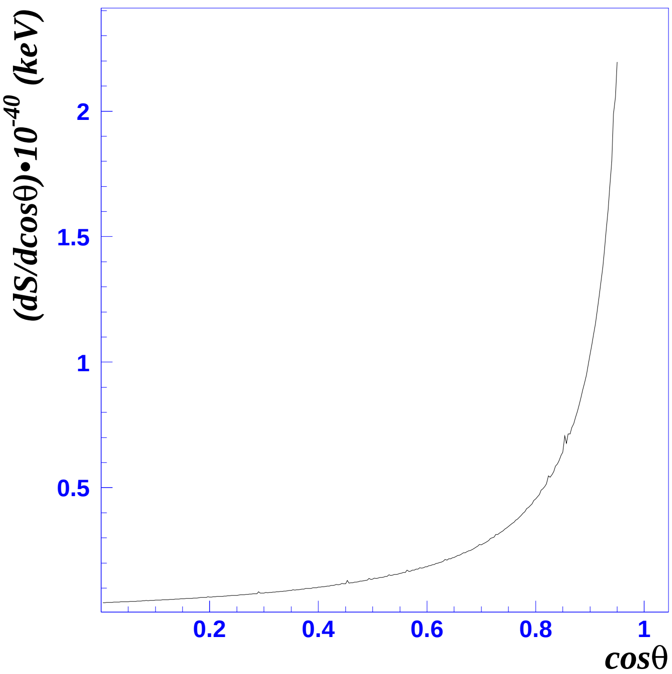

FIG. 3. Angular distribution of total TR energy as a function of $\cos\theta$.

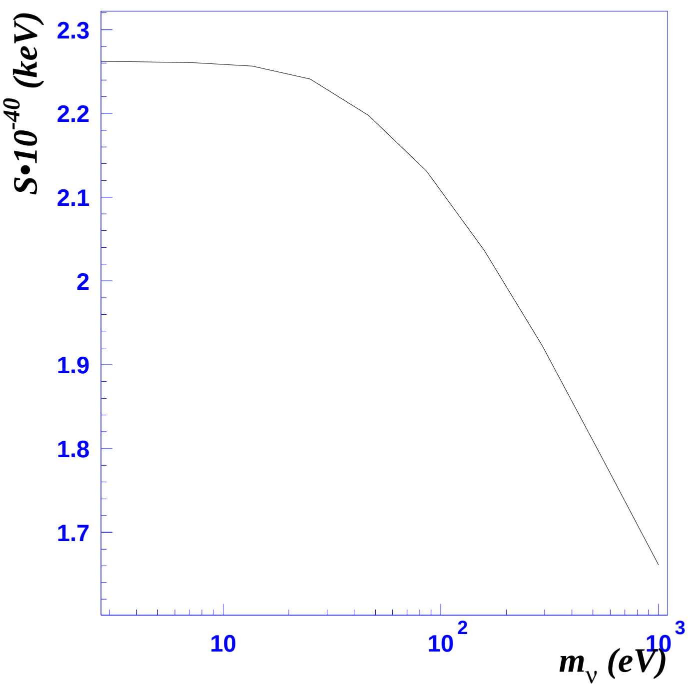

FIG. 4. Total energy in medium-vacuum transition, $n_2 = 0$ and $n_i(\omega) \simeq 1 - \omega_i^2/2\omega^2$ for $\omega \gg \omega_i$ and $E_\nu = 1$ MeV.