EUROPEAN LABORATORY FOR PARTICLE PHYSICS

CERN-EP/98-63
March 25, 1998

On the Detection of Gravitational Waves through their Interaction with Particles in Storage Rings

Daniel Zer-Zion
CERN, CH-1211 Geneve 23
Switzerland

Abstract

It is shown that the interaction between a gravitational wave and ultra-relativistic bunches of particles in storage rings can produce a measurable effect on the non-Euclidean geometry of the space-time manifold of high energy rotating particles. Such an interaction causes simultaneous correlated deflections of bunches at different locations in a collider beam around the storage ring.

The radial deflection of a bunch of particles in a beam caused by a gravitational wave perpendicular to the surface of the ring is predicted to have a frequency equal to twice the revolution frequency of the bunch, and be modulated by the frequency of the gravitational wave.

Using a system of beam position monitors (and possibly a streak camera), every bunch of particles can be monitored and its oscillations reconstructed so that a clear picture of the complete ring can be achieved at any moment.

If the storage ring has two counter-rotating beams, noise effects can be reduced by measuring the difference, at a given point all along the beam, of the relative bunch deflections at both rings. The amplitude and frequency of the gravitational wave (and polarisation, if any) can then be deduced.

Coincidence at different storage rings, with correlated radial deflection amplitudes and frequencies, are also expected. The position of the source can then be deduced.

For gravitational waves with frequencies of the order of 100-1000 Hz and amplitudes of the order of 10^{-20}-10^{-23} the amplitude of the radial deflection can be as large as a milimeter, depending on the quality factor as a gravitational wave antenna and the parameters of the collider.

(To be Submitted to Astroparticle Physics)

1 Introduction

General relativistic gravitational waves (GWs) are ripples in the curvature of space-time that propagate with the speed of light, carry energy and momentum and have well defined polarisation. When passing through a region of space-time a relative acceleration between free particles is induced by GWs (gravitational radiation is explained in detail in reference [1], pp.330-458). GWs are emitted by accelerating masses such as coherent motions of matter (e.g., binary stars systems, supernova explosions) or coherent vibrations of space-time curvature (e.g., black holes).

Measurement of the orbit parameters of the PSR 1913+16 pulsar binary system [2] over a multi-year period provides indirect evidence for the existence of GWs. The decrease in the orbital period of about 75 ms per year, comes about because the system is losing energy (about 10^{32} ergs/s). Since the observed decrease in the period closely matches the value predicted by general relativity, many astronomers regard these observations as being an important manifestation of the emission of GWs.

The direct observation of GWs will provide support for the verification of the existence of the graviton, as the massless spin-two carrier of gravitation. It will verify many predictions of Einstein's general theory of gravitation, e.g. that electromagnetic and GWs propagate with the same speed and the existence of black holes. Their discovery will also probably link many aspects of the Universe in the largest and smallest scales.

Storage rings are machines that can keep particle beams circulating for hours, constantly replacing the energy lost due to synchrotron radiation. They are used to collide one beam of particles with another and are sometimes refered to as colliders. The aim of this paper is to discuss the possibility of detecting GWs through their interaction with high energy rotating particles in storage rings. The effect under study can be considered as the time reversal, on a small scale, of the process which generates GWs.

The paper is organised as follows. In section 2 the geometrical properties of a collider beam as an ultra-relativistic ring are described; section 3 is devoted to the interaction of a GW with particles in rotating rings. In section 4 a description of the method to measure the GW phenomenon as seen by a collider beam is explained in detail, together with applications to present and future storage rings as GWs antennas (in the same section some of the possible noise sources are discussed). In section 5 some conclusions and perspectives are given.

2 Geometrical Properties of a Collider Beam

In order to describe the topological properties of the rotating ring [1] the relation between an inertial frame of reference with cylindrical polar coordinates [2] $\rho_o, \theta_o, z_o, t_o$, and a rotating frame of reference attached to the rotating ring, with coordinates ρ, θ, z, t, is given by

$$\begin{aligned} \rho_o &= \rho,\ z_o = z, \\ t_o &= t,\ \theta_o = \theta + \omega t, \end{aligned} \tag{1}$$

[1] Existing storage rings are built of straight and curved sections. For the present study, as a first approximation, we regard them as being circular in shape.

[2] Geometrodynamical units (i.e., Newton's gravitational constant G = 1, speed of light, c = 1) are used in the derivation of the formulae.

where ω is the constant angular velocity around the z axis. By applying these transformation relations to the expression of the line element in the non-rotating coordinate system

$$ds^2 = -dt_o^2 + d\rho_o^2 + \rho_o^2 d\theta_o^2 + dz_o^2, \tag{2}$$

it becomes, in the rotating system,

$$ds^2 = -\left(1 - \omega^2\rho^2\right) dt^2 + d\rho^2 + \rho^2 d\theta^2 + 2\omega\rho^2 d\theta dt + dz^2. \tag{3}$$

The spatial line element, as seen by a Galilean observer at rest on the rotating ring, is given by [3],

$$dl^2 = d\rho^2 + \rho^2\gamma^2 d\theta^2 + dz^2, \tag{4}$$

where $\gamma \equiv (1 - \omega^2\rho^2)^{-1/2}$. The local time interval in a system at rest on the rotating ring is

$$dt\prime = \gamma^{-1}dt - \omega\rho^2\gamma d\theta, \tag{5}$$

which for $d\theta/dt = -\omega$ gives the expected $dt = \gamma^{-1}dt\prime$ ($t\prime$ is the time measured by an imaginary ideal clock moving with a bunch in the beam).

From eq.(3) the spatial components of Riemann and Ricci tensors and the spatial scalar of curvature, were calculated [4]. For $z = constant$, the spatial scalar of curvature is given by $R = 6\gamma^4\omega^2$. The Gaussian measure of curvature is, therefore, $K \equiv -R/2 = -3\gamma^4\omega^2 = -3\gamma^2(\gamma^2 - 1)/\rho^2$. It follows that the spatial geometry of a rotating ring is non-Euclidean, and for $\gamma >> 1$, $K \approx -3\gamma^4/\rho^2$.

A collider beam can be approximated by a torus shape as

$$\begin{aligned} x_o &= \rho_o \cos\theta_o, \quad y_o = \rho_o \sin\theta_o, \\ z_o^2 &= r^2 - (\rho_o - b)^2, \\ -r &\leq z_o \leq r, \; b - r \leq \rho_0 \leq b + r, \end{aligned} \tag{6}$$

where x_o, y_o, z_o, t_o are the Cartesian coordinates in the non-rotating system, b is the radius of the torus in the xy plane and r the radius of the section perpendicular to the xy plane (in this representation ρ_o measures the distance from the z_o-axis to a point in the torus). The Cartesian coordinates at rest in the rotating system, x, y, z, t are defined by:

$$\begin{aligned} x_o &= x\cos\omega t - y\sin\omega t, \\ y_o &= x\sin\omega t + y\cos\omega t, \\ z_o &= z, \; t_o = t. \end{aligned} \tag{7}$$

From eqs.(3), (4), and (6) the spatial line element as seen by an observer at rest, in relation to the rotating torus is given by

$$dl^2 = \left[1 - \left(\frac{\rho - b}{r}\right)^2\right]^{-1} d\rho^2 + \rho^2\gamma^2 d\theta^2 + dz^2. \tag{8}$$

From eq. (7), the non-zero spatial elements of the Riemann tensor can be obtained. For two dimensions $R_{\theta\rho\theta\rho} = R_{\rho\theta\rho\theta} = -R_{\theta\rho\rho\theta} = -R_{\rho\theta\theta\rho}$, and

$$R_{\theta\rho\theta\rho} = \gamma^4 \left\{ 3\gamma^2\omega^2\rho^2 + \frac{\rho}{r}\left(\frac{\rho-b}{r}\right)\left[1-\left(\frac{\rho-b}{r}\right)^2\right]^{-1} \right\}. \tag{9}$$

The spatial scalar of curvature for the rotating torus is obtained from eqs.(7), (8) and (9), and thus, the Gaussian curvature, K is

$$K = -\gamma^2 \left\{ 3\gamma^2\omega^2 \left[1-\left(\frac{\rho-b}{r}\right)^2\right] + \frac{1}{r\rho}\left(\frac{\rho-b}{r}\right) \right\}. \tag{10}$$

3 Interaction of a GW with a Rotating Ring

For the analysis of the interaction of a GW with a collider beam, we assume that every bunch is concentrated in a region much smaller than the separation between two bunches, and that the energy-momentum and charge densities are constant along the beam. Therefore, due to the macroscopic properties of a GW, a full bunch is considered as a *particle*, and the beam as a rotating ring. In the rest-frame of the beam, the charge and mass densities remain constant and the current density vanishes.

Local deviations from the steady state of high curvature, imposed by the electromagnetic interaction between the accelerating fields and the charged rotating particles, correlated around the ring, will be caused by GWs.

In order to estimate the influence of a GW on the rotating ring we choose the frame where the ring is at rest, and assume:

a) the electromagnetic interactions impose curvature on the rotating ring and keep the ring uniformly rotating. The system is considered before and after a GW interacts with it to be in a state of complete cancellation between the centripetal acceleration of the ring and the electromagnetic forces acting on it (the Gaussian measure of curvature for the ring is that of the torus given in eq.(10));

b) GWs propagate in the z direction and are described by the following transverse-traceless linearised expressions for the metric perturbation (plane wave approximation)

$$\begin{aligned} h_{xx}^{TT} &= -h_{yy}^{TT} = \Re\{\mathcal{A}_{+}e^{-i\omega_g(t-z)}\}, \\ h_{xy}^{TT} &= h_{yx}^{TT} = \Re\{\mathcal{A}_{\times}e^{-i\omega_g(t-z)}\}, \end{aligned} \tag{11}$$

where $\mathcal{A}_+$ and $\mathcal{A}_\times$ represent the dimensionless amplitudes of two independent modes of polarisation, they are directly related to the wave induced displacement of a free particle relative to the origin, to its original displacement from the origin (for details see reference [5]). Therefore the non-zero elements of the Riemann tensor due to GWs are given by

$$\begin{aligned} R_{xtxt} &= -R_{ytyt} = -\frac{1}{2}\frac{\partial^2 h_{xx}^{TT}}{\partial t^2}, \\ R_{xtyt} &= R_{ytxt} = -\frac{1}{2}\frac{\partial^2 h_{xy}^{TT}}{\partial t^2}; \end{aligned} \tag{12}$$

c) the stress-energy tensor for the GWs is given by

$$T_{tt} = T_{zz} = -T_{tz} = \frac{1}{16\pi}\langle \dot{A}_+^2 + \dot{A}_\times^2 \rangle_{time\ avg.}, \tag{13}$$

where the time average is done over a number of wavelengths;

d) the transverse-traceless coordinates and the proper reference frame of the detector coincide.

Under these assumptions the radial displacement of a bunch in the beam can be calculated as a function of time and proper length in relation to the center of mass of the ring.

Let us now limit the discussion to a polarised GW (from now on, the real part of all complex expressions has to be taken),

$$\begin{aligned} A_+ &= \mathcal{A}_+ e^{-i\omega_g(t-z)}, \\ \mathcal{A}_\times &= 0. \end{aligned} \tag{14}$$

In the z direction, the detector is much smaller than the wavelength and, therefore, one can set $z \approx 0$. Thus, on a bunch, a tidal acceleration is produced by the GW,

$$\begin{aligned} \frac{d^2x}{dt^2} &= -R_{xtjt}x^j = -\frac{1}{2}\omega_g^2 \mathcal{A}_+ e^{-i\omega_g t} x, \\ \frac{d^2y}{dt^2} &= -R_{ytjt}x^j = +\frac{1}{2}\omega_g^2 \mathcal{A}_+ e^{-i\omega_g t} y, \\ \frac{d^2z}{dt^2} &= 0. \end{aligned} \tag{15}$$

Therefore, the driving radial acceleration on a mass element due to the GW is given by

$$\begin{aligned} \frac{d^2\xi}{dt^2} &= \frac{x}{\rho_c}\frac{d^2x}{dt^2} + \frac{y}{\rho_c}\frac{d^2y}{dt^2} \\ &= -\frac{1}{2}\omega_g^2 \mathcal{A}_+ \rho_c \cos 2\theta\ e^{-i\omega_g t}, \end{aligned} \tag{16}$$

where ρ_c is the undisturbed radius of the ring and $\rho = \rho_c + \xi$.

The rotating ring is considered to have a natural frequency (i.e., normal mode) of vibration ω_0 and a damping time $\tau_0 >> 1/\omega_0$. The damping time is of the order of the time needed by the beam to transfer the vibrational energy due to the GW into electromagnetic radiation [3]. The equation of motion [4] for a bunch in the rotating ring is, therefore,

$$\ddot{\xi} + \dot{\xi}/\tau_0 + \omega_0^2 \xi = -\frac{1}{2}\omega_g^2 \mathcal{A}_+ \rho_c \cos 2\theta\ e^{-i\omega_g t}, \tag{17}$$

which has a steady state solution

$$\xi = \frac{\frac{1}{2}\omega_g^2 \mathcal{A}_+ \rho_c \cos 2\theta}{\omega_g^2 - \omega_0^2 + i\omega_g/\tau_0} e^{-i\omega_g t}. \tag{18}$$

[3] Transverse oscillations are damped in about 100-200 turns at lepton (e^+e^-) beams by the radiofrequency (RF) system, while in hadronic ($p\bar{p}$) machines, the damping time of a small transverse perturbation is much larger than the life-time of the beam.

[4] At this point one starts to consider the beam as a real relativistic rigid-body [3].

The deflection due to GWs is then expressed by

$$\begin{aligned} \xi &= \alpha_g \cos 2\theta \cos \omega_g t, \\ \alpha_g &= \frac{\frac{1}{2}\omega_g^2 \mathcal{A}_+ \rho_c}{\omega_g^2 - \omega_0^2 + i\omega_g/\tau_0}. \end{aligned} \tag{19}$$

When $\omega_g \approx \omega_0$ the ring is at resonance and the GW causes maximum deflection on the beam; α_g is then given by

$$\alpha_g^{res} = \frac{1}{2} Q \mathcal{A}_+ \rho_c, \tag{20}$$

where $Q \equiv \omega_0 \tau_0 \approx \omega_g \tau_0$ [5].

The mass-density of the beam, η, is

$$\eta = \frac{\gamma M}{2\pi \rho_c}, \tag{21}$$

where M is the total mass of the beam (sum over all the bunches, normally about 10^{10}-10^{11} particles per bunch). The vibrational energy of the ring, due to the absorption of GWs, is given by

$$E_{vibration} = \frac{1}{2} \int_0^{2\pi} \eta \rho_c \dot{\xi}_{max}^2 d\theta = \frac{\gamma M}{2\pi} \int_0^{2\pi} \dot{\xi}_{max}^2 d\theta, \tag{22}$$

which for $\dot{\xi}_{max}^2 = \omega_g^2 \alpha_g^2 \cos^2 2\theta$ is

$$E_{vibration} = \frac{\gamma M \omega_g^2 \alpha_g^2}{8}. \tag{23}$$

This energy is dissipated at a rate $E_{vibration}/\tau_0$, which can be compared with the rate at which the rotating ring absorbs energy from the incoming waves, $E_{vibration}/\tau_0 = -dE/dt = \sigma T^{tz} = (1/32\pi)\sigma \omega_g^2 \mathcal{A}_+^2$. The *cross section* is then given by $\sigma = 4\pi\gamma M \alpha_g^2 / \mathcal{A}_+^2 \tau_0$ which at resonance becomes

$$\sigma_{res} = \pi \rho_c^2 \gamma M \omega_g Q. \tag{24}$$

If the wave is not polarised but remains monochromatic, a similar result is achieved (for a non-monochromatic wave a Fourier decomposition is required).

The change in the proper length of the ring, due to the interaction with the GW, is (the fluctuations in γ are negligible, and $1/\rho \approx 1/\rho_c$),

$$\Delta l = \int_0^{2\pi} dl - 2\pi \rho_c \gamma. \tag{25}$$

The integral has to be solved using the three dimensional line element imposed by the topology of the rotating ring (eq.(3)), $z = constant$),

$$\int_0^{2\pi} dl = \rho_c \gamma \int_0^{2\pi} \left\{ 1 + \left[\frac{1}{\rho_c \gamma} \left(\frac{d\xi}{d\theta} \right) \right]^2 \right\}^{\frac{1}{2}} d\theta, \tag{26}$$

[5] Q depends on the stability of the beam and type of particles being accelerated, therefore, it has different values at different storage rings.

which can be approximated to

$$\int_0^{2\pi} dl \approx 2\pi\rho_c\gamma + \frac{1}{2\rho_c\gamma}\int_0^{2\pi}\left(\frac{d\xi}{d\theta}\right)^2 d\theta, \tag{27}$$

and, in fact, from $(d\xi/d\theta)^2 = 4\alpha_g^2 sin^2 2\theta$, is given by $\Delta l = \pi\alpha_g^2/2\gamma\rho_c$ and, at resonance, becomes

$$\Delta l_{res} = \frac{\pi Q^2 \mathcal{A}_+^2 \rho_c}{8\gamma}. \tag{28}$$

Applying the transformation relations (eq. (1)), in the inertial frame of reference,

$$\xi = \alpha_g \cos\left[2\left(\theta_o - \omega t_o\right)\right]\cos\omega_0 t_o, \tag{29}$$

which shows that the radial fluctuation at a fixed point in the frame where the ring is rotating with angular velocity ω, has a frequency equal to twice the rotation frequency and is modulated by the frequency of the GW, with amplitude α_g. At resonance,

$$\xi_{res} = \frac{1}{2}Q\mathcal{A}_+\rho_c \cos\left[2\left(\theta_o - \omega t_o\right)\right]\cos\omega_g t_o. \tag{30}$$

The delay in the arrival time of a bunch is then given by $\Delta t_o = \gamma\Delta l = (\pi\alpha_g)^2/2\pi\rho_c$, and at resonance,

$$\Delta t_o^{res} = \frac{\pi Q^2 \mathcal{A}_+^2 \rho_c}{8}, \tag{31}$$

Δt_o is the measure of the change in the longitudinal position of a bunch, revolution-by-revolution, as seen by an observer at a fixed point in the inertial frame of reference.

A crucial point in the above analysis is the evaluation of the numerical value of the Q factor. Since the amplitude of the maximal radial deflection of a bunch in a beam, α_g, and Q, are related by $2\alpha_g/Q\rho_c = \mathcal{A}_+$, one can call Q *the amplification factor*. One can easily see that if the possible measurable values of α_g are of the order of 50 μm, and the radius of the storage ring is of the order of 4.2 Km (LEP size rings), then $\alpha_g/\rho_c \approx 12 \times 10^{-9}$, which is many orders of magnitude larger than the expected amplitude of a GW. Therefore, values of $Q \approx 10^{13}$ are needed to have experimental access to amplitudes $\mathcal{A}_+ \approx 10^{-21}$.

It is straightforwards to calculate Q in the rest frame of the beam. Classical electrodynamics, as explained in [7], is used for the present case, as follows. The GW causes a bunch of charge $q = \pm\mathcal{N}e$ and mass $M = \mathcal{N}m_{e,p}$, ($\mathcal{N}$ is the number of particles in the bunch, e is the charge of a single particle in a bunch, and $m_{e,p}$ is the electron or proton mass respectively) to have an acceleration, a, of magnitude $a = \alpha_g\omega_g^2\cos 2\theta$ with a period $T \sim 1/\omega_g$. The energy radiated, E_{rad}, is of the order of

$$E_{rad} \sim \frac{2q^2a^2T}{3} \sim \frac{2q^2\alpha_g^2\omega_g^3\cos^2 2\theta}{3} \tag{32}$$

Since the motion of the bunch under the influence of the GW is harmonic with amplitude $\alpha_g\cos 2\theta$ and frequency ω_g, the mechanical energy of motion, E_0, is of the order of

$$E_0 \sim M\omega_g^2\alpha_g^2\cos^2 2\theta \tag{33}$$

If $E_{rad} << E_0$, the effect of radiation reaction is small, thus

$$E_{rad} << E_0 \rightarrow \mathcal{N}\omega_g\tau_{e,p} << 1 \tag{34}$$

where $\tau_e = \frac{2e^2}{3m_e} = 6.26 \times 10^{-24}$ s, $\tau_p = \frac{2e^2}{3m_p} = 3.4 \times 10^{-27}$ s for electrons or positrons and protons or antiprotons respectively. Normally $\mathcal{N}$ is of the order of $10^{10} - 10^{11}$; thus for all known predicted GW frequencies, the inequality in eq. (34) is satisfied and the problem can be treated based on classical grounds.

The line breadth and level shift of an oscillator are treated in ref. [7] and the results will be directly applied here. The displacement of a bunch from equilibrium is given in eq. (19), when the reactive effects are included, the amplitude of oscillation decreases, while the energy of motion is converted into radiant energy. For $\mathcal{N}\omega_g\tau_{e,p} << 1$, eq. (19) becomes

$$\xi = \xi_0 e^{-\kappa t}, \tag{35}$$

where ξ_0 is ξ at $t = 0$ in eq. (19), and κ is given by

$$\kappa = \frac{\Gamma}{2} \pm i(\omega_g + \Delta\omega_g) \tag{36}$$

$$\Gamma = \mathcal{N}\omega_g^2\tau_{e,p} \tag{37}$$

$$\Delta\omega_g = -\frac{5}{8}\mathcal{N}^2\omega_g^3\tau_{e,p}^2 \tag{38}$$

The constant Γ is called the *decay constant* and $\Delta\omega_g$ is called the *level shift*. The energy of a bunch decays exponentially as $e^{-\Gamma t}$ because of radiation damping.

It is here where the assumption of a damping time $\tau_0 >> 1/\omega_g$ in eq. (17) is justified, since $\tau_0 \equiv 1/\Gamma = 1/\mathcal{N}\omega_g^2\tau_{e,p}$ and, therefore,

$$Q = \omega_g\tau_0 = \frac{1}{\mathcal{N}\omega_g\tau_{e,p}} \tag{39}$$

The type of particles stored (τ_e for electrons or positrons and τ_p for protons or antiprotons) and the number of particles per bunch ($\mathcal{N}$) fix the nominal Q value for a given GW frequency. In Table 1 the numerical values of $\omega_g Q$ at present storage rings and at the LHC are given.

4 Measurement of the Effect

4.1 Description of the Method

The proposed beam position measurement can be performed by the standard instrumentation of modern colliders, used for correcting the orbits of the rotating particles (for the description and applications of beam position monitors see [6] and [8]). A streak camera, of the type described in reference [9] can be used [6] to enable a revolution-by-revolution observation of a single bunch. These detectors, consisting of a system of beam position monitors (and possibly a streak camera), must be synchronised and should be able to store the information long enough

[6]A streak camera is an ultrafast opto-electronic device that generates images of very short light pulses. It uses synchrotron light, produced by the passage of bunches through small wiggler magnets, such that the photon bunch length and longitudinal density distribution corresponds to that of the particle bunch that emitted it.

for short-range logic decisions. This can be implemented, as in calorimeters for high energy physics detectors, using analog delay devices and multiplexers, connected to a trigger system.

For example, the CERN LEP beam orbit measurement system consists [11] of 504 pickups, distributed all around the accelerator. The system measures in a synchronised way the position of a bunch - the data of all pickups are collected together to form a closed orbit measurement or a trajectory measurement.

From the frequency spectrum, caused by the radial oscillations and the position of the bunch along the beam, an event can be isolated and stored on a computer disk and later reconstructed and compared with simulation Monte Carlo programs.

The time delay is a cumulative effect, in the sense that every revolution of a bunch is affected by the interaction with the GW.

The amplitude involved in the analysis of data, coming from bunch positions, is α_g, since the deflection of the beam is given by $\xi = \alpha_g \cos[2(\theta_o - \omega t_o)] \cos \omega_g t_o$, and $\xi_{res}^{max} = Q \mathcal{A}_+ \rho_c / 2$. To some extent, the time delay can be detected from the spectrum seen by the streak camera. Also, from the difference in the radial position for adjacent monitors, the oscillations in the radial position can be reconstructed and the amplitude of the GW calculated.

By observing signals in coincidence coming from rings with counter-rotating beams, the effect can be isolated from different sources of random noise.

Coincidences between similar events at different colliders would support the hypothesis of an interaction with a GW generated by an extra-terrestrial source.

4.2 Present and Future Colliders as GW Antennas

The cross section in eq.(24) is a representative quantity to evaluate the detection possibilities and limitations of this effect. Taking Q from eq. (39) into eq.(24) it becomes

$$\sigma_{res} = \pi \rho_c^2 \frac{r_g}{\mathcal{N} \tau_{e,p}}, \tag{40}$$

where $r_g = \gamma M$ is the gravitational radius of the rotating ring [7].

Another parameter, which should be considered, is the ratio of the time delay to a known length involved in the detection. Since bunches in a beam are synchronised with the RF frequency of the collider, the wavelength of the RF is a good reference for the change in the longitudinal position of a bunch. The longitudinal dislocation of a bunch, relative to the RF wavelength, revolution-by-revolution, is then defined by [8] $\zeta \equiv \Delta l / \lambda_{RF}$.

Currently, LEP [9], SPS, HERA and the Tevatron can be employed in the above measurement, serving as GW antennas. The Large Hadron Collider (LHC), under design and construction at CERN is, in addition to being a remarkably versatile storage ring, a more sensitive facility for the detection of GWs. A potentially interesting mode of LHC operation, not considered here, is with the storage of heavy ions. In Table 1 important values of nominal parameters, for present storage rings and the LHC, are given.

The gravitational radii of these colliders [12], running at nominal energies, are given in Table 2.

[7] In the units used here 1 GeV = 1.33 10^{-52} cm.

[8] Currently, a streak camera at CERN can measure Δl with a presicion of about 300 μm.

[9] The Q value is used for the steady state solution, there is a transient at the begining of the interaction between the GW ant the ring that lasts for about 100-200 turns in electron beams.

Table 1: Nominal storage rings values of parameters involved in GWs analysis.

	LHC	SPS	LEP2	HERA	Tevatron
Particles storaged	pp	p	e^+e^-	ep	$p\bar{p}$
Circumference [Km]	26.66	6.9	26.66	6.34	6.28
Energy [GeV]	7×10^3	450	95.0	e: 26.7 p: 820.0	1×10^3
Relativistic factor (γ)	7461	480	19×10^4	e: 53.4×10^3 p: 874.2	1066
Particles per bunch [10^{10}]	10.5	1.2	41.6	e: 3.65 p: 10.0	p: 15 $\bar{\mathrm{p}}$: 4.5
Number of bunches	2835	4160	4	180	6
Revolution frequency [KHz]	11.245	43.38	11.25	47.35	47.77
RF frequency [MHz]	400	200	352.2	e: 499.7 p: 208.2-52.05	53
Bunch separation [m]	7.5	1.5	6665	30.2	1047
r.m.s. bunch length [cm]	8.1	30	1.8	e: 0.83 p: 8.5	50
$\omega_g Q = \frac{1}{\mathcal{N}\tau_{e,p}}$	2.8×10^{15}	2.45×10^{16}	3.84×10^{11}	e: 4.28×10^{12} p: 2.94×10^{15}	p: 1.96×10^{15} $\bar{\mathrm{p}}$: 6.53×10^{15}

Note that the ratio $\xi_{res}^{max}/\sigma = \mathcal{A}_+/2\pi\rho_c\gamma M\omega$ does not depend on Q and has the interesting property of increasing with the decrease of the frequency and the increase of the amplitude of the GW respectively.

Since GWs are predicted to have a very wide spectrum of frequencies and amplitudes, depending on the scenario at which they are generated [10] with characteristic amplitudes varying from 10^{-12} to 10^{-28} and with characteristic frequencies in the range 10^{-6}- 10^4 Hz (for the calculation of amplitudes and frequencies of GWs from different sources see [1]), the predicted beam deflection and cross section cover a range of many orders of magnitude, depending on the quality factor, $Q = \omega_g\tau_0$, which is an intrinsic property of the ring.

As an example, for an amplitude $\mathcal{A}_+ \approx 10^{-20}$, and a frequency $f_g \approx 100$ Hz, the radial deflection and the cross section, for present storage rings and the LHC, are given in Table 2.

At CERN, the SPS ring is almost completely contained by the LEP ring. Since the damping time of the beams at LEP is very short, the Q value of LEP is much smaller than that of the SPS (oscillations in the transverse plane at proton machines are not damped). Hence, a signal at the SPS could be used to trigger the beam position monitor system of LEP when both signals, from the SPS and LEP, could be compared. The same is true at HERA: proton beam oscillations could trigger the measurement on the electron (or positron) beam.

As a figure of merit it is worth to mention that at LEP the beam position measurement resolution is about 60 μm single observation, and about 8 μm when doing an average over

[10] *burst sources* like black holes and neutron stars formation, supernova explosion; *periodic sources*, like systems of two (e.g. a binary pulsar) or more stars; *primordial waves* originated by a non-isotropic matter density in the Big-Bang. The time to which GWs are confined is of the order of 80 ms, again depending on the way they are generated.

Table 2: Gravitational radii, estimated radial deflection amplitudes for $\mathcal{A}_+ = 10^{-20}$, and cross sections for $f_g = \omega_g/2\pi = 100$ Hz at LHC, SPS, LEP2, HERA and the Tevatron (the Q values are those in Table 1).

	r_g $10^{-38}[cm]$	ξ_{res}^{max} $[\mu m]$	σ_{res} $[mb]$
LHC	2.77×10^4	594	9.18×10^{10}
SPS	298.0	1225	4.87×10^7
LEP2	2.10	0.08	0.95×10^3
HERA	e: 2.33	0.21	6.63×10^3
	p: 196.3	146	3.84×10^7
Tevatron	p: 11.96	98	1.54×10^6
	$\bar{\text{p}}$: 3.59	326	1.54×10^6

224 turns. With the new system coming now into operation at the SPS [11](200 beam position monitors), the resolution will be 20 μm single observation and 3 μm when in orbit mode (average over 100 turns) [10].

4.3 Possible Noise Sources

Noise is a crucial point in any GW detector. The signal is so small that a minor variation of any component of the detector can give rise to a fluctuation that can simulate the effect which is being sought.

Beams accumulated in storage rings are usually very well under control after injection, acceleration and steering to nominal collision energies. Under normal running conditions changes in the orbit parameters are avoided (such a change might for example, induce a change in the beam energy).

Most of the noise sources at storage rings are well known. The most important sources for LEP are as follows (for a detailed study of noise at LEP see [14]):

- Magnetic dipoles:
  - Parasitic currents originated by leakage currents from power supply systems.
  - Thermal behaviour.
  - Sporadic change in the magnetic field of a magnet.
- Circumference of the storage ring:
  - Gravitational deformation of the storage ring [12].
  - Geological deformation.

[11] Currently SPS is operated in a pulse mode. During the 14.4 s of its cycle the full intensity beam is stored at 450 GeV for only 0.5 s before extraction starts. Thus, only 3.5 % of the cycle is available for GW detection. It could also be operated in continuous mode at 310 GeV beam energy.

[12] The Sun-Moon tidal induced acceleration produces a change of several hundreds μm on the circumference of LEP [15].

Although these effects and their consequences are well understood, for the study of GWs a deeper insight will be needed.

5 Conclusion

The possibility of using existing and planned high energy storage rings for detecting GWs has been investigated. After describing the physics of the interaction of gravitational radiation with the stored particle beam, it is found that the effect on the geometry of a rotating ring caused by a GW can be measured using the standard equipment of modern colliders. The design of beam position monitor systems for future colliders (LHC) might include the need for a higher precision in order to detect GWs than is needed for high luminosity in the interaction points.

However, the phenomenon of the interaction of a GW with ultra-relativistic rotating particles could also be studied at present storage rings, although with lower sensitivity.

More detailed analyses, including the development of new simulation programs, are needed in order to understand the technological needs for future storage rings as GWs antennas.

There might be enough time between the rise of the light curve of a supernova (first seen optically) and the arrival of the GWs, emitted a bit later in the same event, to trigger a direct observation of the interaction of the GW with particles in a storage ring. In order to do that one might need a trigger system linking big telescopes and storage ring beam monitor systems[13].

Acknowledgement

The author thanks to:
A.Seidman (Tel-Aviv) for continuous encouragement and stimulation. W.Buchmueller (DESY), R.Klanner (DESY) and G.Wolf (DESY) for useful remarks during his staying at DESY. N.Rosen (Technion) and L.Horwitz (Tel-Aviv), for reading one of the first drafts of the paper and commenting on that.
H. Schmickler (CERN), G. Morpurgo (CERN) and K. Hanke (CERN) for providing very useful information about the functioning of different monitor systems at CERN. D. Plane (CERN) for reading the manuscript and encouragement, and G. Veneziano (CERN) for his critical reading of the manuscript.

Dedication:

This paper is dedicated to the memory of Prof. Nathan Rosen.

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