# FRACTURE FUNCTIONS: FACTORIZATION AND EVOLUTION [a,b]

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Fracture functions and their evolution equation are reviewed. Some phenomenological applications are briefly discussed.

Fracture functions [1] have been introduced to extend the usual QCD improved parton picture of semi-inclusive deep inelastic processes to the low transverse momentum region of phase space, where the target fragmentation contribution becomes important. Trentadue and Veneziano [1] proposed to describe such contribution as a convolution of a new phenomenological distribution, the fracture function, with a hard cross section

$$\sigma_T = \int \frac{dx'}{x'} M^i_{AA'}(x', z, Q^2)\hat{\sigma}_i(x/x', Q^2). \tag{1}$$

The fracture function $M^i_{AA'}(x, z, Q^2)$ represents the probability of finding the parton $i$ in the hadron $A$ with momentum fraction $x$ while observing the hadron $A'$ in the inclusive final state with momentum fraction $z$. In the case in which the momentum transfer $t = |(p_A - p_{A'})^2|$ is measured we define [3] $\mathcal{M}^i_{AA'}(x, z, t, Q^2)$, a $t$-dependent (extended) fracture function.

The same idea of fracture functions implies the existence of a new factorization theorem which allows to write eq. (1). In the case of inclusive DIS one can use OPE but in semi-inclusive processes the straightforward application of OPE fails. The problem comes from the fact that OPE gives a prediction for amplitudes, whereas we need an expansion for cross-sections, i.e. cut amplitudes. A possible way out is to use cut vertices.

The cut-vertex expansion [2] is a generalization of OPE where local operators are replaced by non-local objects, i.e. cut vertices. Such an expansion allows to treat more general processes. Let us consider semi-inclusive DIS in

[a] Talk given at the DIS98 Workshop, Brussels, Belgium, April 4-8, 1998
[b] Work done in collaboration with G. Camici, L. Trentadue and G. Veneziano

$(\phi^3)_6$, that is the scattering reaction $p + J(q) \to p' + X$ where $J = 1/2\ \phi^2$. In the region $p'_t \ll Q$ or equivalently $t \ll Q^2$, the relevant diagrams are those in Fig.1:

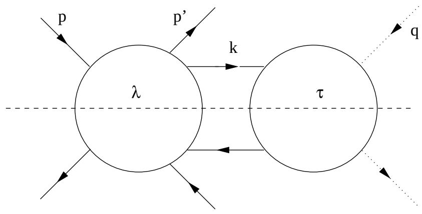


Figure 1: Relevant decomposition in the region $t \ll Q^2$

$$W(p,p',q) = \sum_\tau \int T_\lambda(p,p',k) H_\tau(\hat{k},q) \frac{d^6k}{(2\pi)^6} \tag{2}$$

where, given a vector $k$, $\hat{k} = (k_+, \mathbf{0}, 0)$. Define:

$$v_\lambda(p,p',\bar{x}) = \int T_\lambda(p,p',k)\bar{x}\delta\left(\bar{x} - \frac{k_+}{p_+ - p'_+}\right) \frac{d^6k}{(2\pi)^6} \tag{3}$$

$$C_\tau(x,Q^2) = H_\tau(k^2 = 0, x, q^2) \tag{4}$$

where $\bar{x} = x/(1-z)$. By taking moments we can write

$$W_\sigma(p,p',q) \simeq \sum_\tau v^\sigma_\lambda(p,p') C^\sigma_\tau(Q^2) \equiv v_\sigma(p,p') C_\sigma(Q^2). \tag{5}$$

Here $v_\sigma(p,p')$ is a new cut vertex which contains the long distance dependence of the cross section, whereas $C_\sigma(Q^2)$ is a coefficient function which is calculable as usual in perturbation theory. The expansion is technically obtained [4] by constructing an identity so as to isolate the leading term from the remainder, and care has to be taken to remove the UV divergences hidden in eqs.(3) and (4). Eventually one has to prove that the leading term is *really* leading. This is not obvious since there is no Weinberg theorem for cut amplitudes. Nevertheless, in order to find the leading behavior in the large $Q^2$ limit one can look at the singularities [5] at $p^2$, $p'^2$, $t \to 0$. We find that the leading singularities are given by diagrams of the kind of Fig.1, and so we can say that the cut-vertex expansion really gives the leading contribution. In QCD one has the complication that soft gluon contributions are not suppressed as in $(\phi^3)_6$ by power counting. However, by using gauge invariance, it can been shown that they cancel out [6].

The coefficient function appearing in the cut-vertex expansion is exactly the same as in the inclusive case since it comes from the hard part of the graphs. This means that the $Q^2$ evolution is dictated by the anomalous dimension of the same minimal twist local operator. By using renormalization group we can write eq. (5) in QCD as

$$W_n(z,t,Q^2) = \sum_i \mathcal{M}_n^i(z,t,Q^2)\; C_n^i(1,\alpha_S(Q^2)) \tag{6}$$

where we have defined [3]

$$\mathcal{M}_n^j(z,t,Q^2) \equiv V_n^i(z,t,Q_0^2) \left[ e^{\int_{\alpha_S}^{\alpha_S(Q^2)} d\alpha \frac{\gamma^{(n)}(\alpha)}{\beta(\alpha)}} \right]_{ij} \tag{7}$$

just in terms of the cut vertex $V_n^i(z,t,Q_0^2)$. It follows that the evolution equation for $t$-dependent fracture function is a standard DGLAP equation

$$Q^2 \frac{\partial}{\partial Q^2} \mathcal{M}_{A,A'}^j(x,z,t,Q^2) = \frac{\alpha_S(Q^2)}{2\pi} \int_{\frac{x}{1-z}}^1 \frac{du}{u} P_i^j(u) \mathcal{M}_{A,A'}^i(x/u,z,t,Q^2). \tag{8}$$

In the perturbative region of $t$ we can give a definition [8] of $\mathcal{M}_{AA'}^j(x,z,t,Q^2)$ based on Jet Calculus [7]

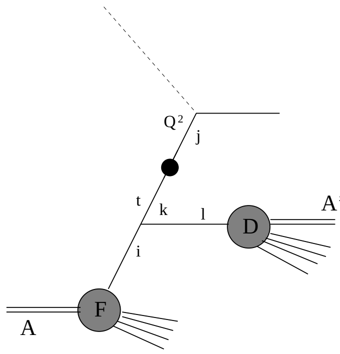


Figure 2: Perturbative definition of the $t$-dependent fracture function

$$\mathcal{M}_{AA'}^j(x,z,t,Q^2) = \frac{\alpha_S(t)}{2\pi t} \int_x^{1-z} \frac{dr}{r} \int_{z+r}^1 \frac{dw}{w(w-r)} F_A^i(w,t) \hat{P}_i^{kl}\left(\frac{r}{w}\right) \times$$
$$\times\, D_{l,A'}\left(\frac{z}{w-r}, t\right) E_k^j(x/r,t,Q^2) \tag{9}$$

where $\hat{P}_i^{kl}$ is the real splitting function and $E_k^j(x,t,Q^2)$ is the evolution kernel from the scale $t$ to $Q^2$. If we define the ordinary fracture function as an integral up to a cut off of order $Q^2$, say $\epsilon Q^2$, the inhomogeneous evolution equation [1] for fracture functions is recovered up to $\log\epsilon$ corrections

$$Q^2 \frac{\partial}{\partial Q^2} M^j_{A,A'}(x,z,Q^2) = \frac{\alpha_S(Q^2)}{2\pi}\int_{\frac{x}{1-z}}^1 \frac{du}{u} P_i^j(u) M^i_{A,A'}(x/u,z,Q^2)$$

$$+ \frac{\alpha_S(Q^2)}{2\pi}\int_x^{\frac{x}{x+z}} \frac{du}{x(1-u)} F^i_A(x/u,Q^2)\hat{P}_i^{jl}(u) D_{l,A'}\left(\frac{zu}{x(1-u)},Q^2\right). \quad (10)$$

Fracture functions are now measured at HERA and the scaling violations observed in experimental data are consistent with the evolution pattern presented here [9].

Fracture functions give the possibility of selecting interesting channels. This fact could be used to test target independence of suppression of the first moment of the polarized proton structure function [10]. Another interesting possibility could be to select a gluon by requiring a proton in the inclusive final state and study correlations with heavy quark production.

What about hadron-hadron scattering? One would like to use HERA data on fracture functions to give predictions for hadron-hadron scattering. However the general claim is that the factorization theorem for diffractive hadron-hadron scattering fails to hold, since the cancellation at work in the inclusive case does not apply here. Nevertheless we believe that further work is needed to asses this conclusion.

Summing up, we can say that fracture functions and their evolution equations are now well understood. We have given them an interpretation based on a formalism which is a direct generalization of OPE. Having proved the factorization theorem, fracture functions can now be reliably used to describe semi-inclusive hard processes in the target fragmentation region.

## References


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