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ON ENERGY LOSS OF NARROW AND DENSE ULTRA-RELATIVISTIC BUNCH IN PLASMA

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Energy balance of a particle bunch propagating through the plasma is considered for the bunches of a high density (compared to plasma) and a small radius (compared to anomalous skin depth). Energy loss to scattering of near-axis plasma electrons is estimated. This component of energy balance is shown to be important but not excessively high for the parameters interesting for high energy physics. In particular, the energy loss to near-axis electrons is proved to remain finite as the bunch radius tends to zero.

Keywords: Plasma wakefield acceleration; Ionisation losses

Energy balance of a particle bunch propagating through the plasma is one of the key questions for plasma-based advanced accelerator techniques [1,2]. Earlier [3], this problem was studied for the bunches of a low density (as compared to that of the plasma) with the most attention paid to beam loading issues. In this paper, we consider the energy balance of a dense (compared to plasma) and narrow (compared to the anomalous skin depth c/ω_p) axisymmetric bunch. In particular, we show that the energy transferred from the bunch to near-axis plasma electrons remains finite as the bunch radius tends to zero (while the bunch length remains finite).

The latter issue is especially important for potential applications of plasma to linear collider for the TeV energy range. In this case, the accelerated beams have to be of extremely small emittances and radii

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to provide acceptable luminosity. The bunch density is much higher than that of the plasma (n_0), though the bunch population should remain much smaller than the number of plasma electrons per $(c/\omega_p)^3$ (otherwise the distortion of the plasma wave would result in too large energy spread of the accelerated bunch).

The total energy balance of such a bunch is composed of two parts: the energy loss due to the acceleration of near-axis plasma electrons and the energy extracted from (or transferred to) remote plasma layers (at radii $\sim c/\omega_p$). The latter does not contain any peculiar to narrow bunch features. It can be easily found in particle-in-cell [3] or finite-difference [4] simulations via calculation of the longitudinal electric field. In the non-linear plasma wave (with the electric field amplitude of the order of $\sqrt{4\pi n_0 mc^2}$), this long-range interaction can provide the energy gain up to $mc\omega_p$ per particle per unit length, where m is the electron mass.

The energy lost to near-axis electrons cannot be calculated so straightforwardly. Fluid codes fail to solve this problem because of essentially non-hydrodynamical motion of plasma electrons near the dense bunch. The correct value of energy losses can, in principle, be obtained with existing particle-in-cell codes, but introduction of the additional length scale (associated with the bunch radius) greatly increases the number of particles required and, thus, prohibits the study of extremely narrow bunches. The aim of this paper is to give a correct estimate of this non-hydrodynamic energy loss for very narrow bunches of very high density. Note that for this estimate it is not necessary to specify the profile of the plasma wave accelerating the bunch. Since the energy acquired by near-axis plasma electrons is much greater than their initial energy (in the plasma wave), the wave profile affects the energy loss only through the change of plasma electron density.

To establish the upper limit of energy losses, we consider an infinitely narrow bunch (composed of N_b particles) propagating through the unperturbed plasma of density n_0. The particles are assumed to be uniformly distributed over the bunch length l. Even for zero radius bunch, the energy loss per unit length (W) turns out to be finite:

$$W = N_b \frac{mc^2}{l} F(L). \tag{1}$$

Here

$$L = \frac{2l\omega_p}{c}\sqrt{\frac{N_b e^2}{mc^2 l}} = \sqrt{\frac{N_b}{\pi n_0 (c/\omega_p)^3} \cdot \frac{l\omega_p}{c}}, \tag{2}$$

$\omega_p = \sqrt{4\pi n_0 e^2/m}$ is the plasma electron frequency and other notation is conventional. The dimensionless functions $F(L)$ are shown in Figure 1 by solid lines. Similar functions for the bunch of a finite width (of the radius c/ω_p, with the particles being uniformly distributed over the bunch cross-section) are shown by dots. The difference between these graphs represents the energy loss to the near-axis plasma layer. It is seen that the energy loss of the narrow bunch can be several times higher than that of a "wide" bunch. The near-axis losses scale as L^2, while the energy transferred to remote plasma layers is proportional to L. Thus, the fraction of near-axis losses increases with the increase of N_b, l, or n_0.

Expression (1) and the function $F(L)$ are obtained as follows. The radial electric field E_r and the azimuthal magnetic field H_φ of the infinitely narrow ultrarelativistic bunch may be approximated by the formula

$$E_r(r) = H_\varphi(r) = \frac{2eN_b}{rl}\exp\left(-\frac{r\omega_p}{c}\right) \tag{3}$$

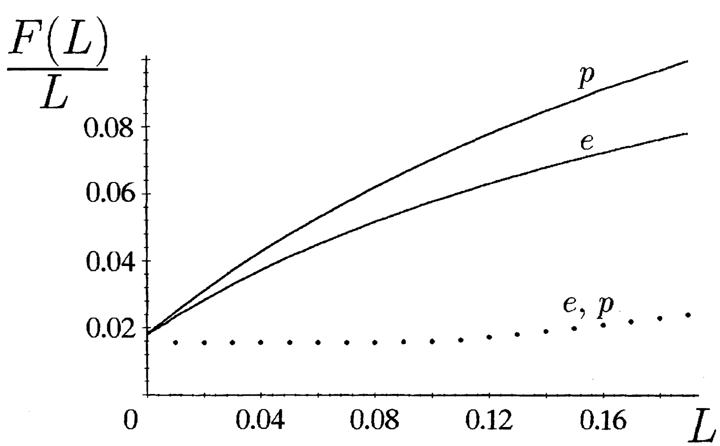

FIGURE 1 The energy loss for electron (e) and positron (p) bunches.

(we use cylindrical coordinates with the z-axis being the direction of bunch propagation). This approximation is good for $r \ll c/\omega_p$, while for $r \gtrsim c/\omega_p$ the screening of the bunch field by the plasma is taken into account only qualitatively through the exponential factor. In particular, we neglect temporal changes of the fields in far region caused by the bunch-driven displacement of plasma electrons. Actually, the law of field screening is somewhat different[5]. However, fully self-consistent treatment of remote plasma layers would not give more accurate value of near-axis losses, but unnecessarily complicates the calculations. It is self-evident that the bunch can be treated as "rigid" and unaffected by the fields appearing in plasma.

Within the considered model, the energy of plasma electrons and the z-component of their generalized momentum are conserved:

$$\gamma mc^2 - e\Phi = \text{const}, \qquad \gamma m v_z - \frac{e}{c}\Phi = \text{const}, \tag{4}$$

where $\Phi(r)$ is the electrostatic potential and the z-component of vector potential at the same time:

$$E_r(r) = H_\varphi(r) = -\frac{\partial \Phi}{\partial r}, \tag{5}$$

γ is the relativistic factor of plasma electrons, and e is the elementary charge ($e > 0$). The total energy loss is

$$W = n_0 \int_0^\infty e\big(\Phi(r_f) - \Phi(r_0)\big) 2\pi r_0 \, \mathrm{d}r_0, \tag{6}$$

where r_0 corresponds to the initial position of a plasma electron (before the bunch disturbs the plasma), and r_f is its final radius (after the bunch passage).

For the electron bunch, we can find r_f for any specified r_0 by merely equating the electron travel time and the duration of field action:

$$\int_{r_0}^{r_f} \frac{\mathrm{d}r}{v_r} = \frac{l}{c} + \frac{1}{c}\int_{r_0}^{r_f} \frac{v_z}{v_r}\mathrm{d}r \tag{7}$$

(the integral on the right-hand side of (7) represents the distance travelled by the electron in z-direction). Since, from (4),

$$v_r = c\frac{\sqrt{2\Delta\Phi}}{1+\Delta\Phi}, \quad v_z = c\frac{\Delta\Phi}{1+\Delta\Phi}, \qquad \text{where } \Delta\Phi = \frac{e\big(\Phi(r) - \Phi(r_0)\big)}{mc^2}, \tag{8}$$

we can rewrite (7) as

$$l = \int_{r_0}^{r_{\mathrm{f}}} \frac{\mathrm{d}r}{\sqrt{2\Delta\Phi}},$$

whence the dependence $r_{\mathrm{f}}(r_0)$ can be easily found numerically. Then numerical integration of (6) yields the formula (1) and the function $F(L)$.

For the positively charged bunch, the energy loss can be found in a similar way with the little complication arising from the oscillatory motion of near-axis plasma electrons (Figure 2). Energy loss for wide bunches is calculated analogously.

The finiteness of the energy loss can be proved analytically as well. For the electron bunch this is evident, since the energy acquired by a plasma electron may be majorized by the function $e(\Phi(\infty) - \Phi(r_0))$, which has the integrable logarithmic singularity at $r_0 = 0$.

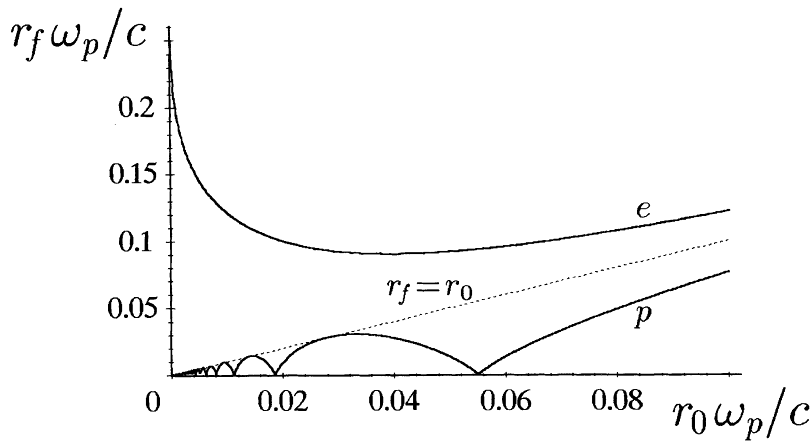

FIGURE 2 The final radius of plasma electrons as the function of their initial radius for electron (e) and positron (p) bunches with $L = 0.1$.

For the positively charged bunch the situation is more complicated, since the infinite energy is acquired by electrons for which

$$\int_0^{r_0} \frac{\mathrm{d}r}{\sqrt{2\Delta\Phi}} = \frac{l}{2k+1} \quad (k = 0, 1, 2, \dots)$$

or

$$r_0 \approx \frac{c}{\omega_\mathrm{p}} \cdot \frac{L}{(2k+1)G}, \quad G = \int_0^1 \frac{\mathrm{d}x}{\sqrt{-\ln x}} = \sqrt{\pi} \tag{9}$$

(here we take into account the logarithmic behaviour of Φ near the axis). The vicinity of each logarithmical singularity (9) makes a finite contribution to W, but there is an infinity of them. To majorize the total energy loss, we consider the contribution ΔW_k of the ring

$$r_k^0 = \frac{Lc}{2\sqrt{\pi}(k+1)\omega_\mathrm{p}} < r_0 < r_k = \frac{Lc}{\sqrt{\pi}(2k+1)\omega_\mathrm{p}}. \tag{10}$$

The final radius r_f is determined from the requirement

$$L = \frac{\sqrt{\pi}(2k+1)\omega_\mathrm{p}}{c} r_0 + \int_0^{r_\mathrm{f}} \frac{\mathrm{d}r \cdot \omega_\mathrm{p}/c}{\sqrt{\ln(r_0/r)}}, \tag{11}$$

whence

$$\frac{\mathrm{d}r_\mathrm{f}}{\mathrm{d}r_0} = -\sqrt{\pi}(2k+1)\sqrt{\ln(r_0/r_\mathrm{f})}.$$

Then

$$\begin{aligned}
\Delta W_k &\approx \frac{4\pi n_0 e^2 N_\mathrm{b}}{l} \int_{r_k^0}^{r_k} r_0 \ln(r_0/r_\mathrm{f}) \mathrm{d}r_0 \\
&= -\frac{m\omega_\mathrm{p}^2 N_\mathrm{b}}{\sqrt{\pi} l(2k+1)} \int_{r_0=r_k^0}^{r_0=r_k} r_0 \sqrt{\ln(r_0/r_\mathrm{f})} \mathrm{d}r_\mathrm{f}, \\
\Delta W_k &< \frac{m\omega_\mathrm{p}^2 N_\mathrm{b} r_k^2}{\sqrt{\pi} l(2k+1)} \int_0^1 \sqrt{\ln(r_k/r_\mathrm{f})}\, \frac{\mathrm{d}r_\mathrm{f}}{r_k} \\
&= N_\mathrm{b}\, mc^2\, \frac{L^2}{2\pi l\,(2k+1)^3}.
\end{aligned} \tag{12}$$

It is easy to show that the contribution of the ring $r_{k+1} < r_0 < r_k^0$ is less than ΔW_k. Since the series

$$\sum_{k=0}^{\infty} \frac{1}{(2k+1)^3}$$

converges, we conclude that the energy loss of the positively charged bunch is finite.

It is interesting to note why the calculation of energy loss in terms of "instant push" gives the erroneous result:

$$p \sim \frac{e|E_r|\,l}{c} \propto \frac{1}{r_0}, \qquad W \propto \int 2\pi r_0 \cdot \frac{p^2}{2m} \mathrm{d}r_0 \propto \int \frac{\mathrm{d}r_0}{r_0} \quad \text{(diverges)}. \tag{13}$$

This is because near-axis plasma electrons rapidly pass to the region of either smaller field (electron bunch) or opposite-sign field (positron bunch). As a result, the estimate (13) turns out to be unacceptably overstated.

The general conclusion of the paper is that the non-hydrodynamic energy losses of a dense bunch travelling in plasma are important but not excessively high for the parameters interesting for high energy physics. To illustrate this conclusion let us estimate the energy loss for the bunch with $N_b = 10^9$ particles. For this bunch to be effectively accelerated in plasma, the condition

$$N_b \ll n_0 \left(\frac{c}{\omega_p}\right)^3 = \left(\frac{mc^2}{4\pi e^2}\right)^{3/2} \cdot \frac{1}{\sqrt{n_0}} \tag{14}$$

should be met, which gives $n_0 \ll 2 \cdot 10^{16}\,\mathrm{cm}^{-3}$ in our case. Also, the bunch length has to be small compared to c/ω_p (to provide small energy spread). Choosing $l = 0.05\,\mathrm{mm}$ and $n_0 = 10^{15}\,\mathrm{cm}^{-3}$ ($c/\omega_p \approx 0.17\,\mathrm{mm}$), we satisfy both conditions. From (2) we find $L \approx 0.15$, which corresponds to near-axis losses of about $75\,\mathrm{MeV/m}$ per particle. Note that this is the average value; actual loss per particle depends on particle position along the bunch, so the particles at the back front of the bunch lose several times greater energy. The plasma of the given density can provide the accelerating gradient up to $3\,\mathrm{GeV/m}$. Thus, non-hydrodynamical energy losses are relatively small, but they should be taken into account in in-depth studies of plasma-based accelerator techniques.

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