New Dimensions at a Millimeter to a Fermi and Superstrings at a TeV

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Abstract

Recently, a new framework for solving the hierarchy problem has been proposed which does not rely on low energy supersymmetry or technicolor. The gravitational and gauge interactions unite at the electroweak scale, and the observed weakness of gravity at long distances is due the existence of large new spatial dimensions. In this letter, we show that this framework can be embedded in string theory. These models have a perturbative description in the context of type I string theory. The gravitational sector consists of closed strings propagating in the higher-dimensional bulk, while ordinary matter consists of open strings living on D3-branes. This scenario raises the exciting possibility that the LHC and NLC will experimentally study ordinary aspects of string physics such as the production of narrow Regge-excitations of all standard model particles, as well more exotic phenomena involving strong gravity such as the production of black holes. The new dimensions can be probed by events with large missing energy carried off by gravitons escaping into the bulk. We finally discuss some important issues of model building, such as proton stability, gauge coupling unification and supersymmetry breaking.

1   Introduction

In a recent paper [1], a general framework for solving the hierarchy problem was proposed not relying on low-energy supersymmetry or technicolor. The hierarchy problem is solved by nullification: in this scenario, gravity becomes unified with the gauge interactions at the weak scale and there is no large disparity between the size of different short distance scales in the theory. As argued in [1], the observed weakness of gravity is then due to the existence of new spatial dimensions much larger than the weak scale, perhaps as large as a millimeter for the case of two extra dimensions. The success of the Standard Model (SM) then implies that, while gravity is free to propagate in the bulk of the extra dimensions, the SM fields must be localised to a 3 spatial dimensional wall at energies beneath the weak scale. While field-theoretic mechanisms for localising the SM fields on a topological defect were suggested, the nature of the theory of gravity above the weak scale was left unspecified in the general framework of [1].

In this letter, we show that the above scenario can be embedded within string theory, which at present offers the only hope for a consistent theory of gravity. The traditional line of thought has been that string theory becomes relevant only at very short distances of order of the Planck length $\sim 10^{-33}$ cm. However various arguments involving unification, supersymmetry breaking or the gauge hierarchy, suggest that it may be relevant at even larger distances. For instance, compatibility of string unification with gauge coupling unification within the minimal supersymmetric standard model [2] implies that the string (or M-theory) scale should be of the order $M_{\mathrm{GUT}} \sim 10^{16}$ GeV, while additional dimensions would show up at even lower energies $\lesssim 10^{15}$ GeV [3, 4]. As another example, low energy supersymmetry breaking within perturbative string theory implies the existence of a large internal dimension whose size determines the breaking scale [5]. The possibility that the string scale is close to the electroweak scale was mentioned in, for example [6, 7, 4]. This is certainly a requirement for a string realization of the scenario proposed in [1].

In this work we show that the only perturbative string theory with weak scale string tension must be a type I theory of open and closed strings with (a) new dimensions much larger than the weak scale ranging from a fermi to a millimeter (b) an $O(1)$ string coupling and (c) SM fields identified with open strings localised on a 3-brane. Aside from providing a specific realization of

the considerations of [1], this construction has the immediate advantage that the localisation of non-gravitational fields to a three dimensional submanifold is automatic and natural. Moreover, our explicit realization will allow us to address a certain number of important theoretical questions arising from this idea, while simultaneously it offers a calculable framework for studying its phenomenological implications.

2 String embedding

Any perturbative description of string theory has two fundamental parameters: the string scale M and a dimensionless coupling λ which controls the loop expansion.* Upon compactification to $D = 4$ dimensions, these parameters can be expressed in terms of the 4D Planck mass M_p, the gauge coupling α_G at the string scale and the compactification volume $(2\pi)^6 V$ of the internal six-dimensional manifold. Imposing the string scale to be at a TeV one can in principle solve for V and λ and, then, trust the solution if $\lambda < 1$. However in the weakly coupled heterotic theory, V and λ drop from one of the two relations and one obtains a prediction for the string scale M_H: $M_H = (\frac{\alpha_G}{8})^{1/2} M_p \simeq 10^{18} \text{GeV}^\dagger$.

The situation changes drastically in the strongly coupled heterotic theory. The latter is described, in the $E_8 \times E_8$ case, by the eleven-dimensional M-theory compactified on a line segment of length πR_{11} [8]. One may now try to identify the M-theory scale M_{11}, which determines the Newton constant of the 11D supergravity, with the electroweak scale. Unfortunately, then, the length of the line segment turns out to be unacceptably large: $R_{11} = \frac{\alpha_G}{2} \frac{M_p^2}{M_{11}^3} \simeq 10^8 \text{km}$!

It only remains to consider the type I theory of open and closed strings which also decribes the strongly coupled heterotic $SO(32)$ string [9]. When compactified down to four dimensions, the gravitional and gauge kinetic terms of the resulting effective four-dimensional theory are, in a self-explanatory notation:

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{\lambda_I^2} V M_I^8 \mathcal{R} + \frac{1}{\lambda_I} V M_I^6 F^2 \right) , \qquad (1)$$

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*Actually λ corresponds to the vacuum expectation value of a dynamical scalar field.
†In fact α_G should be replaced by $k\alpha_G$, where k is the integer Kac-Moody level.

where we ommitted numerical factors. Identifying the coefficient of $\mathcal{R}$ with M_p^2 and that of F^2 with $1/g^2$ yields the relations

$$V^{-1} = \frac{\alpha_G^2}{2} M_p^2 M_I^4 \qquad ; \qquad \lambda_I = \frac{8}{\alpha_G} \left(\frac{M_I}{M_p}\right)^2 , \qquad (2)$$

where in this equation the relevant numerical factors have been included. Taking the type I string scale M_I to be at the TeV, one finds a compactification scale much larger, while the string coupling is infinitesimaly small. Choosing for instance n internal dimensions to have a common radius R_I and the remaining $6 - n$ of the string size, one obtains:

$$R_I^{-1} = \left(\frac{\alpha_G^2}{2}\right)^{1/n} M_p^{2/n} M_I^{1-2/n} \qquad n = 1, \ldots, 6 . \qquad (3)$$

It follows that for $\alpha_G \simeq .1$ the value of the compactification scale varies from 10^{33} GeV, 10^{18} GeV, up to 10^8 GeV for $n = 1, 2$, or 6 large internal dimensions.

One may naively think that such a large compactification scale is irrelevant for low energy physics. However this is not true in string theory due to the presence of winding states in the closed string (gravitational) sector, whose masses are quantized linearly with the radius in string units and are therefore very light. In fact physics is equivalent as if there was a radius $R = 1/(R_I M_I^2)$ which is much larger than the string length and the roles of windings and Kaluza-Klein (KK) momenta are interchanged. On the other hand, in this T-dual theory, open string states which give rise to ordinary non gravitational matter live on D3-branes, since they have only winding modes, identical with the heavy KK modes (3) of the initial theory.

By performing a T-duality to all six compact dimensions, $R \to 1/(RM^2)$ and $\lambda \to \lambda/(RM)$, the relations (2-3) become:

$$V^{-1} = \frac{2}{\alpha_G^2} M_p^{-2} M_I^8 \qquad ; \qquad \lambda = 4\alpha_G$$

$$R^{-1} = \left(\frac{2}{\alpha_G^2}\right)^{1/n} M_I \left(\frac{M_I}{M_p}\right)^{2/n} << M_I . \qquad (4)$$

As a result, the coupling constant of this dual theory is the 4D gauge coupling while the value of the compactification scale varies from 10^{-18} eV, 10^{-3} eV,

up to 10 MeV for $n = 1$, 2, or 6 large internal dimensions. Obviously the case of $n = 1$ is experimentally excluded, $n = 2$ corresponds to two dimensions in the range of 100 microns, while $n = 6$ to six dimensions in the range of .1 fermi.

This setup gives an explicit string realization of the proposal of [1]. In fact, from eq. (4) we have $M_I \sim (\alpha_G^2 M_p^2 R^{-n})^{1/(n+2)} \equiv \alpha_G^{2/(n+2)} M_{p(4+n)}$, where $M_{p(4+n)}$ is the Planck mass in the $(4+n)$ higher dimensional theory. Actually in string theory the presence of the gauge coupling in the above relations lead to a string scale a bit lower that the $(4+n)$-dimensional Planck mass. Taking M_I at the TeV, this leads to somewhat lower values for R when n is small.

An immediate advantage of the string construction is that matter is automatically localized on a 3-brane. The latter can be thought as a thin wall limit of the effective field theory solution. In fact, an important difference here is that all string states living on the brane are delocalized in the extra large dimensions only at energies much higher than the string scale, $E \sim R M_I^2 = 1/R_I$, due to the effect of string winding modes. Above the compactification scale (3) of the dual theory, n extra dimensions open up for matter, as well. Of course this statement should be understood under the assumption than one can naively extrapolate at energies above the string scale, excluding non perturbative effects and the possibility of phase transitions, especially if the theory is not supersymmetric.

It is interesting that, in compactifying to four dimensions, the only way of making the string scale much lower than the (4 dimensional) Planck scale is to have large volume compactifications, with $O(1)$ string coupling. One may wonder in what cases the new dimensions can be kept at the string scale, with an infinitesinal string coupling accounting for the discrepancy between the Planck and weak scales. For the general case of compactifying on a $10 - D$ dimensional manifold, the relations generalizing eqn.(2) read (again omitting numerical factors)

$$V_{10-D}^{-1} = \alpha_D^2 M_{p(D)}^{6-D} M_I^4 \qquad ; \qquad \lambda_I = \frac{1}{\alpha_D} \left(\frac{M_I}{M_{p(D)}} \right)^2 , \qquad (5)$$

It follows that precisely at $D = 6$ the internal volume becomes of order of the string scale, which can be much smaller than the 6D Planck mass by tuning only the string coupling, while keeping the dimensionless gauge coupling to be of order unity. This is consistent with the six-dimensional examples

considered in [7]. For $D > 6$ the compactification scale is smaller than M_I, while for $D < 6$ it becomes bigger and (as we did in the case $D = 4$) one has to go to the T-dual theory in order to describe low energy physics with an effective field theory.

3 Phenomenology

3.1 Accelerator signals and constraints

There are two distinct classes of novel phenomena that occur at a TeV in our framework[‡]:
(1)Production of Regge-excitations
(2)Emission of $(4 + n)$-dimensional gravitons into the extra dimensions.

The existence of Regge-excitations (RE) for all the elementary particles of the standard model is a consequence of having a string theory at a TeV and does not per-se reveal the existence of extra dimensions. In contrast to the RE of ordinary QCD, these states are expected to be relatively weakly coupled and narrow since the string coupling constant is not too large. The ratio of their width to their mass is of order of $\Gamma/m \sim \lambda^2 \sim$ a few per thousand, so they are relatively narrow resonances with well defined mass. The Regge-excitations of the gluon could be produced in gluon-gluon collisions at LHC, showing up as a series of narrow peaks in the gluon-gluon cross section as a function of the gluon pairs' invariant mass. The corresponding amplitude is proportional to:

$$A(s, t) \sim \frac{\Gamma(-s/M_I^2)\Gamma(-t/M_I^2)}{\Gamma(1 - (s + t)/M_I^2)} \,, \tag{6}$$

exhibiting a series of poles corresponding to the RE mass positions.

Next we come to graviton emission into the extra dimensions [10]. The inclusive cross section for single graviton emission is proportional to

$$\sigma(E) \sim \frac{E^n}{M_I^{n+2}} \times (n + 2)^2 \tag{7}$$

[‡]These, together with laboratory and astrophysical constraints on the more general framework proposed in [1] are discussed in detail in ref.[10]

where the M_I dependence is uniquely fixed by the normalization of the graviton fields in the higher dimension theory $g_{AB} = \eta_{AB} + h_{AB}/\sqrt{M_I^{n+2}}$. A $(4+n)$-dimensional graviton contains, in addition to the normal 4-D graviton, graviphotons, Brans-Dicke scalars as well as an antisymmetric tensor. In the present case all of these particles can be emitted into the extra dimensions with equal amplitudes. This leads to the multiplicity factor proportional to $(n+2)^2$ of eq.(7) and enhances the total graviton emission rate.

The cross section (7) is negligible for energies much below M_I but rises abruptly at energies of order M_I, leading to an abundance of events with lots of missing energy, carried by gravitons into the extra dimensions. One way to look for these in hadron colliders is to search for processes with jets+missing energy. Another manifestation of graviton emission into the extra dimensions is that it leads to phenomena analogous to those caused by brehmstrahlung. For example graviton emission, or "gravistrahlung", depletes the beam energy just before a collision takes place. This should be taken into account in the resonant production of narrow states, such as the Regge-excitations of the gluon in gluon-gluon collisions and of other ordinary particles.

Graviton emission will be very important at energies above M_I where it is analogous to Hawking radiation from an excited brane and can be computed using the technology developed, for example, in ref.[11]:

$$\sigma(E) \sim (n+2)^2 \frac{E^n}{M_I^{n+2}} \frac{\Gamma(1 - 2E^2/M_I^2)^2}{\Gamma(1 - E^2/M_I^2)^4} \, . \tag{8}$$

The emmission rate exhibits a sequence of poles associated to the production of RE resonances, as well as a sequence of zeros indicating that the corresponding states are forbidden to propagate in this process [11]. Moreover, it decays exponentially at large energies due to the well known ultraviolet softness of string theory.

There is a qualitative difference between the present D-brane construction and the "thick wall" version of the scenario proposed in [1]. In the latter the binding energy of particles to the walls is typically of order of the weak scale and therefore the particles of the Standard Model can be emitted into the bulk in collisions at TeV energies. This leads to qualitatively different, and even more dramatic, missing energy signatures than those of the D-brane case where only the gravitons migrate in the bulk. In our case, however, the SM particles are open strings stuck on the brane. They do possess winding

modes that feel the bulk, but their mass is very large $\sim RM_I^2$, which ranges from 10^8 GeV for $n = 6$ to 10^{19} GeV for $n = 2$. These winding modes will then be irrelevant to weak scale physics.

Finally a comment on phenomenological constraints on these theories. There are several which are discussed in ref.[10]. The most obvious, but not the most important, comes from the compositeness bounds on the scale of suppression of higher dimension operators, which are at most at 3 TeV for flavor-conserving operators. Such effects are induced by the exchange of the Regge-excitations as well as the graviton. For the former the most dangerous effect comes from the exchange of the RE of the photon between two electrons. It is safely small as long as the RE of the photon are heavier than 300 GeV. The exchange of a $(4 + n)$-dimensional graviton between two electrons induces a higher dimension operator of the form:

$$
\begin{aligned}
\mathcal{O} \quad &\sim \quad \left(\frac{E^n}{M_I^{n+2}} \right) \times \left(\frac{M_I}{E} \right)^{n-2} \times (n+2)^2 (\bar{\psi}\psi)^2 \\
&\sim \quad (n+2)^2 \frac{E^2}{M_I^4} (\bar{\psi}\psi)^2,
\end{aligned}
\tag{9}
$$

where in the first line of the above equation, the first factor has the naive M_I dependence following from the normalisation of h_{AB} and the second results from the sum over the heavy KK excitations of the graviton with mass greater than E which is UV divergent for $n \geq 2$ [12]. Since the largest energy, where the 4-electron vertex is studied accurately, is ~ 100 GeV this is safely small provided the string scale M_I is larger than ~ 1 TeV.

3.2   Proton stability

Every extension of the SM invoking new physics at the electroweak scale must address the issue of the stability of the proton and the abscence of large flavor-violations. An arbitrary effective Lagrangian with all higher dimension operators suppressed by powers of the weak scale is grossly excluded: assuming $O(1)$ coefficients, the standard dimension 6 operators giving proton decay must be suppressed by at least the GUT scale to be safe, while operators contributing to Δm_K and ϵ_K must be suppressed by $\sim 10^3$ and 10^4 TeV respectively. Clearly some mechanism is required to adequately suppress these operators. Since the size of flavor-changing operators is intimately linked to the origin of flavor, the hope is that the same physics which

suppresses the light generation Yukawa couplings also adequately suppresses the FCNC operators. We will therefore not pursue the flavor-changing issue here, and focus on the far more serious problem of proton decay.

Adequately suppressing baryon number violating operators is somehwat easier when the theory above the weak scale is a known field theory as in the case of the MSSM, since only the dangerous dimension 4 operators must crucially be forbidden, and this can be arranged fairly simply (e.g. by imposing R parity). One certainly does not have to impose a symmetry forbidding proton decay altogether. In our case, however, the theory above the weak scale is an unknown string theory, and it is not clear if there is some simple mechansim analogous to imposing R parity which adequately suppresses proton decay without forbidding it. Without knowing the structure of the full theory, it seems safer to look for symmetries that can be imposed on the low energy theory which completely shut off proton decay. Of course, one can imagine that $U(1)_B$ is an exact global symmetry of the theory beneath the string scale, but standard lore suggests that all symmetries in string theory are local, and we will limit ourselves to this possibility. We list a few possibilities below.

A simple possibility [13] is to add a fourth generation whose quarks are assigned baryon number 1 instead of $-1/3$; $U(1)_B$ is then the diag(1/3,1/3,1/3,-1) generator of the flavor $SU(4)$ acting on the quarks. It is easy to see that with this assignement, $U(1)_B$ is anomaly free. Of course, we don't want to gauge the continuous $U(1)_B$ since no massless gauge boson coupled to baryon number has been observed. We can however gauge any discrete Z_q subgroup of this $U(1)_B$, arising for instance if $U(1)_B$ is broken by a scalar field of charge q, implying baryon number is conserved modulo q. Proton decay is completely forbidden as long as $q \neq 1$, and other baryon number violating operators can be enormously suppressed. For instance, in [13], $U(1)_B$ is broken by a field of charge 4/3 in order to allow ordinary Yukawa couplings of the 4th to the rest of the generations, so B is conserved mod 4/3. Since all physical states have integer baryon number this means that in physical processes B is conserved mod 4, and lowest dimension operator invariant under $Z_{4/3}$ but violating $U(1)_B$ is $(QQ)^4 D_c^{\dagger 4}$ has dimension 18, and is certainly safe even when suppressed only by the weak scale.

There are other possibilities which do not require the addition of new chiral matter to the SM. For instance, Ref. [14] gives an anomaly-free discrete "baryon triality" acting as $(1, g^2, g, g^2, g^2)$ on (Q, U^c, D^c, L, E^c), where $g^3 = 1$.

It is easy to check that all operators invariant under the SM and this triality violate baryon number by a multiple of 3 units: the lowest dimension operator violating B by 3 units is 18 dimensional and certainly safe.

3.3 Gauge coupling unification

Since the string scale is close to the weak scale in our scenario, the usual logarithmic evolution of the gauge couplings can not be extrapolated to high scales and the standard picture of gauge coupling unification is lost. On the other hand, there are interesting new possibilities for gauge coupling unification at the weak scale, provided that the SM fields stuck to the brane can propagate in k spatial dimensions with a size r somewhat larger than the string scale. The effective theory above energies r^{-1} but beneath the string scale is a higher dimensional field theory, and since gauge couplings are dimensionful in higher dimensions, they run with a power of energy, rather than logaritmically as in four dimensions. This can also be understood from the four dimensional viewpoint due to the contribution to the running of the towers of KK excitations: at energies $E > 1/r$, we have (at 1-loop)

$$E \frac{dg}{dE} = \frac{b + n(E)b_{KK}}{8\pi^2} \tag{10}$$

where $n(E) \sim (Er)^k$ is the number of KK modes which are lighter than energy E and b_{KK} is the contribution to the beta function from one KK level. Depending on the quantum numbers of the KK towers, the couplings can evolve very quickly above the scale $1/r$ to a unified value at the string scale [15].

3.4 SUSY breaking

In our scenario, there is no longer any need for a "hierarchical" breaking of SUSY, since the string scale is already close to the scale where SUSY should be broken, a feature we find attractive relative to the standard scenario. On the other hand, some of the radii of compactification must be very large compared to the string scale, and while this may have a cosmological explanation, we wish to leave open the possibility that the moduli corresponding to the size of these dimensions are stabilised at large values. Since the potential for the moduli vanish in the SUSY limit, it seems natural to keep the size

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of SUSY breaking for the modulus very much smaller than the TeV scale. This can be done if SUSY is primordially broken only on the brane, with SUSY breaking transmitted to the bulk by gravitational effects. If SUSY is broken maximally on the brane (i.e. $\rho_{brane} \sim M_{st}^4$), and assuming that the transmission to the bulk fields occurs at tree level (therefore proportional to $G_{N(4+n)}$), we can estimate an upper limit the SUSY breaking soft mass for the modulus by dimensional analysis

$$m_{modulus}^2 \sim \frac{G_{N(4+n)}\rho_{brane}}{R^n} \sim \frac{\rho_{brane}}{M_p} \sim (1\text{mm})^{-2}. \tag{11}$$

The spectrum in the bulk is very nearly (at least $N = 2$) supersymmetric, with bose-fermi splittings of at most $\sim (1\text{mm})^{-1} \sim 10^{-4}\text{eV}$. It is reassuring that the modulus mass is at most the inverse size of the compact dimensions for $n = 2$ or smaller for $n > 2$.

The presence of such a light modulus would generate gravitational forces in the millimeter range which could be explored experimentally [16]. Moreover, in the case of $n = 2$ large dimensions, one should take into account all the components of the six-dimensional graviton and of other fields, giving rise to additional scalars and graviphotons. Since the latter contribute to a repulsive force, the gravitational interactions might be drastically modified at (sub)millimeter distances. In particular, if the bulk had $N = 4$ supersymmetry, one would have an exact cancellation of gravitational forces at short distances!

4   Conclusions

As will be discussed in greater detail in [10], the scenario we propose is not experimentally excluded by any lab or astrophysical constraint we are aware of. We briefly discussed some issues of model building such as stabilising the proton through discrete gauge symmetries, SUSY breaking and gauge coupling unification at the weak scale, but it is clear that much work remains to be done to construct a completely realistic model. In particular, the most pressing theoretical issue is to understand the origin of the large size of the extra dimensions, ranging from 5 to 15 orders of magnitude larger than the string scale. Given the prospect of studying quantum gravity at the LHC

and NLC, we feel that future model-building within this framework is well motivated and exciting.

5   Acknowledgements

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