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INTERACTION REGION DIAGNOSTICS IN e^+e^- RING COLLIDERS

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Abstract

We present a review of diagnostic tools intended to control and optimise the beam overlap in the interaction points of both symmetric and asymmetric colliders. Practical applications of some methods adopted to tune the impact parameter at LEP are described. An experiment is proposed to investigate the relation between the coherent tune split between the $\sigma-$ and the $\pi-$modes of oscillation and the incoherent beam-beam parameter under different operational conditions.

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1   Introduction

The control of the transverse impact parameter at which counter-rotating bunches pass each other at the Interaction Points (IP) represents a powerful tool to optimise the overall operational efficiency of the accelerator. The problem is of particular importance in asymmetric colliders where the beams mostly circulate in separate rings and experience different magnetic histories. Significant collision offsets can arise and seriously affect the performance if not properly monitored and corrected. Besides optical machine imperfections, specific modes of operation like bunch-train operation in LEP can generate non-zero collision offsets. Opposite-sign residual vertical dispersion from electrostatic separation schemes and closed orbit perturbations from long range beam-beam effects can combine to generate unwanted offsets. The potential of closed orbit distortions (COD) associated to the beam-beam interaction as diagnostic tool in circular colliders has been thoroughly investigated in[1][2][3].

Beam-beam deflections (BBD) induced on the trajectories of bunches colliding at an offset can be exploited to monitor and control the beam overlap. Adopted for the first time at the SLAC SLC single-pass collider[4][5][6][7][8] the method relies on the reconstruction of orbit angles at the collision point from beam excursion measurements at nearby pick-up monitors.

BBD in storage rings have been experimentally observed at CESR[9] and KEK[10]. A particularly useful application of the method has been developed at LEP[11][12] and is currently adopted to optimise the beam overlap at the four interaction points.

An experiment associating the BBD technique to luminosity measurements is proposed to study the behaviour of the Meller-Yokoya[13] relation between the coherent $\sigma-\pi$ tune split and the incoherent beam-beam parameter under different operational conditions.

2   Notations

In the following we indicate with $(z = x , y)$ the transverse beam coordinates and with δz^* the impact parameter as the difference between the bunch centroid offsets $x_\pm^*$ and $y_\pm^*$ at the IP:

$$\delta z^* \equiv (\delta x^* , \delta y^*) \equiv (x_+^* - x_-^* , y_+^* - y_-^*) . \tag{1}$$

The luminosity for Gaussian bunches colliding at negligible collision angles but with a non vanishing impact parameter is reduced according to

$$L(\delta z^*) = \underbrace{k_{\mathrm{B}} \, f_{\mathrm{rev}} \, L_{\mathrm{X}}}_{\hat{L}} \cdot \exp\left[-\frac{1}{2}\left[\left(\frac{\delta x^*}{\Sigma_{\mathrm{x}}}\right)^2 + \left(\frac{\delta y^*}{\Sigma_{\mathrm{y}}}\right)^2\right]\right] \tag{2}$$

where the luminosity per interaction is

$$L_{\mathrm{X}} = \frac{N_+ N_-}{2\pi\,\Sigma_{\mathrm{x}}\Sigma_{\mathrm{y}}} = \frac{N_+ N_-}{2\pi\,R\,\Sigma_{\mathrm{x}}^2} \tag{3}$$

and the following notations are used:

$$\Sigma_{\mathrm{z}} = \sqrt{\sigma_{\mathrm{z}+}^{*2} + \sigma_{\mathrm{z}-}^{*2}} \;,\;\; R = \frac{\Sigma_{\mathrm{y}}}{\Sigma_{\mathrm{x}}} = \sqrt{\frac{r_+^2 + a\,r_-^2}{1+a}} \;,\;\; r_{\pm} = \left(\frac{\sigma_{\mathrm{y}}^*}{\sigma_{\mathrm{x}}^*}\right)_{\pm} \;,\;\; a = \frac{\sigma_{\mathrm{x}-}^*}{\sigma_{\mathrm{x}+}^*}\;. \tag{4}$$

For **equal bunch sizes** Equ.(2) and (3) simplify as

$$L(\tilde{z}) = \hat{L}\cdot\exp\left(-\frac{\tilde{x}^2 + \tilde{y}^2}{4}\right) \quad,\quad L_{\mathrm{X}} = \frac{N_+\,N_-}{4\pi\,r\,\sigma_{\mathrm{x}}^{*2}} = \frac{N_+\,N_-}{4\pi\left(\varepsilon_{\mathrm{x}}\sqrt{\kappa\,\beta_{\mathrm{x}}^*\,\beta_{\mathrm{y}}^*}\right)_{\pm}}\;. \tag{5}$$

The dimensionless quantities $(\tilde{x}\,,\,\tilde{y})$ are the components of the collision offset $\tilde{z}$ normalised to the transverse rms dimensions $\sigma_{\mathrm{x}}^*, \sigma_{\mathrm{y}}^*$ at each IP

$$\tilde{z} \equiv (\tilde{x}\,,\,\tilde{y}) = \left(\frac{\delta x^*}{\sigma_{\mathrm{x}}^*}, \frac{\delta y^*}{\sigma_{\mathrm{y}}^*}\right) = \frac{\delta z^*}{\sigma_{\mathrm{z}}^*} \tag{6}$$

while the aspect ratio r and the coupling factor $\kappa_{\pm}$ have the known expressions:

$$r = \left(\frac{\sigma_{\mathrm{y}}^*}{\sigma_{\mathrm{x}}^*}\right) = \sqrt{\left(\kappa\,\frac{\beta_{\mathrm{y}}^*}{\beta_{\mathrm{x}}^*}\right)_{\pm}} \quad,\quad \kappa_{\pm} = \left(\frac{\varepsilon_{\mathrm{y}}}{\varepsilon_{\mathrm{x}}}\right)_{\pm}\;. \tag{7}$$

For interaction regions operated in flat beams configurations ($r \ll 1$) small amplitude vertical collision offsets can produce a substantial luminosity reduction due to geometrical (offset) and optical (beam blow-up) effects. For negligible horizontal overlap and blow-up Equ.(5) gives

$$\left(\frac{\Delta L}{\hat{L}}\right)_{\tilde{x}=0} \simeq \begin{cases} -3\% & (\tilde{y} = 1/3) \\ -6\% & (\tilde{y} = 1/2) \\ -22\% & (\tilde{y} = 1) \end{cases} \tag{8}$$

and a non-invasive control of the impact parameter largely contributes to the operational efficiency.

3 Collision optimisation with "Vernier" scans

Large amplitude scans of the impact parameter can be performed to optimise the luminosity at a given IP when operating not too close to the beam-beam limit.

An example is shown in Fig.1 on the use of the "Vernier" scan technique[15] at LEP to compensate collision offsets generated by the vertical beam separation required to avoid unwanted collisions outside the IP in bunch-train operation. The scan is performed at IP2 during a three-bunch per train operation. The maxima of the luminosity measured for each colliding family as a function of the electrostatic bump amplitude define the optimum separator setting at that point. Some coupling to the other interaction points is clearly visible. Besides possible miscrossing from a non perfect closure of the scanning bumps, vertical blow-up confirmed by correlated beam sizes observations mainly contributes to the luminosity loss. The vertical beam blow up inferred from the behaviour of the luminosities at the non-scanned IPs is shown in the fifth picture.

4 Beam-beam deflections

The motion of a *test particle* traversing a Gaussian charge distribution at an offset is expressed in closed form in[16] and revisited in [17][18]. The transverse motion of the charge distribution *centroids*, addressed in[2][7][19][20][21] is generalised to non-Gaussian distributions in[22].

In the Linear Rigid Gaussian Model (LRGM)[20] and for $r \ll 1$ the vertical deflections of the centroids of two Gaussian distributions interacting at a normalised offset $\tilde{z}$ (6) are given by:

$$\Theta_{y\pm}^* = \hat{\Theta}_{\pm}^* \cdot \Re \left[w\left(\frac{\tilde{x} + \mathrm{i}\, r\, \tilde{y}}{d} \right) - \exp\left[-\frac{1}{2}(\tilde{x}^2 - \tilde{y}^2) \right] \cdot w\left(\frac{r\, \tilde{x} + \mathrm{i}\, \tilde{y}}{d} \right) \right] \qquad (9)$$

where w is the complex error function

$$w(z) = e^{-z^2} \left[1 + \frac{2\mathrm{i}}{\sqrt{\pi}} \int_0^z e^{t^2}\, dt \right] = e^{-z^2} \left[1 - \mathrm{erf}(-\mathrm{i}\, z) \right] \ . \qquad (10)$$

The parameter $d = \sqrt{2\,(1 - r^2)}$ accounts for the ellipticity of the electro-magnetic fields (it vanishes for round beams). The kick in the horizontal plane is given by the Imaginary part of (9).
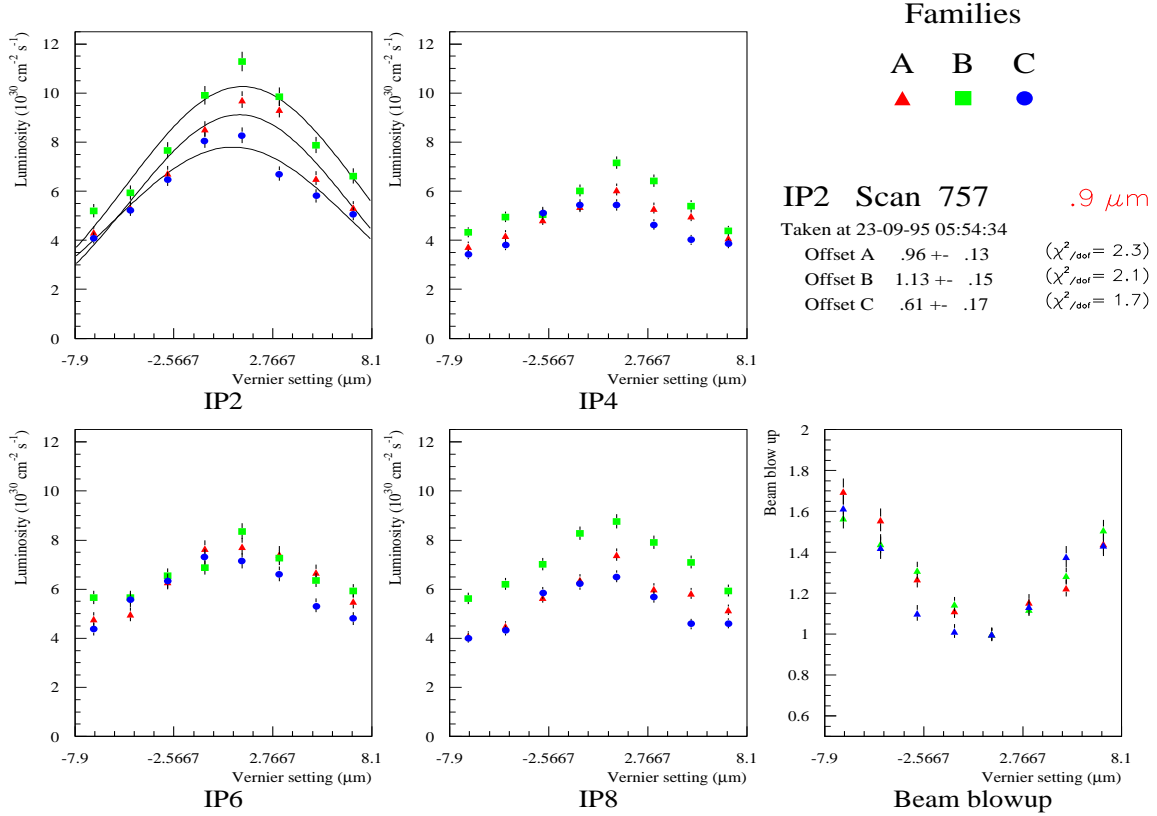
Figure 1: "Vernier" scans performed at the LEP interaction point IP2 to determine the separator setting for optimum overlap in the three-families per train operation mode. The $0.9\,\mu m$ vernier setting is determined from a fit to the luminosity measured for each family. The vertical beam blow up inferred from the luminosity loss at the three non-scanned IPs is shown in the fifth picture.

In the general case of unequal bunch energies and intensities the maximum *centroid deflections* $\hat{\Theta}^*_\pm$ depend on the energy $\gamma_\pm$ of the *deflected* bunch and the population $N_\mp$ of the *deflecting* bunch:

$$|\,\hat{\Theta}^*_\pm\,| = \frac{r_e\sqrt{2\pi}}{\sqrt{\Sigma_x^2 - \Sigma_y^2}}\left(\frac{N_\mp}{\gamma_\pm}\right) \simeq \frac{r_e\sqrt{2\pi}}{\Sigma_x}\left(\frac{N_\mp}{\gamma_\pm}\right) \tag{11}$$

where $r_e \equiv e^2/m_e c^2 = 2.817\,941 \times 10^{-15}$m is the classical electron radius.

The approximation, valid for flat beams ($r \ll 1$), allows for the determination of the convoluted horizontal beam sizes Σ_x **at the interaction** from a measurement of the **maxima** (11) at large bunch separations ($\tilde{x}, \tilde{y} \sim 2.5$).

Another interesting application is provided by the **slopes** of Equ.(9) for impact parameters comparable with the transverse rms beam sizes ($\tilde{x}$, $\tilde{y} \leq 1$)

$$S_{z\pm}^{\mathrm{bbd}} \equiv \left(\frac{\partial\,\Theta_{z\pm}^*}{\partial\,\delta z^*}\right) = \mp 4\pi\,\left(\frac{\Xi_z}{\beta_z^*}\right)_{\pm} \tag{12}$$

directly proportional to the ratio between the coherent beam-beam parameter

$$\Xi_{z\pm} = \frac{r_e}{2\pi}\,\frac{\beta_{z\pm}^*}{\Sigma_z(\Sigma_x + \Sigma_y)}\,\left(\frac{N_\mp}{\gamma_\pm}\right) \tag{13}$$

and the β-functions at the interaction.

A nice example of simultaneous measurement of the deflection slopes experienced by two colliding beams at the SLC is given in[8].

4.1 *BBD* in the vertical plane

As the coherent beam-beam parameter for **equal bunch sizes in the LRGM assumptions** is half the incoherent one

$$\Xi_{z\pm} = \frac{\xi_{z\pm}}{2} = \frac{r_e}{4\pi}\,\frac{\beta_{z\pm}^*}{\sigma_z^*(\sigma_x^* + \sigma_y^*)}\,\left(\frac{N_\mp}{\gamma_\pm}\right) \tag{14}$$

the *BBD* slopes (12) specified in the vertical plane read:

$$S_{y\pm}^{\mathrm{bbd}} = \mp 2\pi\,\left(\frac{\xi_y}{\beta_y^*}\right)_{\pm} = \mp\frac{r_e}{1+r}\,\left(\frac{N_\mp}{\gamma_\pm}\right)\frac{1}{\left(\varepsilon_x\,\sqrt{\kappa\,\beta_x^*\beta_y^*}\right)_{\pm}}\;. \tag{15}$$

In principle different for the two colliding bunches according to their populations and energies, they provide a measurement of the emittances and coupling factors when associated to the maxima (11) once the β-values at the interaction are known.

More directly Equ.(15) provides an independent measurement of the luminosity

$$L = \frac{k_B\,f_{\mathrm{rev}}\,(1+r)}{2\,r_e}\,(N\gamma)_\pm\,\left(\frac{\xi_y}{\beta_y^*}\right)_{\pm} = \frac{k_B\,f_{\mathrm{rev}}\,(1+r)}{4\pi\,r_e}\,(N\gamma)_\pm\,|S_{y\pm}^{\mathrm{bbd}}|\;. \tag{16}$$

without requiring the knowledge of the dynamic β_y^* at the interaction.

4.2 *BBD* measurements at LEP

BBD in the vertical plane are routinely performed at the LEP collider to compensate residual collision offsets from electrostatic separation required to avoid parasitic collisions outside the IPs in the 8-on-8 bunch operation at the Z^0 resonance[12].
Beam-beam deflections are inferred from measurements of the beam excursions at suitable monitors either side of the IP while varying the impact parameter with vertical electrostatic closed bumps. Evaluating the **difference** between the opposite sign deflections to enhance the signal provides the double slope

$$\mathcal{S}_y^{\text{bbd}} = S_{y+}^{\text{bbd}} - S_{y-}^{\text{bbd}} = \mp 4\pi \left(\frac{\Xi_{y+}}{\beta_{y+}^*} + \frac{\Xi_{y-}}{\beta_{y-}^*} \right) \simeq \mp 4\pi \frac{\langle \xi_y \rangle}{\beta_y^*} \tag{17}$$

where

$$\langle \xi_y \rangle = \frac{\xi_{y+} + \xi_{y-}}{2} \simeq \Xi_{y+} + \Xi_{y-} \ . \tag{18}$$

This notation, equivalent to those in[2][7][9], holds for symmetric rings with equal β^*-values and when the beams experience similar tune shifts. For asymmetric rings the beam-beam parameter should be measured for each beam from Equ.(12). At LEP, the luminosity, tune shifts and rms beam sizes are extracted from a fit to the complete scan data. *BBD* measurements at two IPs are shown in Fig.2 as a function of the amplitude of the centroids separation. In particular the luminosity values agree with a numerical estimate using the general expression (16) with the double slope (17). Considering the different scales the slope at IP6 is about twice that at IP2 in agreement with the bunch intensities. This reflects into the average tune shift and luminosity values associated to the measurements.

5 Coherent tune split

Originally investigated at SLAC[23] the beam-beam coupling is at the basis of the performance of any storage ring. Normal modes of coherent oscillations are generated at each bunch collision (π-modes) in addition to the fundamental σ-mode defined by the focusing sequence of the ring magnetic structure. In the unperturbed σ-mode the bunches oscillate in phase at each IP while the π-modes exhibit relative bunch motion. The associated eigenfrequencies spread over a wide range depending on various parameters like the number N_i of interactions per revolution, the shape of transverse distributions, the amplitude of the impact parameter and the bunch populations.

IP 2

$L = 2.8\ 10^{30}\ cm^{-2}s^{-1}$

IP 6

$L = 10.2\ 10^{30}\ cm^{-2}s^{-1}$

$\Delta y_{opt} = -5.6 +- 0.1\ \mu m$
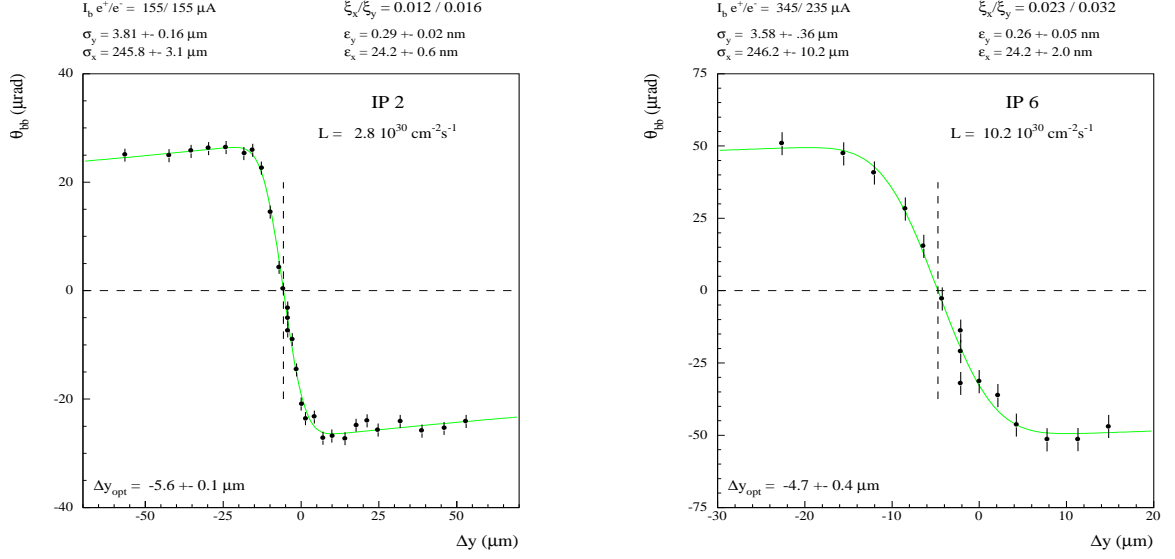
$\Delta y_{opt} = -4.7 +- 0.4\ \mu m$

Figure 2: *Example of BBD scan in the vertical plane at LEP in two different fills. The slope of the measurement with higher intensities (IP6) is about twice the other one (IP2). The luminosity, tune shifts and rms beam sizes are extracted from a fit to the complete scan data. The deflections Θ_{bb} are the algebraic sums of each beam deflection and the fitted luminosity values agree with the use of Equ.(16) with the double slope (17). The zero-crossing of the fit determines the separator settings Δy_{opt} for optimum head-on collision.*

In the simplified case of only one bunch per beam and one interaction per revolution, the relation between tune split and beam-beam parameter reads

$$\cos 2\pi(Q_z^\sigma + \Delta Q_z^{\sigma\pi}) \equiv \cos 2\pi Q_z^\pi = \cos 2\pi Q_z^\sigma - 4\pi\, \Xi_z \sin 2\pi Q_z^\sigma \tag{19}$$

and for $\Xi_z \ll 1$ the tune split is

$$\Delta Q_z^{\sigma\pi} \equiv q_z^\pi - q_z^\sigma \simeq 2\, \Xi_z \ . \tag{20}$$

For **non-rigid equal bunch sizes** Equ.(14) modifies into

$$\Xi_z = \lambda_z(N_i, N_\pm)\, \frac{\xi_z}{2} \tag{21}$$

where the factor

$$\lambda_z(N_i, N_\pm) = \frac{\Delta Q_z^{\sigma\pi}}{\xi_z} \tag{22}$$

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accounts for deviations of the beam transverse dimensions at the interaction from the LRG model. It makes the visible tune difference $\Delta Q_z^{\sigma\pi}$ larger than the LRGM beam-beam parameter ξ_z [13][14][24][25]. Its theoretical value is in the range between $\lambda_z = 2$ (Piwinski[26]) and $\lambda_z = 1$ (Hirata[19], Hofmann-Myers[27]) depending on the model assumed for the transverse distributions of the interacting bunches.

When the beams are separated at the IP the horizontal beam-beam parameter ξ_x is much less reduced than ξ_y as the separation represents a small fraction of the horizontal size. The correction factor (22) is then different in the two planes, according to the aspect ratio and the β-functions at the interaction, which are in turn modified from the nominal value according to the strength of the interaction itself. Furthermore, the factor λ_z has been evaluated for head-on collisions but not for non-zero impact parameters.

5.1   A possible experiment

The above considerations make an experimental investigation on the behaviour of the λ_z parameter quite attractive and enlightening both for flat and round beams.

Experimental approaches have been proposed for LEP[28] and adopted in other storage rings[29]. We suggest a method to measure the Meller-Yokoya parameter (22) by monitoring the $\sigma - \pi$ tune shift and independently derive the beam-beam parameters from the BBD slope (17) and the luminosity (16). If care is taken to collide equal bunch charges the average tune shift $\langle \xi_y \rangle$ from a measurement of the luminosity is a realistic representation of the single bunch tune shift.

The existing facilities for BBD and luminosity measurements, associated to the recently commissioned $\sigma - \pi$ mode on-line detection[30], make this experiment feasible at LEP. The factor (22) should be studied in different operational conditions i.e. as a function of the number of interactions, the bunch intensity and the impact parameter.

6   Collision feedback

The amplitude at position s of the COD generated by a localised kick $\Theta_{z\pm}^*$ associated to a non-zero collision offset δz^* is, from (12)

$$z_\pm(s) = \mp 2\pi \sqrt{\frac{\beta_{z\pm}(s)}{\beta_{z\pm}^*}} \; \frac{\cos\left(|\Delta\mu_{z\pm}(s)| - \pi Q_{z\pm}\right)}{\sin\left(\pi Q_{z\pm}\right)} \cdot \Xi_{z\pm} \cdot \delta z^* \qquad (23)$$

where $\Delta\mu_{z\pm}(s)$ is the phase advances from the IP to the observation point.

The residual centroid offset at the IP in units of the impact parameter

$$\tilde{z}^* = \frac{z^*_\pm}{\delta z^*} = \mp 2\pi \; \Xi_{z\pm} \; \cot(\pi Q_{z\pm}) \tag{24}$$

can be made very small by a proper choice of the working point ($\tilde{z}^* \sim \pm 0.07$ for the LEP 1998 optics). Recording the *COD* amplitude (23) at some selected BPMs provides a signature of the orbit perturbation induced by unwanted collision offsets. Exploiting the symmetry of the *BBD*-driven *COD* w.r.t. the IP and the asymmetry w.r.t. the charge, the sensitivity can be improved by monitoring the sum of the orbit differences (23) at suitable position monitors BPM^j right and left of each IP:

$$\Delta z^j = \left| (z^j_{L+} - z^j_{L-}) \right| + \left| (z^j_{R+} - z^j_{R-}) \right| \; . \tag{25}$$

The BPM offsets are eliminated when the excursions $z^j_{RL\pm}$ are measured as orbit differences w.r.t. an unperturbed one defined either by $\delta z^* = 0$ in a dynamic collision scan or by a **non-colliding bunch**.

7 Outlook

A review of methods suitable to a diagnostics program at the interaction regions of circular symmetric and asymmetric colliders has been presented. Considerations on experimental applications at the LEP collider have been extended aiming at possible applications to asymmetric rings expected to be operational in the future.

References

[1] V. Ziemann, "The Accuracy of Beam-Beam Diagnostics for Circular Colliders"' Proc. Conf. on B-Factories, SLAC-400, Stanford, CA, 6-10 April, 1992.

[2] M. Furman *et al.*, "Closed Orbit Distortion and the Beam-Beam Interaction", SLAC-ABC-49, June 1992.

[3] M. Furman *et al.*, "Beam-Beam Diagnostics from Closed Orbit Distortion", Proc. $15^{t}h$ Int. Conf. on H. E. Accelerators, Hamburg, Germany, July 20-24, 1992.

[4] P.S. Bambade, "Beam Dynamics in the SLC Final Focus System", SLAC-PUB-4227, June 1987.

[5] P.S. Bambade *et al.*, "Observation of beam-beam deflections at the interaction point of the SLAC Linear Collider", Phys. Rev. Lett. **62** (1989) 2949.

[6] W. Koska *et al.*, "Beam-beam deflection as a beam tuning tool at the SLAC linear collider", Nucl. Instr. Meth. **A286** (1990) 32.

[7] P. Raimondi and F.J. Decker, "Flat Beam Spot Sizes Measurement in the SLC Final Focus", Proc. PAC'95, Dallas, Texas, May 1-5, 1995.

[8] F. Rouse *et al.*, "Maintaining micron-size beams in collision at the interaction point of the Stanford Linear Collider", SLAC-PUB-5512 and Proc. PAC'91, San Francisco, CA, May 6-9, 1991.

[9] E. Young, "Observations of the Effects of the Beam-Beam Interaction on the Orbits of Stored Beams in CESR"' Proc. PAC'95, Dallas, Texas, May 1-5, 1995.

[10] Y. Funakoshi *et al.*, "Orbit Feedback System in the KEK B Factory", Proc. Conf. on B-Factories, SLAC-400, Stanford, CA, 6-10 April, 1992.

[11] J. Wenninger, "Measurement of Beam-Beam Deflections at the Interaction Points of LEP", SL Note 96-01 (OP), Jan. 1996.

[12] C. Bovet *et al.*, "Luminosity Optimisation using Beam-Beam Deflections at LEP", Proc. EPAC'96, Sitges, Spain, June 1996.

[13] R. E. Meller and R. H. Siemann, "Coherent Normal Modes of Colliding Beams", IEEE Trans. NS-28, 2431 (1981).

[14] K. Yokoya *et al.*, "Tune Shift of Coherent Beam-Beam Oscillations", Part. Acc., **27**, 181, (1990).

[15] M. Böge *et al.*, "Measurements of Collision Offsets and Difference in Vertical Dispersion at the LEP Interaction Points" Proc. EPAC'96, Sitges, Spain, June 1996.

[16] M. Bassetti and G.A. Erskine, "Closed Expression for the Electrical Field of a Two-dimensional Gaussian Charge", CERN-ISR-TH/80-06, March 1980.

[17] Y. Okomoto and R. Talman, "Rational approximation of the complex error function and the electric field of a 2D Gaussian charge distribution", AIP Conference Proc. 153 (1985) 789.

[18] R. Talman, "Multiparticle Phenomena and Landau Damping", Cornell Report CBN 80-13 (1980).

[19] K. Hirata, "Coherent Betatron Oscillation Modes due to Beam-Beam Interaction", Nucl. Instr. Meth. **A269** (1988) 7.

[20] E. Keil, "Beam-Beam Dynamics", CERN SL/94-78 (AP), Sept. 1994.

[21] K. Hirata and E. Keil, "Linear Beam-Beam Resonances due to Coherent Dipole Motion", CERN SL/95-117 (AP), Dec. 1995.

[22] V. Ziemann, "Beyond Bassetti and Erskine: Beam-Beam Deflections for non-Gaussian Beams", SLAC-PUB-5582, June 1991.

[23] The SPEAR Group. "Beam-Beam Coupling in SPEAR", Proc. IXth Int. Conference on H.E. Accelerators, Stanford, CA, May 2-7, 1974.

[24] R. Talman, "Measurement, Analysis and Modification of Multiparticle Phenomena in Accelerators", CNLS-84/610, May 1984, Cornell University and Proc. AIP Fermilab Conference, 1984.

[25] Y.I. Alexahin, "On the Landau Damping and Decoherence of Transverse Dipole Oscillations in Colliding Beams", CERN-SL-96-64 AP, August 1996.

[26] A. Piwinski, "Observations of Beam-Beam Effects in PETRA", IEEE Trans. NS-26, 4268 (1979).

[27] A. Hofmann and S. Myers, "Evaluation of the Absolute LEP Luminosity from Measurement of the Coherent Beam-Beam Tune Split", LEP Note 604, April 1988.

[28] R. Littauer and M. Placidi, "Beam-Beam Modes in Q-monitoring", LEP Note 555, Feb. 1986.

[29] E. Young *et al.*, "Collisions of Resonantly Coupled Round Beams at the Cornell Electron-Positron Storage Ring (CESR)", Proc. PAC 97, Vancouver, 1997.

[30] O. Berrig *et al.*, "Real-time monitoring of beam-beam modes at LEP", Contributed to EPAC 98, Stokholm, 22-26 June 1998.