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# Resonant production of gamma rays in jolted cold neutron stars

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## Abstract

Acoustic shock waves passing through colliding cold neutron stars can cause repetitive superconducting phase transitions in which the proton condensate relaxes to its equilibrium value via coherent oscillations. As a result, a resonant non-thermal production of gamma rays in the MeV energy range with power up to  $10^{52\pm 1}$  erg/s can take place during the short period of time *before* the nuclear matter is heated by the shock waves.

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The recent discovery of the afterglow associated with the gamma ray bursts (GRB) [1] has provided convincing evidence that the GRB originate at redshift  $z \sim 1$  from an event that releases of order  $10^{52}$  erg in photons (assuming the spherical symmetry of emission). A number of theoretical attempts to explain the phenomenon invoked a small energetic fireball [2]. Although many astrophysical events, for instance, neutron star collisions, can release the required amount of energy, it is extremely difficult to reconcile the optical thickness of a fireball with the short duration and the high energy of the gamma ray burst it is supposed to produce. In this Letter we describe a new mechanism for the gamma-ray production by the colliding cold neutron stars prior to the eruption of a fireball. We comment on the possible connection of this phenomenon with the observed GRB.

We will show that powerful acoustic shock waves passing through a neutron star can cause repetitive superconducting phase transitions during which the relaxation of the order parameter is accompanied by a resonant non-thermal production of photons with energies of order MeV and the total power up to  $10^{52\pm 1}$  erg  $s^{-1}$ . These gamma-rays are copiously produced due to a coherent motion of the superconducting proton condensate. Such oscillations can be powered by the density waves generated, for example, in the collision of two neutron stars. The production of gamma rays continues until the medium is heated to the temperature comparable to  $T_c$ , the superconducting phase transition temperature.

When two neutron stars collide, the kinetic energy is transferred into the shock waves that propagate through the nuclear matter and eventually dissipate their energy into heat. The density variations in a relativistic shock wave are much slower than the time scale of nuclear interactions. The interior of the neutron star is a superconductor because it contains the superfluid proton condensate [3]. An acoustic wave excited by the initial impact of a collision, or by some precursory tidal effects, causes large variations of density in some macroscopic domains of the size of order the wavelength. The proton energy gap depends on the density [4] as shown in Fig. 1; the shock wave can drive it to zero in some formerly superconducting regions, or vice versa. Suppose a surge (or an ebb) of density

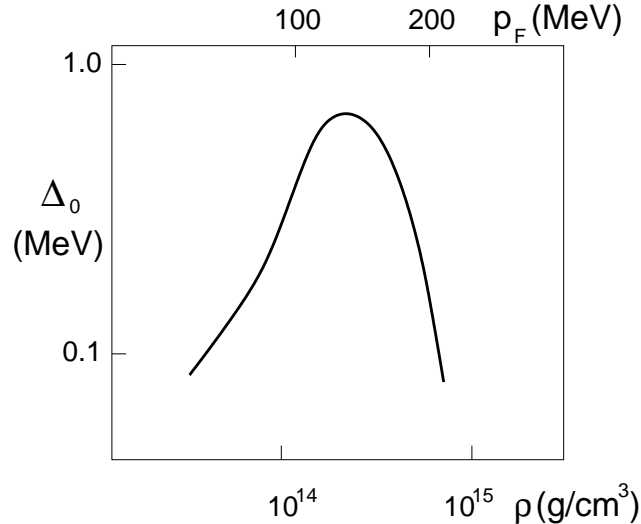


Figure 1: Proton energy gap  $\Delta_0$  at zero temperature as a function of density  $\rho$  and the Fermi momentum  $p_F$  of protons; based on Ref. [4].

destroyed superconductivity in some macroscopic region of the star. At a later time, as the sound wave moves on, the original density is restored and the phase transition to a superconducting phase takes place. The proton condensate  $\phi$  changes from zero to the value  $\phi_0$ , which minimizes the free energy. This relaxation process is described by the generalized, time-dependent Ginzburg-Landau (GL) theory [5]. In nuclear matter  $T_c \sim 0.5$  MeV. At low temperature  $T \ll T_c$ ,  $\phi$  oscillates until it settles in the minimum (Fig. 2).

Coherent oscillations of the order parameter result in a non-thermal production of electromagnetic radiation. We will see that such emission is very effective because it occurs through a parametric resonance that produces some very large occupation numbers of photons in the MeV energy band. A resonant emission of Bose particles by a coherently oscillating scalar field has been studied intensely in quantum field theory, in particular, in application to cosmology [6]. We will adopt a very similar approach to a semiclassical description of the gamma-ray emission via parametric resonance.

Eventually, the star is heated to the temperature above  $T_c$  by the acoustic shock waves. At this time the proton condensate is destroyed, the resonant emission of photons ceases, and the fireball erupts.

Now we estimate the energy associated with the coherent motion of the proton condensate. As shown in Fig. 1, the energy gap depends sharply on density. Let us consider a propagation of an (ultra-relativistic [7]) acoustic shock wave of sufficiently high amplitude through the interior of the neutron star. Density waves are characterized by the time scale  $\tau_a \sim 10^{-7} \dots 10^{-3}$  s [8], and are very slow in comparison to the relaxation time of the proton condensate which is of order MeV, or  $10^{-20}$  s. The surge of density in the acoustic wave converts the protons from the superconducting to the normal state. As the density subsides, the superconducting transition takes place. The energy change associated with proton condensation in the volume  $V$  is [9]

$$\delta E = V \frac{m_* p_F}{4\pi^2} \Delta_0^2, \quad (1)$$

where  $m_*$  is the effective proton mass in nuclear matter,  $m_* \approx 0.6m_p$  [4];  $\Delta_0$  is the proton energy gap; and  $p_F$  is the Fermi momentum. For density  $\rho \sim 10^{14} \dots 10^{15}$  g cm $^{-3}$ ,  $p_F \sim 10^2$  MeV and  $\Delta_0 \lesssim 0.8$  MeV [4]. This yields  $\delta E/V \sim 10^3 (\text{MeV})^4$ .

The proton condensate must dissipate energy  $\delta E$  in each cycle of the acoustic ‘‘pumping’’ that repeat after time  $\tau_a$ . Therefore, the power dissipated by the condensate in volume  $V \sim (4\pi/3)R^3$ , where  $R \approx 10$  km is the radius of a neutron star, is

$$P = \frac{\delta E}{\tau_a} \approx \left( \frac{0.1 \mu\text{s}}{\tau_a} \right) 10^{52} \text{ erg s}^{-1}. \quad (2)$$

Analysis of the oscillation spectra of the neutron stars [8] gives the range of values for the periods of various eigenmodes:

$$10^{-2} \mu\text{s} \lesssim \tau_a \lesssim 0.1\text{s} \quad (3)$$

for magnetic field  $\sim 10^{12}$  G. A larger magnetic field would give rise to modes with shorter periods  $\tau_a$ . Clearly, the higher-frequency modes provide more frequent pumping, forcing the condensate to dissipate more energy.

If a sizeable fraction of the energy (2) stored in the motion of the proton condensate at the onset of a phase transition is radiated away with gamma rays, the power of the

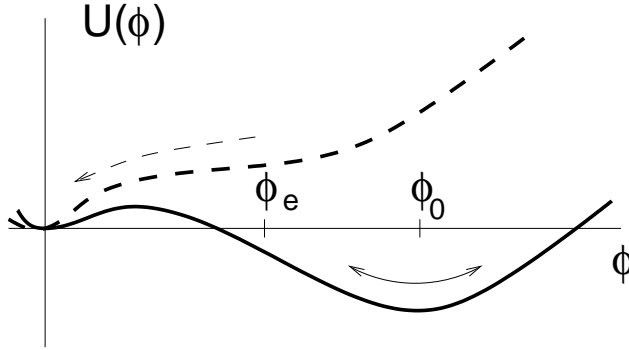


Figure 2: After the phase transition, the order parameter oscillates around the minimum of the effective potential.

resulting signal is consistent with that of observed GRB for the range of  $\tau_a$  given by (3). We will now describe the resonant emission of photons by the condensate.

Superconducting condensed-matter systems out of equilibrium display a number of interesting phenomena related to time dependence of the order parameter. However, in the case of a neutron star, the specific relative values of parameters, the MeV energy scales, and the sharp dependence of the energy gap on density make it difficult to find a close condensed-matter analogue. Nevertheless, one can use the general formalism developed for nonequilibrium superconductivity.

In the presence of a magnetic field, the phase transition is first order [3]; it proceeds through nucleation of bubbles that grow and coalesce. This process is described by a time and coordinate dependent GL order parameter  $\phi(x, t)$  [5]. For simplicity, we will neglect the effects of magnetic field on the motion of the proton condensate after the nucleation of the initial bubbles, whose  $t = 0$  profile,  $\phi(x, 0)$ , serves as the initial condition for the equation of motion (4) written below. This is a good approximation as long as the magnetic field  $H$  is well below the critical value  $H_c$ . If the potential barrier (Fig. 2) is much smaller than  $\delta E/V$  in equation (1) (which again requires  $H \ll H_c$ ), the maximal value of the field inside the critical bubble,  $\phi_e \equiv \max_x \phi(x, 0)$  is not close to  $\phi_0$ . The subsequent relaxation of the order parameter  $\phi$  (assumed to be real) is described by a partial differential equation [5]

$$\ddot{\phi} + \frac{8\varepsilon_F}{3c}\dot{\phi} - \frac{2\varepsilon_F}{3cm_*}\nabla^2\phi + \frac{\partial U(\phi)}{\partial\phi} = 0. \quad (4)$$

Here  $\phi(x, t) = (n_p/2m_*)^{1/2} \left( \frac{\Delta(x, t)}{\Delta_0} \right)$  is the order parameter proportional to the energy gap  $\Delta(x, t)$ ;  $\Delta_0$  is the equilibrium value (Fig. 1) that corresponds to  $\phi_0 = (n_p/2m_*)^{1/2} \approx 20$  MeV. (Note that we have included the proton mass  $m_*$  in the normalization of  $\phi$ , so as to make its dimension one in mass units; the GL order parameter is often normalized differently [5, 9], in which case  $m_*$  reappears in the gauge coupling.)  $\varepsilon_F$  is the Fermi energy;  $c = (28\zeta(3)/3\pi^3)\varepsilon_F/T_c$ ;  $n_p = Y_p\rho/m_p$  is the total number density of protons, which make up a fraction  $Y_p \approx 0.03$  of all baryons. The GL potential can be written as  $U(\phi) = -a\phi^2 + (b/2)\phi^4$ .

According to Ref. [5], equation (4) is valid for  $T \ll T_c$ , which is the regime we are considering. The same equation is also valid [5] for  $T \approx T_c$  if the characteristic frequency  $\omega$  of time variations of the order parameter is greater than the energy gap,  $\omega > \Delta_0$ . This condition is automatically satisfied for nuclear matter, where we find  $\omega \sim 3$  MeV, while  $\Delta_0 < 0.8$  MeV (Fig. 1).

Equation (4) has a solution  $\phi \propto \sin(\omega t)$  oscillating around the minimum of the effective potential  $U(\phi)$ . These oscillations are damped by the “friction” term in equation (4) that contains  $(\dot{\phi})$ . The effect of damping depends on the relative magnitudes of  $8\varepsilon_F/3c \approx 7.4T_c \approx 4.2\Delta_0$  and the frequency of oscillations  $\omega$ . The Fermi energy  $\varepsilon_F = p_F^2/2m_* \approx 0.2$  to 20 MeV for  $\rho = 10^{14}$  to  $10^{15}$  g cm<sup>-3</sup>. The oscillation frequency is determined by the effective mass  $\omega = \sqrt{U''(\phi_0)}$  around the minimum  $\phi_0 = \sqrt{a/b}$  of  $U(\phi)$ . Requiring that  $U(\phi_0) = -\delta E/V$  from equation (1), one obtains  $\omega = \Delta_0\sqrt{2m_*p_F}/\pi\phi_0 \approx 3$ –5 MeV for  $\Delta_0 = 0.4$ –0.8 MeV. Since  $(8\varepsilon_F/3c)/\omega \approx 0.6$ , neither the first, nor the second term in equation (4) can be neglected in our case.

A coherent state of photons is described by the electromagnetic field  $A_\mu$ . The Cooper pair of protons has charge  $2e$ . In the unitary gauge, in which the scalar field  $\phi$  is real, the coupling of the gauge field to  $\phi$  is  $\mathcal{L}_{int} = (2e)^2\phi^2(A_\mu)^2$ .

When  $\phi$  oscillates around the minimum, the effective photon mass,  $\sqrt{2(2e)^2\phi^2}$ , is

time-dependent. This causes a parametric resonance manifest in the appearance of exponentially growing solutions for the gauge field  $A_\mu$  interpreted as a copious non-thermal particle production (*cf.* Ref. [6]). We will assume a simple form for the oscillations of the field  $\phi$  around the minimum,  $\phi = \phi_0 + \Phi \sin(\omega t)$ ,  $\Phi \ll \phi_0 \approx 20$  MeV, and neglect the back-reaction of photons on the motion of  $\phi$ . To see which modes are amplified, we consider a Fourier decomposition of  $A_\mu(x, t)$ . The equation of motion for a mode  $A_\mu^{(k)}(t)$  with the wavenumber  $k$  is

$$\frac{\partial^2}{\partial t^2} A_\mu^{(k)} + [k^2 + 2(2e)^2 \phi_0^2 + 4(2e)^2 \phi_0 \Phi \sin(\omega t)] A_\mu^{(k)} = 0. \quad (5)$$

The Mathieu equation (5) has exponentially growing solutions  $A^{(k)} \propto \exp\{\mu_k \omega t/2\}$  that describe a copious production of the gamma-quanta with momenta in some spectral band. The stability chart (see, *e. g.*, Ref. [10]) is used to determine the values of the momenta for which the resonant production takes place. The corresponding instability band is usually specified [10] in terms of a parameter  $a_k \equiv 4[(k^2 + 8e^2 \phi_0^2)/\omega^2]$ . If the parameter  $q \equiv 32e^2 \phi_0 \Phi/\omega^2$  is small, only a narrow band around  $a_k = l^2$ , where  $l$  is some integer number, is in resonance; the best-amplified mode has  $k = \omega/2$  and  $\mu_k = q/2$ . If, however,  $q > 1$ , the parametric resonance is broad. A detailed discussion of particle production through the parametric resonance can be found in Ref. [6].

For the proton condensate inside a neutron star,  $\phi_0 \sim 20$  MeV and  $\omega \sim 5$  MeV. If the amplitude of oscillations  $\Phi$  is between 1 and 10 MeV, then  $q \approx 2$ –20, respectively. This corresponds to a broad and, hence, very efficient resonance. The exponentially growing modes satisfy the resonance condition

$$k^2 = \frac{1}{4} \omega^2 a_k - 8e^2 \phi_0^2, \quad (6)$$

where the resonant values of  $a_k$  and the corresponding exponents  $\mu_k$  can be read off the instability chart [10]. For the ranges of the parameters quoted above, a broad band is in resonance; the exponents  $\mu_k$  can take relatively large values. This signals a copious production of gamma rays, whose occupation numbers increase exponentially already

during the first few oscillations of the field  $\phi$ . As the amplitude of the oscillations subsides, the system enters into a narrow-resonance regime, in which only the modes around  $k = \omega/2 \sim$  a few MeV are further amplified. Given the efficiency of the resonance and the absence of thermal excitations at  $T \ll T_c$  that could dissipate the energy stored in the condensate, it is reasonable to assume that a substantial fraction of the power  $P$  in equation (2) is transferred to photons.

Because the electrons are highly degenerate, the gamma quanta cannot decay via the pair production  $\gamma \rightarrow e^+e^-$  unless the final-state electrons have energy in excess of the Fermi momentum  $p_F \gtrsim 100$  MeV. Therefore, a decay of the MeV-energy gamma photons is forbidden by the Pauli exclusion principle (applied to the final-state electrons). Compton scattering off the electrons and protons near the Fermi surface is kinematically suppressed but is not forbidden. It increases the temperature of the electrons and makes the spectrum of the gamma rays somewhat more thermal by the time they reach the surface of the star. Comptonization preserves the number of photons. It can cause some de-coherence effects that take place over the distances of order  $10^{-9}$  cm, much greater than the wavelength. A coherent wave-packet passing through neighbouring regions with oscillating scalar condensate may undergo further amplification. The thin outer crust of a neutron star is optically opaque. It is not clear, however, whether it can withstand the photon pressure corresponding to the energy in equation (2) and prevent the gamma rays from leaving the star.

The duration  $\tau$  of resonant photo-production depends on how soon the nuclear matter is heated to the temperature of order  $T_c$ . It would seem that the detailed studies of coalescing neutron stars [11] predict a typical time scale of a few milliseconds for the acoustic shock waves to heat the nuclear matter. However, for technical reasons, these simulations [11] started with non-zero initial temperatures to reduce the sensitivity of their results to small variations of the internal energy at  $T = 0$ . In addition, these analyses do not take into account the possible cooling effects of the resonant gamma-emission which, as seen from equation (2), can be very significant and can delay the eruption of a thermal fireball. Therefore, one can take 1 ms as a very conservative lower



bound on the duration of the resonant production of gamma rays. If a more detailed analysis shows that  $\tau$  can be as long as 1 s, the mechanism we described can explain the origin of the observed GRB. If  $\tau \ll 1$  s, one can look for a signature of resonant emission during the first milliseconds of each GRB, assuming they originate from some events that involve vibrating cold neutron stars.

In summary, we have described a mechanism that can lead to a short burst of gamma rays with power up to  $10^{52\pm 1}$  erg s<sup>-1</sup> from a jolted neutron star (for instance, in the event of a neutron star collision). The resonant emission of gamma rays is a result of a non-thermal photo-production by the acoustically-powered coherent oscillations of the superconducting proton condensate. The duration of the pulse is uncertain and depends on the details of the heat transport. It is possible that the observed gamma-ray bursts, or at least the signal during the short time interval in the beginning of some bursts, may be explained by this mechanism.

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