

# Gauge Unification and the Supersymmetric Light Higgs Mass

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We consider general supersymmetric models with: a) arbitrary matter content; and, b) gauge coupling Unification near the String scale  $\sim 10^{17}$  GeV, and derive the absolute upper limit on the mass of the lightest Higgs boson. For a top-quark mass  $M_t = 175$  GeV, and depending on the supersymmetric parameter  $\tan\beta$ , this mass bound can be as high as  $\sim 200$  GeV.

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Low-energy supersymmetry [1] is a key ingredient of the candidate best qualified models to supersede the Standard Model (SM) at energies beyond the TeV range. The extensive experimental search of the (super)-partners of SM elementary particles predicted by supersymmetry (SUSY) has been so far unsuccessful, challenging [2], with the rise of experimental mass limits, the naturalness and relevance of SUSY for the electroweak scale physics.

In this context, the sector of the theory responsible for electroweak symmetry breaking has a special status. While all superpartners of the known SM particles can be made heavy simply by rising soft SUSY-breaking mass parameters in the model, the Higgs sector necessarily contains a physical Higgs scalar whose mass does not depend sensitively on the details of soft masses but is fixed by the scale of electroweak symmetry breaking. This important fact follows simply from the spontaneous breaking of the electroweak gauge symmetry [3,4] and it is not specific to supersymmetric models. More precisely, the general statement is that some Higgs boson must exist whose mass-squared satisfies  $m_h^2 \leq \lambda v^2$ , where  $v$  is the electroweak scale ( $v = 174.1$  GeV) and  $\lambda$  is the dimensionless quartic coupling of some Higgs state in the model. In other words, the mass of the light Higgs can be made heavy only at the expense of making the coupling  $\lambda$  very strong. The role of supersymmetry is to fix  $\lambda$  in some models. For example, in the Minimal Supersymmetric Standard Model (MSSM), which includes two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}, \quad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}, \quad (1)$$

to give masses to quarks and leptons,  $\lambda$  is related to the  $SU(2)_L \times U(1)_Y$  gauge couplings ( $g$  and  $g'$  respectively) and the following (tree-level) bound on the mass of the lightest Higgs boson holds

$$m_h^2 \leq M_Z^2 \cos^2 2\beta, \quad (2)$$

where  $\tan\beta = \langle H_2^0 \rangle / \langle H_1^0 \rangle$ . This represents a very stringent prediction which, as is well known, gets significantly relaxed when radiative corrections to  $\lambda$  are included [5–7]. These corrections depend logarithmically on the soft-masses and push the upper mass limit for the lightest Higgs boson of the MSSM up to 125 GeV (for a top-quark mass  $M_t = 175$  GeV).

The fact that  $\lambda$  is calculable in terms of gauge couplings in the MSSM is due to supersymmetry and to the fact that the superpotential does not contain cubic terms of the form  $W = h_X X H_i H_j$  (with  $i, j = 1, 2$ ) as no  $X$  field exists with the appropriate quantum numbers to form a gauge invariant object. In extended models, the presence of such fields and couplings modifies the quartic Higgs self-interactions which can ultimately have an impact on the tree-level upper bound on  $m_h^2$ , which receives corrections proportional to  $h_X^2 v^2$ . The Yukawa coupling  $h_X$  is unknown but asymptotically non-free, and so it can be bounded from above if it is further required to remain in the perturbative regime up to some large energy scale (where GUT, String or Planck physics takes over).

As there is no reason to believe that low-energy supersymmetry is realized in nature in the form of the MSSM, the experimentalist willing to test SUSY via Higgs searches would like to know what is the absolute general upper limit that should be reached on  $m_h$  to rule out low-energy SUSY. This is the important question we set ourselves to answer in this letter.

1. To give a precise answer we have to assume that all couplings in the theory remain perturbative up to a very large energy scale. This is particularly well motivated in low-energy SUSY models from the successful unification of gauge couplings in the simplest MSSM model, the best (indirect) evidence we have so far for Supersymmetry. In considering extensions of the MSSM, we will always require that this remarkable feature is not spoiled. The addition of extra gauge singlets or complete  $SU(5)$  representations are the natural possibilities for extended models where minimal unification is automatically preserved (at the one-loop level). In our search for the upper Higgs

mass limit we do not restrict ourselves to these possibilities but consider other options. Although splitted  $SU(5)$  representations are difficult to arrange in GUT models, they can easily arise in string models and may even help in solving the mismatch between the MSSM unification scale and the String scale [degenerate full  $SU(5)$  multiplets do not modify (at one-loop) the unification scale].

We assume that any model must contain at least the two minimal Higgs doublets  $H_{1,2}$  required to give quarks and leptons their masses. In principle, more than two Higgs doublets could be involved in  $SU(2)_L \times U(1)_Y$  breaking, but in such a case a rotation in field space can be made so that only  $H_{1,2}$  have non-zero vacuum expectation values. Additional higher Higgs representations are generally very constrained by  $\rho = M_W^2/M_Z^2 \cos^2 \theta_W \simeq 1$ . In addition, if they contribute significantly to the  $W^\pm$  and  $Z^0$  masses, the Higgs bound, sensitive to the doublet contribution, gets weaker and, in addition, other light Higgses appear in the spectrum [4]. It is thus conservative to assume that  $\langle H_{1,2}^0 \rangle$  are responsible for all the breaking and thus, the known  $Z^0$  mass fixes  $\langle H_1^0 \rangle^2 + \langle H_2^0 \rangle^2 = v^2$ .

To maximize the upper bound on  $m_h$  we next assume that the model also contains extra chiral multiplets with the appropriate quantum numbers to give couplings of the form  $W = h_X X H_i H_j$ . Thus,  $X$  can only be a singlet ( $S$ ) or a  $Y = 0, \pm 1$  triplet ( $T_Y$ ). From the gauge-invariant trilinear superpotential

$$W = \lambda_1 H_1 \cdot H_2 S + \lambda_2 H_1 \cdot T_0 H_2 + \chi_1 H_1 \cdot T_1 H_1 + \chi_2 H_2 \cdot T_{-1} H_2, \quad (3)$$

the tree-level mass bound follows (we correct here a normalization error in Refs. [8,9]):

$$m_h^2/v^2 \leq \frac{1}{2}(g^2 + g'^2) \cos^2 2\beta + (\lambda_1^2 + \frac{1}{2}\lambda_2^2) \sin^2 2\beta + 4\chi_1^2 \cos^4 \beta + 4\chi_2^2 \sin^4 \beta. \quad (4)$$

The different dependence of the various terms with  $\tan \beta$  will make them important in different regimes. In particular we already anticipate that, in the large  $\tan \beta$  region, the  $\chi_2$  contribution will be crucial for the upper limit.  $S$  and  $T_0$  have the same dependence while it can be shown that the effect of  $\lambda_1$  is always be more important than that of  $\lambda_2$ . For this reason we will not take into account the possible effect of  $T_0$  representations.

**2.** As the next step, one should impose triviality bounds on the extra couplings entering (4) by assuming they do not reach a Landau pole below the unification scale. As stressed in [10,11], large values of the gauge couplings slow down the running of Yukawa couplings with increasing energy and so they can be larger at the electroweak scale. To get numerical values for these upper bounds we then have to specify further the particle content of the model, imposing gauge coupling unification and making the gauge couplings as large as possible. For the renormalization group (RG) analysis we consider

that all superpartners can be roughly characterized by a common mass  $M_{SUSY}$  below which the effective theory is just the SM and, restricted by naturalness, we take  $M_{SUSY} = 1$  TeV. No such constraint should be imposed on extra matter in vector like representations, like  $(5 + \bar{5})$   $SU(5)$  pairs, which could be present at intermediate scales. By setting their masses down to 1 TeV we are enhancing their effect on the running of gauge couplings, which being stronger will also tend to increase the  $m_h$  bound.

To achieve unification with only one scale  $M_{SUSY}$  fixed to 1 TeV is not completely trivial. When the MSSM is enlarged by one singlet  $S$  and a pair  $\{T_1, T_{-1}\}$  (to cancel anomalies) the running  $g_1^2 = 5g'^2/3$  and  $g^2$  meet at  $M_X \sim 10^{17}$  GeV. Interestingly enough, this is closer to the Heterotic String scale than the MSSM unification scale. Of course,  $g_3^2$  fails to unify unless extra matter is added. This can be achieved, for example, by adding 4  $(3 + \bar{3})$  [ $SU(2)_L \times U(1)_Y$  'singlet quark' chiral multiplets] or one  $(3 + \bar{3})$  plus one  $SU(3)_c$  octet. In addition to this, one can still have one  $(5 + \bar{5})$   $SU(5)$  pair, which will not change the unification scale. The unification of the couplings is shown in Figure 1 (solid lines).

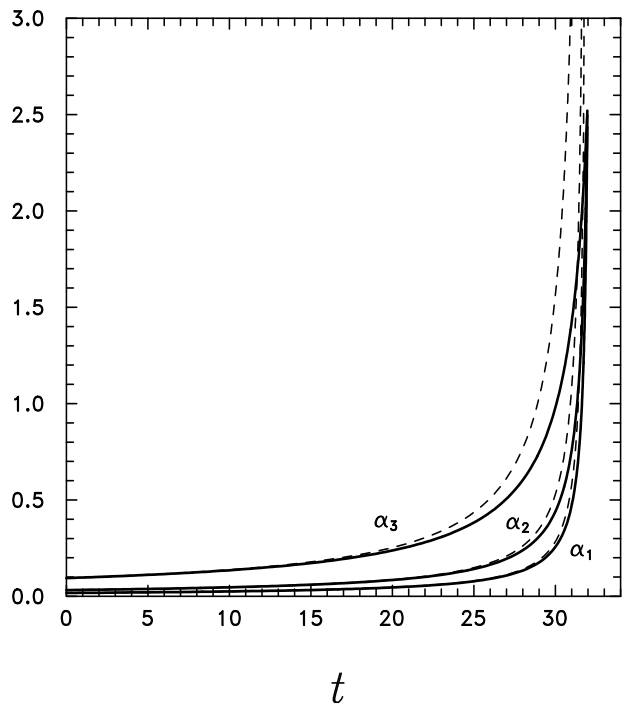


FIG. 1. Running  $\alpha_i$ 's ( $= g_i^2/4\pi$ ) for the model discussed in the text (solid lines) and upper perturbative limit (dashed lines) with  $t = \log(Q/M_{SUSY})$ .

For comparison, dashed lines show the running couplings when their beta functions are chosen in such a way that all couplings reach a Landau pole at the unification scale. In this case the low-energy couplings are fully determined by the 'light' matter content of the model,

which determines the RG beta functions. This behaviour is dubbed non-perturbative unification [12] and was long ago proposed as an alternative to conventional unification, with the attractive feature of having less sensitivity of the low-energy couplings to high-energy physics (In addition, the choice of  $k_i$  normalization factors for gauge coupling unification is now immaterial). The dashed lines can be considered as the perturbative upper limit on the gauge couplings and comparison with the solid lines shows that our model is close to saturation and represents a concrete realization of the most extreme scenario to maximize the  $m_h$  bound. The emphasis here should lie in this fact rather than in the plausibility or physics motivation of the model per se.

The particular model we use serves the purpose of illustrating the fact that the Higgs mass bound can be saturated. The model includes exotic representations with non-canonical charge assignment, which can nevertheless appear in string models.  $(3, 1)_0$  and  $(\bar{3}, 1)_0$ 's can appear in general embeddings [13] of the Standard Model group other than the usual embeddings in grand unified groups like  $SU(5)$  or  $E_6$ . On the other hand,  $SU(2)$  triplets are possible if the  $SU(2)$  Kač-Moody level is larger than 1 (as we have seen, the effect of this on the unification condition is not important in the limiting case of non-perturbative unification). While these triplets are the key ingredient to go beyond the Higgs mass limits of the MSSM, the additional representations included to ensure unification are not uniquely determined. Different representations of exotic matter at intermediate scales, as e.g. color octets with canonical charge assignment, could equally well give correct unification. In such cases, the lightest Higgs can well be much heavier than in the MSSM even if the general upper limit is not reached.

Having optimized in this way the most appropriate running gauge couplings, we turn to the running of  $\lambda_1$  and  $\chi_{1,2}$ . The relevant RG equations can be found in Refs. [8,9]. Inspection of this set of coupled equations teaches that the low-energy values of each Yukawa are maximized when other Yukawas (and self-interactions among  $T_Y$ 's and  $S$ ) are shut off. So, we consider each coupling in turn (setting the others to zero, which is a RG-invariant condition) and compute its maximum value at 1 TeV as a function of  $\tan\beta$  (which influences the top and bottom Yukawa couplings entering the RGs) for  $M_t = 175$  GeV. This physical top-quark mass is related to the Yukawa coupling  $h_t$  by the  $\overline{\text{MS}}$  relation  $h_t v \sin\beta/\sqrt{2} = M_t/[1 + 4\alpha_s(M_t)/3\pi]$ . Plugging the maximum values of  $\lambda_1$ ,  $\chi_1$  or  $\chi_2$  in the tree-level bound (4) we obtain then three different upper limits on the tree-level Higgs mass. To add the important radiative corrections we follow the RG method, as explained e.g. in Refs. [14–16], which includes two-loop RG improvement and stop-mixing effects.

**3.** Before presenting our results, it is worth discussing in more detail the bound presented in Eq. (4). If the extra fields responsible for the enhancement of  $m_h$  sit at 1 TeV, should their effect not decouple from the low-energy

effective theory? Indeed, in a simple toy model with an extra singlet  $S$  coupled to  $H_1 \cdot H_2$  as in (3), when a large supersymmetric mass is given to  $S$ , the  $F$ -term contribution ( $\sim \lambda_1^2$ ) to the Higgs doublet self-interactions is cancelled by a tree diagram that interchanges the heavy singlet, thus realizing decoupling. If, on the other hand, we lower the mass scale of the extra fields  $S$  and  $T_Y$  to the electroweak scale to avoid decoupling, it is generically the case that more than one light Higgs appear in the spectrum. A complicated mixed squared-mass matrix results whose lightest eigenvalue does not saturate the bound (4). Is then this mass limit simply a too conservative overestimate of the real upper limit? It is easy to convince oneself that, in the presence of soft breaking masses, the perfect decoupling cancellation obtained in the large SUSY mass limit does not take place (we are assuming here that SUSY masses, if present for the extra matter, are not larger than 1 TeV) and the final lightest Higgs mass depends in a complicated way on these soft mass-parameters. The interesting outcome is that soft-masses can be adjusted in order to saturate the bound (4) and so, the numbers we will present can be reached in particular models and no limits lower than these can be given without additional assumptions (which we will not make here, in the interest of generality).

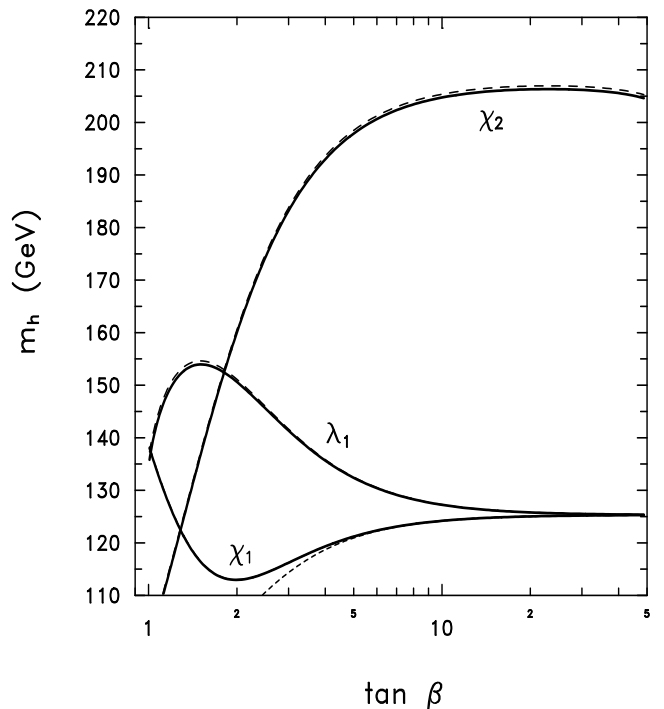


FIG. 2. Radiatively corrected upper bounds on  $m_h$  when different Yukawa couplings are present in the model and for different assumptions on the running of gauge couplings. The short-dashed line gives the upper bound in the MSSM.

The final bounds, with radiative corrections included, are presented in Figure 2. Solid lines are the mass lim-

its when the particular model described previously is assumed to determine the running of the gauge couplings. Long-dashed lines are instead obtained when all gauge couplings reach a Landau pole at  $M_X \sim 10^{17} \text{ GeV}$  and show that the particular model we used practically saturates the absolute bounds.

Below  $\tan \beta \sim 1$  (beyond  $\tan \beta \sim 60$ ),  $h_t$  ( $h_b$ ) reaches a Landau pole below  $M_X$  and that region is thus excluded. Lines labeled ‘ $\lambda_1$ ’ show the effect of the singlet coupling  $\lambda_1$ . The maximal value of this coupling depends on  $\tan \beta$  through  $h_t$ . For small  $\tan \beta$ ,  $h_t$  is large and forces  $\lambda_1$  to be small. With increasing  $\tan \beta$ ,  $h_t$  becomes smaller and  $\lambda_1$  can get larger values. This effect, combined with the  $\sin^2 2\beta$  dependence of the  $\lambda_1$ -contribution to  $m_h^2$  explains the shape of these lines. At large  $\tan \beta$ , the  $\lambda_1$  contribution to the mass limit shuts off and the MSSM limit is recovered. [A limit similar to the  $\lambda_1$ -bound has been recently obtained [11] in the MSSM with a singlet field  $S$  and pairs of  $SU(5)$  ( $5 + \bar{5}$ ) saturating the gauge couplings.] The same happens for the lines labeled ‘ $\chi_1$ ’ which give the  $\chi_1$ -bound. The upper triviality bound on this Yukawa coupling is basically independent of  $\tan \beta$ . The interplay between the minimal ( $\cos^2 2\beta$ ) piece of the bound (4) and the  $\chi_1$  piece explains the shape of these lines. It is interesting how the effect of the  $\chi_2$  coupling is instead more important for large  $\tan \beta$ , where it reinforces the minimal contribution, providing the absolute upper limit. This is given by the lines labeled ‘ $\chi_2$ ’ and can be as large as 205 GeV. We remark that these lines assume that only one of the couplings  $\lambda_1$ ,  $\chi_1$  or  $\chi_2$  is non-zero. If two couplings differ from zero simultaneously the bound is reduced.

4. In conclusion, we calculate a numerical absolute upper limit on the mass of the lightest supersymmetric Higgs boson for any model with arbitrary matter content compatible with gauge coupling unification around (and perturbativity up to) the String scale. With this assumption, we show that this light Higgs mass can be as high as  $\sim 200$  GeV, significantly heavier than previously thought. The model saturating this bound has asymptotically divergent gauge couplings and points toward non-perturbative unification. Besides being of obvious interest to the experimentalists, this result has interest for theorists too. If Higgs searches reach the MSSM bounds without finding a signal for a Higgs boson, this could be taken, if one is willing to stick to low-energy supersymmetry, as evidence for additional matter beyond the minimal model. Without stressing the point too much, this could be welcome to reconcile the unification scale with the String scale.

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