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A Note on Softly Broken MQCD

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Abstract: We consider generic MQCD configurations with matter described by semi-infinte D4-branes and softly broken supersymmetry. We show that the matter sector does not introduce supersymmetry breaking parameters so that the most relevant supersymmetry breaking operator at low energies is the gaugino mass term. By studying the run-away properties of these models in the decoupling limit of the adjoint matter, we argue that these softly broken MQCD configurations fail to capture the infrared physics of QCD at scales below the gaugino mass scale.

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1. Introduction and Conclusions

The modeling of gauge dynamics via brane configurations has provided a nice geometric picture of some very non-trivial infrared phenomena of gauge theories, like the dynamical generation of superpotentials, confinement and gaugino condensation in $N = 1$ theories, and the complete Seiberg-Witten effective action in $N = 2$ theories. A recent review with an extensive list of references is [1]. The systematics of the engineering of gauge theories in the world-volume of complicated brane configurations was initially developed in [2]. The advantage of the brane set up at weak coupling is that the structure of the classical moduli space of vacua is given a geometric interpretation in terms of the rules of brane dynamics in, say, weakly coupled type IIA string theory. Quantum corrections in the weak coupling regime include bending effects of the branes [3], D-instanton corrections [4], and other effects which comprise a set of "force rules" of brane dynamics [5]. An efficient way of summing up all these corrections was proposed by Witten in [3]. Roughly speaking, one takes the IIA string theory to strong coupling, and tries to calculate non-perturbative effects in terms of the perturbation theory of the dual theory. In this case the dual theory is M-theory, and one is confined to the infrared domain, where it admits a description in terms of eleven dimensional supergravity. At strong coupling, the quantum corrections like D-instanton effects are summed up into the background geometry of the M-theory solitonic configuration; a single twisted M-theory five-brane in the simplest examples.

From the point of view of the low energy field theory, changing the string coupling keeping the gauge coupling fixed, corresponds to turning on a tower of perturbatively irrelevant operators. These operators decouple in the infrared as long as we are in the weak coupling regime, but in general there is no guarantee that they continue to do so in the strong coupling regime. As a result, one discovers that the supergravity description in terms of the M5-brane (the MQCD regime) depends explicitly on the radius of the new dimension R, in addition to the low energy scale of gauge theory Λ_{QCD}. Holomorphic observables (protected by supersymmetry) exactly match the gauge theory ones, but the R dependence shows up in higher derivative interactions [6] and in massive states in theories with a mass gap [7].

Soft supersymmetry breaking is introduced geometrically by rotations of the branes in the type IIA picture at weak coupling [8,9,10], or by rotations of the asymptotic boundary conditions of the M5-brane in the M-theory picture [7]. [1] In the light of the previous comments, it is to be expected that the loss of supersymmetry constraints makes the matching of MQCD and true QCD physics less succesful. Still, MQCD in itself is an interesting theory whose supersymmetry breaking dynamics is worth study. In addition, it has been shown in [16] that some non-trivial vacuum physics, like level crossing phenomena, can be detected geometrically in MQCD.

In this note we add some comments on the interpretation of non-supersymmetric brane configurations. First, we show that, in a general class of models, many of the supersymmetry breaking deformations naively present in the type IIA brane picture (rotation angles), cannot be realized smoothly in the M-theory limit. The requirement that the five-brane configurations are smooth, restricts some of the possible rotation angles. In particular, in models with semi-infinite D4-branes representing hypermultiplet couplings, the asymptotic orientation of the D4-branes in eleven dimensional space does not introduce any new supersymmetry breaking couplings. Therefore, in this class of models all soft parameters are associated to rotations of the NS-branes of type IIA, or in field theory terms, to soft parameters in the $N = 2$ vector multiplet sector. The most relevant supersymmetry breaking operator at low energies is then the gaugino mass term $\mathcal{L}_{\rm soft} = m_\lambda {\rm Tr}\, \lambda\lambda$. It is important to emphasize that this result applies to models with supersymmetry breaking rotations restricted to NS branes and semi-infinite D4-branes. In principle, models with matter described by D6-branes could have a larger parameter space and non trivial rotations of the D6-branes could be important in generating soft parameters in the flavour sector. However, experience with the $N = 2$ and $N = 1$ models shows that qualitative features of the Higgs phases like the existence of run-away vacua, can be established at the level of the eigenvalues of the meson matrices, which are correctly captured by semi-infinite D4-branes.

The interpretation of this result depends on the extent to which the brane configuration captures the infrared behavior of the theory. On general grounds, at scales below the gaugino mass, radiative corrections would induce a mass for the squarks as well, supressed only by some powers of the coupling. The fact that MQCD represents the strong coupling regime makes it possible for this squark mass to be incorporated into the brane geometry,

[1] Discussions of various issues concerning soft supersymmetry breaking in SQCD and MQCD can be found in refs. [11,12,13,14,15].

even if it does not appear in the asymptotic boundary conditions of the brane configuration. In other words, we would like to know if the brane contains information down to the deep infrared domain, at energies $E < m_\lambda$, or only captures QCD physics down to an intermediate scale $m_\lambda < E < \Lambda_{QCD}$.

We investigate this question by examining the run-away vacua of $N = 1$ super QCD for $N_f < N_c$, and the possibility of vacuum stabilization by a squark mass. In these models, say the $N_f = N_c - 1$ case, the Affleck-Dine-Seiberg superpotential [17]

$$W_{ADS} = \frac{\Lambda_1^{2N_c+1}}{\det(\widetilde{Q}Q)}$$

induces a potential that pushes the vacuum expectation value of the squarks to infinity. If a small squark mass would be added to the bare lagrangian or generated dynamically at some scale, it would stop the run-away. As pointed out in ref. [12], such vacua are physically very interesting, as one can argue for chiral symmetry breaking with the standard vectorlike pattern of QCD.

A geometric criterion for run-away in the brane configurations of M-theory was introduced in ref. [18] (see also ref. [19]). Here we study the impact of supersymmetry breaking rotations on the runway behavior of such configurations. We analyze the renormalization group flows of the soft breaking parameters in the limit where one decouples the adjoint matter of the $N = 2$ vector multiplet. The result is that run-away behavior is not affected by soft supersymmetry breaking, and the models with $N_f < N_c$ show no stable vacuum after a gaugino mass is introduced.

This result means that these brane configurations fail to capture the deep infrared region of the field theory, even at the level of the vacuum structure. In order to do so, one presumably would need to incorporate non trivial backgrounds, i.e. D6-branes, in the determination of soft parameters of the flavour sector.

2. Regularity constraints on fivebrane configurations

We consider general IIA brane configurations combining NS-branes and finite or semi-infinite D4-branes, the latter representing only flavour degrees of freedom. These models support product gauge groups, each factor with a rank given by the number of D4-branes suspended between contiguous NS-branes. Each of the NS-branes could have a high multiplicity, allowing for the introduction of non-trivial Landau-Ginzburg interactions [20]. We shall assume that a general configuration with softly broken supersymmetry has been obtained by asymptotic rotation of the branes involved from a completely degenerate configuration of the Coulomb branch, i.e. the complete M-theory fivebrane has the structure $M_4 \times \mathbf{C}(z_\alpha)$, with M_4 the standard $3+1$ dimensional Minkowski space, and $\mathbf{C}(z_\alpha)$ a punctured complex plane. The interpretation of the punctures is that the region around them is mapped to asymptotic infinity in the M-theory space-time $R^{1,9} \times S^1_R$, approaching the type IIA configuration. We denote the embedding coordinates by $X(z,\overline{z})$, with $X = (\vec{X}, X^{10})$, and $\vec{X} = (X^4, X^5, X^6, X^7, X^8, X^9)$. The background metric in the relevant seven-dimensional space is

$$ds^2 = (dX)^2 = (d\vec{X})^2 + (dX^{10})^2\,, \tag{2.1}$$

so that the minimal area equations for the embedded surface $X(z,\overline{z})$ are equivalent to the harmonic condition $\partial_z \partial_{\overline{z}} X(z,\overline{z}) = 0$, plus the Virasoro constraints $T_{zz} = (\partial_z X)^2 = 0$.

The general ansatz compatible with the harmonicity is $X(z,\overline{z}) = \sum_\alpha \frac{1}{2}\,(X_\alpha(z) + \text{c.c.})$, and

$$X_\alpha(z) = x_\alpha + P_\alpha \log{(z - z_\alpha)} + \sum_{m\neq 0} C_{\alpha,m}\,(z - z_\alpha)^m \tag{2.2}$$

and the asymptotic conditions at the punctures are specified by the coefficients in this expansion. We can restrict them by physical considerations. First, if we define n_α to be the wrapping number at z_α of the M5 around the circle X^{10}, we can choose the normalization of the z coordinate such that the monodromy under $(z - z_\alpha) \to (z - z_\alpha)e^{2\pi i}$ is given by $X^{10} \to X^{10} + 2\pi R n_\alpha$. Then using the gauge freedom to shift $X(z,\overline{z})$ by a single harmonic function we can reduce X^{10} to the simplest form compatible with the monodromy:

$$X^{10}_\alpha(z) = -iR\,n_\alpha \log{(z - z_\alpha)} \tag{2.3}$$

Since the wrapping number around X^{10} is basically the D4-brane number in this construction, the integers n_α are interpreted as the number of D4-branes from the "left" minus the number from the "right" when the puncture $z_\alpha = z_a$ is associated to an NS-brane in the type IIA limit. In the following, it will be convenient to distinguish the index $\alpha = (a, k)$, between the punctures associated to asymptotic NS-branes, with index a, and the punctures associated to semi-infinite D4-branes, with index k. For the latter case, the number n_k is interpreted simply as the number of D4-branes sticking out of the puncture.

Now, in the IIA limit, an asymptotic NS-brane with multiplicity h_α is a plane in $\vec{X}$-space, with an affine representation

$$\vec{X} \sim \mathrm{Re}\,(\vec{C}\,y). \tag{2.4}$$

Here, $\vec{C}$ is a complex vector whose real and imaginary parts generate the plane, and y is a complex variable encoding the affine parameters. The multiplicity is obtained by setting $(z - z_\alpha)^{h_\alpha} = 1/y$, so that the z-plane around z_α covers the y plane h_α times. Therefore the remaining non-trivial embedding function is given by

$$\vec{X}_\alpha(z) = \vec{x}_\alpha + \vec{P}_\alpha \log{(z - z_\alpha)} + \sum_{n\neq 0} \vec{C}_{\alpha,n}\,(z - z_\alpha)^n \tag{2.5}$$

where $\vec{C}_{\alpha,n}$ vanishes for $n < -h_\alpha$. The logarithmic terms in the z_a punctures, corresponding to NS-branes, reflect the "bending effect" discussed in [3]. The D4-branes ending on the NS-branes behave as sources of codimension two in the NS-brane world-volume, with charges $\vec{P}_a$. For a semi-infinite D4-brane all complex vectors $\vec{C}_{k,n}$ vanish and only the logarithmic terms remain, because the semi-infinite D4-branes do not have a plane at infinity in the ten-dimensional internal space. Indeed, taking $\vec{C}_{k,n}$ real and non-vanishing would not describe a plane, and moreover the monodromy around the X^{10} direction would not act trivially on the resulting semi-infinite D4-brane. So all complex vectors $\vec{C}_{k,n}$ vanish for semi-infinite D4-branes.

Finally, the most important constraint is obtained by requiring that all the "momenta" $\vec{P}_\alpha$ in the non-compact directions are real, i.e. that the monodromy under $(z - z_\alpha) \to (z - z_\alpha)e^{2\pi i}$ only affects X^{10} and leaves $\vec{X}$ invariant. This is a condition for the embedding to be smooth.

Now it is straightforward to write explicitly the Virasoro constraints in terms of the vectors $B_{\alpha,n} = nC_{\alpha,n} + \delta_{n,0}P_\alpha$, which are just the Fourier components of $\partial_z X_\alpha(z)$. They read:

$$\begin{aligned} &B_{\alpha,n} \cdot B_{\beta,m} = 0\;, \quad \alpha \neq \beta \\ &\sum_m B_{\alpha,m} \cdot B_{\alpha,n-m} = 0, \end{aligned} \tag{2.6}$$

or, in terms of the previous variables:

$$\begin{aligned} &\vec{C}_{\alpha,n} \cdot \vec{C}_{\beta,m} = \vec{C}_{\alpha,n} \cdot \vec{P}_\beta = \vec{P}_\alpha \cdot \vec{P}_\beta - R^2 n_\alpha n_\beta = 0\;, \qquad \alpha \neq \beta \\ &\delta_{n,0}\left(\vec{P}_\alpha^2 - R^2 n_\alpha^2\right) + 2n\vec{P}_\alpha \cdot \vec{C}_{\alpha,n} + \sum_m m(n-m)\vec{C}_{\alpha,m} \cdot \vec{C}_{\alpha,n-m} = 0 \end{aligned} \tag{2.7}$$

Our main result stems now from the fact that semi-infinite D4-branes do not have a plane at infinity in $\vec{X}$-space, so that $\vec{C}_{k,n} = 0$, and the last equation simplifies dramatically for the flavour punctures: $\vec{P}_k^2 = R^2 n_k^2$. Combining this equation with $\vec{P}_k \cdot \vec{P}_\ell = R^2 n_k n_\ell$, we obtain that all $\vec{P}_k$ vectors must be collinear. Using the $SO(6)$ isometries of the $\vec{X}$-space we can reduce one of the $\vec{C}$ to canonical form, say $\vec{C}_{\infty,1} = (1, -i, 0, 0, 0, 0)$. Then, the previous equations, together with the real character of P_α, imply that no momentum vector $\vec{P}_k$ lies in the (X^4, X^5) plane, and therefore the semi-infinte D4-branes can be conventionally taken to point in the X^6 direction: $\vec{P}_k = (0, 0, \eta\, R\, n_k, 0, 0, 0)$, with $\eta = \pm 1$, and there is still a full $SO(3)$ isometry remaining in the (X^7, X^8, X^9) directions. The only remaining general rule is that all the asymptotic NS-planes are orthogonal to the X^6 direction (i.e. $C^6_{a,n} = 0$), due to the equation $\vec{P}_k \cdot \vec{C}_{a,n} = 0$.

The fact that all $\vec{P}_k$ vectors are parallel and conventionally aligned along the X^6 direction, means that there are no new supersymmetry breaking couplings induced by the flavour sector. In other words, the flavour sector, represented here by semi-infinite D4-branes, is overconstrained and it does not introduce extra soft supersymmetry breaking parameters. Thus, in these models, breaking supersymmetry by making a generic rotation of the branes corresponds, in field theory terms, to introduce soft supersymmetry breaking parameters in the $N = 2$ vector multiplet sector only. At sufficient low energies, the most relevant supersymmetry breaking mechanism is that of giving a tree-level mass to the gauginos.

On the other hand, the fact that all flavour D4-branes point in a given direction means that their relative distances define vacuum parameters of the model. This is analogous to the fact that the gauge coupling is a modulus of the theory for a brane configuration representing a conformally invariant fixed point, as the NS-branes "bend" in a parallel fashion asymptotically [3]. In the supersymmetric limit these parameters are associated to the eigenvalues of the meson matrix [18], and this is therefore the natural interpretation as well upon supersymmetry breaking. This allows us to study the run-away phenomenon after supersymmetry breaking, just by looking at the positions of the flavour D4-branes in the transverse hyperplane.

It is worth to make a comment on the supersymmetric case. Supersymmetry requires that the embedding is holomorphic with respect to some complex structure of a six-dimensional internal (sub-) manifold. Our (arbitrary) choices of orientation fix this almost completely. Indeed we should set $v = X^4 + iX^5$ and the direction in which we have oriented the semi-infinite D4-branes implies that $s = (X^6 \pm i X^{10})/R$, or better $t = \exp(-s)$. Choosing the plus sign, we obtain that supersymmetry, with our choices of axes, requires $P^6_\alpha = R n_\alpha$. When we have $N = 1$ supersymmetry, we set $w = X^8 + iX^9$, $X^7 = 0$, whereas with $N = 2$ also $w = 0$. Finally, when the values of the parameters are such that we have $N = 2$ supersymmetry, the corresponding Seiberg-Witten curve fixes the values of the punctures z_α.

Since the supersymmetry soft breaking does not depend on the matter content of the curve, we can restrict ourselves to consider the simplest models without losing generality. We thus consider the curve describing (in type-IIA language) two NS-branes with N_c D4-branes suspended between them and N_f semi-infinite D4-branes. By convention we set the two NS-branes at $z = 0$ and $|z| = \infty$ and we keep fixed the NS-brane at $|z| = \infty$ (corresponding to the vector $\vec{C}_{\infty,1}$). r of the N_f D4-branes are at $z = z_+$ and $N_f - r$ at $z = z_-$. The curve is

$$\begin{aligned} v &= \frac{(z-z_+)(z-z_-)}{z} - m_f + \frac{\epsilon}{\bar{z}} \,, & w &= \frac{\zeta}{z} + \frac{\bar{\lambda}}{\bar{z}} \,, \\ t &= \frac{z^{N_c}}{(z-z_+)^r (z-z_-)^{N_f - r}} \,, & X^7 &= 2\sqrt{\epsilon} \log \left| \frac{z}{\Lambda_2} \right| \end{aligned} \tag{2.8}$$

where $z_+ z_- \epsilon + \zeta\lambda = 0$. The complex parameters ζ and λ are combinations of the Cs whereas the real parameter ϵ is a combination of the Ps. We have chosen the x_α so that $v|_{z=z_\pm, \epsilon=0} = -m_f$.

Notice that the curve is invariant under $\zeta \leftrightarrow \lambda$ with $w \leftrightarrow \bar{w}$. Since λ is a soft supersymmetry breaking parameter and ζ is the parameter breaking $N = 2$ to $N = 1$, the previous symmetry instructs us (consistently) to consider only cases where the supersymmetry soft breaking scale is much smaller than the scale at which supersymmetry is broken to $N = 1$. We solve the relation between the parameters by setting $\zeta = \mu z_+ z_-$ and $\lambda = -\epsilon/\mu$. The $N = 1$ breaking parameter μ is related to the mass of the adjoint chiral multiplet by $\mu = C_\zeta \ell_p^2 m_{Adj}/R$, and the punctures $z_\pm$ depend on $z_\pm = z_\pm(N_c, N_f, r, \Lambda_2, m_f)$ and are fixed by the comparison of the $N = 2$ curve (i.e. the $\varepsilon, \mu \to 0$ limit) with the corresponding Seiberg-Witten curve [18].

3. Run-away and decoupling of adjoint matter in softly broken curves

In the supersymmetric case, it is well known [18] that the curve eq. (2.8) does not lead to run-away in the decoupling limit of the adjoint chiral multiplet (i.e. $|\mu| \to \infty$) when $N_f > N_c$ if $m_f = 0$, and when $r = 0$ if $m_f \neq 0$. This is the correct behaviour known from field theory [17].

It is interesting to see what happens in the decoupling limit of the adjoint chiral multiplet with soft supersymmetry breaking. We first consider the massless case $m_f = 0$.

Recall [18] that in this case $z_\pm = c_\pm \Lambda_2$ and that the matching between the $N = 2$ and $N = 1$ scales reads $(\Lambda_1)^{3N_c - N_f} = (m_{Adj})^{N_c} (\Lambda_2)^{2N_c - N_f}$. Since our curve describes soft supersymmetry breaking, the relevant scale after we decouple the adjoint chiral multiplet is Λ_1, while the supersymmetry breaking scale is much smaller. This implies that in the limit $|\mu| \to \infty$ we have $\Lambda_2 \sim \mu^{-N_c/(2N_c - N_f)}$.

Let us now consider the issue of run-away. For $r \neq 0$ we should look if $w|_{z_\pm}$ and $v|_{z_\pm}$ run-away or remain finite in the $\mu \to \infty$ limit. For $r = 0$ only the puncture z_- is to be considered. The conditions to stop the run-away are

$$v|_{z_\pm} \sim \frac{\epsilon}{\Lambda_2} < \infty \ , \qquad\qquad w|_{z_\pm} \sim \mu \Lambda_2 - \frac{\epsilon}{\bar{\mu} \bar{\Lambda}_2} < \infty \tag{3.1}$$

in the $|\mu| \to \infty$ limit. From the first condition it is evident that only if ϵ/Λ_2 is kept finite in the limit, then there is not run-away induced by the supersymmetry breaking terms. But if ϵ/Λ_2 is finite, the second equation in (3.1) implies that $w|_{z_\pm}$ has the same limit as in the supersymmetric case. It follows that run-away can be stopped *only* in the cases in which it is also stopped in absence of soft supersymmetry breaking. Moreover ϵ should vanish at least as fast as Λ_2 in this limit.

As discussed in the introduction, the fact that the soft supersymmetry breaking encoded in the curves is not able to stop the run-away, implies that MQCD captures QCD physics only down to an intermediate scale $m_\lambda < E < \Lambda_{QCD}$, and not below.

It is possible to determine the scaling of ϵ in terms of μ and Λ_2 by requiring that the various components of the curve have a meaningful physical interpretation in the $|\mu| \to \infty$ limit.

In our notation, the world-sheet parameter, z, has dimensions of energy. This means that z has to be scaled as we take decoupling limits, like $|\mu| \to \infty$. In particular, since the softly broken curves are constructed as deformations of the $N = 2$ curve, the interesting features of the curve occur at $z \sim O(\Lambda_2)$ as long as $|\mu|, |\epsilon| \ll |\Lambda_2|$. However, in the $|\mu| \to \infty$ limit, the right angle limit of the NS'-brane is a singular configuration where the original curve factorizes in components. The relevant scale of the curve now is the $N = 1$ dynamical scale Λ_1, and one can easily check that $z \sim O(\Lambda_1)$ only captures the NS component of the degenerate $N = 1$ curve.

In general, in order to isolate different structures in the curve surviving in the $|\mu| \to \infty$ limit, we write $z = \alpha u$, with $u \sim O(\Lambda_1)$. Now if α is chosen of order $O(1)$, in the $|\mu| \to \infty$ limit only the component of the NS-brane at $|z| = \infty$ (the non-rotated brane) remains; This component is supersymmetric: $v = u$, $w = 0$, $t \sim u^{N_c - N_f}$ and $X^7 = 0$. If α is chosen of order $O(|\Lambda_2/\Lambda_1|)$ in the limit we remain with the matter component of the curve: $v = \epsilon|\Lambda_1|/(|\Lambda_2|\bar{u})$, $w = 0$, $t \sim u^{N_c}/(u - c'_+|\Lambda_1|)^r(u - c'_-|\Lambda_1|)^{N_f - r}$ and $X^7 = 0$. Finally for $\alpha = O(|\mu(\Lambda_2)^2/(\Lambda_1)^2|)$ the component of the rotated NS'-brane remains. In this case $v = \epsilon|\Lambda_1|^2/[|\mu(\Lambda_2)^2|\bar{u}]$, $w \sim |\Lambda_1|^2/u$, $t \sim (u)^{N_c}$ and $X^7 = 0$. From the last expression for v we see that to stabilize this component in the $|\mu| \to \infty$ limit we need to set $\epsilon \sim \mu(\Lambda_2)^2$. More precisely we set

$$\epsilon = |\mu(\Lambda_2)^2| \, f_\epsilon(|m_\lambda|, |\Lambda_1|, R, \theta) \tag{3.2}$$

where by m_λ we indicate the effective supersymmetry breaking mass scale, to be identified at first order with the gaugino mass. The real function f_ϵ must vanish for $m_\lambda = 0$ and depends also on the phases of all complex quantities which we denoted collectively by θ; this dependence is constrained by the $U(1)$ symmetries.

In this way, in the limit $|\mu| \to \infty$, the first two components of the curve are supersymmetric, whereas the third component, the one associated to the rotated NS'-brane, is not:

$$\begin{array}{llllll}
\alpha \sim O(1) : & v = u \ , & w = 0 \ , & t \sim u^{N_c - N_f} \ , & X^7 = 0 \\
\alpha \sim O\left(\left|\dfrac{\Lambda_2}{\Lambda_1}\right|\right) : & v = 0 \ , & w = 0 \ , & t \sim \dfrac{u^{N_c - N_f}}{(u - c'_+)^r (u - c'_-)^{N_f - r}} \ , & X^7 = 0 \\
\alpha \sim O\left(\left|\dfrac{\mu(\Lambda_2)^2}{\Lambda_1}\right|\right) : & v = \dfrac{|\Lambda_1|^2 f_\epsilon}{\bar{u}} \ , & w = \dfrac{|\Lambda_1|^2 c'_+ c'_-}{u} \ , & t \sim u^{N_c} \ , & X^7 = 0 \ .
\end{array} \tag{3.3}$$

This is of course what we expected to happen from the discussion in the previous section.

In the case $m_f \neq 0$ and $r = 0$ (see also ref. [21]), the analysis is much simpler since it turns out that the curve remains in a single component and ϵ is constant in the $|\mu| \to \infty$ limit. Thus we have $\epsilon = g_\epsilon(|m_\lambda|, |m_f|, |\Lambda_1|, R, \theta)$, where g_ϵ vanishes if either of m_λ or m_f vanishes. The curve is given by eq. (2.8) with $r = 0$, $\lambda = 0$, $\zeta = -c_+ m_f$, $z_+ = 0$, $z_- = -m_f$ and $|\Lambda_2| \to |\Lambda_1|$.

It is interesting to notice that expanding $\epsilon(m_\lambda, m_f, m_{Adj}, \Lambda_1, R)$ in positive powers of m_λ for very small m_λ, and in the $|m_{Adj}| \to \infty$ limit, the $U(1)$ charges imply that the most general form for ϵ is [2]

$$\epsilon = m_\lambda R^2 \left\{ \left[(\Lambda_1)^{3N_c - N_f} (m_f)^{N_f} \right]^{\frac{1}{N_c}} + D\, m_{Adj} (\Lambda_2)^2 \right\} \; + \text{ c.c. } + \text{ higher orders} \tag{3.4}$$

where D is an adimensional constant. This gives the explicit form of f_ϵ and g_ϵ to first order in m_λ.

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