# $AdS_6$ Interpretation of 5D Superconformal Field Theories

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## Abstract

We explore the connection of anti-de-Sitter supergravity in six dimensions, based on the exceptional $F(4)$ superalgebra, and its boundary superconformal singleton theory. The interpretation of these results in terms of a D4-D8 system and its near horizon geometry is suggested.

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Recently, a close connection between superconformal field theories in $d$ dimensions and anti-de-Sitter supergravities in $d + 1$ dimensions has emerged [1]-[24]. The original proposed duality between world-volume theories of $p$-branes systems and their nearly horizon geometry in $AdS_{p+2}$ space-time [2] has further been interpreted as a correspondence between superconformal $p+1$ world-volume theories with $N$ (Poincaré) supersymmetries and $AdS_{p+2}$ supergravity with $2N$ supersymmetries. In addition, the global symmetries of the former correspond to gauge symmetries of the latter [3, 4, 5].

Apart from its physical interpretation, these recent developments have been inherited by some peculiar properties of a class of unitary representations of the $O(p+1, 2)$ conformal group, the so called singletons [25], which have the properties that they allow propagation of massless degrees of freedom not on the $AdS_{p+2}$ bulk but rather on its boundary $\partial AdS_{p+2} = \tilde{M}_{p+1}$, where $\tilde{M}_{p+1}$ is a certain completion of Minkowski space. Bulk degrees of freedom turn out to be composite of singletons. This fact being closely related to the group property of decomposition of tensor products of singleton representations [3]. When this state of affairs is enlarged to incorporate supersymmetry, then a quite remarkable relation is discovered between BPS $p$-brane world-volume dynamics and gauged superalgebras.

The relevant superalgebras for $p = 1, 2, 3$ and $5$ are members of infinite sequences of superalgebras, usually denoted by $OSp(N/M)$ and $SU(N/M)$, the orthosymplectic and unitary series [26]. However, in the particular case of a six-dimensional anti-de-Sitter space, there is an isolated superalgebra, $F(4)$, whose bosonic part is $SO(5, 2) \times SU(2)$. It is the aim of the present work to study this case.

It turns out that the gauged $F(4)$ theory [27] has on its boundary a $N = 2$ superconformal field theory which is a superconformal fixed point of a five-dimensional Yang-Mills theory. The relevant singleton representation of $F(4) \times G$ on the supergravity side is just a 5d hypermultiplet in a symplectic representation of the flavour group $G$ at the fixed point of the Yang-Mills theory.

Five-dimensional superconformal field theories with some flavour symmetry group $G$ will correspond in the six dimensional bulk to the gauged $F(4)$ theory [27], coupled to matter vector multiplets gauging the group $G$. [1] These theories are based on a non-linear sigma-model [27, 28]

$$R^+ \times \frac{SO(4,k)}{SO(4) \times SO(k)}, \qquad (1)$$

where the diagonal $SU(2)$ in $SO(4) = SU(2) \times SU(2)$ is the gauged R-symmetry of the world-volume theory and we have additional gauge bosons corresponding to the flavour symmetry $G$ ($k = \dim G$) and a neutral $U(1)$ vector in the gravitational multiplet. The latter is absorbed by an antisymmetric tensor $B_{\mu\nu}$ to give a "massive" two-form [27]. The gauged bulk theory has a $G$-invariant anti-de-Sitter vacuum for a fixed value of the *dilaton* and all matter scalars vanishing [27]. The gauged $F(4)$ theory, in the Poincaré limit, corresponds to the $d = 6$, $(1,1)$ theory (with 16 supercharges).

One may ask why no superconformal extension exists for the $(2,2)$ theory, contrary to the other cases in diverse dimensions. This can be understood from the fact that the $N = 4$, 5d theory in Minkowski space-time admits only vector multiplets and the latter are not conformal multiplets in five dimensions. In other words, there is no candidate for a supersingleton multiplet for a hypothetical maximally extended theory. One may also ask what is the correspective of this statement in terms of brane world-volume, in the spirit of [2]. $n$ D4-branes in type IIA realize on their world-volume a $U(n)$ Yang-Mills theory which flows in the IR to a free (and not superconformal) theory of vector multiplets. The near horizon geometry of the D4-branes is not related to $AdS_6$. The regime of validity of the supergravity solution (small curvature and small dilaton) in the large $n$ limit does not cover the IR region, as discussed in [7].

However, in the $N = 2$ case the 5d hypermultiplets are conformal invariant and there

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[1]The Lagrangian for gauged supergravity coupled to matter vector multiplets has not yet been constructed.

is a candidate supergravity multiplet for the corresponding $F(4)$ superalgebra. Non-trivial superconformal fixed points in $N = 2$ Yang-Mills theories in 5d were found in [29]. A common characteristic of these theories is that the global symmetries are enhanced at the fixed point. The R-symmetry for $N = 2$ five-dimensional Yang-Mills theories is $SU(2)_R$ and we do not expect that it is enhanced at the superconformal point: it becomes the bosonic $SU(2)$ subgroup of the superalgebra $F(4)$. We will consider Yang-Mills theories with gauge group $USp(2n)$ with matter in the antisymmetric representations and $N_f$ fundamentals ($N_f \leq 7$): the additional global symmetry is $SU(2) \times SO(2N_f) \times U(1)_I$, where $U(1)_I$ is associated with the current $*(F \wedge F)$, which exists and is conserved in five dimensions. For generic values of the parameters, these theories are IR free. However, if we tune the parameters (in particular the *bare* coupling constant) in such a way that the effective coupling constant diverges at the origin of the Coulomb branch, we find non trivial fixed points, where the global symmetry is enhanced to $SU(2) \times E_{N_f+1}$ ($E_5 = Spin(10), E_4 = SU(5), E_3 = SU(3) \times SU(2), E_2 = SU(2) \times U(1), E_1 = SU(2)$) [29]. Other fixed points with $U(1)$ or no global symmetry at all were constructed in [30]. The theory at the fixed point is a superconformal theory of interacting hypermultiplets, with a global symmetry $SU(2) \times E_{N_f+1}$. The global symmetry quantum numbers are carried by instantons, which are particles in five dimensions and are the only states charged under $U(1)_I$ [29]. Instantons can become massless exactly when the coupling constant diverges. If we give mass to the adjoint we get another series of fixed points with $E_{N_f+1}$ global symmetry.

Evidence for non-trivial fixed points for other simple gauge groups coupled to matter in various representations can be found in [31]. There is a reason for having discussed the $USp(2n)$ case. It is the case which admits a brane realization in terms of D4 and D8-branes. The 5d fixed points were indeed originally found by analyzing a D4-D8-brane configuration [29]. We briefly review the construction. Consider type I′ on $S^1/\mathbf{Z}_2$. There are two orientifold planes and we will consider $2N_f$ D8 branes coinciding with one of them. The $USp(2n)$ Yang-Mills theory, with exactly the matter content described above,

is obtained on the world-volume of $2n$ D4-branes living at the same orientifold. The global symmetry of the D4-branes theory is $SU(2) \times SO(2N_f) \times U(1)_I$. The $SU(2)$ factor comes from the space-time Lorentz group; it is the less relevant factor in our discussion, and disappears if we give a mass to the matter in the antisymmetric representation. $SO(2N_f)$ is the gauge symmetry of the $N_f$ D8-branes nine-dimensional world-volume theory; it is a subgroup of the $SO(32)$ type I gauge group. $U(1)_I$ corresponds to the $U(1)$ vector field of the type IIA theory. The enhancement of global symmetry can be understood using the duality with the $SO(32)$ heterotic string [29]. In nine dimensions, T-duality connects the $SO(32)$ and $E_8 \times E_8$ heterotic strings. The type I′ backgrounds with enhanced symmetry correspond to the points in the moduli space of the $SO(32)$ heterotic string on $S^1$ where there is an enhancement of symmetry to $E_8 \times E_8$, or subgroups. The $U(1)_I$ charge is the winding number of the dual heterotic string and the perturbative enhancement of space-time symmetry takes place at points in the moduli space where heterotic winding modes become massless. In the previous examples, $SO(2N_f) \times U(1)_I$ is enhanced to $E_{N_f+1}$ .

For our purpose the details of the type I′ theory are irrelevant: we are describing a system of $2n$ D4-branes in the background of $2N_f$ D8-branes at a nine-dimensional orientifold plane. The value of the dilaton at the orientifold diverges at the point in moduli space where we expect enhanced symmetry [32, 29]; the heterotic winding modes are D0-branes in type I′ and therefore become massless, providing the extra gauge bosons needed to fill the adjoint of $E_{N_f+1}$ [33, 34]. The value of the dilaton at the orientifold is, for the five-dimensional theory on the D4-branes, the effective coupling constant at the origin of the Coulomb branch, which is therefore tuned to infinity. The D0-branes are instantons for the D4-branes theory. This gives the picture of a non-trivial superconformal fixed point, with relevant degrees of freedom corresponding to instantons, obtained by tuning to zero the inverse coupling constant [29].

It is amusing to notice that the enhancement of the flavour symmetry $O(2N_f) \times O(2) \to$

$E_{N_f+1}$ at the superconformal fixed point is the compact version of the enhancement of the T-S duality group to the U duality group $O(10-d, 10-d) \times O(1,1) \rightarrow E_{11-d(11-d)}$, obtained by replacing $N_f \rightarrow 10 - d$ [35].

The spinor representations of $SO(2N_f)$ $(SO(10 - d, 10 - d))$, appearing in the decomposition of the adjoint of $E_{N_f+1}$ $(E_{11-d(11-d)})$ and providing the missing vector bosons of the enhanced symmetry, can be obtained in the brane description by quantizing the modes corresponding to the D0-D8 open strings [33, 34].

Let us examine the implications of a $AdS_6$ supergravity description for the superconformal fixed points with $E_{N_f+1}$ global symmetry. The D4-D8 system has a global symmetry $SU(2)_R \times SU(2) \times E_{N_f+1}$. For simplicity, we do not consider the $SU(2)$ factor in the following analysis: the corresponding superconformal theory is obtained as a limit of the same $USp(2n)$ Yang-Mills theory with a mass term for the multiplet in the antisymmetric representation. These fixed points will correspond to the gauged $F(4)$ supergravity [27] coupled to matter vector multiplets in the adjoint of $E_{N_f+1}$. The superconformal theory at the boundary is a theory of singleton hypermultiplets transforming in a symplectic representation of $E_{N_f+1}$. The massless bulk supermultiplets can be identified with bilinear composite operators on the boundary, corresponding to the supermultiplets of global currents [3]. The multiplets are classified according to the maximal subgroup $USp(4) \times O(2) \subset O(5,2)$, i.e. by the energy level $E_0$ and a $USp(4)$ representation.

Let us analyse in details the case $N_f = 6$. The singleton hypermultiplets transform in the fundamental of $E_7$, which decomposes under $SO(12) \times O(2)$ as $\mathbf{56} = (\mathbf{12}, 2) + (\mathbf{32}, 1)$. The hypermultiplets contain the fermion $(E_0 = 2)$ and scalar $(E_0 = 3/2)$ fields, $\psi^A, A_i^A$, where $A$ is an index in the $\mathbf{56}$ of $E_7$ and $i$ is a index in the $\mathbf{2}$ of $SU(2)$. The scalars obey the reality condition: $(A_i^A)^* = \epsilon_{ij} \Omega_{AB} A_j^B$ (where $\Omega_{AB}$ is the $E_7$ antisymmetric tensor). The total number of states is $4 \times 56$. The following bilinears belong to the energy-momentum tensor supermultiplet, which contains $2^6$ states and is related to the graviton supermultiplet

in $AdS_6$,

$$\text{graviton}\,(E_0 = 5):\quad \text{conserved traceless energy-momentum tensor}$$

$$\text{gravitinos}\,(E_0 = 9/2):\quad \gamma\partial A_i^A \gamma_\mu \psi^B \Omega_{AB},$$

$$SU(2)_R\,\text{currents}\,(E_0 = 4):\quad A_i^A \overleftrightarrow{\partial}_\mu A_j^B \Omega_{AB},$$

$$\text{antisymmetric tensor and singlet vector}\,(E_0 = 4):\quad \psi^A \sigma_{\mu\nu} \psi^B \Omega_{AB},$$

$$\text{fermions}\,(E_0 = 7/2):\quad A_i^A \psi^B \Omega_{AB},$$

$$\text{singlet scalar}\,(E_0 = 3):\quad A_i^A A_j^B \epsilon_{ij} \Omega_{AB}. \tag{2}$$

The global current supermultiplet, related to the $AdS_6$ vector multiplets, with $2^4 \times \dim G$ states, contains

$$E_7\,\text{currents}\,(E_0 = 4):\quad A_i^A \overleftrightarrow{\partial}_\mu A_j^B \epsilon_{ij} T_{AB}^I + \psi^A \gamma_\mu \psi^B T_{AB}^I,$$

$$\text{fermions}\,(E_0 = 7/2):\quad A_i^A \psi^B T_{AB}^I,$$

$$\text{scalars}\,(E_0 = 3):\quad A_i^A A_j^B \sigma_{ij}^a T_{AB}^I, \qquad (\dim SU(2), \dim E_7),$$

$$(E_0 = 4):\quad \psi^A \psi^B T_{AB}^I, \qquad (1, \dim E_7), \tag{3}$$

where $T_{AB}^I$ are the $E_7$ (symmetric) matrix generators in the fundamental representation. Note that the total number of scalars is $4\dim G + 1$, in agreement with (1).

The last set of scalars, singlets under $SU(2)_R$ and in the adjoint of $E_7$, are the highest components (highest number of $\theta$ and, as a consequence, also highest conformal dimension) of the global current supermultiplet and therefore correspond to supersymmetry preserving deformations of the superconformal point. Having conformal dimension 4, they correspond to relevant deformations, which break superconformal invariance (and also the global symmetry $E_7$). Going to the Cartan subalgebra of $E_7$, we find $N_f$ parameters $t_i$ ($i = 0, ..., N_f - 1$). $t_0 = 1/g^2$ corresponds to turning on the inverse coupling constant, breaking the global symmetry to $SO(2N_f) \times U(1)_I$, which is appropriate for the Yang-Mills

theory with non-infinite coupling. The other parameters $m_i$ corresponds to masses for some of the quarks and also break, partially or totally, $SO(2N_f)$. The other scalars in the global current multiplet, having lower dimension and therefore not being the highest components of their supermultiplet, are supersymmetry breaking deformations.

We see that also the scalar in the supergraviton multiplet, which is a singlet of the R and global symmetries, is not the highest component of its supermultiplet and therefore it is a deformation which breaks supersymmetry. In particular, the *dilaton* does not correspond to the coupling constant of the Yang-Mills theory. It is interesting to consider what happens to this singlet scalar in superconformal theories in different dimensions. In $D = 4$, it is the highest component in its multiplet (which is the graviton multiplet for $N = 4$, a tensor multiplet for $N = 2$ and an hypermultiplet for $N = 1$ [8]). In all the cases, it has dimension 4. It corresponds to a marginal supersymmetric deformation, which can be identified with the coupling constant: the theory has indeed a line of fixed points. In $D = 3$ and 6, the singlet is not the highest component of the supermultiplet and, therefore, it is a relevant deformation which breaks supersymmetry and it cannot be identified with the coupling constant. On the other hand, the (inverse) coupling constant is an irrelevant deformation in $D = 3$, which, therefore, does not belong to the massless multiplets in $AdS$, but to some massive KK mode. In $D = 6$, the (inverse) coupling constant is a relevant parameter and belongs to some global current multiplet, as in the $D = 5$ case.

Note that the five dimensional Yang-Mills theories considered in this paper have been constructed originally in terms of D4 and D8-branes systems. Also, we have argued an interpretation of their fixed point superconformal field theories (in the large $n$ limit) in terms of boundary singleton theories of $F(4)$ supergravities in $AdS_6$. These two facts give evidence that a solitonic D4-D8-branes configuration preserving eight supercharges, with a near horizon geometry described by a $F(4)$ gauged six dimensional supergravity theory (with 16 supercharges) should exist. In fact, since D8-branes are only known to exist in

the "massive" type IIA supergravity [36, 37], it may be natural to consider such solitonic solutions in this ten dimensional supergravity. Such a relation between the $F(4)$ and the massive IIA supergravities is also suggested by the fact that they are the only known cases where a Higgs mechanism takes place, where a massless two-form absorbs the degrees a freedom of a gauge boson to become massive. As a result, the relations between $p$-branes world-volumes and $AdS_{p+2}$ theories that were already known for $p = 3$ in type IIB and $p = 2, 5$ in eleven dimensional supergravity would be completed for $p = 4$ in the massive type IIA.

Although the connection between $F(4)$ and massive type IIA supergravities seems to suggest that the gauged six-dimensional supergravity, dual to the five-dimensional fixed point, can be obtained as the near horizon geometry of a configuration with D4 and D8 branes, it cannot be excluded that the brane configuration with $AdS_6$ near-horizon geometry is instead realized in a different set-up. A chain of dualities, for example, transforms the D4-D8 system into a fivebrane wrapped around a circle in the $E_8 \times E_8$ heterotic string. In this heterotic description, the full series of $E_{N_f+1}$ global symmetries, which would be harder to get in a massive type IIA compactification, are more likely to be manifest as space-time fields, as discussed for six-dimensional (1,0) theories with $E_8$ symmetry in [15]. The five-dimensional fixed points discussed in this paper are indeed the reduction to 5d (with additional Wilson line) of the six-dimensional (1,0) theory.

It would be interesting to find the right solution. Besides giving evidence for the $AdS$/CFT correspondence, it would provide an explicit KK reduction from ten or eleven dimensions to six, and the KK modes would give information on the spectrum of conformal operator of the fixed point theory.

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