# PREHEATING THE UNIVERSE IN HYBRID INFLATION

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One of the fundamental problems of modern cosmology is to explain the origin of all the matter and radiation in the Universe today. The inflationary model predicts that the oscillations of the scalar field at the end of inflation will convert the coherent energy density of the inflaton into a large number of particles, responsible for the present entropy of the Universe. The transition from the inflationary era to the radiation era was originally called reheating, and we now understand that it may consist of three different stages: preheating, in which the homogeneous inflaton field decays coherently into bosonic waves (scalars and/or vectors) with large occupation numbers; backreaction and rescattering, in which different energy bands get mixed; and finally decoherence and thermalization, in which those waves break up into particles that thermalize and acquire a black body spectrum at a certain temperature. These three stages are non-perturbative, non-linear and out of equilibrium, and we are just beginning to understand them. In this talk I will concentrate on the preheating part, putting emphasis on the differences between preheating in chaotic and in hybrid inflation.

## 1 Introduction

At the end of inflation all the energy density is in the homogeneous zero mode of the inflaton field. The Universe is in a vacuum-like state with zero temperature and vanishing particle and entropy densities. The problem of reheating is how to convert all this coherent energy into a state of thermalized relativistic particles. The original analysis [1] assumed the perturbative decay of the inflaton into bosons and fermions, as if the inflaton were already an ensemble of decoherent *particles*. Reheating ended when the total decay rate of the inflaton was of the order of the expansion rate of the Universe, $\Gamma \sim H$, while the total energy of the inflaton field decayed exponentially fast into other particles. As a consequence, the final reheating temperature only depended upon $\Gamma$. We understand today that, for certain parameter ranges, there is a new decay channel that is non-perturbative,[2] due to the coherent oscillations of the inflaton field, which induces stimulated emission of bosonic[a] particles into energy bands with large occupation numbers. The modes in these bands can be understood as Bose condensates, and they behave like classical waves. The backreaction of these modes on the homogeneous inflaton field and the rescattering among themselves produce a state that is far from thermal equilibrium and may induce very interesting phenomena, such as non-thermal phase transitions [3] with production of topological defects, a stochastic background of gravitational waves,[4] production of heavy particles in a state far from equilibrium, which may help GUT baryogenesis [5] or constitute today the dark matter in our Universe.[6] These classical waves eventually reach a state of turbulence where, hopefully, decoherence will occur and thermalization will follow, although these stages are not yet fully understood, either analytically or numerically.

The period in which particles are produced via parametric resonance is called *preheating*.[2] The idea is relatively simple, the oscillations of the inflaton field induce mixing of positive and negative frequencies in the quantum state of the field it couples to. In the language of quantum fields in curved space, creation and annihilation operators mix via Bogoliubov transformations,[b] $a_k = \alpha_k \bar{a}_k + \beta^* \bar{a}^\dagger_{-k}$, and with every oscillation of the inflaton field, new particles are produced, $n_k \equiv \langle \bar{0}| a_k^\dagger a_k |\bar{0}\rangle = |\beta_k|^2$. In the case of chaotic inflation, with a massive inflaton $\phi$ coupled to a massless scalar field $\chi$, the evolution equation for the Fourier modes, $\ddot{X}_k + \omega_k^2 X_k = 0$, with $X_k = a^{3/2}(t)\chi_k$ and $\omega_k^2 = k^2/a^2(t) + g^2\phi^2(t)$, can be cast in the form of a Mathieu equation, with coefficients $A = k^2/4a^2m^2 + 2q$ and $q = g^2\Phi^2/4m^2$, where $\Phi$ is the amplitude and $m$ is the frequency of inflaton oscillations, $\phi(t) = \Phi(t)\sin mt$. For certain values of the parameters $(A, q)$ there are exact solutions that grow exponentially with time, and each mode $k$ belongs to an instability band of the Mathieu equation.[c] These instabilities can be interpreted as coherent "particle" production with large occupation numbers. One way of understanding this phenomenon is to consider the energy of these modes as that of a harmonic oscillator, $E_k = |\dot{X}_k|^2/2 + \omega_k^2|X_k|^2/2 = \hbar\omega_k(n_k + 1/2)$. The occupation number of level $k$ can grow exponentially fast, $n_k \sim \exp(2\mu_k mt) \gg 1$, and these modes soon behave like classical waves. It is analogous to the well-known mechanism of generation of density perturbations during inflation.[1] The parameter $q$ during preheating determines the strength of the resonance. It is possible that the model parameters are such that parametric resonance does *not* occur, and then the usual perturbative approach would follow, with decay rate $\Gamma$. In fact, as the Universe expands, the growth of the scale factor and the decrease of the amplitude of inflaton oscillations shifts the values of $(A, q)$ along the stability/instability chart of the Mathieu equation, going from broad resonance, for $q \gg 1$, to narrow resonance, $q \ll 1$, and finally to the perturbative decay of the inflaton. Parametric resonance will stop whenever the inflaton decay is dominated by the perturbative decay, $qm < \Gamma$, or when the instability modes are redshifted away from the (last) narrow resonance band, $q^2m < H$.

## 2 Preheating in hybrid inflation

Until recently, preheating had been studied in chaotic and new inflation only,[9] where the end of inflation occurs when the rate of expansion is of order the mass of the field, $m \sim H$, and the inflaton starts to oscillate around the minimum of its potential, inducing particle production.[d] In a recent paper,[10] we have studied the case of hybrid inflation, where the end of inflation is triggered by the symmetry breaking of another scalar field coupled to the inflaton, and not

[a] Fermions can also be parametrically amplified, but their occupation numbers are constrained, $n_k \leq 1$, by Pauli's exclusion principle.

[b] For an introduction to particle production in strong external fields, see Grib et al.[7]

[c] For a recent comprehensive review on preheating after chaotic inflation see Kofman [8] and references therein.

[d] Although production of classical "waves" is more appropriate, we will nevertheless use the terminology of "particle" production with large occupation numbers.

by slow-roll.[11] This fact alone gives hybrid inflation several advantages with respect to chaotic inflation. First, we can consider models of inflation at low-energy scales, say the electroweak scale, which nevertheless give the correct amplitude of temperature fluctuations in the CMB. Second, we may have an effective frequency of oscillation of one or both of the scalar fields being much larger than the rate of expansion. As we will see, this induces a very efficient and long-lived narrow resonance, where many oscillations occur in one Hubble time, and a large number of particles are produced before their momenta are redshifted by the expansion of the Universe. Preheating can be very efficient in this case. Third, at the stage of rescattering, gravitational waves are produced with wavelengths at most of order the size of the horizon at that time, large enough today to be detected at gravitational wave interferometers such as LIGO.

There are two fields in hybrid models, $\phi$ drives slow-roll inflation and $\sigma$ triggers its end through a symmetry-breaking potential,

$$V(\phi,\sigma) = \frac{1}{4\lambda}(M^2 - \lambda\sigma^2)^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{2}g^2\phi^2\sigma^2\,. \tag{1}$$

These fields may couple to yet another one, $\chi$, with couplings $(h_1^2\,\phi^2 + h_2^2\,\sigma^2)\chi^2/2$. Preheating depends very strongly on the model parameters. During inflation, the symmetry-breaking field $\sigma$ has a large mass, due to its coupling to $\phi$, and is fixed at $\sigma = 0$. The inflaton $\phi$ slow-rolls down its effective potential $V(\phi) = V_0 + m^2\phi^2/2$, driving inflation and producing the metric fluctuations that later will give rise to temperature anisotropies in the CMB and density perturbations for large-scale structure. The amplitude of temperature anisotropies on the scale of the horizon, as seen by COBE,[12] constrains the parameters of the model to satisfy

$$\frac{g}{\lambda\sqrt{\lambda}}\,\frac{M^5}{m^2 M_{\rm P}^3} \simeq 3.5\times 10^{-5}\,, \tag{2}$$

$$n - 1 \simeq \frac{\lambda}{\pi}\,\frac{m^2 M_{\rm P}^2}{M^4} < 0.2\,. \tag{3}$$

Contrary to the case of chaotic inflation, these constraints leave plenty of freedom to chose the model parameters. One of the advantages of hybrid inflation is that the rate of expansion at the end of inflation, $H \sim M^2/\sqrt{\lambda}M_{\rm P}$, could be in a wide range of scales, from just below Planck scale, as in models of supergravity hybrid inflation,[15] all the way to the electroweak scale. In terms of the rate of expansion, COBE constraints can be written as $g = 2\times 10^{-4}(n-1)\,M/H$ and $n - 1 = 2m^2/3H^2 < 0.2$.

When the inflaton $\phi$ falls below $\phi_c = M/g$, the field $\sigma$ triggers the end of inflation via a sudden or "waterfall" spontaneous symmetry breaking. This process occurs because of the exponential growth of quantum fluctuations. In most models of hybrid inflation, the mass-squared of the $\sigma$ field changes from large and positive to large and negative in much less than one $e$-fold.[e] It can then be shown that all tachyonic modes grow at approximately the same speed and thus the $\sigma$ field can be described as a homogeneous field.[10] The behaviour of $\phi$ and $\sigma$ after the end of inflation therefore follows the homogeneous field equations, and depends crucially on the ratio of couplings,

$$\frac{g}{\sqrt{\lambda}} \simeq (n-1)\,\frac{M_{\rm P}}{M}\,10^{-4}\,. \tag{4}$$

Depending on whether $g^2 \ll \lambda$, $g^2 \gg \lambda$ or $g^2 \sim \lambda$, the oscillations around the minimum will occur mainly along the $\phi$ field, the $\sigma$ field, or both, respectively. Due to their mutual couplings, the resonant production of particles could be very efficient or completely suppressed, depending on this ratio.[10] Another important factor is the relation of the frequency of oscillations to the rate of expansion. In chaotic inflation this was fixed by the end of slow-roll condition, $m \sim H$.

[e] There are hybrid models with two stages of inflation where this condition is not satisfied.[13,14]

However, in hybrid inflation we have the freedom to lower the rate of expansion with respect to the mass $M$ and thus we can consider models where the field oscillates many times, as much as $10^8$ times, in one Hubble time.[10] Thus, the resonant modes remain for a long time in their instability bands and may produce large occupation numbers before they are redshifted away by the expansion.[9] In particular, in hybrid preheating, the narrow resonance may be long-lived, $q^2\bar{m} > H$, even for very small parameters $q \ll 1$, thanks to $\bar{m} \gg H$. This is a fundamental difference with respect to chaotic inflation. Let us now consider in some detail the three different cases mentioned above.

In the first case, $g^2 \ll \lambda$, the frequency of oscillations of the $\phi$ field is $\bar{m} \simeq gM/\sqrt{\lambda} \gg m$ and, depending on the model parameters, can be much greater than the rate of expansion. For $\bar{m} \sim H$, the symmetry-breaking field soon settles at its minimum $\sigma_0$, giving large masses to any field it couples to, while the amplitude of inflaton oscillations quickly decays with the expansion of the Universe. As a consequence, there is essentially no production of either $\phi$ or $\sigma$ particles.[10] However, if a massless scalar (or vector) field $\chi$ couples to $\phi$, and not to $\sigma$ (for whatever symmetry reason), it is possible to produce $\chi$-particles in the broad resonance regime, for a wide range of couplings. This is the situation that most resembles that of chaotic inflation; the growth parameter is $\mu \sim 0.13$ and backreaction sets in after about 20 oscillations of the $\phi$ field. In the other case, $\bar{m} \gg H$, the amplitude of oscillations of $\phi$ after the end of inflation remains large, of order $\phi_c$, during many oscillations, driving explosive particle production even in the narrow resonance, for $\mu_k \ll 1$. Since the rate of expansion is so small, the modes remain in the narrow resonance for a long time, and backreaction on the inflaton oscillations occurs before the modes are redshifted away from the resonance.[10] This is very different from the behaviour in chaotic inflation. However, even if $\chi$ particles are quickly produced, we still do not have a significant production of $\phi$ or $\sigma$ particles before $\chi$-production backreacts on $\phi$.

In the case $g^2 \gg \lambda$, the situation described above is reversed: it is now the symmetry-breaking field that oscillates around its minimum, while the inflaton field becomes negligible. The frequency of $\sigma$ oscillations is $\bar{m} = \sqrt{2}\,M$ and, as before, it can be much larger than the rate of expansion. For $\bar{m} \sim H$, the inflaton settles at $\phi = 0$ while $\sigma$ oscillates with ever-decreasing amplitude. In this case, there is an insignificant production of either $\phi$ or $\sigma$ particles, due to their large effective masses.[10] Furthermore, if a field $\chi$ couples to either $\phi$ or $\sigma$ its production via parametric resonance is suppressed, because of the small amplitude of $\phi$ oscillations or of the effective mass induced by $\sigma_0$, respectively. It follows that preheating in this case is very inefficient. On the other hand, for $\bar{m} \gg H$, explosive particle production occurs for all fields, but mainly for $\sigma$ particles. Even though the resonance is not particularly broad, $q \sim 6$, the growth parameter is very large, $\mu \sim 0.3$, and there is very efficient preheating in just a few oscillations, before backreaction sets in. At the same time, production of $\phi$ and $\chi$ particles is possible, although not as efficiently as for $\sigma$.[10] Note that preheating in this model could occur even in the absence of extra fields $\chi$, simply due to the self-coupling of the symmetry-breaking field.

Finally, in the case $g^2 \sim \lambda$, typical of certain models of hybrid inflation in supergravity,[15] the two homogeneous fields oscillate with similar amplitudes and frequencies around the global minimum ($\phi = 0, \sigma = \sigma_0$). Since they are coupled, their frequencies and amplitudes vary rather chaotically and it takes many oscillations for their behaviour to stabilize around a periodic oscillation. During the chaotic motion, there can be no parametric resonance since this effect requires a periodic behaviour. By the time the oscillations become periodic, the amplitude has decreased so much that not even the narrow resonance can be excited. Thus, for $\bar{m} \sim H$, there is no particle production in either of the three fields.[10] However, for $\bar{m} \gg H$, there are many oscillations in one Hubble time and eventually the motion becomes periodic while the amplitude of oscillations is still large. This results in a mild production of $\phi$ and $\sigma$ particles and an explosive production of $\chi$ particles, if coupled only to the $\phi$ field.[10]

It should be noted that, in hybrid preheating, the equation that describes the instability growth of modes $\chi_k$ is not exactly of the Mathieu type. Even if the $\chi$ field couples only to one of the fields, $\phi$ or $\sigma$, the fact that the two homogeneous fields follow coupled equations implies that the frequencies of oscillations of each field will vary strongly with time, unless the other field is fixed at its minimum. This does no preclude parametric resonance, it simply modifies the stability/instability chart. In some of the cases we studied,[10] the analysis with the Mathieu equation is a reasonably good approximation; in others, one has to compute the corresponding spectrum of instability bands. This is particularly important for the case $g^2 \sim \lambda$, where both fields oscillate simultaneously in a rather chaotic way and their frequencies of oscillation depend explicitly on the value of the other field, which changes within one oscillation.

## 3 Phenomenological consequences

The processes by which the classical waves with large occupation numbers produced at preheating decohere and become an ensemble of relativistic particles in thermal equilibrium is still uncertain. Numerical lattice simulations have been performed to try to understand the process of backreaction and rescattering of those waves among themselves and with the inflaton background.[16] however, no one has yet been able to compute the final reheating temperature of a model in which preheating was important. It could be that, in the end, the reheating temperature is precisely the one computed from the usual perturbative analysis, $T_{\rm rh} \sim 0.1\sqrt{\Gamma M_{\rm P}}$. However, the reheating temperature is not the only observable one can consider; in fact, preheating has opened the door to very exotic phenomena that might have novel experimental signatures with which to test our models of inflation. The interesting period is that after backreaction and before thermalization, where large numbers of particles (or waves) rescatter off themselves in a state far from equilibrium. Among these new phenomena, there are non-thermal phase transitions [3] with production of topological defects; generation of gravitational waves;[4] or production of heavy particles in a state far from equilibrium, which may help GUT baryogenesis [5] or constitute the dark matter in our Universe.[6]

I will concentrate here on the production of a stochastic background of gravitational waves, which has specific signatures in the case of hybrid preheating. The idea is the following: the collisions among coherent waves of particles during rescattering radiates a reasonable fraction of energy, typicaly $\sim 10^{-5}$, in the form of gravitational waves. Their spectrum today depends on the details of rescattering and can be computed only numerically in lattice simulations.[4] However, the low-frequency end of the spectrum, for wavelengths of order the size of the horizon at rescattering, can be computed analytically since in that case it is dominated by the gravitational bremsstrahlung associated with the scattering of $\chi$ particles off the inflaton condensate, with the corresponding "evaporation" of inflaton particles. Taking into account that the occupation numbers of $\chi$ particles are typically of order $n_k(\chi) \sim 10^2 h_1^{-2}$ at the end of rescattering,[9] and assuming that decoherence and thermalization occurred immediately after this stage, one can estimate the fraction of energy density in gravitational waves today [4]

$$\Omega_{\rm gw}(\omega)h^2 \sim \Omega_{\rm rad}h^2 \, \frac{\bar{m}^2}{h_1^2 M_{\rm P}^2}\,\frac{\omega}{H_{\rm rs}}\Big(\frac{g_0}{g_*}\Big)^{1/3}\,, \tag{5}$$

where $\Omega_{\rm rad}h^2 \simeq 4\times 10^{-5}$ and $\Omega_{\rm gw}(\omega) \equiv (\rho_c^{-1} d\rho_{\rm gw}/d\ln\omega)_0$. On the other hand, the wavelengths $2\pi/k_{\rm rs}$ of the gravitational wave spectrum are redshifted to $\lambda \simeq 0.5(M_{\rm P}H_{\rm rs})^{1/2}k_{\rm rs}^{-1}$ today.[4] In particular, the largest wavelength today corresponds to the size of the horizon at rescattering, or $k_{\rm rs} \sim H_{\rm rs}$, and therefore the minimum frequency in the spectrum is $f_{\rm min} \simeq 2\times 10^2\ (H_{\rm rs}/{\rm TeV})^{1/2}$ Hz, which is right in the centre, for $H_{\rm rs} \sim 1$ TeV, of the range of frequencies detectable by LIGO, $10\,{\rm Hz} \leq f \leq 10^4$ Hz. In chaotic inflation, the fraction in gravitational waves today is typically [4] of order $\Omega_{\rm gw}h^2 \sim 10^{-12}$ at the minimum frequency $f_{\rm min} \sim 10^6$ Hz, which lies outside the range of frequencies and expected sensitivity of LIGO. The large rate of expansion at rescattering turns out to be a disadvantage for detecting gravitational waves from preheating in chaotic inflation. One would require a much lower rate, of order a few TeV, to be within the range of LIGO.

Let us consider now a concrete hybrid inflation model with a low rate of expansion at the end of inflation, $H \sim 10^2$ TeV. Such a model could be consistent with COBE observations with very natural parameter values, $g^2 \sim 0.01, \lambda \sim 1, M \sim 10^{12}$ GeV and $m \sim 1$ TeV. This model is of the first type, $g^2 \ll \lambda$, and has a large frequency of oscillations of the inflaton field, $\bar{m} \sim 10^6 H$. There will be explosive production of $\chi$ particles even in the narrow resonance, $q \sim 10^4 h_1^2 \sim 10^{-3}$, with a very small growth parameter $\mu \simeq q/2$. Backreaction in this model occurs at $\bar{m}t_1 \simeq (1/4\mu)\ln(10^6\bar{m}(\bar{m}t_1)^3/h_1^5 M_{\rm P})^{1/2} \sim 33/q$, which is still below one Hubble time, $t_1 \sim H^{-1}/30$, after the end of inflation. The occupation numbers of $\chi$ particles at this stage can be estimated[9] as $n_k(\chi) \simeq 3\times 10^2 h_1^{-2} q^{-1/4} \sim 30\,h_1^{-5/2}$. Therefore, the estimate of Eq.(5) should be modified with an extra factor $10h_1$ in the denominator. Taking $H_{\rm rs} \sim 10$ TeV at the end of rescattering, the fraction of energy density in gravitational waves today is $\Omega_{\rm gw}h^2 \sim 4\times 10^{-10}$ at the minimum frequency $f_{\rm min} \sim 600$ Hz, just in the appropriate range for detection by LIGO.

We conclude that preheating in hybrid inflation has many features that differs from the usual chaotic or new inflation type of preheating, and makes it phenomenologically attractive: inflation may occur at a much lower scale, and still give the desired amplitude of temperature anisotropies in the CMB; there could be many oscillations of the inflaton field per Hubble time after inflation, which allows for a long-lived narrow resonance and very efficient preheating; and it is possible, in certain models, to generate a stochastic background of gravitational waves in a range accesible to observations.

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