

CERN-TH/97-205
OUTP-98-21P

Hierarchies of R -violating Interactions from Family Symmetries

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ABSTRACT

We investigate the possibility of constructing models of R -violating $LQ\bar{D}$ Yukawa couplings using a single $U(1)$ flavour-symmetry group and supermultiplet charge assignments that are compatible with the known hierarchies of quark and lepton masses. The mismatch of mass and current eigenstates inferred from the known charged-current mixing induces the propagation of strong phenomenological constraints on some R -violating couplings to many others. Applying these constraints, we look for flavour-symmetry models that are consistent with different squark-production hypotheses devised to explain the possible HERA large- Q^2 anomaly. The $e^+d \rightarrow \tilde{t}$ interpretation of the HERA data is accommodated relatively easily, at the price of postulating an extra parity. The $e^+s \rightarrow \tilde{t}$ interpretation of the events requires models to have only small (2,3) mixing in the down quark sector. The $e^+d \rightarrow \tilde{c}$ mechanism cannot be accommodated without large violations of squark-mass universality, due to the very strong experimental constraints on R -violating operators. We display a model in which baryon decay due to dangerous dimension-five operators is automatically suppressed.

CERN-TH/97-205
OUTP-98-21P
March 1998

1 Introduction

Although the minimal supersymmetric Standard Model (MSSM) has dominated the phenomenological studies of supersymmetric signals [1], it has long been known that the symmetries of the Standard Model allow additional dimension-four couplings which may lead to interesting baryon- and lepton- number-violating processes [2]. These couplings are expected to be present in the low energy Lagrangian, unless forbidden by a symmetry such as R parity [3]. The complete set of such terms in the superpotential is:

$$\lambda L_i L_j \bar{E}_k + \lambda' L_i Q_j \bar{D}_k + \lambda'' \bar{U}_i \bar{D}_j \bar{D}_k$$

where the $L(Q)$ are the left-handed lepton (quark) superfields, and the $\bar{E}, (\bar{D}, \bar{U})$ are the corresponding right-handed fields. The symmetries of the model imply that there are 45 operators in total. However, there are many experimental constraints on these operators and their combinations, of which the most stringent comes from proton stability and excludes the simultaneous presence of certain products of $LQ\bar{D}$ and $\bar{U}\bar{D}\bar{D}$ couplings [4]. In addition, experimental constraints from the non-observation of modifications to Standard Model processes, or of possible exotic processes, gives bounds for most of the operators [5] and some combinations involving pairs of fermion generations. On the other hand, possible strong limits on R -parity violating interactions from cosmological arguments [6] can be avoided in various schemes [7], including the case of electroweak baryogenesis [8].

The large number of R -violating couplings complicates the systematic discussion of the phenomenological implications of these constraints. To date, most phenomenological analyses have assumed the dominance of a single operator, arguing that the Yukawa couplings of the Standard model display just such a property. In flavour-symmetry models, the dominant operator is naturally specified in the quark and lepton *current* basis. It is plausible to assume that mass mixing will induce non-zero coefficients for operators related to the dominant one. In Section 2 of this paper, we pursue this argument and compile the corresponding implications of some severe upper limits on particular R -violating interactions.

There have been many attempts to understand quark and lepton masses and the mixing angles between mass and current eigenstates using models for family symmetries [9]. Some of these reproduce successfully the qualitative features of fermion masses and mixings, and so provide plausible frameworks for analyzing the possible hierarchy of R -violating interactions [10]. In this paper we consider models based on a single $U(1)$ family symmetry, with fermion charges constrained by the observed hierarchy of fermion masses and mixing angles [11]. Such models are discussed in Section 3, where problems arising from symmetric mass matrices and from constraints on products of operators are emphasized.

As a specific application of this analysis, we look in Section 4 for models that might accommodate the proposed R -violating interpretations [12] of the possible HERA large- Q^2 anomaly [13, 14]. Of the 45 operators mentioned earlier, 9 could in principle lead

to resonant squark production at HERA. Of these, only the λ'_{121} , λ'_{131} and λ'_{132} cases survived an initial confrontation with other experimental constraints [12]. The suggestion that the apparent HERA excess may be due to single sparticle production via some R -violating couplings may not be gaining support [15]. Nevertheless, our analysis gives an indication which of the proposed mechanisms may be compatible with $U(1)$ family-symmetry models. Within the framework of the most symmetric schemes describing fermion masses, we find the bounds on R -violating couplings are so strong that such schemes do not lead to a significant excess of HERA events over the Standard Model prediction. However, in more general schemes we find that the λ'_{131} interpretation is easy to accommodate, whereas the λ'_{121} interpretation has difficulties with squark mass universality. We indicate how to construct a model consistent with the λ'_{132} interpretation, though we do not present a specific example. We also show how the structure of the quark and lepton mass matrices would be strongly constrained by the confirmation of such an R -violating signal.

2 R Violation and Family Symmetries

In order to obtain a realistic form for the quark and lepton masses and mixing angles, it is necessary to have non-diagonal forms for the mass matrices in the current basis. Diagonalising the mass matrix then implies that the mass eigenstates are mixtures of the current eigenstates. Attempts to make sense of the pattern of fermion masses and mixing angles often start with a family symmetry in the current basis which, when exact, allows only the third generation of quarks and leptons to acquire mass. Spontaneous breaking of this symmetry then allows other entries of the mass matrix to be non-zero. If the breaking is weak, these entries will be small, offering an explanation for the observed hierarchy of fermion masses and mixing angles.

If R parity is violated, such a symmetry would have important implications for R -violating operators, since couplings with different family structures would also appear with different powers of the family symmetry-breaking parameter. This is consistent with the common assumption that a single R -violating operator dominates. However, this assumption would apply in the current quark and lepton basis, and in the mass-eigenstate basis there would be several operators corresponding to the original dominant one in the current basis. Any given family-symmetry model would make characteristic predictions for the pattern of these related operators. Since there are stringent bounds on some of R -violating operators, particularly on those involving the first family and on some combinations that mix families, an analysis of such sub-leading operators in the mass-eigenstate basis may provide the most stringent bounds on the operators related by mass mixing. In addition, there could also be further contributions due to operators that are sub-dominant in the current basis, with strengths given by powers of the family symmetry-breaking parameter that are calculable in any given model.

The relation between the forms of the mass matrix in the current and the mass eigenstate

basis is given by

$$\begin{aligned}
M'_u &= V_u^L \cdot M_u^{Diag} \cdot (V_u^R)^\dagger \\
M'_d &= V_d^L \cdot M_d^{Diag} \cdot (V_d^R)^\dagger \\
M'_\ell &= V_\ell^L \cdot M_\ell^{Diag} \cdot (V_\ell^R)^\dagger
\end{aligned} \tag{1}$$

where $V_{u,d,\ell}^{L,R}$ are the unitary matrices relating the left- and right-handed u , d and ℓ current eigenstates to their mass eigenstates. We use the notation $\mathcal{L}_{mass} = \bar{\Psi}'_L M' \Psi'_R$ for all mass terms, so that the V_L are given by diagonalising $M' M'^\dagger$, whilst the V_R are obtained by diagonalising $M'^\dagger M'$. Only information on the entries of the Cabibbo-Kobayashi-Maskawa (CKM) product matrix

$$V^{CKM} = V_u^{L\dagger} V_d^L \tag{2}$$

is provided by experiments to date.

In general, one can construct models where the quark mixing is either in the up sector, or in the down sector, or both. In the class of models studied in this paper, in which the mass matrices have small off-diagonal entries generated by spontaneous breaking of a family symmetry, one may obtain useful connections between the mixing matrices and the elements of the mass matrices in the current basis by perturbative expressions for the off-diagonal elements, which are given in the Appendix. From this general analysis, it may be seen that in the specific case of the CKM mixing matrix the leading-order contribution comes from the d -quark mass-matrix elements that lie above the diagonal in our representation. As a result, we have little experimental input to guide us in constructing models for the elements below the diagonal. However, it has been noted for some time that a phenomenologically successful relationship results if one assumes a ‘‘texture zero’’ in the (1,1) position *and* symmetry between the (1,2) and (2,1) matrix elements [16]. In this case one finds the relation

$$|V_{ud}| = \left(\frac{m_d}{m_s} + \frac{m_u}{m_c} + 2\sqrt{\frac{m_d m_u}{m_s m_c}} \cos \phi \right)^{\frac{1}{2}}$$

where ϕ is the usual CP-violating phase in the CKM matrix. The fact that this relation works well is the only phenomenological indication we have for a symmetric structure of the mass matrices, and it may just be accidental. Nevertheless, we think it a useful starting point for our analysis, so we consider first a simple model capable of yielding this form and accommodating the remaining fermion masses and mixing angles [11].

The model consists of a single $U(1)$ family symmetry with the same charges for the left- and right-handed states, as shown in Table 1, where, e.g., the choice $a_i = (-4, 1, 0)$ gives an acceptable pattern for the mass matrices. Suppressing unknown numerical factors and

	Q_i	\bar{U}_i	\bar{D}_i	L_i	\bar{E}_i	H_2	H_1
$U(1)$	a_i	a_i	a_i	b_i	b_i	$-2a_3$	wa_3

Table 1: *Assignments of $U(1)$ charges.*

phases, which are all expected to be of order unity, with these charge assignments the up-quark mass matrix takes the form

$$M^{up} = \begin{pmatrix} \epsilon^8 & \epsilon^3 & \epsilon^4 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^4 & \epsilon & 1 \end{pmatrix}$$

The down-quark mass matrix has a similar form, but with a different expansion parameter $\bar{\epsilon} \approx \sqrt{\epsilon}$. Since the up and down sectors have similar structures, mixing is present in both sectors, though it may be larger in the down sector, simply because $\bar{\epsilon} > \epsilon$. For the mass matrices of [11] that we consider initially, one finds the following expressions for the quark mixing matrices ¹:

$$V_u^{L,R} \approx \begin{pmatrix} 1 & \epsilon & 2\epsilon^4 \\ -\epsilon & 1 & \epsilon \\ \epsilon^2 & -\epsilon & 1 \end{pmatrix}, \quad V_d^{L,R} \approx \begin{pmatrix} 1 & \bar{\epsilon} & 2\bar{\epsilon}^4 \\ -\bar{\epsilon} & 1 & \bar{\epsilon} \\ \bar{\epsilon}^2 & -\bar{\epsilon} & 1 \end{pmatrix}$$

using the second-order perturbation-theory formulae given in the Appendix.

We now discuss the importance of this mixing for R -parity violation. The most relevant experimental constraints are those on the operators $L_1 Q_1 \bar{D}_1$ and $L_1 Q_3 \bar{D}_3$, for which $\lambda'_{111} \leq 0.002$ from nuclear $\beta\beta$ decay [17] and for squark and gluino masses of 200 GeV, while $\lambda'_{133} \leq 0.001$ from bounds [18] on Majorana neutrino masses, again assuming masses of 200 GeV for the sparticles. ² However, operators related to these operators by mass mixing are also strongly constrained by these bounds. Consider first the relations

$$\begin{aligned} (L_1 Q_1 \bar{D}_1)' &= L_1 Q_1 \bar{D}_1 + \epsilon L_1 Q_2 \bar{D}_1 + 2\epsilon^4 L_1 Q_3 \bar{D}_1 + \dots \\ (L_1 Q_3 \bar{D}_3)' &= L_1 Q_3 \bar{D}_3 - \bar{\epsilon} L_1 Q_3 \bar{D}_2 + \bar{\epsilon}^2 L_1 Q_3 \bar{D}_1 + \dots \end{aligned} \quad (3)$$

where the notation $()'$ denotes effective operators in a current-eigenstate basis. We see that the operators $L_1 Q_2 \bar{D}_1$ and $L_1 Q_3 \bar{D}_1$ mix with $L_1 Q_1 \bar{D}_1$, so their coefficients are constrained to be the appropriate mixing coefficient (ϵ^{-1} and $(1/(2\epsilon^4))$, respectively) times the bound on λ'_{111} . Similarly, the coefficients of the operators $L_1 Q_3 \bar{D}_2$ and $L_1 Q_3 \bar{D}_1$ are constrained to be less than $\bar{\epsilon}^{-1}$ and $\bar{\epsilon}^{-2}$ times the bound on λ'_{133} , respectively.

We display below, as an example, matrices of upper limits on $L_1 Q_j \bar{D}_k$ operators. These limits follow from the mixing in this particular model, combined with the experimental

¹Lepton mixing is discussed in the next section.

²The quoted bound is clearly only approximate, as the exact value depends on soft parameters [19].

upper bounds for sfermion masses of 200 GeV [5, 17, 18, 20, 21, 22]. We first look at the bounds that arise from mixing with the λ'_{111} operator, tabulating the direct experimental bounds in cases where they are stronger than those originating from the λ'_{111} mixing:

$$L_{1jk}^{(\text{from } 111)} < \begin{pmatrix} 0.002 & 0.009 & 0.04 \\ 0.038(0.009) & 0.03 & 0.3 \\ 0.07 & 0.56 & 0.001 \end{pmatrix}$$

In certain entries we have two values, because bounds on $eu\bar{d}$ terms involve mixing in the up-quark sector, whereas bounds on $\nu_e d\bar{d}$ terms involve mixing in the down-quark sector. Then we repeat the analysis for mixing with the λ'_{133} coupling:

$$L_{1jk}^{(\text{from } 133)} < \begin{pmatrix} 0.002 & 0.04 & 0.04(0.02) \\ 0.07 & 0.03(0.02) & 0.02(0.004) \\ 0.02 & 0.004 & 0.001 \end{pmatrix}$$

and finally we gather all the best limits for the matrix elements in this particular model:

$$L_{1jk}^{\text{best}} < \begin{pmatrix} 0.002 & 0.009 & 0.04(0.02) \\ 0.02(0.009) & 0.03(0.02) & 0.02(0.004) \\ 0.02 & 0.004 & 0.001 \end{pmatrix}$$

Here, the bound on the (2,1) entry arises from constraints on λ'_{121} from $K \rightarrow \pi\nu\bar{\nu}$ [21], in the case that $V_{12,21}^{CKM}$ arises predominantly from the down-quark sector. At this stage we have not yet taken into account other bounds, especially bounds on products of R -violating couplings that pose even stricter constraints [23, 24]. As an example for this Ansatz, the couplings $L_1 Q_2 \bar{D}_1$ and $L_1 Q_1 \bar{D}_2$ appear at such an order in the family-symmetry breaking that the strong bound on the product of these couplings from contributions to Δm_K [23] is not satisfied. We shall return to this and related issues at a later stage.

It should be noted that the model of [11] has mixing in both the up and down sectors. Indeed, the (2,3) entry of the down mass matrix is $\bar{\epsilon} = 0.23$, which is much larger than V_{23}^{CKM} . Thus, to obtain viable mass matrices in this example, one needs a suppression of the mixing in $|V_{cb}| = (a' \frac{m_s}{m_b} + a \frac{m_c}{m_t} + 2\sqrt{aa' \frac{m_s m_c}{m_b m_t}} \cos \phi)^{\frac{1}{2}}$ [11]. This case, in which the mixing between states is much larger than would have been estimated just using the appropriate CKM mixing matrix element, serves as a healthy reminder of the potential importance of the details of the underlying model for fermion masses when drawing implications for R -violating phenomena.

3 Exploring Hierarchies of R -Violating Interactions

We now consider the effect of the $U(1)$ symmetry on the pattern of allowed R -violating interactions [10]. We first recall that possible sets of quark and lepton charges leading to

correct mass hierarchies are given [11] by:

Case 1: $a_i = b_i = (-4, 1, 0)$, where a_i and b_i are the quark and lepton charges respectively, and

Case 2: $a_i = (-4, 1, 0)$, $b_i = (-\frac{7}{2}, \frac{1}{2}, 0)$.

In **Case 1**, where leptons and quarks have the same charges, one needs an additional symmetry in order to eliminate dimension-four nucleon-decay operators. This may be done simply by imposing an anomaly-free flavour-independent baryon parity [25], under which the fields transform as

$$Z_3 : (Q, \bar{U}, \bar{D}, L, \bar{E}, H_1, H_2) \rightarrow (1, a^2, a, a^2, a^2, a^2, a) \quad (4)$$

This allows only the lepton-number-violating operators, while forbidding baryon-number-violating ones ³.

In **Case 2**, which is motivated by constraints on HERA-friendly models, the lepton charges of the first two generations are half-integers. One might at first think that the residual \tilde{Z}_2 symmetry of the $U(1)$ forbids the $L_{1,2}Q\bar{D}$ operators. However, it is straightforward to combine this \tilde{Z}_2 with a normal Z_2^M matter parity, so as to allow these terms while also forbidding the $\bar{U}\bar{D}\bar{D}$ terms. This is possible if $\tilde{Z}_2 \times Z_2^M$ is broken to a residual Z_2 by a field Φ that is odd under both symmetries. In this case, $\bar{U}\bar{D}\bar{D}$ is forbidden, because it transforms as $(+, -)$ under $\tilde{Z}_2 \times Z_2^M$. Similarly, $L_{1,2}Q\bar{D}$ transforms as $(-, -)$ and is also forbidden, but it occurs at $\mathcal{O}\left(\frac{\Phi}{M}\right)$ through the term $(\Phi LQ\bar{D})/M$.

Let us now pass to the charges of the R -violating operators. The first thing to notice is that the form of the mass matrices only determines the relative charges of the operators, not their absolute charges. To see this, note that the symmetric structure of M^{up} is unchanged if we add a family-independent constant to the charges of the \bar{U} fields. This shows that, as we have already mentioned, the charge normalization of our operators is undetermined by the mass structure.

However, anomaly cancellation must be imposed. With the general charge assignment given in the first line of Table 2, the coefficients of the $SU(3)^2 \times U(1)$, $SU(2)^2 \times U(1)$ and $U_Y(1) \times U(1)$ anomalies are proportional to $A_{3,2,1}$, where

$$\begin{aligned} A_3 &= 2 \sum a_i + \frac{3}{2}w_1 + \frac{3}{2}w_2 \\ A_2 &= \frac{3}{2} \sum a_i + \frac{1}{2} \sum b_i + \frac{1}{2}a_3(w-2) \\ A_1 &= \frac{11}{6} \sum a_i + \frac{3}{2} \sum b_i + \frac{1}{2}a_3(w-2) + 4w_1 + w_2 + 3w_3 \end{aligned} \quad (5)$$

We demand that these should vanish up to a Green-Schwarz term [27], i.e., $A_3 : A_2 : A_1 = 1 : 1 : 5/3$. The effect of this is shown in Table 2: in the first row we have a generic

³A flavour-dependent generalisation of this symmetry has been discussed in [26]. In this case, consistent solutions were found containing only a subclass of operators violating lepton number ($LL\bar{E}$) and baryon-number ($\bar{U}\bar{D}\bar{D}$). In this way, it was possible to have both lepton and baryon number violation without disturbing proton stability. However, we do not pursue such models here.

charge assignment where the flavour-independent pieces w_i are to be chosen such that the anomaly cancellation conditions are satisfied. Imposing these conditions and reabsorbing a_3 in the definitions of the charges, one obtains the charges shown in the second row of Table 2, where $a'_i \equiv a_i - a_3$, and $b'_i \equiv b_i - b_3$ are of the same form as discussed above, i.e., $a'_i = (-4, 1, 0)$, $b'_i = (-4, 1, 0)$ in Case 1 and $b'_i = (-\frac{7}{2}, \frac{1}{2}, 0)$ in Case 2.

	Q_i	\bar{U}_i	\bar{D}_i	L_i	\bar{E}_i	H_2	H_1
$U(1)$	a_i	$a_i + w_1$	$a_i + w_2$	b_i	$b_i + w_3$	$-2a_3$	wa_3
$U(1)$	a'_i	$a'_i + w_1$	$a'_i - w_1$	b'_i	$b'_i - w_1$	$-w_1$	w_1

Table 2: Assignments of flavour symmetry charges, before and after imposing anomaly cancellation.

We now discuss the possible hierarchies of R -violating operators in the two cases.

Case 1: In this case, the charges of the operators $O_{ijk} \equiv L_i L_j \bar{E}_k$ and $L_i Q_j \bar{D}_k$ are the same, and depend only on the values of i, j, k , and not on their order, as given in Table 3. We note that the constraints on the operators $L_1 Q_1 \bar{D}_1$ from nuclear $\beta\beta$ decay and on $L_1 L_3 \bar{E}_3$ from bounds on Majorana neutrino masses constrain the choice of the charge w_1 . Since the exact constraint depends on the magnitude of the expansion parameter for the R -violating couplings, we need to consider what the constraints are on this expansion parameter.

ijk	111	121	122	222	131
$U(1)$	$-12 - w_1$	$-7 - w_1$	$-2 - w_1$	$3 - w_1$	$-8 - w_1$
ijk	133	333	223	233	123
$U(1)$	$-4 - w_1$	$-w_1$	$2 - w_1$	$1 - w_1$	$-3 - w_1$

Table 3: Operator charges in Case 1.

In the case of the mass matrices, it was suggested in [11] that the mixing between Higgs fields carrying different $U(1)$ quantum numbers was responsible for filling in the remaining elements of the mass matrix. In this case the expansion parameters ϵ and $\bar{\epsilon}$ are as given in [11], with M_2, M_1 being the mass scales of the heavy Higgs fields H_2, H_1 that mix with the light Higgses responsible for electroweak breaking. The scales of the vacuum expectation values $\langle \theta \rangle, \langle \bar{\theta} \rangle$ are bounded from below by $(1/\sqrt{192\pi})M_{string}$, the scale of the $U(1)$ symmetry breaking. Hence M_2 and M_1 are bounded from below by $\epsilon^{-1}\theta$ and $\bar{\epsilon}^{-1}\theta$, respectively. In the case of R violation, mixing between the operators $LL\bar{E}$ or $LQ\bar{D}$ proceeds through heavy lepton or heavy quark mixing rather than through heavy Higgs exchange. If the former are much heavier than the Higgs states, the corresponding expansion parameter ϵ' will be much smaller. The limiting case occurs when they have string-scale masses, corresponding to

$$\epsilon' = \epsilon \frac{M_2}{M_{string}} \geq \frac{\langle \theta \rangle}{M_{string}} = \frac{1}{\sqrt{192\pi}} \approx 0.02 \quad (6)$$

Taking this lower limit for the expansion parameter and using the constraint $\lambda'_{111} \leq 0.002$ from nuclear $\beta\beta$ decay, we find that $|-12 - w_1| \geq 2$, whilst the constraint $\lambda'_{133} \leq 0.001$ from bounds [18] on Majorana neutrino masses indicates that $|-4 - w_1| \geq 2$.

Next, we note that the magnitudes of the couplings in Table 3 are symmetric in the three indices ijk . This implies, for example, that at this level the λ'_{121} and λ'_{112} couplings should have similar magnitudes. This must be made consistent with with the constraint $(L_1 Q_2 \bar{D}_1) \cdot (L_1 Q_1 \bar{D}_2) \leq 4 \cdot 10^{-9}$, which arises from bounds on Δm_K [23]. In the present context, this constraint indicates that the relevant charge $|-7 - w_1|$ has to be large, and we reach our first HERA-unfriendly conclusion: in this case the $e^+d \rightarrow \bar{c}$ interpretation of the HERA data would become untenable.

Thirdly, a related problem is that some couplings to muons would have comparable magnitudes to those listed in Table 3. For example, the magnitude of the λ'_{211} coupling would be comparable to that of the λ'_{121} coupling. However, certain products of couplings involving electrons and muons have to be extremely suppressed [5]. For 200 GeV sfermions,

$$\begin{aligned}\lambda_{231}\lambda_{131} &\leq 2.8 \cdot 10^{-6} \\ \lambda'_{1k1}\lambda'_{2k2} &\leq 3.2 \cdot 10^{-6} \\ \lambda'_{1k1}\lambda'_{2k1} &\leq 2 \cdot 10^{-7} \\ \lambda'_{11j}\lambda'_{21j} &\leq 2 \cdot 10^{-7}\end{aligned}\tag{7}$$

Using the form of the mixing matrices for Case 1:

$$V_\ell^{L,R} \approx \begin{pmatrix} 1 & \bar{\epsilon}/3 & 2\bar{\epsilon}^4 \\ -\bar{\epsilon}/3 & 1 & \bar{\epsilon} \\ \bar{\epsilon}^2 & -\bar{\epsilon} & 1 \end{pmatrix}$$

we shall see later that these bounds are so severe as to rule out any possible HERA-friendly model of this simple type. Note that, in order to obtain correct lepton masses within this Ansatz, a factor of ~ 3 is needed in the (22) element of the mass matrix, and this factor also enters in the mixings.

Other strong constraints on products of couplings are the following:

$$\begin{aligned}\lambda_{1j1}\lambda_{1j2} &\leq 2.8 \cdot 10^{-6} \\ \lambda'_{i13}\lambda'_{i31} &\leq 3.2 \cdot 10^{-7} \\ \lambda'_{i12}\lambda'_{i21} &\leq 4 \cdot 10^{-9},\end{aligned}\tag{8}$$

which are particularly stringent in the model under consideration. Using these bounds, one finds

$$\begin{aligned}\lambda'_{i13} &\leq 6 \cdot 10^{-4} \\ \lambda'_{i12} &\leq 6 \cdot 10^{-5},\end{aligned}\tag{9}$$

together with corresponding bounds for permutations of the indices.

If the reported apparent excess of HERA events at large Q^2 were due to production of a single squark by an R -violating coupling, one would need

$$\begin{aligned}\lambda'_{121,131} &\approx 0.04/\sqrt{\mathcal{B}} \\ \lambda'_{132} &\approx 0.3/\sqrt{\mathcal{B}}\end{aligned}$$

where \mathcal{B} is the branching ratio of the decay $\tilde{q} \rightarrow e^+q$. We see immediately from (9) that the first two possibilities cannot be realised in this model. Moreover, we see from (3) that $\lambda'_{132} \approx \lambda'_{133}/\bar{\epsilon} \leq 0.004$, and infer the third possibility cannot be realised either.

What happens to the remaining couplings? From the mixing discussed above, we have:

(i) $\lambda'_{111} = \lambda'_{112}/\bar{\epsilon} \leq 3 \cdot 10^{-4}$, which is a stronger bound than the one from neutrinoless $\beta\beta$ decay,⁴

(ii) $\lambda'_{222} = \lambda'_{221}/\bar{\epsilon} \leq 3 \cdot 10^{-4}$,

(iii) $\lambda'_{223} = \lambda'_{213}/\bar{\epsilon} = \lambda'_{312}/\bar{\epsilon} \leq 3 \cdot 10^{-4}$ or $\lambda'_{223} = \lambda'_{213}/\epsilon = \lambda'_{312}/\epsilon \leq 0.0013$.

The constraints here are very strict because the experimental bound on λ'_{312} is more severe than that on λ'_{213} : since we require these two terms to have the same charge, we must take the stricter limit. We also have

(iv) $\lambda'_{233} = \lambda'_{223}/\bar{\epsilon} \leq 0.0013(0.006)$, for expansion parameters $\bar{\epsilon}$ and ϵ respectively,

(v) $\lambda'_{333} = \lambda'_{233}/\bar{\epsilon} \leq 0.006(0.025)$.

In each of these cases, R violation may be manifest in hadron-hadron colliders. For sfermion masses of 100 GeV and $\lambda \geq 10^{-6}$, the lightest supersymmetric particle is expected to decay inside the accelerator. The above constraints allow couplings that are significantly larger than this lower bound. Through a suitable choice of w_1 , the couplings that are more severely constrained can be made small, while some others can be of importance for collider physics, though none can be very large in this type of model with symmetric mass matrices. Hence, single-squark production via an R -violating coupling is suppressed, and the best signal would be squark-pair production followed by R -violating decay.

Case 2:

In this case, the charges of the operators depend on the flavour-symmetry charge of the singlet field Φ that we have introduced. This does not affect the relative magnitudes of the R -violating couplings, since Φ appears in all terms. However, this charge and the vacuum expectation value of Φ do provide a possible source of suppression for the R -violating couplings. We take as an indicative value $a_\Phi = 1/2$: the corresponding subclasses of $LL\bar{E}\Phi$ and $LQ\bar{D}\Phi$ operators with integer flavour charge appear in Tables 4 and 5.

In this model, the lepton mass matrix takes the form

⁴We use the down-quark mixing parameter, which is larger, since the operator we compare with differs only in the index of \bar{D} .

ijk($LL\bar{E}$)	121	122	133	233
$U(1)$	$-6 - w_1$	$-2 - w_1$	$-3 - w_1$	$1 - w_1$

Table 4: Integer $LL\bar{E}$ charges for Case 2.

ijk($LQ\bar{D}$)	111	121	122	131	123	133
$U(1)$	$-11 - w_1$	$-6 - w_1$	$-1 - w_1$	$-7 - w_1$	$-2 - w_1$	$-3 - w_1$
ijk(LQD)	211	221	222	231	223	233
$U(1)$	$-7 - w_1$	$-2 - w_1$	$3 - w_1$	$-3 - w_1$	$2 - w_1$	$1 - w_1$

Table 5: Integer $LL\bar{D}$ charges for Case 2: these charges remain the same if j and k are interchanged.

$$V_\ell^{L,R} \approx \begin{pmatrix} 1 & \bar{\epsilon}^2 & 0 \\ -\bar{\epsilon}^2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

which is independent of the value chosen for w_1 . What are the predictions for the strength of the R -violating couplings in this model? As in the previous model, we have:

$$\begin{aligned} \lambda'_{i13} &\leq 6 \cdot 10^{-4} \\ \lambda'_{i12} &\leq 6 \cdot 10^{-5} \end{aligned} \tag{10}$$

and so again there is no possibility to explain the HERA events, essentially for the same reasons as in Case 1. From the charges of Table 5, we find that $\lambda'_{211} = \lambda'_{113}$, $\lambda'_{133} = \lambda'_{213}$ and $\lambda'_{123} = \lambda'_{212}$. Since $\lambda'_{113} = \lambda'_{123}/\bar{\epsilon} \leq 3 \cdot 10^{-4}$, we obtain the slightly stronger bound $\lambda'_{i13} = \lambda'_{i31} \leq 3 \cdot 10^{-4}$. For the remaining couplings we have the following bounds:

- (i) $\lambda'_{111} = \lambda'_{112}/\bar{\epsilon} \leq 3 \cdot 10^{-4}$,
- (ii) $\lambda'_{122} = \lambda'_{121}/\bar{\epsilon} \leq 3 \cdot 10^{-4}$,
- (iii) $\lambda'_{222} = \lambda'_{221}/\bar{\epsilon} \leq 3 \cdot 10^{-4}$,
- (iv) $\lambda'_{223} = \lambda'_{213}/\bar{\epsilon} \leq 0.0013$,
- (v) $\lambda'_{233} = \lambda'_{223}/\bar{\epsilon} \leq 0.006$.

The difference from the previous solution is that the couplings $L_3 Q_j \bar{D}_k$ are absent, and thus constraints from them are evaded. However, the model remains restrictive, as all the quarks of the same generation have the same charge. Therefore, the strict bounds on products of operators still constrain strongly individual couplings. Nevertheless, we see from the limits above that several possibilities exist for R -violating squark decays within hadron-hadron collider detectors.

We see therefore that there are four problems that do not allow an explanation of the HERA events within the framework of these models. **First**, the quarks and leptons of the same generation have the same charges, so the $L_i L_j \bar{E}_k$ and $L_i Q_j \bar{D}_k$ couplings are

subject to the same bounds. **Secondly**, the choice of symmetric mass matrices makes the last two equations of (8) difficult to satisfy, because Q_i and \bar{D}_i have the same charge and hence each factor involved has the same suppression, so that one cannot arrange to satisfy the inequality while keeping one coupling large. **Thirdly**, the model has large mixing in the (1,2) down-quark sector, making the last of eqs (8) difficult to satisfy. **Finally**, the large (1,2) mixing in the charged lepton sector may not be reconciled with bounds on products of couplings that involve electrons and muons.

Thus we see that the combination of the various R -violating bounds with simple family symmetries produces strong constraints on a variety of R -violating couplings. For the case of the family symmetry leading to the symmetric mass matrix (2) these constraints imply that R violation does not give rise to anomalous events at HERA at a significant rate.

4 HERA-Friendly Textures of R -Violating Couplings

In this section we explore modifications of the simple $U(1)$ family structure, which may be able to accommodate an R -violating interpretation of the apparent excess of events at HERA. As we have stressed, a major problem in building a model to accommodate the HERA events lies in the need to satisfy the bounds (8) while keeping large one of the individual couplings involved in these products. This leads us to consider models with the (1,2) mixing entirely in the up-quark sector and to deviate from the symmetric mass-matrix structure.

4.1 Asymmetric Flavour Textures

Once one gives up on the symmetric form, the pattern of masses is insufficient to constrain the $U(1)$ charge structure, so there are many new possibilities. Here we present just one viable choice to illustrate the options, but we certainly do not claim any uniqueness. We start with the charge assignment $(-4,1,0)$ for the quark doublets of the model discussed above, and modify the up- and down-quark singlet charges to achieve the desired structure. In order to reduce the arbitrariness, we also choose to generate both the up- and down-quark mass matrices with the same expansion parameter, as would be the case if the non-renormalisable terms $Q_i\bar{D}_i H_2\theta/M$ arise through heavy-quark mixing.

With this Ansatz, a suitable choice for the up-quark singlet charges is $(-5,1,0)$, which gives

$$M^{up} = \begin{pmatrix} \epsilon^9 & 4\epsilon^3 & \epsilon^4 \\ 4\epsilon^4 & \epsilon^2 & \epsilon \\ \epsilon^5 & \epsilon & 1 \end{pmatrix} \quad (11)$$

A value of $\epsilon \approx 1/20$ gives an acceptable charm mass. Note we have been forced to assume

an enhancement factor of 4 in the (1,2) and (2,1) elements in order to accommodate the mixing needed to generate $V_{12}^{CKM} \approx V_{u12}^L = 4\epsilon \approx 0.2$. The mass eigenvalues are $1, \epsilon^2$ and $16\epsilon^5$, from which we see that the factor of 16 which appears from the coefficients in the off-diagonal entries compared to the solution of [11] is compensated by the additional power in the expansion parameter.

The choice of down-quark charges is dictated by the requirement that we keep the (1,2) mixing small. A suitable choice for the charges of the singlet down quarks is (7,-3,1), which gives the structure

$$M^{down} = \begin{pmatrix} \epsilon^3 & \epsilon^7 & \epsilon^3 \\ \epsilon^8 & \epsilon^2 & \epsilon^2 \\ \epsilon^7 & \epsilon^3 & \epsilon \end{pmatrix} \quad (12)$$

The eigenvalues scale as $\epsilon, \epsilon^2, \epsilon^3$, and $V_{d13}^L \approx V_{d31}^L \approx \epsilon^2$. Moreover, $V_{d23,32}^L \approx \epsilon = 0.05$, so we do not require the cancellations that were needed in [11] (remember that in this case $V_{d23,32}^L \approx \bar{\epsilon} = 0.23$). Note that this choice has the advantage of reducing the bottom mass through an ϵ factor, putting us in the small- $\tan\beta$ regime. This phenomenological choice of charges does not yet ensure anomaly cancellation, but at a later stage, when we also have a good phenomenological choice for the lepton mass matrix, we will discuss what flavour-independent charges have to be added in order to cancel anomalies. These additional charges will not modify the hierarchy of couplings, nor the relative magnitudes of the R -violating couplings of a given type.

The key point of this model is its large $U(1)$ charge difference between the relevant $LQ\bar{D}$ couplings:

$$a_{L_1 Q_2 \bar{D}_1} - a_{L_1 Q_1 \bar{D}_2} \rightarrow 15 \quad (13)$$

$$a_{L_1 Q_3 \bar{D}_1} - a_{L_1 Q_1 \bar{D}_3} \rightarrow 10 \quad (14)$$

which leads to large relative suppressions of these operators, though mixing in the (1,2) and (1,3) down sectors will close this gap. Consider first the operators appearing in (13). The mixing of the left-handed down quarks is given by the form of $V_{12,21}^L$ in the Appendix, and the second-order term dominates with $m_{13}^d m_{32}^d / (M_3 M_2) \approx \epsilon^3 = 1.3 \cdot 10^{-4}$. The mixing of \bar{D}_1, \bar{D}_2 is given by $m_{21}^d / M_2 \approx m_{31}^d m_{23}^d / (M_3 M_2) \approx \epsilon^6$. Taking the same expansion parameter as for the masses, consistent with (6), the net suppression is $\epsilon^3 \epsilon^6 \approx 2 \cdot 10^{-12}$. This is more than sufficient to satisfy the bound on $L_1 Q_1 \bar{D}_2$ while allowing the $L_1 Q_2 \bar{D}_1$ to have a coefficient large enough to give the HERA events. Similarly, one may check that it is possible to for the $L_1 Q_3 \bar{D}_1$ operator to be relevant for HERA, without inducing an $L_1 Q_1 \bar{D}_3$ coupling with an unacceptable value.

4.2 Lepton Flavour Violation

We now turn to the assignment of lepton charges in such a model. Given the bounds of (7), if any coupling involving an electron is large, the corresponding coupling involving

muons should be small. The simplest solution is to choose charge assignments so that the $U(1)$ charge of the (1,2) entry of the lepton mass matrix is half-integer. In this case, a residual Z_2 symmetry forbids the (1,2) mass term. The choice of charges is restricted by the anomaly cancellation conditions. These are:

$$\begin{aligned}
A_3 &= (a_{Q_1} + a_{Q_2} + a_{Q_3}) + \frac{1}{2}(a_{\bar{U}_1} + a_{\bar{U}_2} + a_{\bar{U}_3}) + \frac{1}{2}(a_{\bar{D}_1} + a_{\bar{D}_2} + a_{\bar{D}_3}) \\
A_2 &= \frac{3}{2}(a_{Q_1} + a_{Q_2} + a_{Q_3}) + \frac{1}{2}(a_{L_1} + a_{L_2} + a_{L_3}) + \frac{1}{2}(a_{H_1} + a_{H_2}) \\
A_1 &= \frac{1}{6}(a_{Q_1} + a_{Q_2} + a_{Q_3}) + \frac{4}{3}(a_{\bar{U}_1} + a_{\bar{U}_2} + a_{\bar{U}_3}) + \frac{1}{3}(a_{\bar{D}_1} + a_{\bar{D}_2} + a_{\bar{D}_3}) \\
&\quad + \frac{1}{2}(a_{L_1} + a_{L_2} + a_{L_3}) + (a_{\bar{E}_1} + a_{\bar{E}_2} + a_{\bar{E}_3}) + \frac{1}{2}(a_{H_1} + a_{H_2})
\end{aligned} \tag{15}$$

where by a_{F_i} we denote the charge of particle F in the i^{th} generation. We see from A_3 and A_2 that, for integer quark charges, a natural solution of the conditions has the sums of $a_{\bar{E}_i}$ and a_{L_i} integers. Thus, we need two of the $a_{\bar{E}_i}$ and a_{L_i} to be half-integers. We see for example, that the choice $a_{L_1} = 9/2, a_{L_2} = -1, a_{L_3} = -1/2, a_{\bar{E}_1} = -1/2, a_{\bar{E}_2} = -1, a_{\bar{E}_3} = -1/2$ generates a viable mass hierarchy. Here we have chosen the (3,3) charge to be the same as that of the down-quark, in order to give $b - \tau$ unification. Clearly this pattern of charges is not the only viable choice, but it indicates how things may work. The corresponding lepton mass matrix in this solution is:

$$M^L = \begin{pmatrix} \epsilon^4 & 0 & \epsilon^4 \\ 0 & \epsilon^2 & 0 \\ \epsilon & 0 & \epsilon \end{pmatrix} \tag{16}$$

and there is (1,3) mixing, but no (1,2) or (2,3) mixing. The eigenvalues of this mass matrix are $\epsilon, \epsilon^2, \epsilon^4$, consistent with the measured values for $\epsilon = 0.05$.

	Q_i	\bar{U}_i	\bar{D}_i	L_i	\bar{E}_i	H_2	H_1
$U(1)$	a'_i	$a'_{\bar{U}_i}$	$a'_{\bar{D}_i}$	a'_{L_i}	$a'_{\bar{E}_i}$	0	0
$U(1)$	a'_i	$a'_{\bar{U}_i} + w_1$	$-1 + a'_{\bar{D}_i} - w_1$	$-1 - a'_{L_i}$	$a'_{\bar{E}_i} - w_1$	$-w_1$	$1 + w_1$

Table 6: *Flavour charges in models with asymmetric mass matrices.*

We denote by $a'_{Q_i} \equiv (-4, 1, 0)$, $a'_{\bar{U}_i} \equiv (-5, 1, 0)$, $a'_{\bar{D}_i} \equiv (7, -3, 1)$, $a'_{L_i} \equiv (9/2, -1, -1/2)$ and $a'_{\bar{E}_i} \equiv (-1/2, -1, -1/2)$ the choices of generation-dependent charges in this model. The top line of Table 6, which includes these and the corresponding charges for the Higgs multiplets, is not anomaly-free. It is easy now to satisfy the anomaly-matching conditions, allowing for additional family-independent components of the $U(1)$ charge, which do not affect the mass matrix structure. An anomaly-free solution is obtained by adding charges as indicated in the second line of Table 6, where the variable w_1 is an integer.

4.3 Nucleon Stability

We now demonstrate that nucleon decay graphs due to combinations of $LQ\bar{D}$ and $\bar{U}\bar{D}\bar{D}$ interactions may be eliminated in this HERA-friendly example by imposing an anomaly-free discrete gauge symmetry. In the specific model discussed above, where the first- and third-generation leptons have half-integer charges under the flavour symmetry, we can again combine the residual \tilde{Z}_2 symmetry of the $U(1)$ [11] with a normal Z_2^M matter parity, where $\tilde{Z}_2 \times Z_2^M$ is broken to a diagonal Z_2 by a field Φ that is odd under both symmetries. Then the couplings $\bar{U}\bar{D}\bar{D}$ and $L_2Q\bar{D}$ transform as $(+, -)$ under the symmetry and are forbidden. Renormalisable $L_{1,3}Q\bar{D}$ couplings which transform as $(-, -)$ are also forbidden, but effective couplings of this type may occur at $\mathcal{O}\left(\frac{\Phi}{M}\right)$ through the term $(\Phi LQ\bar{D})/M$.

Such underlying Z_2 symmetries can be illustrated in the context of a GUT group, if desired. Consider, for example, the Pati-Salam gauge group $SU(4) \times SU(2)_L \times SU(2)_R$ [28]. In models based on this group, the fermionic fields belong to either the 4 or the $\bar{4}$ representations of $SU(4)$, and no trilinear R -violating term is invariant under the symmetry. However, invariants can be constructed by introducing an adjoint field Σ [29].⁵ If Φ has a half-integer charge, the fact that it is in the adjoint of $SU(4)$ means that all baryon-number-violating operators are forbidden in any order, whilst the terms $L_{1,3}Q\bar{D}\Phi$ have integer charge and are therefore allowed. Moreover, no effective terms $L_{1,3}\bar{E}H_2\Phi$, which could cause problems with the lepton mass hierarchies, are allowed, as they are not invariant under the extended gauge group.

4.4 Hierarchy of R -Violating Interactions in a HERA-Friendly Model

We now consider the effect of the $U(1)$ symmetry on the pattern of allowed R -violating interactions in the models that were motivated by the HERA events. The charges of the operators depend on the half-integer charge of the field Φ under the flavour symmetry. This does not affect the relative magnitudes of the R -violating couplings, since Φ appears in all terms. However, this charge and the vacuum expectation value of Φ do provide a possible source of suppression for the R -violating couplings. The corresponding subclasses of $LL\bar{E}\Phi$ and $LQ\bar{D}\Phi$ operators with integer flavour charge, before introducing mixing effects, appear in Tables 7 and 8. We have used the charges of Table 6, imposing anomaly cancellation. Here we have taken Φ to have $U(1)$ charge $1/2$, but its actual value can be re-absorbed in the definition of w_1 .

Let us first consider the possibility that the possible excess HERA events are due to the $L_1Q_3\bar{D}_2$ operator with $\lambda' > 0.3/\sqrt{B}$, which will be the dominant operator if $w_1 = 0$. The relative suppression of the $L_1Q_3\bar{D}_3$ operator is ϵ^4 . Of course, mixing effects in the

⁵A study of the string origin of non-renormalisable operators in this model has been presented in [30].

(2,3) sector of the down-quark mass matrix re-introduce an $L_1 Q_3 \bar{D}_3$ operator at order ϵ^2 , via the mixing of right-handed down quarks. In the type of models with a small $V_{d_{2,3}}^R$ mixing therefore, this solution may in principle be accommodated. However, for our specific choice of charges, we see that the operator $L_1 Q_3 \bar{D}_2$ has the same charge as $L_1 Q_1 \bar{D}_3$, which is bound by charged-current universality [20] to be ≤ 0.04 for a squark mass of 200 GeV. Hence the $L_1 Q_3 \bar{D}_2$ interpretation of the HERA data is not realisable in this specific model. However, this is rather accidental for this particular example, and need not be the case in general.

ijk($LL\bar{E}\Phi$)	122	131	133	231
$U(1)$	$1 - w_1$	$2 - w_1$	$2 - w_1$	$-4 - w_1$

Table 7: Integer $LL\bar{E}\Phi$ charges, ignoring mass mixing.

ijk($LQ\bar{D}\Phi$)	111	112	113	121	122	123
$U(1)$	$6 - w_1$	$-4 - w_1$	$-w_1$	$11 - w_1$	$1 - w_1$	$5 - w_1$
ijk($LQ\bar{D}\Phi$)	131	132	133	311	312	313
$U(1)$	$10 - w_1$	$-w_1$	$4 - w_1$	$1 - w_1$	$-9 - w_1$	$-5 - w_1$
ijk($LQ\bar{D}\Phi$)	321	322	323	331	332	333
$U(1)$	$6 - w_1$	$-4 - w_1$	$-w_1$	$5 - w_1$	$-5 - w_1$	$-1 - w_1$

Table 8: Integer $LQ\bar{D}\Phi$ charges, ignoring mass mixing.

We now look at the possibilities that the HERA events arise from the $L_1 Q_{2,3} \bar{D}_1$ operators. We see from Table 6 that, in the absence of mixing, the relative suppressions of the $L_1 Q_1 \bar{D}_1$ and $L_1 Q_3 \bar{D}_3$ operators would have been enough to make these cases viable. When mixing effects are included, the possible effects of unknown phases should be taken into account when comparing with bounds. In the specific case that the HERA events are due to an $L_1 Q_2 \bar{D}_1$ coupling, we have no problem with the $L_1 Q_3 \bar{D}_3$ operator, but there is a potential difficulty with $\beta\beta$ decay, due to mixing with the $L_1 Q_1 \bar{D}_1$ operator:

$$\left| \lambda_{112} V_{u_{12}}^L \left(\frac{200 \text{ GeV}}{m_{\tilde{u}_L}} \right)^2 \right| < 4 \cdot 10^{-3} \left(\frac{m_{\tilde{g}}}{1 \text{ TeV}} \right)^{1/2} \quad (17)$$

where $m_{\tilde{g}}$ is the gluino mass. Given (a) that the V^{CKM} mixing arises from the up sector in our framework ⁶, and (b) that the bounds from the Tevatron indicate that the branching ratio of \tilde{c}_L to fermions can not be close to unity in the context of this interpretation, implying that λ'_{121} has to be larger than 0.04, we see that this solution is not naturally accommodated. It might be possible if $m_{\tilde{u}_L}$ is significantly larger than $m_{\tilde{c}_L}$,

⁶Even in the case that the $V_{12,21}^{CKM}$ mixing arises from the down-quark sector, squark mass universality violation is required in order to evade bounds from $K \rightarrow \pi\nu\bar{\nu}$.

but this requires a violation of squark-mass universality that is potentially dangerous for flavour-changing neutral interactions.

On the other hand, if the HERA events are due to a $L_1 Q_3 \bar{D}_1$ coupling, there is no problem with the $L_1 Q_1 \bar{D}_1$ operator, but a problem could in principle appear with the $L_1 Q_3 \bar{D}_3$ coupling that is bounded from limits on neutrino Majorana masses [18]. However, the relevant (3,1) mixing term is small, indicating that in this case the unknown coefficients may be such that the bounds are easily accommodated. What about the other couplings? For $L_1 Q_3 \bar{D}_1 \approx 0.04$, the model predicts that $L_1 Q_2 \bar{D}_1 \approx L_3 Q_1 \bar{D}_2 \approx 0.002$, while all other couplings are very suppressed. Indeed, looking at the charges, we see that the next larger couplings are suppressed by ϵ^4 as compared to $L_1 Q_3 \bar{D}_1$. Mixing effects are also suppressed, except for the operators $L_1 Q_1 \bar{D}_1 \approx L_1 Q_2 \bar{D}_1 (4\epsilon) \approx 0.0004$ and $L_3 Q_2 \bar{D}_2 \approx L_3 Q_1 \bar{D}_2 (4\epsilon) \approx 0.0004$, which are within the allowed range.

Finally, note that we do not have any mixing between $L_1 Q_j \bar{D}_k$ and $L_2 Q_j \bar{D}_k$ couplings (the later are forbidden by the symmetry), so the dangerous product combinations that violate lepton flavour are also absent.

In the light of the above discussion, we conclude that, of the valence quark production mechanisms via $L_1 Q_2 \bar{D}_1$ and $L_1 Q_3 \bar{D}_1$ couplings, the second possibility seems to be favoured. It should be possible to make a model with a coupling $L_1 Q_3 \bar{D}_2$ sufficiently large to explain the HERA data, although we have not displayed one here.

5 Baryon Decay via Dimension-Five Operators

We saw earlier on that the experimental absence of baryon decay imposed important constraints on possible models, which are most easily evaded by imposing a baryon parity symmetry that forbids the dangerous $\bar{U} \bar{D} \bar{D}$ couplings. However, this is not the end of the story, since models may also contain dimension-five operators that would generate proton decay at an unacceptable level. The most dangerous among these operators are the operators $[QQQL]_F$ and $[QQQH_1]_F$, the latter in the presence of $LQ\bar{D}$ couplings.⁷ These operators can lead to fast proton decay via loop diagrams. In the case of $[QQQL]_F$ operators that involve the two lightest generations, the constraint on the coupling η of any such operator is $\eta \leq 10^{-7}$. This bound has some flexibility, since the magnitude of the loop diagrams depends on details of the sparticle spectrum, but this possibility is not crucial for the subsequent discussion of models. In the case of $[QQQH_1]_F$ operators with couplings η' , fast proton decay may occur if they are present simultaneously with $LQ\bar{D}$ operators with generic coefficients λ' . The product of the corresponding couplings is constrained: $\eta' \lambda' \leq 10^{-10}$. Since an R -violating interpretation of the HERA events requires either $L_1 Q_2 \bar{D}_1$ or $L_1 Q_3 \bar{D}_1 \approx 0.04$, it is clear that we have to worry about the $[QQQH_1]_F$ operator as well.

⁷The lepton-number-violating operators $[Q\bar{U}\bar{E}H_1]_F$, $[Q\bar{U}L^*]_D$ and $[Q\bar{U}L^*]_D$ are dangerous in the presence of $\bar{U} \bar{D} \bar{D}$ ones.

We now analyse the dimension-five operator charges in the different cases discussed in previous sections, to see whether they are large enough for the suppression by powers of small quantities to be sufficient. How small the terms actually are depends on the expansion parameter, as we have already discussed in a previous section.

The $QQQH_1$ operators are easily dealt with, even though the baryon stability requirements seem to be more severe for them. The reason is that these operators transform as $(+, -)$ under the $\tilde{Z}_2 \times Z_2^M$, and are thus forbidden. What about the $QQQL$ operators? The $QQQL_{1,3}$ operators are not present, because they transform as $(-, +)$ under $\tilde{Z}_2 \times Z_2^M$. However, the operators $QQQL_2$ are allowed. These are dangerous, because proton decay may occur via the modes $p \rightarrow \bar{\nu}_{1,2,3}\pi^+$ and $p \rightarrow \bar{\nu}_{1,2,3}K^+$. Let us look at the flavour charges of these operators. We recall that colour antisymmetrisation implies that all the quark flavour indices cannot be identical. The operators that are not suppressed enough by quark mixing parameters have the following charges in the model that could explain the HERA events:

$$\begin{aligned}
a_{Q_1Q_1Q_2L_2} &= -9 \\
a_{Q_1Q_2Q_2L_2} &= -4 \\
a_{Q_1Q_1Q_3L_2} &= -10 \\
a_{Q_1Q_2Q_3L_2} &= -5
\end{aligned}
\tag{18}$$

where for the lepton charge we used the anomaly-free choice of Table 6.

We infer that we do not need any further underlying symmetry in order to suppress these couplings adequately. However, even in models where this suppression does not occur, there could be some GUT symmetry that forbids the offending $QQQL$ operators⁸. This would be an interesting constraint on GUT model-building, but should not be taken as a serious obstacle to constructing HERA-friendly models.

6 Concluding Comments

We have discussed the implications of a single $U(1)$ abelian flavour symmetry for the possible hierarchies of R -violating couplings. The relations between the Standard-Model Yukawa couplings and R -violating couplings depend on the choice of model charges, so the observed hierarchies of quark and lepton masses do not lead to a unique specification of the dominant R -violating couplings. However, we have identified certain general features of such a framework, highlighting the importance of mass mixing between current eigenstates. We have identified various interesting possibilities for hadron-hadron collider phenomenology that are consistent with this mixing and the available experimental constraints. Within this general approach, we have searched specifically for simple

⁸Moreover, in string-derived GUT models, string selection rules may lead to the vanishing of operators that are invariant under field-theory symmetries. In such models, it is possible to construct realistic fermion mass matrices while having maximal proton stability [31].

consistent models that lead to the favoured R -violating scenarios for the explaining the possible excess in the HERA data.

Our results may be summarised as follows:

- Flavour symmetries lead us to expect a hierarchy in the R -violating couplings, analogous to that observed for the known fermion masses. These hierarchies can be consistent with a squark-production interpretation of the HERA data (if required), as well as with the various other experimental constraints on the couplings.
- The simplest charge assignments lead to unified, and thus more predictive, forms for the mass matrices. For the case of equal charges for up and down quarks and leptons of a given generation, the symmetry together with bounds from products of R -violating couplings implies that there should be no significant anomalous events at HERA coming from such couplings. If we wish to accommodate such anomalous events, we are forced to depart from this picture. Schemes with asymmetric charges and different assignments for up quarks, down quarks and leptons give rise to larger splittings between different operators.
- Some of the charge assignments considered forbid large coefficients of dimension-five operators that are potentially dangerous for baryon stability. In schemes where this is not true, such terms would need to be forbidden by further GUT symmetries.

One can consider relaxing various of our conditions, for example by introducing a higher level of asymmetry in the mass matrices, invoking multiple $U(1)$ flavour symmetries, etc., and in such models the predictions can be further altered. Moreover, additional zero couplings may be expected when one goes to a specific GUT/string construction. However, it is interesting that it is possible to construct phenomenological models with a single $U(1)$ flavour symmetry that are compatible with attempts to explain the reported excess of HERA data by R -violating squark production, albeit at a price. In order to constrain the possible schemes, and perhaps rule some out, more experimental data are required.

Appendix

Using second-order perturbation theory, it is easy to derive the mixing elements for a generic mass matrix [32, 33], where m stands for the off-diagonal contributions and M for the diagonal part. The left-handed mixing is given by [33]

$$V_{ij}^L = -\frac{(m_{ij}M_j + m_{ji}^*M_i)}{M_i^2 - M_j^2} + \frac{(m_{ik}M_k + m_{ki}^*M_i)(m_{kj}M_j + m_{jk}^*M_k)}{(M_i^2 - M_j^2)(M_k^2 - M_j^2)} - \frac{m_{ik}m_{jk}^*}{(M_i^2 - M_j^2)}$$

and V_{ij}^R is given by a corresponding expression, substituting the mass matrix by its hermitian conjugate. In the case that the ratio of m_{ij} to m_{ji}^* is considerably larger than the ratio M_i/M_j , the mixing elements are given by:

$$V_{12}^L = +\frac{m_{12}}{M_2} - \left[\frac{m_{13}m_{32}}{M_3M_2} \right] + \dots$$

$$\begin{aligned}
V_{21}^L &= -\frac{m_{12}^*}{M_2} + \left[\frac{m_{32}^* m_{13}^*}{M_2 M_3} \right] + \dots \\
V_{13}^L &= +\frac{m_{13}}{M_3} + \left(\frac{m_{12} m_{23} M_2}{M_3^3} \right) + \left[\frac{m_{12} m_{32}^*}{M_3^2} \right] + \dots \\
V_{31}^L &= -\frac{m_{13}^*}{M_3} + \frac{m_{12}^* m_{23}^*}{M_2 M_3} + \left[-\frac{m_{32} m_{12}^*}{M_3^2} \right] + \dots \\
V_{23}^L &= +\frac{m_{23}}{M_3} + \left(\frac{m_{12}^* m_{13} M_2}{M_3^3} \right) + \left[\frac{m_{21} m_{31}^*}{M_3^2} \right] + \dots \\
V_{32}^L &= -\frac{m_{23}^*}{M_3} - \frac{m_{12} m_{13}^*}{M_2 M_3} + \left[-\frac{m_{31} m_{21}^*}{M_3^2} \right] + \dots
\end{aligned} \tag{19}$$

In the above expressions, terms in brackets mark contributions which involve mass entries below the diagonal. These terms, as well as the ones in parentheses, can in most cases be neglected. However, they may become relevant if the mass matrices have texture zeroes.

In the case of the full V_{CKM} matrix, one has

$$\begin{aligned}
V_{12}^{CKM} &= +\frac{m_{12}^d}{M_2^d} - \frac{m_{12}^u}{M_2^u} + \dots \\
V_{21}^{CKM} &= -\frac{m_{12}^{d*}}{M_2^d} + \frac{m_{12}^{u*}}{M_2^u} + \dots \\
V_{23}^{CKM} &= +\frac{m_{23}^d}{M_3^d} - \frac{m_{23}^u}{M_3^u} + \frac{m_{13}^d m_{12}^{u*}}{M_3^d M_2^u} - \frac{m_{13}^u m_{12}^{d*}}{M_3^u M_2^d} + \dots \\
V_{32}^{CKM} &= -\frac{m_{23}^{d*}}{M_3^d} + \frac{m_{23}^{u*}}{M_3^u} - \frac{m_{12}^d m_{13}^{d*}}{M_2^d M_3^d} + \frac{m_{12}^d m_{13}^{u*}}{M_3^u M_2^d} + \dots \\
V_{13}^{CKM} &= +\frac{m_{13}^d}{M_3^d} - \frac{m_{13}^u}{M_3^u} - \frac{m_{23}^d m_{12}^u}{M_3^d M_2^u} + \frac{m_{12}^u m_{23}^u}{M_3^u M_2^u} + \dots \\
V_{31}^{CKM} &= -\frac{m_{13}^{d*}}{M_3^d} + \frac{m_{13}^{u*}}{M_3^u} + \frac{m_{12}^{d*} m_{23}^d}{M_2^d M_3^d} - \frac{m_{12}^{u*} m_{23}^u}{M_3^u M_2^d} + \dots
\end{aligned} \tag{20}$$

These formulae are used in the text in conjunction with specific parametric forms for the off-diagonal terms in the up- and down-quark mass matrices.

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