

# SINGLE-BUBBLE OPEN INFLATION: AN OVERVIEW

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**Abstract.** The idea that the universe might be open is an old one, and the possibility of having an open universe arise from inflation is not new either. However, a concrete realization of a consistent single-bubble open inflation model is known only recently. There has been great progress in the last two years in the development of models of inflation consistent with observations in such an open universe. In this overview I will describe the basic features and the phenomenological consequences of such models, making emphasis in the predictions of the CMB temperature anisotropies that differ from ordinary inflation.

## 1. Introduction

The idea that the universe might be open is an old one, see e.g. [1]. Early attempts to accommodate standard inflation in an open universe [2] failed to realize that in usual inflation homogeneity implies flatness [3], due to the Grishchuk-Zel'dovich effect [4]. The possibility of having a truly open universe arise from inflation is not new either, see [5], via the nucleation of a single bubble in de Sitter space. However, a concrete realization of a consistent model is known only recently, the single-bubble open inflation model [6, 7]. Soon afterwards there was great progress in determining the precise primordial spectra of perturbations [8-19], most of it based on quantum field theory in spatially open spaces. Simultaneously there has been a large effort in model building [7, 20, 21, 22] and constraining the existing models from observations of the temperature power spectrum of cosmic microwave background (CMB) anisotropies [21-24].

In this review talk I will concentrate in model building and constraints from CMB anisotropies. We will describe the nature of the various primordial perturbations and give the corresponding spectra, without deriv-

ing them from quantum field theoretical arguments. The interested reader should find this in the literature. We will then compute the corresponding angular power spectra of temperature anisotropies in the CMB. Furthermore, we will give a review of the different single- and multiple-field open inflation models and constrain their parameters from present observations of the CMB anisotropies. Sometimes this is enough to rule out some of the models. Finally, we will describe how future observations of the CMB temperature and polarization anisotropies might be able to decide among different inflationary models, both flat and open inflation ones.

## 2. Single-bubble open inflation

The simplest inflationary model consistent with an open universe arises from the nucleation of a bubble in de Sitter space [5], inside which a second stage of inflation drives the spatial curvature to *almost* flatness. The small deviation from flatness at the end of inflation will be amplified by the subsequent expansion during the radiation and matter eras. The present value of the density parameter is determined from  $|1 - \Omega_0| \sim \exp(-2N_e) \times 10^{56}$ , where  $N_e \leq 65$  is the required number of e-folds during inflation (inside the bubble), in order to give an open universe today. A few percent change in  $N_e$  could lead to an almost flat universe or a wide open one.

This simple picture could be realized in the context of a single-field scalar potential [6] or in multiple-field potentials [7, 20, 21, 22], as long as there exists a false vacuum during which the universe becomes homogeneous and then one of the fields tunnels to the true vacuum, creating a single isolated bubble. The space-time inside this bubble is that of an open universe [25, 5]. Although single-field models can in principle be constructed, they require a certain amount of fine tuning in order to avoid tunneling via the Hawking-Moss instanton [7]. The problem is that one needs a large mass for successful tunneling and a small mass for successful slow-roll. For that reason, it seems more natural to consider multiple-field models of open inflation [7, 20, 21, 22], where one field does the tunneling, another drives slow-roll inflation inside the bubble, and yet another may end inflation, like in the open hybrid model [22]. Such models account for the large scale homogeneity observed by COBE [26] and are also consistent with recent determinations of a low density parameter [27, 28, 29].

The quantum tunneling can be described with the use of the bounce action formalism developed by Coleman-DeLuccia [25] and Parke [30] in the thin wall approximation, valid when the width of the bubble wall is much smaller than the radius of curvature of the bubble. This only requires that the barrier between the false and the true vacuum be sufficiently high,  $U_0 \gg \Delta U = U_F - U_T$ . In this case we can write the radius of the bubble

in terms of dimensionless parameters  $a$  and  $b$  [24],

$$R_0 H_T = [1 + (a + b)^2]^{-1/2} \equiv [1 + \Delta^2]^{-1/2}, \quad (1)$$

$$a \equiv \frac{\Delta U}{3S_1 H_T}, \quad b \equiv \frac{\kappa^2 S_1}{4H_T}, \quad (2)$$

where  $\kappa^2 \equiv 8\pi G$  and  $S_1$  is the surface tension of the bubble wall, which can be computed as  $S_1 = \int_{\sigma_F}^{\sigma_T} d\sigma [2(U(\sigma) - U_F)]^{1/2}$ . Here  $\sigma$  is the tunneling field. Since  $S_1 \sim U_0/M \sim M(\Delta\sigma)^2$  for a mass  $M$  in the false vacuum, the parameter  $a \simeq (\Delta U/U_0) M/H_T$ , which characterizes the degeneracy of the vacua, can be made arbitrarily small by tuning  $U_T \simeq U_F$ . On the other hand, the parameter  $b \simeq (\Delta\sigma/M_{\text{Pl}})^2 M/H_T$ , which characterizes the width of the barrier, is not a tunable parameter and could be very large or very small depending on the model.

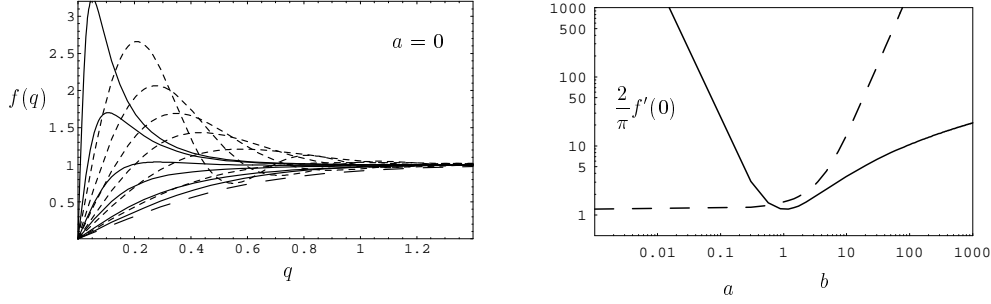
In order to prevent collisions with other nucleated bubbles (at least in our past light cone) it is necessary that the probability of tunneling be sufficiently suppressed. For an open universe of  $\Omega_0 > 0.2$ , this is satisfied as long as the bounce action  $S_B > 6$ , see Ref. [5]. This imposes only a very mild constraint on the tunneling parameters  $a$  and  $b$ , as long as the energy density in the true vacuum satisfies  $U_T \ll M_{\text{Pl}}^4$ , see Ref. [24].

### 3. Primordial perturbation spectra

There are essentially two kinds of primordial perturbations in open inflation, quantum mechanical and classical or semiclassical. In the first category we have the usual scalar and tensor metric perturbations [9, 10, 16], modified in the context of an open universe due to bubble nucleation, as well as supercurvature [8, 23, 10] and bubble wall perturbations [12, 13, 14, 17], which are specific to open inflation. The other category, which appears in the context of two-field open inflation [7], contains classical and semiclassical effects due to tunneling to different values of the inflaton field [18, 19]. All these perturbations create anisotropies in the CMB which distort the angular power spectrum on large scales (low multipoles). On smaller scales, at about one degree separation (multipoles  $l \sim 200$ ), the (geometrical) effect of an open universe is to shift the acoustic peaks of the temperature power spectrum to higher multipoles [31], but the primordial spectrum there is essentially that of a flat universe.

#### 3.1. Scalar and tensor perturbations.

Soon after the proposal of a single-bubble open inflation model [6], the primordial spectrum of scalar perturbations was computed and found to be identical to the flat case except for a prefactor that depends on the



*Figure 1.* The left panel shows the spectral function  $f(q)$  in (4) as a function of  $q$  for  $a = 0$  and  $b = 0.05, 0.1, 0.2, 0.3, 0.5, 1.0$  (continuous lines, from top to bottom), and  $b = 2, 5, 10, 20, 50, 200$  (dotted lines, from bottom to top). It is clear that  $f(q)$  is linear in  $q$  at  $q = 0$ . The dashed line corresponds to  $f(q) = \tanh \pi q/2$ . The right panel shows the sharp increase in the slope  $f'(q = 0)$  as we increase or decrease  $b$  away from one (continuous line) and as we increase  $a$  (dashed line). The minimum slope corresponds to  $b \simeq 1, a = 0$ , *i.e.* for almost degenerate vacua.

bubble geometry [10],

$$\mathcal{P}_{\mathcal{R}}(q) = A_S^2 f(q), \quad A_S^2 = \frac{\kappa^2}{2\epsilon} \left( \frac{H_T}{2\pi} \right)^2. \quad (3)$$

The function  $f(q)$  depends on the tunneling parameters  $a$  and  $b$ , see Eq. (2),

$$f(q) = \coth \pi q - \frac{z^2 \cos \tilde{q} + 2qz \sin \tilde{q}}{(4q^2 + z^2) \sinh \pi q}, \quad (4)$$

where  $\tilde{q} = q \ln((1+x)/(1-x))$  and

$$x = (1 - R_0^2 H_T^2)^{1/2} = \Delta (1 + \Delta^2)^{-1/2}, \quad (5)$$

$$z = (1 - R_0^2 H_T^2)^{1/2} - (1 - R_0^2 H_F^2)^{1/2} = 2b (1 + \Delta^2)^{-1/2}, \quad (6)$$

see Eq. (1). The function  $f(q)$  is linear at small  $q$ , and approaches a constant value  $f(q) = 1$  at  $q \geq 2$ , see Fig. 1. Here  $q$  is the effective momentum in an open universe, determined from  $q^2 = k^2 - 1$ . For scalar perturbations, the effect of  $f(q)$  on the temperature power spectrum is almost negligible, and therefore the tilt of the scalar spectrum is approximately given by the same formula as in flat space [32],

$$n_S - 1 \equiv \frac{d \ln \mathcal{P}_{\mathcal{R}}(k)}{d \ln k} \simeq -6\epsilon + 2\eta, \quad (7)$$

in the slow-roll approximation,

$$\epsilon = \frac{1}{2\kappa^2} \left( \frac{V'(\phi)}{V(\phi)} \right)^2 \ll 1, \quad \eta = \frac{1}{\kappa^2} \left( \frac{V''(\phi)}{V(\phi)} \right) \ll 1. \quad (8)$$

On the other hand, the primordial spectrum of tensor or gravitational waves' anisotropies took much longer to evaluate. There was for some years the open problem that the observed power spectrum presented an infrared divergence at  $q = 0$  in an open universe [33]. It was clear that a physical regulator was necessary. But this is precisely the role played by the bubble wall; recent computations have shown that in the presence of the bubble the tensor primordial spectrum is given by [16]

$$\mathcal{P}_g(q) = A_T^2 f(q), \quad A_T^2 = 8\kappa^2 \left(\frac{H_T}{2\pi}\right)^2. \quad (9)$$

It is the shape of the function  $f(q)$ , see (4) and Fig. 1, which gives a finite physical observable, as I will explain in the following section. Here  $q$  is the effective momentum for tensor modes, defined by  $q^2 = k^2 - 3$ . While for scalar modes the presence of this function  $f(q)$  in the primordial spectrum becomes irrelevant for observations, for tensor modes the slope of this function at  $q = 0$  is an important ingredient in the final value of the predicted power spectrum at low multipoles [24].

### 3.2. Supercurvature and bubble wall modes.

Apart from the usual continuum ( $q^2 \geq 0$ ) of scalar and tensor modes, generalized to an open universe, in single-bubble inflationary models there is a new type of quantum fluctuations which are purely geometrical, due to the boundary conditions associated with the presence of the bubble wall in open de Sitter. These modes are discrete modes, that appear whenever the mass of the scalar field in the false vacuum is smaller than the rate of expansion,  $m^2 < 2H^2$ .

The supercurvature mode was first postulated in Ref. [8] from purely mathematical arguments related to homogeneous random fields. Only later did concrete models of single-bubble open inflation [9] prove its existence in the primordial spectrum of scalar fluctuations. Such a mode corresponds to an infinite wavelength ( $k^2 = 0, q^2 = -1$ ) mode, and thus received its name of *supercurvature* mode [the curvature scale corresponds to the eigenmode with eigenvalue  $k^2 = 1, q^2 = 0$ ]. These authors computed its amplitude as [10, 23]

$$A_{SC}^2 = \frac{\kappa^2}{2\epsilon} \left(\frac{H_F}{2\pi}\right)^2 = A_S^2 \frac{H_F^2}{H_T^2}, \quad (10)$$

where  $A_S^2$  is given by Eq. (3).

However, this is not the only discrete supercurvature mode possible in single-bubble models. As realized in Ref. [12], there are also scalar fluctuations of the bubble wall with  $k^2 = -3, q^2 = -4$ , which could in principle have its imprint in the CMB insotropies. Such modes were also discussed

in Ref. [11] and their amplitude computed in Ref. [13, 14], based on field theoretical arguments,

$$A_W^2 = \frac{3\kappa^2}{2z} \left( \frac{H_T}{2\pi} \right)^2 = A_S^2 \frac{3\epsilon}{z}, \quad (11)$$

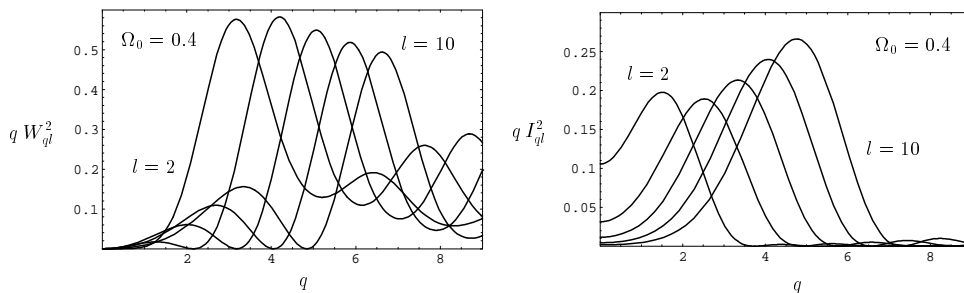
where  $z$  is given by Eq. (6) and  $A_S^2$  is the scalar amplitude (3). Such modes contribute as transverse traceless curvature perturbations [13, 12], which nevertheless behaves as a homogeneous random field [15]. It was later realized [17] that the bubble wall fluctuation mode is actually part of the tensor primordial spectrum, once gravitational backreaction is included.

### 3.3. Semiclassical perturbations and quasi-open inflation.

All single-field models of open inflation predict the above primordial spectra of anisotropies: a continuum of scalar and tensor modes and the discrete supercurvature and bubble wall modes. However, as mentioned in the introduction, it is difficult to construct such models without a certain amount of fine tuning [7], and thus multiple-field models were considered [7, 20, 21, 22]. However, a large class of two-field models do not lead to infinite open universes, as it was previously thought, but to an ensemble of very large but finite inflating ‘islands’. The reason is that the quantum tunneling responsible for the nucleation of the bubble does not occur simultaneously along both field directions and equal-time hypersurfaces in the open universe are not synchronized with equal-density or fixed-field hypersurfaces [7, 18]. The most probable tunneling trajectory corresponds to a value of the inflaton field at the bottom of its potential; large values, necessary for the second period of inflation inside the bubble, only arise as localized fluctuations. The interior of each nucleated bubble will contain an infinite number of such inflating regions of comoving size of order  $\gamma^{-1}$ , where  $\gamma$  depends on the parameters of the model, see below. Each one of these islands will be a quasi-open universe. We may happen to live in one of those patches of comoving size  $d \leq \gamma^{-1}$ , where the universe appears to be open. This new effect, semiclassical in origin, was recently discussed in Ref. [19] and the amplitude of the associated fluctuation in the CMB was computed as

$$A_C = \frac{3}{2} \frac{H_T^2}{m_T^2} \gamma, \quad \gamma = \frac{2}{3} \frac{m_F^2}{H_F^2} + \frac{1}{8} H_F^2 R_0^4 (m_T^2 - m_F^2), \quad (12)$$

where  $R_0$  is the radius of the bubble at tunneling (1), and the eigenvalue  $\gamma$  was computed in the thin-wall approximation [19]. As we shall see in the next section, many of the present models are quasi-open. This does not mean that they are not good cosmological models. If the co-moving size of the inflating islands is sufficiently large, then the resulting classical



*Figure 2.* Windows functions for the scalar (left panel) and tensor (right panel) CMB power spectra  $l(l+1)C_l/A^2$ , see Eqs. (13) and (14), for the first few multipoles,  $l = 2, 4, 6, 8, 10$ .

anisotropy may be unobservable. This will prove rather constraining for the models.

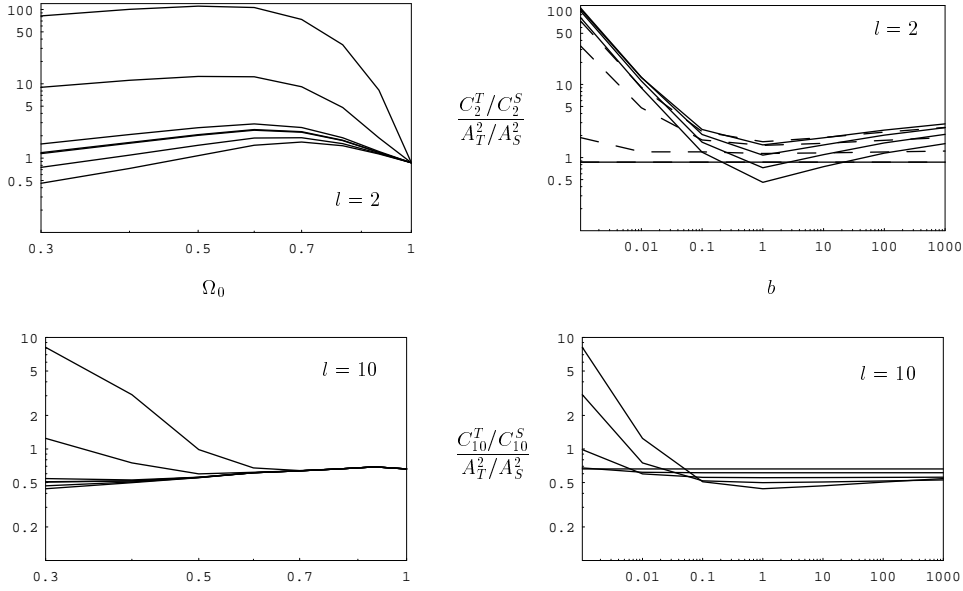
#### 4. CMB temperature anisotropies

Quantum fluctuations of the inflaton field  $\phi$  during inflation produce long-wavelength scalar curvature perturbations and tensor (gravitational waves) perturbations, which may leave their signature in the CMB temperature anisotropies, when they reenter the horizon. Temperature anisotropies are usually given in terms of the two-point correlation function or power spectrum,  $C_l$ , defined by an expansion in multipole number  $l$ . We are mainly interested in the large scale (low multipole number) temperature anisotropies since it is there where gravitational waves and the discrete modes could become important. After  $l \sim 30$ , the tensor power spectrum drops down [34, 35] while the density perturbation spectrum increases towards the first acoustic peak, see Ref. [36]. On these large scales the dominant effect is gravitational redshift via the Sachs-Wolfe effect [37]. The scalar and tensor components of the temperature power spectrum can be written as

$$l(l+1)C_l^S = \int_0^\infty dq A_S^2 f(q) W_{ql}^2, \quad (13)$$

$$l(l+1)C_l^T = \int_0^\infty dq A_T^2 f(q) I_{ql}^2, \quad (14)$$

where  $W_{ql}$  and  $I_{ql}$  are the corresponding window functions, see Ref. [24], which depend on the particular value of  $\Omega_0$ . We have plotted these functions in Fig. 2 for a typical value,  $\Omega_0 = 0.4$ , and the first few multipoles. Note that the scalar window functions grow as a large power of  $q$  at the origin, so that the scalar power spectrum is rather insensitive to the ‘hump’ in the function  $f(q)$ , as mentioned above. On the other hand, the tensor window



*Figure 3.* The ratio of tensor to scalar contribution to the CMB power spectrum  $C_l^T/C_l^S$ , normalized to the corresponding amplitudes, for the quadrupole and tenth multipole, for  $a = 0$ , as a function of  $\Omega_0$  (for  $b = 0.001, 0.01, 0.1, 1, 10, 100, 1000$  from top to bottom) on the left panels, and as a function of the tunneling parameter  $b$  (for  $\Omega_0 = 0.3 - 1.0$ ) on the right panels.

functions are singular at  $q = 0$ , and only the linear dependence of  $f(q)$  at the origin prevents the existence of the infrared divergence found in [33]. Furthermore, since the functions  $qI_{ql}^2$  are not negligible near the origin, the tensor power spectrum turns out to be very sensitive to the ‘hump’ in the spectral function  $f(q)$ , see Fig. 1.

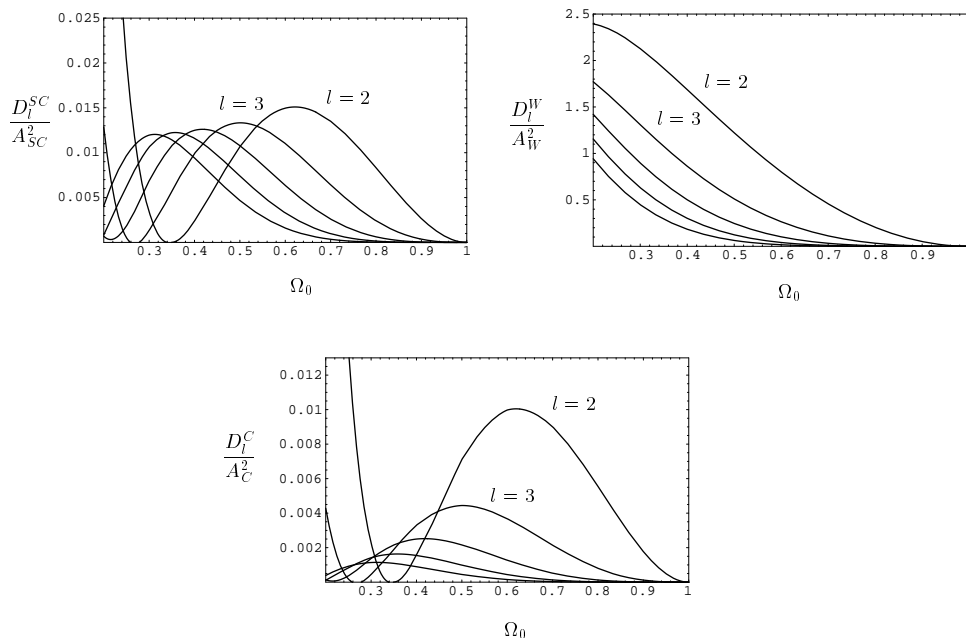
The ratio of tensor to scalar contribution to the temperature power spectrum is a fundamental observable in standard inflation, which depends on the slow-roll parameters (8) and provides a consistency check of the theory [32]. In single-bubble open inflation such a ratio depends not only on the slow-roll parameters but also on the tunneling parameters (2) and on the value of  $\Omega_0$ , see Ref. [24],

$$R_l = \frac{C_l^T}{C_l^S} \simeq f_l(\Omega_0, a, b) [1 - 1.3(n_S - 1)] 16 \epsilon. \quad (15)$$

We have plotted this ratio in Fig. 3 for the quadrupole and tenth multipole of the power spectrum, as a function of  $\Omega_0$  and the tunneling parameter  $b$ .

In the ideal case in which the gravitational wave perturbation can be disentangled from the scalar component in future precise observations of





*Figure 4.* CMB power spectrum  $l(l+1) C_l/A^2$ , normalized to the corresponding amplitude, for the supercurvature, bubble wall and semiclassical fluctuations, as a function of  $\Omega_0$ , for the first few multipoles  $l = 2, 3, \dots$ . Note the dip in the supercurvature and semiclassical power spectra at different values of  $\Omega_0$  for different multipoles, due to accidental cancellations.

the CMB power spectrum [38-42], one might be able to test this relation for a given value of  $\Omega_0$ . This would then constitute a check on the tunneling parameters  $a$  and  $b$ . Such prospects are however very bleak from measurements of the temperature power spectrum alone, with the next generation of satellites, see e.g. [43, 39]. At most one can expect to impose constraints on the parameters of the model from the absence of a significant gravitational wave contribution to the CMB. However, taking into account also the polarization power spectrum, together with the temperature data, one expects to do much better, see Refs. [44, 41] for the case of flat models. Hopefully similar conclusions can be reached in the context of open models, and CMB observations may be able to check the generalized consistency relation (15) with some accuracy.

The supercurvature, bubble wall and semiclassical modes also contribute to the temperature power spectrum. Their contribution is plotted in Fig. 4, normalized to the corresponding amplitude, see Eqs. (10), (11) and (12), as a function of  $\Omega_0$  for the first few multipoles. Note the dip in the spectrum at certain values of  $\Omega_0$  due to accidental cancellations [21, 22, 24]. This does

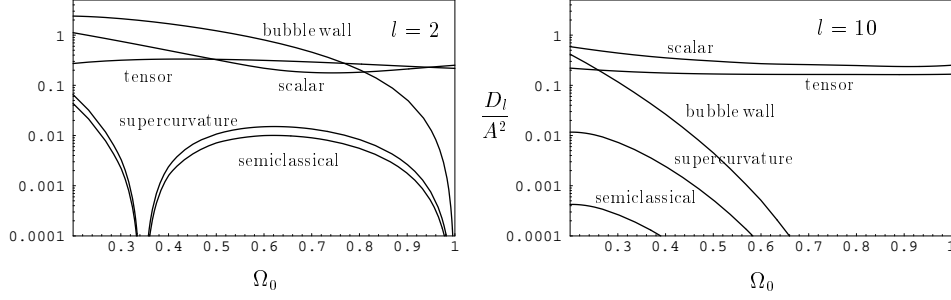


Figure 5. Quadrupole (left panel) and tenth multipole (right panel) of the CMB power spectra, normalized to the corresponding amplitude,  $l(l+1)C_l/A^2$ , for the tensor, scalar, supercurvature, bubble wall and semiclassical primordial spectra. We are assuming the minimal contribution from tensors ( $a = 0, b = 1$ ).

not affect the bounds since higher multipoles will fill in those gaps. The relative importance of the different components of the power spectrum is crucial in order to derive bounds on the model parameters. We have plotted in Fig. 5 the quadrupole and tenth multipole of the CMB power spectrum for each mode, normalized to their corresponding amplitudes. Assuming that the observed quadrupole only comes from the scalar component, one can deduce the following constraints, see Ref. [24, 19],

$$H_T = \sqrt{\pi\epsilon} 5 \times 10^{-5} M_{\text{Pl}}, \quad (16)$$

$$H_F < 10 H_T, \quad (17)$$

$$\epsilon < z/3, \quad (18)$$

$$\gamma < \frac{m_T^2}{H_T^2} 3 \times 10^{-4}. \quad (19)$$

The third expression accounts for both the tensor and the bubble wall constraints, since the bubble wall fluctuation is actually part of the tensor spectrum [17] and gives the largest contribution at low multipoles. The constraints coming from higher multipoles are significantly weaker, see Fig. 5. One could argue that the quadrupole is going to be hidden in the cosmic variance [42] and thus only the constraints at higher multipoles, say  $l = 10$ , should be imposed. However, polarization power spectra may one day be used to get around cosmic variance [45] and we may be able to extract information about the scalar and tensor components at low multipoles. We will therefore take the conservative attitude that consistent models of open inflation should satisfy the bounds coming from the quadrupole, (16)-(19).

## 5. Model building

In this section we will review the different single-bubble open inflation models present in the literature, and use the CMB observations to rule out some of them and severely constrain others. We will see that open inflation models could be as predictive as ordinary inflation, in the sense that they also can be ruled out if they are in conflict with observations.

As mentioned in the introduction, single-field models of open inflation [6] require some finetuning in order to have a large mass for successful tunneling and a small mass for slow-roll inside the bubble [7]. Even if such a model can be constructed from particle physics, it still needs to satisfy the constraints coming from observations of the CMB anisotropies. These models lack both supercurvature and semiclassical anisotropies, by construction. However, they generically produce too large tensor modes at low multipoles, where it is dominated by the bubble wall fluctuations. The reason is that in these models the tunneling parameter  $b$  that characterizes the width of the tunneling barrier (2) is extremely small and it is very difficult to satisfy the bound (18) due to the bubble wall fluctuations, unless one does extreme finetuning of the parameters. Let us analyze a typical example, which is a variant of the new inflation type [6]. Tunneling occurs from a symmetric phase at  $\sigma = 0$  to a value  $\sigma_b$  from which the field slowly rolls down the potential towards the symmetry breaking phase at  $\sigma = v \sim M_{\text{GUT}} \sim 10^{15}$  GeV. The fact that we have a finite number of e-folds,  $N = 60$ , requires  $\sigma_b \sim v \exp(-\alpha N) \ll v$ , where  $\alpha \simeq 2m_T^2/3H_T^2$ . The rate of expansion in the false vacuum is of order that in the true vacuum,  $H_T = (8\pi V(0)/3M_{\text{Pl}}^2)^{1/2} \sim 3 \times 10^{-6} M_{\text{Pl}}$ , which implies  $\epsilon \sim 10^{-3}$ . Taking a typical mass in the false vacuum to be  $M \sim M_{\text{GUT}}$ , we find

$$b_{\text{single}} \simeq \left( \frac{\sigma_b}{M_{\text{Pl}}} \right)^2 \frac{M}{H_T} \sim 5 \times 10^{-9}, \quad (20)$$

which gives  $z \sim 2b \sim 10^{-8}$ , an extremely small number that makes it impossible to satisfy the bubble wall constraint,  $\epsilon < z/3$ , see Eq. (18).

In other words, the simplest single-field models of open inflation [6] are not only fine tuned but actually produce too large gravitational wave anisotropies in the CMB on large scales to be consistent with observations. Perhaps more complicated models could still be fine tuned to agree with observations.

### 5.1. Coupled and uncoupled two-field models.

In this section we shall consider a class of two-field models [7] with a potential of the form

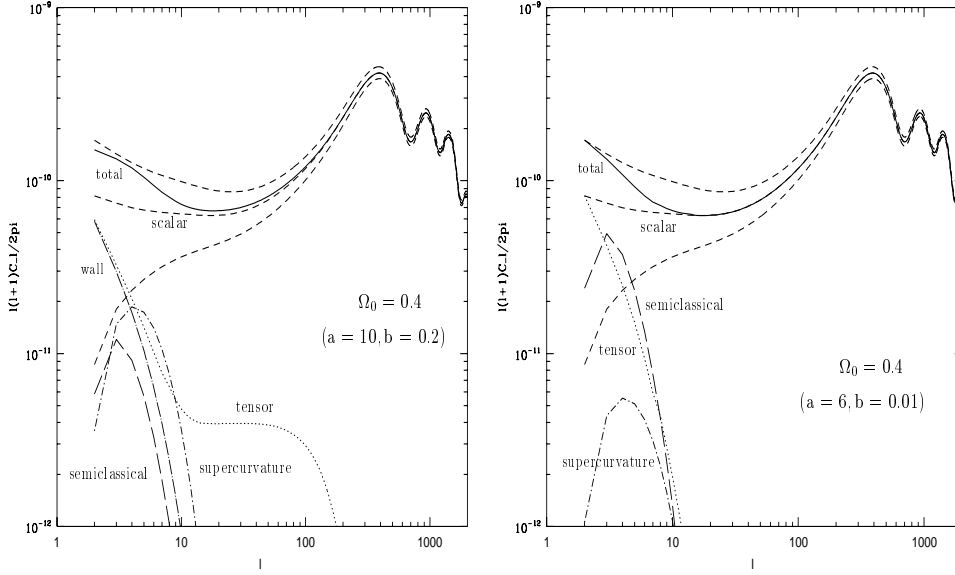
$$V(\sigma, \phi) = V_0(\sigma) + \frac{1}{2}m^2\phi^2 + \frac{1}{2}g^2\sigma^2\phi^2. \quad (21)$$

Here  $V_0$  is a non-degenerate double well potential, with a false vacuum at  $\sigma = 0$  and a true vacuum at  $\sigma = v$ . When  $\sigma$  is in the false vacuum,  $V_0$  dominates the energy density and we have an initial de Sitter phase with expansion rate given by  $H_F^2 \approx 8\pi V_0(0)/3M_{\text{Pl}}^2$ . Once a bubble of true vacuum  $\sigma = v$  forms, the energy density of the slow-roll field  $\phi$  may drive a second period of inflation. However, as pointed out in Ref. [7], the simplest two-field model of open inflation, given by (21) with  $g = 0$  and  $m \neq 0$ , is actually a quasi-open one because equal-time hypersurfaces, defined by the  $\sigma$  field after nucleation, are not synchronized with equal-density hypersurfaces, determined by the slow-roll of the  $\phi$  field during inflation inside the bubble. In order to suppress this effect it was argued [7] that a large rate of expansion in the false vacuum compared to the true vacuum,  $H_F \gg H_T$ , could prevent the  $\phi$  field from rolling outside the bubble and distorting the equal-density hypersurfaces inside the bubble. However, this would induce [23] a large supercurvature mode anisotropy in the CMB, which would be incompatible with observations. A careful analysis [18, 19] shows that indeed the effect is important at low multipoles. In this model,  $m_F = m_T = m$ , the supercurvature eigenvalue  $\gamma = 2m_F^2/3H_F^2$ , and in order to satisfy (19) it requires  $H_F > 50H_T$ , which is incompatible with the supercurvature constraint,  $H_F < 10H_T$ . Therefore, this model seems to be ruled out.

In order to construct a truly open model, Linde and Mezhlumian suggested taking  $m = 0$  and  $g \neq 0$ , i.e. the ‘‘coupled’’ two-field model [7]. In this way, the mass of the slow-roll field vanishes in the false vacuum, and it would appear that the problem of classical evolution outside the bubble is circumvented. However, as we showed in Ref. [19], this is not exactly so, and actually the whole class of models (21) leads to quasi-open universes, which are constrained by CMB observations. Let us work out those constraints in detail. We will assume a tunneling potential like [7]

$$V_0(\sigma) = V_0 + \frac{1}{2}M^2\sigma^2 - \alpha M\sigma^3 + \frac{1}{4}\lambda\sigma^4. \quad (22)$$

For  $\alpha = \sqrt{\lambda}$  we have  $\sigma_c = 2M/\sqrt{\lambda}$  and  $\phi_c = M/g$ . The field can tunnel for  $\phi < \phi_c$ . The constant  $V_0 \simeq 2.77M^4/\lambda$  has been added to ensure that the absolute minimum, at  $\phi = 0$  and  $\sigma_0 \simeq 1.3\sigma_c$ , has vanishing cosmological constant. After tunneling, the field  $\phi$  moves along an effective potential  $V(\phi) = m^2\phi^2/2$ , where the effective mass varies only slightly from tunneling to the end of inflation,  $m \simeq 1.3g\sigma_c$ . This potential drives a period of chaotic inflation with slow-roll parameters  $\epsilon = \eta = 1/2N_e = 1/120$ . Substituting into (16) we find  $H_T = 6.32m = 8 \times 10^{-6}M_{\text{Pl}}$ , and therefore  $g\sigma_c = 10^{-6}M_{\text{Pl}}$ . The rate of expansion in the false vacuum is determined from  $H_F^2/H_T^2 =$



*Figure 6.* The complete angular power spectrum of temperature anisotropies for the coupled two-field model (left panel) for  $\Omega_0 = 0.4$  and  $(a = 10, b = 0.2)$ , and the open hybrid model (right panel) for  $\Omega_0 = 0.4$  and  $(a = 6, b = 0.01)$ . We show also the individual contributions from the scalar, tensor, supercurvature, semiclassical and bubble wall modes, as well as the expected cosmic variance (dashed lines) for  $1/3$  of the sky coverage from the future CMB satellites. Note that the bubble wall mode is responsible for the large growth of the tensor contribution at low multipoles. Only the scalar modes remain beyond about  $l = 50$ , where they grow towards the acoustic peaks.

$1 + 4ab$ , where  $b = (4\pi\sqrt{2}/3\lambda)M^3/H_T M_{\text{Pl}}^2$ , which gives

$$M = \frac{(1 + 4ab)^{1/2}}{4b} H_F \geq \sqrt{2} H_F, \quad (23)$$

the last condition arising from preventing the formation of the bubble through the Hawking-Moss instanton, see Ref. [7]. Furthermore, taking  $m_F = 0$  in the eigenvalue  $\gamma$  gives (12)

$$A_C = \frac{3}{16} \frac{H_F^2}{H_T^2} (R_0 H_T)^4 = \frac{3(1 + 4ab)}{16[1 + (a + b)^2]^2} < 4 \times 10^{-4}. \quad (24)$$

From the supercurvature mode condition (17),  $(1 + 4ab)^{1/2} < 10$ , together with (23) we find the constraint  $b < 1$ . From Eq. (24), we realize that having nearly degenerate vacua,  $a \ll 1$ , is not compatible with observations. Satisfying (24) would require  $b \ll 1$  and  $a \gg 1$ . However, for these values of the parameters we expect large tensor contributions, see Fig. 3. So

there should be a compromise between the different mode contributions. For details, see Ref. [46].

We have shown in the left panel of Fig. 6a the complete temperature power spectrum for a coupled two-field model having  $a = 10$  and  $b = 0.2$ , for  $\Omega_0 = 0.4$ , which is consistent with observations. It has contributions from all the modes: scalar, tensor, supercurvature, semiclassical and bubble wall. Note however that the bubble wall mode is in fact included in the sharp growth of the tensor contribution at small multipole number, as emphasized in Ref. [17] and shown explicitly in Fig. 6a, and should not be counted twice. Although it is in principle possible to construct a model consistent with observations, the parameters of the model are not very natural. In this simple model (22) one would expect that quantum tunneling would occur soon after the critical point,  $\phi = \phi_c$ , where  $a \ll 1$  and  $b \sim 1$ . In that case, instead of an infinite open universe we would actually live in a finite quasi-open one [19]. In order to suppress the associated semiclassical anisotropy we had to choose other values of the parameters. As can be seen from Fig. 6a, there exists a range of parameters for which all contributions to the CMB anisotropies are compatible with observations.

### 5.2. Supernatural open inflation.

An attractive scenario for open inflation is the model of a complex scalar field with a slightly tilted mexican hat potential, where the radial component of the field does the tunneling and the pseudo-Goldstone mode does the slow-roll. This model was called “supernatural” inflation in Ref. [7], because the hierarchy between tunneling and slow-roll mass scales is protected by the approximate global  $U(1)$  symmetry. Expanding the field in the form  $\Phi = (\sigma/\sqrt{2}) \exp(i\phi/v)$ , where  $v$  is the expectation value of  $\sigma$  in the broken phase, we consider a potential of the form  $V = V_0(\sigma) + V_1(\sigma, \phi)$ , where  $V_0$  is  $U(1)$  invariant and  $V_1$  is a small perturbation that breaks this invariance. It is assumed that  $V$  has a local minimum at  $\Phi = 0$  which makes the symmetric phase metastable. We shall consider a tilt in the potential of the form  $V_1 = \Lambda^4(\sigma)G(\phi)$  where  $\Lambda$  is a slowly varying function of  $\sigma$  which vanishes at  $\sigma = 0$ . For definiteness we can take  $G = (1 - \cos \phi/v)$ . The idea is that  $\sigma$  tunnels from the symmetric phase  $\sigma = 0$  to the broken phase  $\sigma_0 = v$ , landing at a certain value of  $\phi$  away from the minimum of the tilted bottom. Once in the broken phase, the potential  $V_1$  cannot be neglected, and the field  $\phi$  slowly rolls down to its minimum, driving a second period of inflation inside the bubble. An attractive feature of this model is that depending on the value of  $\phi$  on which we end after tunneling, the number of  $e$ -foldings of inflation will be different. Hence it appears that in principle we can get a different value of the density parameter in each nucleated bubble. As we shall see, however, the picture is somewhat

different. For  $V_1(\phi) = \Lambda^4(1 - \cos \phi/v)$  we find the slow-roll parameters  $\epsilon = (1/2\kappa^2 v^2) \cot^2 \phi/2v \ll 1$  and  $\eta = \epsilon - 1/2\kappa^2 v^2$ . From the constraint on the spectral tilt,  $n_S - 1 = -4\epsilon - 1/\kappa^2 v^2 > -0.2$ , we find that necessarily  $\kappa^2 v^2 > 5$ , which means that the vev of  $\sigma$  is  $v \simeq M_{\text{Pl}}$ . We are again in a situation similar to the single-field models, where we need some extreme fine tuning to prevent the Hawking-Moss instanton from forming the bubble, see Ref. [7]. Indeed, for a generic tunneling potential like (22) we have  $V_0 \simeq M^2 \sigma_0^2/2$  and thus  $H_F \simeq 2M \sigma_0/M_{\text{Pl}} \geq M$ . Under this conditions the tunneling does *not* occur along the Coleman-DeLuccia instanton, which is necessary for the formation of an open universe inside the bubble. The only way to prevent this is by artificially bending the potential so that it has a large mass at the false vacuum. In Ref. [7] it was proposed a way to lower the minimum at the center of the Mexican hat via radiative corrections from a coupling of the  $U(1)$  field  $\Phi$  to another scalar  $\chi$ . For certain values of the coupling constant,  $g^4 = 32\pi\lambda$ , it is possible to make the two minima, at  $\sigma = 0$  and  $\sigma_0$ , exactly degenerate. The tunneling potential is then

$$V_0(\sigma) = \frac{\lambda}{2}(\sigma_0^2 - \sigma^2)\sigma^2 + \lambda\sigma^4 \ln \frac{\sigma}{\sigma_0}, \quad (25)$$

where  $\sigma_0 = M/\sqrt{\lambda} \simeq M_{\text{Pl}}$ . The associated tunneling parameters become  $a = 0$  and  $b = (\sigma_0/M_{\text{Pl}})^2 M/H_T \simeq M/H_T$ , which can be large. As emphasized in Ref. [19] there is a supercurvature mode in this model, associated with the massless Goldstone mode, which induces both supercurvature and semiclassical perturbations. Because of the different normalization of the supercurvature mode in supernatural inflation,  $H_F \rightarrow 2R_0^{-1}$ , the supercurvature constraint (17) should read in this case

$$R_0 H_T > 0.2, \quad (26)$$

which is not trivially satisfied. On the other hand, the eigenvalue  $\gamma = R_0^2 m_T^2/2$  for the Goldstone mode [19] induces a large semiclassical perturbation (19) unless

$$R_0 H_T < 0.024. \quad (27)$$

It is clear that these two constraints cannot be accommodated simultaneously, and thus the model is incompatible with observations. One still has to make sure [46] that a slight modification of the potential does not give a certain range of parameters in which the model works.

### 5.3. Induced gravity open inflation.

This model was proposed in Ref. [20] as a way of avoiding the problems of classical motion outside the bubble. In the false vacuum the inflaton field is trapped due to its non-minimal coupling to gravity, with coupling  $\xi$ . When

the tunneling occurs it is left free to slide down its symmetry breaking potential  $V(\varphi) = \lambda(\varphi^2 - \nu^2)^2/8$ . The expectation value of the inflaton at the global minimum gives the Planck mass today,  $M_{\text{Pl}}^2 = 8\pi\xi\nu^2$ . The model is parametrized by  $\alpha = 8U_F/\lambda\nu^4$ , which determines the value of the stable fixed point in the false vacuum,  $\varphi_{\text{st}}^2 = \nu^2(1 + \alpha)$ , as well as the difference in rates of expansion in the false and true vacua,  $H_F^2 = H_T^2(1 + \alpha)/\alpha$ , and the slow-roll parameters,  $\epsilon = 8\xi/(1 + 6\xi)\alpha^2$ ,  $\eta = 8\xi(1 - \alpha)/(1 + 6\xi)\alpha^2$ , see Ref. [21].

We will assume a tunneling potential for the  $\sigma$  field of the type

$$U(\sigma) = \frac{1}{4}\lambda'\sigma^2(\sigma - \sigma_0)^2 + \mu U_0\left[1 - \left(\frac{\sigma}{\sigma_0}\right)^4\right], \quad (28)$$

where  $\sigma_0 = M\sqrt{2/\lambda}$ ,  $U_0 = M^4/4\lambda'$  and  $\mu \ll 1$  for the thin wall approximation to be valid. This form of the potential gives a tunneling parameter  $b = (2\pi/3\lambda)M^3/H_T M_{\text{Pl}}^2$ , which determines the relation between the mass of the  $\sigma$  field in the false vacuum and the rate of expansion there,  $M = H_F(1 + 1/\alpha)^{1/2}/\mu b$ . Thanks to  $\mu \ll 1$ , we can have  $M \gg H_F$  for values of  $b \geq 1$ , which induces gravitational wave anisotropies that are well under control.

Furthermore, the induced gravity model seems to be truly open, since the inflaton field  $\varphi$  is static in the false vacuum and thus there is no supercurvature mode associated with classical motion outside the bubble, see Ref. [19]. Therefore the constrain (19) does not apply, and there exists for this model a range of parameters for which all contributions to the CMB anisotropies are compatible with observations, see Ref. [24]. However, the instanton may not take you to  $\varphi_{\text{st}}$  in the true vacuum, but to a different value, closer to the minimum of the potential,  $\varphi = v$ . In that case, the number of  $e$ -folds is smaller than expected and so is the value of  $\Omega_0$ . Such effects should be taken into account for the determination of the model parameters, see Ref. [46].

#### 5.4. Open hybrid inflation.

This model was proposed recently [22] in an attempt to produce a significantly tilted scalar spectrum in the context of open inflation to be in agreement with large scale structure [47]. It is based on the hybrid inflation scenario [48, 49], which has recently received some attention from the point of view of particle physics [50-54], together with a tunneling field which sets the initial conditions inside the bubble.

In this model there are three fields: the tunneling field  $\sigma$ , the inflaton field  $\phi$  and the triggering field  $\psi$ . The tunneling occurs like in the coupled model of section 5.1 with potential

$$U(\sigma, \phi) = V_0 + \frac{\lambda}{4}\sigma^2(\sigma - \sigma_0)^2 + \frac{1}{2}g^2(\phi^2 - \phi_c^2)\sigma^2 + U_0, \quad (29)$$



where  $\sigma_0 = 2M/\sqrt{\lambda}$ ,  $\phi_c = M/g$ ,  $V_0 \simeq 2.77M^4/\lambda$  to ensure that at the global minimum we have vanishing cosmological constant, and  $U_0$  is the vacuum energy density associated with the triggering field. We satisfy  $V_0 \ll U_0$ . If the  $\sigma$  field tunnels when  $\phi = 3\phi_c/4$ , then  $\Delta U = U_F - U_T \simeq V_0/2 \simeq m^2\phi_c^2/4$ . After that the inflaton field will slow-roll down the effective potential  $U = U_0 + m^2\phi^2/2 \simeq U_0$  driving hybrid inflation, until the coupling to  $\psi$  triggers its end. The model is parametrized by  $\alpha = m^2/H^2$ , see Refs. [22, 24, 46], in terms of which the spectral tilts can be written as  $n_S - 1 = 2\alpha/3 - 2\epsilon$  and  $n_T = -2\epsilon$ . At tunneling we can write  $V_0 \simeq m^2\phi_c^2/2 = 8m^2\phi_T^2/9$ , so that the slow-roll parameter  $\epsilon = (\alpha/3)9V_0/16U_T = 3\alpha ab/2$ , where  $4ab = \Delta U/U_T \simeq V_0/2U_T$ , see (2), and thus

$$n_S = 1 + \frac{2\alpha}{3} \left(1 - \frac{9ab}{2}\right). \quad (30)$$

In order for open hybrid to have a large tilt we require *both* a large  $\alpha$  and a small value of  $ab$ . As we will show, this is not going to be possible due to the set of conditions (16-19) from the CMB.

For that purpose, we should compute the tunneling parameter  $b = (4\pi\sqrt{2}/3\lambda)M^3/H_T M_{\text{Pl}}^2 \simeq (V_0/4U_T)H_T/M$ , which requires  $M = 2aH_T > 2H_T$ , and thus  $a > 1$ . Since both  $H_T < M$  and  $V_0 \ll U_T$ , we expect  $b \ll 1$ , which will induce large tensor anisotropies at low multipoles. This is a generic feature of open hybrid models. The tensor amplitude is related to the scalar one by  $A_T^2 = 16\epsilon A_S^2 = 8A_S^2(n_S - 1)9ab/(2 - 9ab)$ . In order to satisfy the CMB constraints we require

$$H_F^2/H_T^2 = 1 + 4ab < 100, \quad (31)$$

$$\epsilon = \frac{3\alpha ab}{2} < \frac{2b}{3[1 + (a+b)^2]^{1/2}}, \quad (32)$$

$$A_C = \frac{3(1 + 4ab)}{16[1 + (a+b)^2]^2} < 4 \times 10^{-4}, \quad (33)$$

$$M = \frac{2a}{(1 + 4ab)^{1/2}}H_F > 2H_F. \quad (34)$$

Since  $a > 1$  requires  $b < V_0/8U_T \ll 1$ , we can use the third constraint to get the bound  $a > 5$ , which then imposes (through the second constraint) that  $\alpha < 4/9a(1 + a^2)^{1/2} \simeq 1/2a^2 < 1/50$ . This means that the scalar tilt (30) cannot be significantly larger than 1, as was the aim of Ref. [22].

We have plotted in Fig. 6b the complete angular power spectrum of temperature anisotropies for the open hybrid model, in the case  $\Omega_0 = 0.4$  and  $(a = 6, b = 0.01)$ . We have also included the cosmic variance uncertainty [40]

$$\sigma_l = \left[ \frac{2}{(2l + 1)f_{\text{sky}}} \right]^{1/2} C_l, \quad (35)$$

where we have chosen  $f_{\text{sky}} = 1/3$  as the typical fraction of the sky covered by the future satellites MAP [38] and PLANCK [39]. In order to prevent the tensor contribution from exceeding the cosmic variance we had to reduce the scalar spectral tilt to  $n = 1.006$ , which is essentially scale invariant and may not be enough to allow consistency with large scale structure [47]. Furthermore, as we decrease in  $\Omega_0$  it will be necessary to have scalar spectra which are closer and closer to scale invariance, in order to reduce the tensor contribution. In any case, there exists for this model a range of parameters for which all contributions to the CMB anisotropies, see e.g. Fig. 6b, are compatible with observations.

## 6. Conclusions

Single-bubble open inflation is an ingenious way of reconciling an infinite open universe with the inflationary paradigm. In this scenario, a symmetric bubble nucleates in de Sitter space and its interior undergoes a second stage of slow-roll inflation to almost flatness. In the near future, observations of the microwave background with the new generation of satellites, MAP and Planck, will determine with better than 1% accuracy whether we live in an open universe or not. It is therefore crucial to know whether inflation can be made compatible with such a universe. Single-bubble open inflation models provide a natural scenario for understanding the large scale homogeneity and isotropy. Furthermore, these inflationary models generically predict a nearly scale invariant spectrum of density and gravitational wave perturbations, which could be responsible for the observed CMB temperature anisotropies. Future observations could then determine whether these models are compatible with the observed features of the CMB power spectrum. For that purpose it is necessary to know the predicted power spectrum with great accuracy. Open models have a more complicated primordial spectrum of perturbations, with extra discrete modes and possibly large tensor anisotropies. In order to constrain those models we have to compute the full spectrum for a large range of parameters.

In this review we have shown that the simplest single-field models of open inflation are not only fine tuned, but actually ruled out because they induce too large tensor anisotropies in the CMB, which is incompatible with present observations. On the other hand, two-field models generically do not lead to infinite open universes, as previously thought, but to an ensemble of very large but finite inflating ‘islands’. Each one of these islands will be a quasi-open universe. We may happen to live in one of those patches, where the universe *appears* to be open. This new effect, semiclassical in origin, was recently discussed in Ref. [19] where it was found that many of the present models are in fact quasi-open. This does not mean that they

are not good cosmological models. If the co-moving size of the inflating islands is sufficiently large, then the resulting semiclassical anisotropy may be unobservable. We have shown however that such a component imposes very stringent constraints on the models. Most of them have a narrow range of parameters for which they are compatible with observations.

It is perhaps worth mentioning here some alternative proposals (not single-bubble) for the generation of an open universe in the context of inflation. First of all, the group of Roma [55] proposed a model based on higher order gravity that induces bubble nucleation and later percolation, resulting in a distribution peaked at  $\Omega_0 \simeq 0.2$ . Perhaps the most striking recent results are those of Hawking-Turok [56] and Linde [57], who claim that an open inflationary universe could have been created directly from the vacuum, without the intermediate de Sitter phase.

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