On N=8 Supergravity on AdS_5 and N=4 Superconformal Yang-Mills theory

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We discuss the spectrum of states of IIB supergravity on $AdS_5 \times S^5$ in a manifest $SU(2, 2/4)$ invariant setting. The boundary fields are described in terms of $N = 4$ superconformal Yang-Mills theory and the proposed correspondence between supergravity in AdS_5 and superconformal invariant singleton theory at the boundary is formulated in an $N = 4$ superfield covariant language.

February 98

1. Introduction.

In recent times, it has been conjectured that there is a close connection between certain supergravity theories in AdS_{p+2} and p-brane dynamics on the $p+1$ world-volume[1,2,3,4,5,6,7].

In certain limits, this relation has a plausible validity of application and it implies, in a more algebraic setting, that certain field and string theories, based on the same superalgebra, are *dual* to each other. This was first conjectured in [2], and further studied in [8,9,10]. In recent papers [11,12], a more precise definition for this duality was given, using a *functional approach* to Green function correlators, in analogy to the relation between string theory and target space dynamics.

In a recent set of papers, many properties of the possible interconnections started to be explored in three main directions: comparing theories of different p-branes in different dimensions with the nearly horizon supergravity theory in AdS_{p+2} [8], computing correlators in the *boundary* theories to get informations on the spectrum of the *bulk* theory [11,12] or *viceversa*, using the group-theoretical relation of gauge fields in anti-de-Sitter space with *singletons* in the boundary theories [13,14].

In particular, it was shown in [13,14] that *massless gauge* fields in AdS_n correspond to *composite boundary excitations* which, in a field theory framework, correspond to the existence of some *conformal operators* in the boundary theory whose scaling dimension do not get renormalized.

More recently, these expectations have been confirmed by dynamical calculations in $N=4$ superconformal $U(n)$ Yang-Mills theory for the case of D3 branes [11,12] where horizon geometry is related to $N=8$ supergravity in AdS_5. Yang-Mills theory is the gauged $SU(4)-R$ symmetry of the four-dimensional boundary theory [13,14,15].

It is the aim of the present paper to expand further this setting by providing, as promised in previous papers, a manifest $N=4$ superconformal invariant formulation of the *composite boundary excitations* as well as of the *field theory formulation* of the proposed duality, as recently suggested in various papers.

The paper is organized as follows. In section 2, we give some basic facts on certain gauge fields on AdS_5, by showing that for these fields, quite independently of supersymmetry, there is a reasonable definition of *mass*, which coincides with what comes out from the supergravity literature. In section 3, we show the relevant *particle* multiplet of $SU(2,2/4)$ superalgebra and the *field representations*. This in particular allows to check the spectrum

of K-K excitations of type IIB on $AdS_5 \times S^5$ in a manifest $N = 4$ formalism. As an output, we show that fields with different masses but the same spin (including the elementary excitations), apparently coming from different towers of operators, are actually coming from the same $N = 4$ supertower. In section 4, we give an $N = 4$ manifest formulation of the *functional approach* proposed in [11,12] to relate the bulk supergravity theory to the boundary conformal field theory. The appendix summarizes some properties of conformal superfields.

2. Unitary representations of particle states in AdS_5.

Elementary particles in AdS_{d+1} are classified by unitary, irreducible representations of $SO(d, 2)$. To classify the states [16,17,18], $SO(d, 2)$ is decomposed with respect to its maximal compact subgroup $SO(d) \times SO(2)$. Representations with positive energy are highest weight representations classified by the highest weights; that is, once again by $SO(2) \times SO(d)$ quantum numbers, the minimal energy E_0 and the spin. Thus, for d = 4, $SO(4) = SU(2) \times SU(2)$, and irreducible representations are classified by three labels, two spins and the energy. We denote such representations by $D(E_0, J_1, J_2)$.

An important object, which will enter in the definition of a particle mass in AdS_5 is the quadratic Casimir of $SO(4, 2)$ which takes the value,

$$E_0(E_0 - 4) + 2J_1(J_1 + 1) + 2J_2(J_2 + 1). \tag{2.1}$$

Unitary representations lie within two unitary conformal bounds [19,20]: 1) $E_0 \geq 2 + J_1 + J_2$ (if $J_1 J_2 \neq 0$) and 2) $E_0 \geq 1 + J$ (if $J_1 J_2 = 0$). It was shown in [13,14] that these bounds are precisely saturated by massless particles in anti-De Sitter (1) and massless particles at the boundary (2).

Let us now consider gauge fields of interest in AdS_5; they are vectors, symmetric tensors and antisymmetric tensors. Imposing that the laplacian operator substains a gauge symmetry for massless tensor fields, we define *mass* in terms of the value of the Casimir operator, shifted so as to be zero in the massless case. The result is [14],

. Vector field: A_μ

$$D(E_0, \frac{1}{2}, \frac{1}{2}): \qquad m^2 = C_I = (E_0 - 1)(E_0 - 3) \tag{2.2}$$

3

. Symmetric tensor: $g_{\mu\nu} = g_{\nu\mu}$

$$D(E_0, 1, 1): \qquad m^2 = C_I - 8 = E_0(E_0 - 4) \tag{2.3}$$

. Antisymmetric tensor: $A_{\mu\nu} = -A_{\nu\mu}$

$$D(E_0, 1, 0) \oplus D(E_0, 0, 1): \qquad m^2 = C_I = (E_0 - 2)^2 \tag{2.4}$$

while for scalar fields, as usual,

. Scalars: ϕ

$$D(E_0, 0, 0): \qquad m^2 = C_I = E_0(E_0 - 4) \tag{2.5}$$

and for fermions,

. Fermions of spin 3/2: ψ_μ

$$D(E_0, 1, \frac{1}{2}) + D(E_0, \frac{1}{2}, 1); \qquad m = E_0 - 2 \tag{2.6}$$

. Fermions of spin 1/2: λ

$$D(E_0, 0, \frac{1}{2}) + D(E_0, \frac{1}{2}, 0); \qquad m = E_0 - 2 \tag{2.7}$$

It is not surprising that all the K-K excitations, both massless and massive, for type IIB supergravity on $AdS_5 \times S^5$ [15,21,22] verify these formulae, for different values of E_0. Apparent different towers are due to the fact that supersymmetry relates particles with the same value of (J_1, J_2) but different E_0 and $SU(4)$ assignment.

The unitarity bound $E_0 = 2 + J_1 + J_2$ corresponds to *conserved currents* in the boundary conformal field theory [14]. It corresponds to *massless* gauge fields in AdS_5; therefore we get that

$$A_\mu: \qquad D(3, \frac{1}{2}, \frac{1}{2}); \qquad g_{\mu\nu}: \qquad D(4, 1, 1); \qquad A_{\mu\nu}: D(3, 1, 0) \oplus D(3, 0, 1). \tag{2.8}$$

For scalars, since there is no gauge symmetry, there is no unique definition of masslessness, but for the special value $m^2 = 0$ we have $E_0 = 4$; this state will be associated to the (complex) dilaton, which, as expected, has the same E_0 of the graviton.

The other exceptional representation is the singleton representation which saturates the other bound $E_0 = 1 + J$. It essentially corresponds to a topological theory which lives at the boundary of AdS_5. For $J = 1$, this gives the singleton Maxwell theory discussed in [13]. $J = 0, 1/2$ are the singletons discussed in the Anti-de Sitter literature [16,17,18,10,23]; they corresponds to the irreducible representations $D(1, 0, 0), D(3/2, 1/2, 0)$ and $D(2, 1, 0)$ of $O(4, 2)$. These fields are the building blocks of $N = 4$ super Yang-Mills theory on M_4. It is the D3 brane world-volume theory.

3. Supermultiplets and superfields of $SU(2,2/4)$.

In $N = 4$ conformal supersymmetry, with fields defined at the boundary of AdS_5, we have two basic multiplets, which are related to massless particles at the boundary (supersingleton) and massless particles in the bulk (supergraviton).

As proposed in [13], following the conjecture of [2], the supergraviton multiplet is a *composite boundary operator* of singleton fields at the boundary. It corresponds to gauge covariant bilinear composites, described by a supermultiplet of currents at the boundary, as explained in [13].

Let us give the $D(E_0, J_1, J_2)$ components of the multiplets:

. Supersingleton multiplet,

$$D(1,0,0|6) + D(\frac{3}{2}, \frac{1}{2}, 0|4) + D(\frac{3}{2}, 0, \frac{1}{2}|\bar{4}) + D(2,1,0|1) + D(2,0,1|1), \qquad (3.1)$$

where the fourth index in the D symbol is the $SU(4)$ representation. The supermultiplet contains the field content of an $N = 4$ vector multiplet, a gauge field, four complex fermions in the **4** of $SU(4)$ and six scalars in the **6** of $SU(4)$.

. Supergraviton multiplet,

$$D(4,1,1|1) + D(\frac{7}{2}, 1, \frac{1}{2}|\bar{4}) + D(\frac{7}{2}, \frac{1}{2}, 1|4) + D(3, \frac{1}{2}, \frac{1}{2}|15) + D(3,1,0|6_c)$$
$$+ D(3,0,1|\bar{6}_c) + D(\frac{7}{2}, \frac{1}{2}, 0|4) + D(\frac{7}{2}, 0, \frac{1}{2}|\bar{4}) + D(\frac{5}{2}, \frac{1}{2}, 0|20) + D(\frac{5}{2}, 0, \frac{1}{2}|\overline{20}) \qquad (3.2)$$
$$+ D(2,0,0|20_R) + D(3,0,0|10) + D(3,0,0|\overline{10}) + D(4,0,0|1) + D(4,0,0|\bar{1}),$$

which contains the graviton, eight gravitinos in the $\mathbf{4} + \bar{\mathbf{4}}$ of $SU(4)$, fifteen vectors which gauge $SU(4)$, twelve antisymmetric tensors in the $\mathbf{6}_c$, forty-eight spin $1/2$ fermions in the $\mathbf{4} + \bar{\mathbf{4}} + \mathbf{20} + \overline{\mathbf{20}}$ and forty-two scalars in the $\mathbf{20}_R + \mathbf{10} + \overline{\mathbf{10}} + \mathbf{1} + \bar{\mathbf{1}}$.

As anticipated, we see that there are spin $1/2$ and spin 0 particles having different *masses*, since they have different E_0. However, the masses are completely fixed by $N = 4$ supersymmetry. To read the quantum number it suffices to associate these fields to the $N = 4$ conformal supercurrent multiplet $J_{[AB][CD]}$ (with $A = 1, ..., 4$)[1], whose complete component expansion was given in [24], and whose $N = 4$ superfield was given in [25].

The multiplet starts with the traceless symmetric combination of $Tr\phi_{\{l}\phi_{m\}}$ (with $i = 1, ..., 6$). Since ϕ_l is a field with $E_0 = 1$, this operator has $E_0 = 2$. As for the other

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[1] $J_{[AB][CD]}$ stands for the expression $J_{\{[AB],[CD]\}} - \frac{1}{24}\epsilon_{ABCD}J_{[EF][GH]}\epsilon^{EFGH}$.

scalars, those with $E_0 = 3$ in the **10** of $SU(4)$ are θ^2 components of this superfield, while the singlet dilaton with $E_0 = 4$ is a θ^4 component, as shown in [24,25]. In an analogous way, the spin $1/2$ with $E_0 = 5/2$ in the $\mathbf{20}_c$ is a θ component while the spin $1/2$ in the **4** with $E_0 = 7/2$ is a θ^3 component. Finally, the vector and antisymmetric tensors are in the θ^2 component and the graviton in the θ^4. The components with from five to eight θ are actually derivatives of lower components.

In [12], some K-K tower of scalars in the spectrum of type IIB on $AdS_5 \times S^5$ have been identified with BPS composite fields in the $N = 4$ Yang-Mills theory, as a check of the proposed duality of this theory with the world-volume theory of the D3 branes. It is not difficult at this point to perform the analysis of the spectrum in an $N = 4$ manifestly covariant way and to identify also the composite fields associated with gauge fields, symmetric and antisymmetric tensors and fermions. The K-K excitations of type IIB on $AdS_5 \times S^5$ are precisely given by taking $SU(n)$ gauge invariant polynomials of the singleton supermultiplets, whose superfield expression is a chiral superfield strenght $W_l = \Gamma_l^{[AB]} W_{[AB]}$. The massless multiplet in AdS_5 corresponds to

$$J_{lm} = Tr W_{\{l} W_{m\}}, \qquad (3.3)$$

which is the *supercurrent* multiplet of [25], and it corresponds to the supergraviton multiplet we just discussed in detail. The singleton superfield W_l, which is Lie algebra valued in $SU(n)$, as well as the supercurrent superfield J_{lm}, are both chiral and satisfy some additional constraints, that can be found in [25].

If we read the tower of K-K excitations in [22], we see that, for a given (J_1, J_2) they satisfy the main formulae given in eq. (2.2), (2.3), (2.4), (2.5), (2.6) and (2.7), but with different values of E_0 as given by the $N = 4$ superfield multiplication.

Let us briefly check the massless multiplet and the first excitations. The conformal weight and the $SU(4)$ representation for the components of the massless multiplet was given above. Using formula (2.5), for the three type of scalars with $E_0 = 2, 3, 4$ we expect to find in the supergraviton multiplet scalars in the $\mathbf{20}_R$ of $SU(4)$ with $m^2 = -4$, scalars in the **10** with $m^2 = -3$ and singlet scalars with $m^2 = 0$. This is indeed the result obtained in [22], as can be easily checked by looking at table III in that reference and at the various figures where states in the supergraviton multiplet are encircled. Using formulae (2.2), (2.3), since we have vectors and tensors with $E_0 = 3$, we reproduce the massless gauge fields which fill the adjoint of $SU(4)$ and an antisymmetric tensor in the $\mathbf{6}_c$ of $SU(4)$ with

$m^2 = 1$ found in [22] in the supergraviton multiplet. Using eq. (2.6), (2.7), also the masses for the fermions are easily shown to agree with the ones found in [22]; in particular it is clear that the two spin $1/2$ fermions in the $\mathbf{20}_c$ and $\mathbf{4}$ have different masses since they have different E_0.

The massive K-K excitations can be obtained by considering higher degree polynomials of W_l. The $N = 4$ superfield multiplication gives immediatly the dimension E_0 and the $SU(4)$ representation. It can be easily checked that the full tower of K-K states can be obtained in this way. We simply notice, as a check, that, in complete agreement with the formulae which gives the masses in terms of E_0, all the masses for the scalar K-K modes in [22] have the form $E_0(E_0 - 4)$ for some E_0, as in formula (2.5), the masses for the tensors have the form $(E_0 - 2)^2$ for some E_0, as in formula (2.3), the masses for vectors have the form $(E_0 - 1)(E_0 - 3)$ for some E_0, as in formula (2.2), and so on.

4. On the CFT/AdS connection.

In order to apply the $N = 4$ formalism to the proposal of the CFT/AdS equivalence, we recall the field representations of the $SU(2, 2/4)$ algebra. They are the Weyl superfield and the gravity gauge potential superfield.

The Weyl superfield is a chiral conformal multiplet W which satisfies a set of constraints given in [25]. Let us just remember that the analogous multiplet in $N = 1$ is $W_{\alpha\beta\gamma}$ and in $N = 2$ is $W_{\alpha\beta}$ whose first components are the gravitino and graviphoton field strenght, respectively. In the $N = 4$ case, the W multiplet is a dimension 0 chiral superfield, whose first component is a complex scalar, the conformal dilaton ϕ_0.

This superfield has a *prepotential* $V_{[AB][CD]}$ of conformal dimension -6 whose relation to the Weyl multiplet is [25],

$$W = \bar{D}^8 D^{4[AB][CD]} V_{[AB][CD]}. \tag{4.1}$$

This relation is in fact telling us the usual story that the conformal supergravity gauge potential $V_{[AB][CD]}$ is *conjugate* to the *supercurrent* multiplet $J_{[AB][CD]}$. It has the same value for the Casimir since $E_0 \to E_0 - 4$.

We then see that the proposed coupling $\int d^4x \phi_0 F_{\mu\nu}^2$ [11,12] is now replaced by the $N = 4$ manifest superconformal coupling [25],

$$\int d^4x d^{16}\theta V^{[AB][CD]} J_{[AB][CD]}. \tag{4.2}$$

This is the manifest $N = 4$ supergravity coupling of the $N = 4$ conformal supergravity field to the *conformal operator* on the boundary. The dilaton coupling

$$\int d^4x \phi_0 F_{\mu\nu}^2, \tag{4.3}$$

considered in [11,12] comes, for example, from the θ^{12} component of V multiplied by the θ^4 component of W to match the θ^{16} integrand.

Note that $V^{[AB][CD]}$ has a very large negative conformal dimension (-6) but, due to its large gauge symmetry, in the Wess-zumino gauge, it starts with the θ^{12} component which is precisely ϕ_0.

There is, therefore, a manifest $N = 4$ setting of the CFT/AdS proposal made in [11]: if the value at the boundary of the conformal supergravity field V, that couples to the conformal supercurrent J, is extended to a field $\hat{V}$ in the bulk of AdS_5, the CFT generating functional with sources V is identified with the supergravity action evaluated on $\hat{V}$ in the bulk:

$$Z(V_{[AB][CD]})_{CFT} = \langle e^{i \int d^4x d^{16}\theta V^{[AB][CD]} J_{[AB][CD]}} \rangle_{CFT} = S_{AdS}(\hat{V}^{[AB][CD]}). \tag{4.4}$$

$Z(V_{[AB][CD]})$ is the generating functional for the CFT correlators of superconformal currents. The previous formula is therefore a non-trivial statement which allows in principle to compute CFT Green functions and OPE using tree-level supergravity theory. Conformal field theory correlators in $N = 4$ superconformal Yang-Mills theory are highly constrained by OPE techiques. OPE in four-dimensional conformal invariant quantum field theories were widely studied in the seventies [26,27,28,29,30] and, more recently, further generalized to extended superconformal field theories including the $N = 4$ case [31,32].

The $E_0 = 2$ scalar is the last component of $V^{[AB][CD]}$ which couples to the $E_0 = 2$ composite in $J_{[AB][CD]}$. This is obvious from superfield multiplication. In an analogous way, there is a $E_0 = 1$ scalar coupled to the $E_0 = 3$ scalar composite.

Appendix A. Useful formulae for supercurrents and conformal supergravity fields.

In this appendix we summarize some results of refs. [24,25] concerning supercurrents and their coupling to the conformal supergravity fields.

For the sake of simplicity, we will consider the two cases of $N = 1$ and $N = 4$ conformal supergravity, the former because of its simplicity, being related to $N = 1$ supergravity, and the latter because of its relevance in the present context.

Let us start with the $N = 1$ case [33]. The superconformal algebra in this case $U(2, 2/1)$ [34,35] and the supercurrent is a vector real superfield $J_{\alpha\dot{\alpha}}$ satisfying the constraint $D^{\alpha} J_{\alpha\dot{\alpha}} = 0$. It can be shown that this implies that the only field components of $J_{\alpha\dot{\alpha}}$ are $(A_{\mu}, J_{\mu\alpha}, T_{\mu\nu})$, i.e. an axial current, a vector spin current and a symmetric tensor which are conserved conformal fields,

$$
\begin{aligned}
\partial^{\mu} A_{\mu} = 0 \qquad & A_{\mu} : D(3, \tfrac{1}{2}, \tfrac{1}{2}) \\
\partial^{\mu} J_{\mu\alpha} = \gamma^{\mu} J_{\mu\alpha} = 0 \qquad & J_{\mu\alpha} : D(\tfrac{7}{2}, 1, \tfrac{1}{2}) + D(\tfrac{7}{2}, \tfrac{1}{2}, 1) \\
T_{\mu\mu} = \partial^{\mu} T_{\mu\nu} = 0 \qquad & T_{\mu\nu} : D(4, 1, 1)
\end{aligned}
\tag{A.1}
$$

The above is precisely the massless representation of supergravity on AdS_5 with an $O(2)$ gauge group, related to the $U(1)$ R-symmetry of the boundary theory [36].

The Weyl supermultiplet is a chiral $(\tfrac{3}{2}, 0)$ representation of $SL(2, C)$ $W_{\alpha\beta\gamma}$ with conformal weight 3/2. It can be expressed as $W_{\alpha\beta\gamma} = \bar{D}^2 D_{\{\beta} \partial_{\alpha\dot{\alpha}} V_{\gamma\}}^{\dot{\alpha}}$, with a *prepotential* $V_{\alpha\dot{\alpha}}$ (of weight -1) which corresponds to the gauge potential of $N = 1$ conformal supergravity [37]. The *current* coupling is therefore in this case,

$$
\int d^4 x d^4 \theta V^{\alpha\dot{\alpha}} J_{\alpha\dot{\alpha}}
\tag{A.2}
$$

In this case we take as *singleton multiplet* the $N = 1$ Yang-Mills multiplet $D(2, 1, 0|0) + D(3/2, 1/2, 0|3/2) + D(3/2, 0, 1/2| - 3/2)$ (where the fourth index is the $U(1)$ charge) and $J_{\alpha\dot{\alpha}}$ can be written as a composite field as

$$
J_{\alpha\dot{\alpha}} = Tr W_{\alpha} W_{\dot{\alpha}},
\tag{A.3}
$$

where W_{α} is the chiral field strength of the singleton (Yang-Mills) multiplet[2].

In the $N = 4$ case, the singleton superfield is a chiral $N = 4$ superfield $W_{[AB]}$ which satisfies an additional constraint [25]. In terms of $W_{[AB]}$ ($A = 1, ..., 4$) which can also

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[2] In this case, one can have additional singletons, the chiral multiplets, with $D(E_0, 0, 0|E_0) + D(E_0, 1/2, 0|E_0 - 3/2)$.

be written (as we did in section 4) as $W_l, (l = 1, ..., 6)$ (vector of $SU(4) = O(6)$) the supercurrent multiplet J is a dual superfield given by,

$$J_{mn} = Tr(W_l W_n - \frac{1}{6}\delta_{mn}W^p W_p), \qquad (A.4)$$

i.e. in the $\mathbf{20}_R$ of $SU(4)$.

The Weyl multiplet is a chiral superfield which admits a *prepotential* V_{lm} such that

$$W = \bar{D}^8 D^{4lm} V_{lm}, \qquad (A.5)$$

where V_{lm} has conformal weight -6. The current coupling in the $N = 4$ case is then [25]

$$\int d^4x d^8\theta d^8\bar{\theta} V^{lm} J_{lm}. \qquad (A.6)$$

For completeness we give here the explicit expression of $J_{lm} = J_{ABCD}$ as a composite of singleton superfields. This was obtained in [24]. The components of the multiplet are $(g_{\mu\nu}, \psi_\mu^A, A_{\mu B}^A, A_{\mu\nu}^{AB}, \lambda_A, \chi_C^{AB}, \phi, e_{AB}, d_{CD}^{AB})$ i.e. a symmetric tensor current, a vector spin current in the $\mathbf{4} + \bar{\mathbf{4}}$, a vector current in the adjoint of $SU(4)$, an antisymmetric in the $\mathbf{6}_c$, fermions λ and χ in the $\mathbf{4}$ and $\mathbf{20}$, scalars ϕ, e and d in the $\mathbf{1}$, $\mathbf{10}$ and $\mathbf{20}_R$. In terms of the $N = 4$ supermultiplet components, $(F_{\mu\nu}, \psi_A, \phi_{AB})$ (where ψ_A are the four spinors rotated by $SU(4)$ and ϕ_{AB}, subject to the constraint $\phi^{AB} = (\phi_{AB})^* = \frac{1}{2}\epsilon^{ABCD}\phi_{CD}$, are the scalars in the $\mathbf{6}$ of $SU(4)$), the explicit expression is (formulae (A.4) and (A.6) in [24])

$$g_{\mu\nu} = \frac{1}{2}\{\delta_{\mu\nu}(F_{\rho\sigma}^-)^2 - 4F_{\mu\rho}^- F_{\nu\rho}^- + h.c.\} - \frac{1}{2}\bar{\psi}^A \gamma_{\{\mu}\partial_{\nu\}}^{\leftrightarrow}\psi_A + \delta_{\mu\nu}(\partial_\nu\phi^{AB})(\partial_\rho\phi_{AB})$$
$$\qquad\quad - 2(\partial_\mu\phi^{AB})(\partial_\nu\phi_{AB}) - \frac{1}{3}(\delta_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu)(\phi^{AB}\phi_{AB}),$$

$$\psi_\mu^A = -(\sigma F^-)\gamma_\mu\bar{\psi}^A + 2i\phi^{AB}\partial_\mu^{\leftrightarrow}\psi_B + \frac{4}{3}i\sigma_{\mu\lambda}\partial_\lambda(\phi^{AB}\psi_B),$$

$$A_{\mu B}^A = \phi^{AC}\partial_\mu^{\leftrightarrow}\phi_{CB} + \bar{\psi}^A\gamma_\mu\psi_B - \frac{1}{4}\delta_B^A\bar{\psi}^C\gamma_\mu\psi_C,$$

$$A_{\mu\nu}^{AB} = \bar{\psi}^A\sigma_{\mu\nu}\bar{\psi}^B + 2i\phi^{AB}F_{\mu\nu}^+, \qquad\qquad\qquad\qquad (A.7)$$

$$\lambda_A = \sigma F^-\psi_A,$$

$$\chi_C^{AB} = \frac{1}{2}\epsilon^{ABDE}(\phi_{DE}\psi_C + \phi_{CE}\psi_D),$$

$$\phi = (F_{\mu\nu}^-)^2,$$

$$e_{AB} = \psi_A\psi_B,$$

$$d_{CD}^{AB} = \phi_{CD}^{AB} - \frac{1}{12}\delta_C^{[A}\delta_D^{B]}\phi^{EF}\phi_{EF}.$$

10

The currents are conserved and satisfies,

$$\partial_\mu g_{\mu\nu} = 0 \qquad \partial_\mu \psi_\mu^A = 0,$$
$$g_{\mu\nu} = g_{\nu\mu} \qquad \gamma_\mu \psi_\mu^A = 0, \qquad\qquad (A.8)$$
$$g_{\mu\mu} = 0 \qquad \partial_\mu A_{\mu B}^A = 0.$$

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