

CERN-TH/98-49
UCLA/98/TEP/1
February 1998

GAUGE FIELDS AS COMPOSITE BOUNDARY EXCITATIONS

Sergio Ferrara* and Christian Frønsdal**

**CERN, Geneva, Switzerland.*

***Physics Department, University of California, Los Angeles, CA 90090-1547.*

ABSTRACT. We investigate representations of the conformal group that describe “massless” particles in the interior and at the boundary of anti-de Sitter space.

It turns out that massless gauge excitations in anti-de Sitter are gauge “current” operators at the boundary. Conversely, massless excitations at the boundary are topological singletons in the interior. These representations lie at the threshold of two “unitary bounds” that apply to any conformally invariant field theory. Gravity and Yang-Mills gauge symmetry in anti-De Sitter is translated to global translational symmetry and continuous R -symmetry of the boundary superconformal field theory.

Introduction.

In recent times, new evidence [1,2,3,4] has emerged for a possible connection between brane dynamics in M and string theories and certain supergravity theories in anti-de Sitter geometries [5]. More specifically, it has been conjectured that a new “duality” [1] may occur between the p -brane world volume theory and anti-de Sitter supergravity in Ad_{p+2} , with the same underlying superalgebra [6,7,8].

If this duality is to work, not only the superalgebra must be the same but also “representations” on states and on fields should be dual to each other.

In this paper we argue that, due to the particular nature of the $SO(p+1, 2)$ conformal group, it is possible to make such correspondence in a meaningful group-theoretical way.

This fact was anticipated [4] when saying that the “massless” graviton, the “massless” gravitino and the “massless” gauge fields of anti-De Sitter space are composite operators, belonging to the “supercurrent multiplet” [28] in the boundary theory.

It is the aim of this paper to further explore this conjecture by identifying the representations of $SO(p+1, 2)$ that characterize both theories and making the correspondence more precise.

It turns out, to no surprise, that representation theory has the power to chart the physical degrees of freedom, as well as gauge modes, in both theories. A duality dictionary emerges between “current operators” on the boundary (brane) and elementary massless excitations in the anti-De Sitter bulk. More importantly, not only the excitations, but also their interactions are predicted by this correspondence, because of the symmetries inherent to the dual theories.

The paper is organized as follows: In Section 1 we review some basic aspects of the particle and field representations of $SO(4, 2)$ acting as the conformal group of space time. The same representations will appear when $SO(4, 2)$ acts on anti-De Sitter space.

Our analysis will be largely confined to the $p = 3$ case [9] although many properties are valid for generic p .

In Sections 2 a catalogue of gauge fields are given together with their gauge properties. In Sections 3 and 4 we identify the representations of $SO(n-1, 2)$ that appear, and in Section 5 we summarize the models of compositeness of massless particles that arise naturally in the context. Finally, the last sections make some suggestions for future investigations.

1. Some Properties of Conformal Fields on Minkowski Space.

In this section we recall some properties of conformal fields in Minkowski space $M_4 = SO(4, 2)/IO(3, 1) \otimes O(1, 1)$.

For what follows we will be interested in “current fields” namely, representations labelled (E_0, j_1, j_2) with $j_1 = j_2 = \frac{s}{2}$. We limit our discussion to bosonic currents.

These representations are associated with “conserved currents” of any spin for $E_0 = 2 + s$. This is the first unitarity bound; it can be derived from the requirement of unitarity

of the theory, by looking at the 2-point function of irreducible $SO(4, 2)$ symmetric traceless tensors $T_{\alpha_1 \dots \alpha_N}$ satisfying supplementary conditions [10] on the six-cone [11]

$$y^\mu T_{\mu\mu_1 \dots \mu_{N-1}} = \partial^\mu T_{\mu\mu_1 \dots \mu_{N-1}} = 0.$$

An explicit realization of these tensors can be given in any conformal invariant free field theory, in terms of free massless fields on M_4 , carrying representations with $j_1 j_2 = 0$ and $E_0 = 1 + j$. Massless representations are at the unitarity bound for representations with $j_1 j_2 = 0$.

In the specific case of $N = 4$ Yang-Mills theory, the conformal multiplet contains massless fields in the representation [12]

$$D(1, 0, 0) \oplus D(\frac{3}{2}, \frac{1}{2}, 0) \oplus D(\frac{3}{2}, 0, \frac{1}{2}) \oplus D(2, 1, 0) \oplus D(2, 0, 1).$$

The tensor currents are bilinears in these fields.

In 5-dimensional anti-De Sitter space the same group $SO(4, 2)$ plays the role of space time isometry group. The representations with $E_0 = 2 + j_1 + j_2$ correspond to “massless particles”. In particular, the graviton, the gravitino and the vector fields correspond to

$$D(4, 1, 1), D(\frac{7}{2}, 1, \frac{1}{2}) \oplus D(\frac{7}{2}, \frac{1}{2}, 1), D(3, \frac{1}{2}, \frac{1}{2})$$

respectively. The appearance of these representations in the (4-dimensional) boundary theory, at the interacting level, is ensured by the global symmetries which give the conserved currents with spin 2, 3/2, and 1. These symmetries are global, superconformal invariance, which in particular implies the existence of 15 vector currents [13] in the adjoint representation of $SU(4)$ [4].

We see that in five-dimensional anti-De Sitter space time these representations give particles and symmetries such that consistent interactions are possible only as anti-De Sitter $N = 8$ supergravity with $SU(4)$ Yang-Mills.

Interestingly enough, this gives a “duality” between “currents” on the p -brane world volume and “massless fields” in AdS_5 space-time together with their geometric interactions.

Note that this would also explain why we only get massless particles up to spin 2 in the anti-De Sitter bulk. It has to do with the fact that it is not possible, in an interacting theory on the world-volume, to have conserved tensors of spin higher than two, so that $E_0 = 2 + s$, with $s > 2$, is possible in free field theory only.

It is interesting to notice that the interpretation of “massless particles” ($E_0 = 1 + j$) in the boundary conformal theory is that of “singletons” in the corresponding anti-De Sitter description. At the level of excitations we may think of massless particles in the bulk as bound states of two singletons, an idea that was put forward long ago [14]. Apparently the proposed “duality” between brane dynamics and anti-De Sitter supergravity gives a dynamical framework in which to study this idea of compositeness.

It is amazing to observe that among these massless representations, a special role is played by spin-one constituents (photons on the bulk) and spin-one singletons (photons on the boundary). They correspond to $D(3, \frac{1}{2}, \frac{1}{2})$ and $D(2, 1, 0)$ respectively. For both of

them the Casimirs of $SO(4, 2)$ vanish,* giving to them a particular role with respect to gauge symmetry, something that will be discussed later.

2. Catalogue of Gauge Fields in AdS_n .

Gauge fields are of two basic types: (1) Massless fields that carry excitations observable in the bulk. (2) Topological, singleton type gauge theories that describe oscillations on the boundary. Some issues will be discussed in the setting of anti-De Sitter spaces of arbitrary dimensions, but when this becomes too general for the convenience of the exposition we shall specialize to the case of greatest interest: space time dimensions $n = 4$ and $n = 5$.

Instead of coordinates in the strict sense we shall use the parameters y_0, \dots, y_n of the hyperboloid

$$y^2 := y_0^2 - y_1^2 - \dots - y_{n-1}^2 + y_n^2 = R^2 > 0.$$

in a $n + 1$ -dimensional pseudo-euclidean space. All the fields will be taken to be defined in $y^2 > 1$, by fixing the degree of homogeneity to the most convenient value. This allows the boundary at infinity to be identified with the cone $y^2 = 0$, and the degree of homogeneity then coincides with the degree of the field in the radial variable

$$r = \sqrt{y_1^2 + \dots + y_{n-1}^2}$$

as $r \rightarrow \infty$; a field that is homogeneous of degree N will behave as r^N near the boundary at infinity. The space time symmetry group is $SO(n - 1, 2)$ in the bulk. On the boundary, that can be identified with either AdS_{n-1} or $n - 1$ -dimensional Minkowski space, it acts as the conformal group of that space.

The angle in the plane of (y_0, y_n) is identified with the time, and the associated generator of rotations, with the energy. The function

$$y_+ := y_0 + iy_n = Y e^{it}, \quad Y := \sqrt{y_0^2 + y_n^2} \quad (2.1)$$

carries one unit of energy.

2.0. Scalar and spinor gauge fields in AdS_n .

The set of solutions of the scalar wave equation on AdS_n normally includes the modes of an irreducible, highest weight representation of $SO(n - 1, 2)$. But there is a very special

* The values of the three $SO(4, 2)$ Casimir operators, for the tensor representations $D(E_0, s/2, s/2)$, are given in the following expressions [10],

$$\begin{aligned} C_1 &= J_{ij} J^{ij} = 2s(s + 2) + 2E_0(E_0 - 4), \\ C_2 &= \epsilon_{ijklmn} J^{ij} J^{kl} J^{mn} = 0, \\ C_3 &= J_{ij} J^{jk} J_{kl} J^{li} = s(s + 2)[E_0(E_0 - 4) + 3]. \end{aligned}$$

case, when the mass parameter is fixed to the value that makes some of the solutions logarithmic, in which the situation is quite different. In this case there appears a subspace of gauge modes and the representation induced on it is non-decomposable. The gauge modes are characterized only by a slower fall-off at spatial infinity, and not by any local property. For this reason there is no local, gauge invariant interaction. Interactions consistent with unitarity are possible only at the boundary at infinity, where the gauge modes disappear. In other words, one has a topological gauge theory. This phenomenon was first detected in the case of 4 space time dimensions [15]. The general case of n -dimensional space time was discussed recently by us, though we were primarily concerned with the 5-dimensional case [4].

There is a spinorial analogue [16]. In four dimensions the scalar and spinor topological gauge fields were dubbed singletons, after the peculiarly degenerate representations of $SO(3,2)$ that are carried by the space of one-particle states. But it is the topological nature of the gauge field that gives these theories their appeal, and we shall use the term singleton exclusively for this type of topological field theories.

The singletons associated with scalar and spinor fields have a very important role to play. The scalar case has been explained too recently for another review at this place. The spinor case has been analyzed in detail only in 3 [17,18,19] and 4 [16] dimensions, but this will not deter us from making use of them, since as we shall see that the information that is required is obtained easily from representation theory.

2.1 Vector gauge fields on AdS_n .

To construct a vector field that transforms irreducibly under AdS_n one imposes all possible invariant conditions on it,

$$(y^2 \partial^2 - \kappa)A = 0, \quad y \cdot A = 0, \quad \partial \cdot A = 0, \quad (2.2)$$

and

$$y \cdot \partial A = NA. \quad (2.3)$$

The parameters κ and N will be fixed presently.

The essential feature that characterizes all gauge theories is the existence of invariant, uncomplemented (precisely: not invariantly complemented) subspaces. In the case at hand such a subspace exists whenever the parameters are such that the system (2.2), (2.3) admits solutions of the form

$$A_\mu = (y^2 \partial_\mu - ay_\mu)\Lambda, \quad (2.4)$$

for some value of the parameter a . One easily checks that this is the case if and only if

$$\kappa = (N+1)(N+n-2), \quad a = N-1.$$

Since the degree of homogeneity is arbitrary it will be convenient to choose it so that $\kappa = 0$; this gives us two possibilities,

$$(1) \quad N = -1, \quad (2) \quad N = 2 - n.$$

(1) This possibility was analyzed in [4], in the case $n = 5$. It leads to a topological gauge theory on AdS_5 . The fields fall off as r^{-1} at infinity and on the boundary they give the excitations of ordinary, four dimensional, conformally invariant Maxwell theory. The theory is “massless on the boundary”.

(2) This is the theory that, in a dimension higher than 4, is usually called massless; to be more precise we shall say that it is “massless in the bulk”. The 5-dimensional case was examined by Gunaydin and Marcus [12].

Both theories have meaningful boundary values at infinity. As we said, this manifold can be identified with the cone $y^2 = 0$. In the case $n = 5$ it is the familiar Dirac cone in six dimension, the conformal completion of Minkowski space. In this four-dimensional setting both gauge theories were discussed in [20], where the second type of gauge theory was called “current type”. Indeed, it is a fascinating fact that the boundary values of this type of field, that describes massless particles in five dimensions, are of the type of a current, and thus naturally associated with a composite operator, in four dimensions.

2.2. Tensor gauge fields in AdS_n .

The procedure is the same: One requires

$$(y^2 \partial^2 - \kappa)g = 0, \quad g_{\mu\nu} = g_{\nu\mu}, \quad g_{\mu\mu} = 0, \quad y \cdot g = 0, \quad \partial \cdot g = 0, \quad (2.5)$$

and

$$y \cdot \partial g = N g.$$

A gauge field,

$$g_{\mu\nu} = (y^2 \partial_\mu - a y_\mu) \Lambda_\nu + (\mu, \nu) \quad (2.6)$$

satisfies all these equations if and only if

$$\kappa = N(N + n - 1), \quad a = N.$$

Again we set $\kappa = 0$ and find two possibilities,

$$(1) \quad N = 0, \quad (2) \quad N = 1 - n.$$

(1) This field theory has been examined in 4 dimensions, where it is linearized AdS_4 gravity [21], hence there (in four dimensions) it is massless in the bulk. It is part of the $OSp(8/4)$ massless graviton supermultiplet [22].

(2) The five-dimensional case was examined in [12] and interpreted in terms of massless, 5-dimensional gravitons; it is a part of the massless $SU(2, 2/4)$ graviton supermultiplet.

Fields of other tensorial structure also admit gauge subspaces, see [21].

3. Modes and representations.

Physical states are associated with those unitary representations of $so(n-1, 2)$ that have energy spectra bounded from below; that is, highest weight representations. For every field theory we must identify a set of solutions of the field equations on which the natural (geometric) action of the group induces a representation of this type. In a true gauge theory this representation is not induced on the field modes themselves, but on equivalence classes, the ignorable modes being gauge modes. The field representation is thus non-decomposable. We shall now identify these non-decomposable representations. It will be done by examination of the modes of lowest energy (highest weight).

3.0. Scalar and spinor fields.

The scalar field mode (solution of the scalar field equation) of lowest energy has the form

$$\left(\frac{1}{y_+}\right)^{E_0}.$$

It is the highest weight vector of the representation $D(E_0, \vec{0})$ with lowest energy E_0 and weight zero on $so(n-1)$. As one applies energy raising operators to this mode one finds that, if $\frac{n-1}{2} - E_0 = 1$, then there appears a mode of the form

$$y^2 \left(\frac{1}{y_+}\right)^{E_0+2} f(\vec{y}).$$

Since y^2 is an invariant, this mode belongs to an invariant subspace of field modes that fall off more quickly at infinity. Its lowest energy is $E_0 + 2$ and the total representation is

$$D(E_0, \vec{0}) \rightarrow D(E_0 + 2, \vec{0}).$$

Spinor fields show a similar phenomenon. The representations are, in the case of low dimensional space times, as follows

$$\begin{aligned} n = 3, & \quad D\left(\frac{1}{2}, 0\right) \rightarrow D\left(\frac{3}{2}, 0\right), \\ n = 4, & \quad D\left(1, \frac{1}{2}\right) \rightarrow D\left(2, \frac{1}{2}\right), \\ n = 5, & \quad D\left(\frac{3}{2}, \frac{1}{2}, 0\right) \rightarrow D\left(\frac{5}{2}, 0, \frac{1}{2}\right). \end{aligned}$$

The notation for the weights is explained below.

3.1. Vector fields.

The simplest highest weight vector field mode is

$$A \cdot z = z_+(y_+)^N. \tag{3.1}$$

It is homogeneous, harmonic and divergenceless, but not transverse ($y \cdot A \neq 0$). The simplest transverse mode is

$$A \cdot z = (\vec{y} z_+ - \vec{z} y_+)(y_+)^{N-1}. \tag{3.2}$$

This is the highest weight vector of the representation

$$D(E_0, F), \quad E_0 = -N, \quad (3.3)$$

where the number E_0 is the $o(2)$ weight, the lowest energy, and F is the highest weight of the fundamental representation of $so(n-1)$. This is not always the only highest weight mode, but it will be useful to examine it. First, we notice that it is a gauge mode if and only if $N = -1$:

$$A \cdot z = (\vec{y} z_+ - \vec{z} y_+)(y_+)^{-2} = z \cdot \partial(-\vec{y}/y_+). \quad (3.4)$$

So we must distinguish two cases.

(1) The case $N = -1$. The mode (3.2) is pure gauge, but there is another highest weight mode,

$$A \cdot z = (\vec{y} \wedge \vec{z})(y_+)^{-2}, \quad (3.5)$$

that is not. Actually, this is not an absolute highest weight, but is highest weight modulo a subspace of gauge fields. Applying the energy lowering operators to (3.5) we obtain (3.4); which, being a gauge mode, can not give us back (3.5) on the application of energy raising operators. So we have a representation that includes

$$D(2, F \wedge F) \rightarrow D(1, F). \quad (3.6)$$

Now we may hope that the quotient representation $D(2, F \wedge F)$ (the one that is of interest!) is unitary; that can be true only in very low dimension. Consider the simplest cases.

Space time dimension 3. The group is $SO(2, 2)$. This case was investigated by Gunaydin [17] and also in [18,19]. There are several versions of three-dimensional Maxwell theory, but all of them contain $D(2, 0)$. (The fundamental representation of $o(2)$ is 2-dimensional and the representation $F \wedge F$ is the identity representation.) Among them, one is massless in the bulk; others are topological and related to conformal field theories in two dimensions [19].

Space time dimension 4. The irreducible representations of $so(3)$ are labelled by a half-integer j and the highest weight is j . The representation (3.6) becomes

$$D(2, 1) \rightarrow D(1, 1); \quad (3.7)$$

it describes the physical and gauge sectors of one version of Maxwell theory on AdS_4 [23].

Space time dimension 5. The representations of $so(4) = su(2) \otimes su(2)$ have highest weights j_1, j_2 , labelled by two half-integers. The representation (3.6) becomes

$$[D(2, 1, 0) \oplus D(2, 0, 1)] \rightarrow D(1, \frac{1}{2}, \frac{1}{2}), \quad (3.8)$$

which is the representation of $so(4, 2)$ associated with electrodynamics *in 4 dimensions!* In five dimensions it is a topological singleton gauge theory [4]. On the boundary at infinity

it becomes conformal Maxwell theory in a four-dimensional space that is the conformal completion of both Minkowski and anti De Sitter space.

Higher dimension. We suspect that both factors in the representation (3.6) are non-unitary for $n > 6$.

(2) The case $N = 2 - n$. This case is simpler, for now the mode of absolutely lowest energy, (3.2), is “physical” (not a gauge field). What may happen, however, is that the space for which (3.3) is a cyclic vector (for the natural action of $so(n - 1, 2)$ on the field) may include gauge modes. Actually, by a law of nature, this usually does happen whenever it can happen. To understand when it can happen, we must turn to the theory of highest weight representations (next section); to find out if it actually does happen in a particular context we must do some calculations. In fact, it is easy to see that the gauge mode in question is (2.4) with

$$\Lambda = (y_+)^{N-1}.$$

We discuss the lowest dimensions.

Space time dimension 3. Here $N = -1$, which reduces to the case already discussed, and which explains the proliferation of versions of three-dimensional Maxwell theory [19].

Space time dimension 4. Now $N = -2$. The representation generated from (3.2) is

$$D(2, 1) \rightarrow D(3, 0). \tag{3.9}$$

The theory is an alternative version of anti De Sitter electrodynamics [23]. To get a theory that is conformally invariant one must combine (3.9) with (3.7).

Space time dimension 5. Here $N = -3$ and the mode (3.2) is the highest weight vector of

$$D(3, \frac{1}{2}, \frac{1}{2}) \rightarrow D(4, 0, 0).$$

This is the representation of massless Maxwell theory in five dimensions; that is, it is massless in the bulk. At infinity it becomes the current type gauge theory already mentioned in [20].

4. Non-decomposable representations.

To get an overview of the possibilities, and to understand the structural relationship between gauge theories in various dimensions, we must consult the theory of non-decomposable, highest weight representations of $so(n - 1, 2)$. It is possible to deal with all dimensions together, in a uniform manner, but since the picture changes radically with n , especially for the low values of n that are of primary interest, we shall consider each case in turn. We will here consider the 3, 4 and 5-dimensional cases relevant for string, membrane and three-brane horizons, respectively.

4.1. Space time dimension 3. The group $SO(2, 2)$ is not simple, and its representation theory reduces to that of $SO(2, 1)$ [17]. The most important representation contains 9 subfactors, including the identity representation [19]. The associated gauge theories include

singletons that extend left and right moving conformal fields from the two-dimensional boundary to the interior. The associated superconformal algebras were discussed in [24] and [12]. We shall forego a detailed description.

Gravity in three dimensions is locally trivial but very interesting globally [25].

4.2. Space time dimension 4. The group $SO(3, 2)$ is of rank 2, so that an accurate picture can be given of its weight lattice. In Fig. 1 the coordinates are the energy E and the spin j . To each point with coordinates (E_0, j) ($2j$ a non-negative integer) there is a highest weight representation with weights in an area limited by straight lines rising at 45 degrees, and the vertical axis. The two dotted lines are perpendicular to the roots and pass through the point $(\frac{3}{2}, -\frac{1}{2})$, the coordinates of one half the sum of the positive roots. The following irreducible, highest representations are unitary.

$$D(E_0, j), E_0 \geq j + 1, j \geq 1;$$

$$D(E_0, j), E_0 \geq j + \frac{1}{2}, j = 0, \frac{1}{2}.$$

The rule that governs non-decomposable representations is very simple. In order that one irreducible, highest weight module extend another, two conditions must be fulfilled. (1) First, it is necessary that their highest weights be related to each other by an element of the group of transformations, the Weyl group, that is generated by the reflections through the dotted lines (the Weyl planes). It is clear that, in most cases this implies that one or the other lies below the limit imposed by unitarity. (2) The difference between the two highest weights must belong to the lattice generated by the noncompact roots. The possibilities include

$$D(\frac{1}{2}, 0) \rightarrow D(\frac{5}{2}, 0), \quad D(1, \frac{1}{2}) \rightarrow D(2, \frac{1}{2}); \quad (4.1)$$

these are the singletons, topological gauge theories (scalar and spinor fields) in four dimensions, the first topological gauge theories to be discovered [15]. Vector gauge theories (electrodynamics) make use of

$$D(2, 1) \rightarrow D(3, 0), \quad D(2, 1) \rightarrow D(1, 1), \quad (4.2)$$

both of which have been mentioned already. Massless spinor fields are described by $D(\frac{3}{2}, \frac{1}{2})$; there is no extension and no gauge theory. Rarita-Schwinger theory is a gauge theory of ordinary massless particles in the bulk.

Linearized gravity in four dimensions [21] is a gauge theory that includes tensor gauge fields. It employs the following non-decomposable representations

$$[D(3, 2) \oplus D(-1, 1)] \rightarrow D(0, 2), \quad D(3, 2) \rightarrow D(4, 1). \quad (4.3)$$

The relative positions of the highest weights vis-à-vis the Weyl planes can be observed in the figure.

4.3. Space time dimension 5. The weight lattice for $so(4, 2)$ can not be illustrated so easily by a plane figure, but Fig. 2 is an attempt. The representations of vector gauge

theories are here shown, with arrows indicating extensions. There is no general theory of multiple extensions, but here is an example that is worth noticing. The representation that is carried by the physical and the gauge sector of five-dimensional Maxwell theory is fairly simple,

$$D_1 = D(3, \frac{1}{2}, \frac{1}{2}) \rightarrow D(4, 0, 0). \quad (4.4)$$

Conformal electrodynamics, on the four-dimensional boundary, contains

$$D_2 = (D(2, 1, 0) \oplus D(2, 0, 1) \oplus \text{Id}) \rightarrow D(1, \frac{1}{2}, \frac{1}{2}). \quad (4.5)$$

But in the extension of this latter theory, to a vector gauge theory of singleton type on AdS_5 , one finds [4] the monstrous representation

$$D_2 \rightarrow D_1 \quad (4.6)$$

that contains all six of the representations related among themselves by the Weyl group. Here the modes of five-dimensional electrodynamics appear as gauge modes of the singleton vector gauge theory. The non-existence of gauge invariant local interactions of these singletons in the bulk can thus be interpreted as a basic incompatibility between the two types of vector gauge excitations.

An observation that we find even more fascinating is the fact that the photons of five-dimensional electrodynamics have boundary values at infinity that have the characteristics of currents rather than of massless field. Currents are composite operators and this points to a type of duality between composite states on the boundary and massless fields in the bulk.

There is also the extension

$$D(1, 0, 0) \rightarrow D(3, 00);$$

It is associated with a scalar singleton field in five dimensions that has recently been examined [4]. The spinorial analogue has not yet been studied, but it is clear that it carries the representation

$$D(\frac{3}{2}, \frac{1}{2}, 0) \rightarrow D(\frac{5}{2}, 0, \frac{1}{2})$$

and/or its helicity conjugate. On the boundary the gauge sector disappears and there remains the representation associated with massless spin- $\frac{1}{2}$ particles in four dimensions, which is not a gauge theory.

5. Composite operators and particles.

Representation theory points to interesting properties of composite operators. The first example is the observation [14] that the singleton representations of AdS_4 are related to the massless representations of the same group by

$$(D(\frac{1}{2}, 0) \oplus D(1, \frac{1}{2})) \otimes (D(\frac{1}{2}, 0) \oplus D(1, \frac{1}{2})) = \bigoplus_s D(s+1, s),$$

where the sum runs over all spins: $s = 0, \frac{1}{2}, 1, \dots$. The direct sum $D(\frac{1}{2}, 0) \oplus D(1, \frac{1}{2})$ extends to a representation of the AdS_4 supersymmetry algebra $\text{osp}(4/1)$, and the direct sum on the right to a direct sum of massless representations of the same algebra.

In general one has

$$D(E_0, \dots) \otimes D(E'_0, \dots) = D(E_0 + E'_0, \dots) \oplus \dots$$

In higher dimensions the interesting cases are, first,

$$(1) \quad D(2, F \wedge F)^{\otimes 2} = D(4, \vec{w}) \oplus \dots$$

The first term is massless in the bulk if the space time dimension is 5; in particular,

$$D(2, 1, 0) \otimes D(2, 0, 1) = D(4, 1, 1) \oplus \dots$$

Since the Maxwell field on the boundary extends to a singleton in the bulk, this gives a picture of 5-dimensional gravitons composed of singletons.

Next,

$$(2) \quad D(2, F \wedge F) \otimes D(n-2, F) = D(n, F) \oplus \dots$$

The irreducible representations on the right hand side are not massless in dimension n . The same applies to

$$(3) \quad D(n-2, F)^{\otimes 2} = D(2n-4, \vec{0}) \oplus \dots$$

So far we did not indicate a substructure for five-dimensional photons. (Two four-dimensional massless vector representations yield $D(4, 0, 0)$, which is pure gauge.) Nevertheless, there is a way. Half integral spin singletons tend to have lower energy than singletons with integral spins. In particular we have

$$(4) \quad D(\frac{3}{2}, \frac{1}{2}, 0) \otimes D(\frac{3}{2}, 0, \frac{1}{2}) = D(3, \frac{1}{2}, \frac{1}{2}).$$

Massless bosons in five dimensions are composites of spin- $\frac{1}{2}$ singletons.

6. Further outlook.

A remarkable property seems to be a general characteristic of the most interesting gauge theories: they have a strong affinity to the vacuum. Many are “zero center” modules, by which we mean that all the (super) Casimir operators take the value zero. The simplest way to detect that a highest weight representation has zero center is to verify that its highest weight is related to that of the trivial representation (zero) by an element of the Weyl group, or by the existence of an extension that involves the trivial representation, as in (4.5). Examples of zero center modules are electrodynamics with its super symmetric and superconformal extensions, and $N = 6$ supergravity. A more familiar example is the

scalar field in 2 dimensions. The field theories associated with zero center modules (“spontaneously generated field theories” [26]) all contain the zero mode that is so important in 2-dimensional conformal field theory. The physical implications of this, in dimensions higher than two, are not yet clear.

It should be pointed out that the injunction against observing singletons in the bulk is valid within the framework of conventional, perturbative quantum field theory. At the price of departing slightly from that context, to explore alternate methods of quantization, it is possible view propagating and interacting massless particles as 2-singleton bound states with zero binding energy [27].

Finally, we notice that, since the $U(N)$ Yang-Mills theory on the boundary can be compared, for large N , to the anti-De Sitter bulk theory with radius $R^2/\alpha' = \sqrt{4\pi gN}$, the study of the spectrum of the former should give information about the spectrum of the other [1]. This is in strict analogy with (matrix) M theory [29], the spectrum of which is related to that of 11-dimensional supergravity; in a very recent paper [30] some progress was made in this direction.

Acknowledgements.

S.F. is supported in part by DOE under grant DE-FG03-91ER40662, Task C, and by EEC Science Program SCI*-CI92-0789 (INFN-Frascati).

References.

- [1] J. Maldacena, hep-th/9711200.
- [2] K. Sfetsos and K. Skenderis, hep-th/9711138.
- [3] P. Claus, R. Kallosh and A. van Proeyen, hep-th/9711161; P. Claus, R. Kallosh, J. Kumar, P. Townsend and A. van Proeyen, hep-th/9801206.
- [4] S. Ferrara and C. Frønsdal, hep-th/ 9712239.
- [5] See, e.g., A Salam and E. Sezgin, “Supergravity in Diverse Dimensions.” (World Scientific) (1989) Vols. I and II.
- [6] M.P. Blencowe and M.J. Duff, Phys. Lett. **B203** (1988) 229; Nucl. Phys. **B310** (1988) 389; M.J. Duff, Class. Quant. Gravity, **5** (1988) 189. E. Bergshoeff, M.J. Duff, C.N. Pope and E. Sezgin, Phys. Lett. **B199** (1988) 69.
- [7] H. Nicolai, E Sezgin and Y Tanii, Nucl. Phys **B305** (1988) 483.
- [8] C. W. Gibbons, P. K. Townsend, Phys. Rev. Lett. **71** (1993) 3754.
- [9] M. Gunaydin, L. J. Romans, N. P. Warner, Phys. Lett. **154B** (1985) 268.
- [10] S. Ferrara, A. F. Grillo and R. Gatto, Ann. Phys. **76** (1973) 161.
- [11] G. Mack, A. Salam, Am. Phys. **53** (1969) 174.
- [12] M. Gunaydin and N. Marcus, Class. Quant. Grav. **2** (1985) L11; M. Gunaydin and D. Minic, hep-th/9702047.
- [13] P. Howe, K. Stelle and P. Townsend, Nucl. Phys. **B192** (1081) 332.
- [14] M. Flato and C. Fronsdal, Lett.Math.Phys. **2** (1978) 421; Phys.Lett. **97B** (1980) 236.
- [15] M. Flato and C. Fronsdal, J. Math. Phys. **22** (1981) 1100.
- [16] C. Fronsdal, Phys. Rev. D **26** (1982) 1988. W. Heidenreich, N.C. **84** (1980) 220.
- [17] M. Gunaydin, Proc. Trieste conf. *Supermembranes and Physics in 2+1 dimensions*, Ed’s M. J. Duff, C. N. Pope, E. Sezgin, World Scientific, 1990, p 442; M. Gunaydin, B. E. W. Nilsson, G. Sierra and P. K. Townsend, Phys. Lett. **B176** (1986) 45.
- [18] M. Flato and C. Fronsdal, Lett. Math. Phys., **20** (1990) 65.
- [19] C. Fronsdal, Ann. Phys. **22** (1991) 27.
- [20] B. Binengar, C. Fronsdal and W. Heidenreich, J. Math. Phys. **24** (1983) 2828.
- [21] C. Fronsdal and W. Heidenreich, J. Math. Phys. **28** (1987) 215.
- [22] B. de Wit and H. Nicolai, Nucl. Phys. **B208** (1982) 323.
- [23] C. Fronsdal, Phys. Rev. D **12** (1975) 3819.
- [24] M.Gunaydin, B.E.W. Nilsson, G. Sierra and P. Townsend, Phys Lett. **176B** (1986) 45.
- [25] S. Deser, R. Jackiw and G. ’t Hooft, Ann.Phys. **152** (1984) 220. G.Horowitz and D.Welch, Phys.Rev.**71** (1993) 328. N.Kaloper, Phys.Rev.D **48** (1993) 2598. A.Achicano and P. Townsend, Phys.Lett. **B180** (1986) 89. E. Witten, Nucl. Phys. B311 (1998) 46.
- [26] M. Flato and C. Fronsdal, *Spontaneously generated Field Theories, Zero-Center Modules, Colored Singletons and the virtues of N = 6 Supergravity*. In “Essays in Supersymmetry”, (C. Fronsdal, Ed.) Reidel 1986.
- [27] M. Flato and C. Fronsdal, Phys. Lett. **B172** (1986) 412; J. Geom. Phys., **6** (1988) 294; *idem* **6** (1989) 293. C. Fronsdal, proc. Colloque Rideau, Paris, January 1995.
- [28] S. Ferrara and B. Zumino, N.P. **B87** (1975) 207.
- [29] T. Banks, W. Fischler, S. Shenker and L. Susskind, hep-th/9610043.
- [30] G. T. Horowitz and H Ooguri, hep-th/9802116.