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On LEP Performance Limits in 1999 and 2000

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A first estimate of the performance of LEP expected in 1999 (2000) is presented taking into account the configuration of LEP in this period when all the upgrades will have been implemented. Based on first-order theory the scaling laws with energy of the different limitations are given and the available peak luminosity as a function of energy is calculated for two extreme cases of the beam optics. An estimate of the average luminosity is presented.

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1. Introduction

The LEP upgrade will be concluded for the start-up in 1999 with all the major hardware in place and no further significant investment planned.

In the shutdown 1998/99, the last four superconducting Nb-film modules will be installed and the four large “12 kW” cryoplants will be upgraded to a higher cooling power. The higher-order mode couplers will be connected to external loads of higher rating by either rigid coaxial lines or to cables allowing a higher power than initially foreseen.

This note examines various effects which limit the performance of LEP as far as peak luminosity and energy are concerned and tries to give a first estimate of the peak luminosity as a function of energy. It is based on work started at the Chamonix Workshop 1997 and done in order to understand which upgrade is required for the cryoplants, and in response to requests by the LEP200 Working group [1] and various CERN committees such as SPC and LEPC [2].

The note examines fundamental limits. The more technical problems on the road to higher energy are scrutinized in the companion paper [3] where the individual subsystems are examined and, if required, the necessary steps for improvement are proposed and listed.

Section 2 gives the well-known formulae for the luminosity and, for convenience, the scaling laws of the different limits. As can be seen there, first-order theory is used, e.g. beam blow-up by beam-beam effects is neglected, etc.

Section 3 gives the required rf voltage for the two types of optics considered and the concurrent accelerating gradient in the Nb-film cavities.

The limits are examined one by one independently of each other in section 4 while section 5 gives a synopsis of the results comparing them with the requested luminosity by the physics community [1,2].

In section 4, only operation with eight bunches per beam is considered. The results for four-bunch operation are obtained by simple scaling and are given in section 5.

The terms luminosity and energy have the meaning of peak luminosity and beam energy respectively throughout the note.

It is expected that further work and experiments in LEP will allow refining and improving these estimates. Hence, this note should be considered a first educated guess.

2. Formulae and scaling laws

Using first order theory and neglecting all intensity or beam-beam driven blow-up effects the peak luminosity and the beam-beam tune-shifts are given by

$$L = \frac{k_b I_b^2}{4\pi e^2 f_0 \sigma_x \sigma_y} \qquad (1)$$

$$\xi_i = \frac{r_e}{2\pi e f_0} \frac{I_b \beta_i}{\gamma \sigma_i (\sigma_x + \sigma_y)} \qquad \text{with } i = x,y \qquad (2)$$

where f_0 is the revolution frequency, I_b the bunch current, σ_x/σ_y the rms beam sizes at the intersection points which in the limit of vanishing vertical dispersion are given by

$$\sigma_x = \sqrt{\beta_x \varepsilon_{x0} / (\kappa + 1)} \qquad (3a)$$

$$\sigma_y = \sqrt{\beta_y \varepsilon_{x0} \kappa / (\kappa + 1)} \qquad (3b)$$

with ε_{x0} being the uncoupled emittance which is proportional to $1/J_x$, β_x/β_y the beta values at the interaction points and $\kappa = \varepsilon_y/\varepsilon_x$ where ε_x and ε_y are the horizontal and vertical emittance respectively in the presence of coupling. We further define the function Ω which depends only on the optics and not on beam energy. The latter is described by the Lorentz factor γ.

Since we consider only flat beams at the interaction point

$$\sigma_x >> \sigma_y$$

and

$$\kappa << 1$$

the optics function is defined by

$$\Omega = \sigma_x \sigma_y / \gamma^2 = (\kappa\, \beta_x\, \beta_y)^{1/2} (\varepsilon_{x0} / \gamma^2). \qquad (4)$$

We further define the total current in both beams

$$2I_0 = 2k_b I_b \qquad (5)$$

assuming equal positron and electron beam currents.

From these basic relationships, it is possible to derive the following scaling laws.

2.1 Limit by total beam current $2I_0$

$$L \sim (2I_0)^2/(k_b\gamma^2\Omega) \tag{6}$$

This applies e.g. when $2I_0$ is limited by the rf power P_{rf} or the synchrotron radiation power. The total power emitted by synchrotron radiation P_s is equal to P_{rf} in case of a perfect match between klystrons and cavities. With the energy loss per turn $U_0 \sim \gamma^4$ we get the limit by maximum available P_{rf} or maximum admissible P_s

$$L \sim P_i^2/(k_b\gamma^{10}\Omega) \qquad i = s \text{ or rf} \tag{7}$$

Mismatch between klystrons and cavities is taken into account in section 4.

2.2 Beam-beam limit

Since the vertical beam-beam parameter (2)

$$\xi_y = \frac{r_e}{2\pi e f_0} \cdot \frac{I_b\beta_y}{\gamma^3\Omega} \tag{8}$$

is limited to a maximum value, we get as scaling law for the luminosity at the beam-beam limit

$$L \sim k_b\xi_y^2\gamma^4\Omega/\beta_y^2 \tag{9}$$

Since $\qquad \xi_x = \xi_y(\kappa\beta_x/\beta_y)^{1/2}$

one observes that $\qquad \xi_x < \xi_y$

for the parameters used in this note. It is shown in section 4.9 that it is sufficient to consider only the vertical limit.

2.3 Limit due to beam-induced higher-order mode losses

In the simplest case that no coherent addition of the fields induced by the individual bunches takes place, the higher-order mode power lost by the beam in a component is given by

$$P_{hom} \sim k(\sigma_s)(2I_0)I_b \tag{10}$$

If this power cannot be exceeded, the scaling law becomes

$$L \sim P_{hom}/(k(\sigma_s)\,\gamma^2\Omega) \tag{11}$$

where $k(\sigma_s)$ is the longitudinal loss factor of the beam in this particular component in units of V/C.

Note that the P_{hom} may have its maximum at a particular energy during the ramp because there the bunch length σ_s could become shorter than at collision energy. A careful control of σ_s by the wigglers and Q_s during ramping is thus mandatory.

Since the beam-induced losses in the cryomodules are expected to scale according to (10) and since the available cooling power per module P_{cm} for the dynamic load is limited we get (see section 4.7)

$$L \sim (P_{cm} - b\,\gamma^8) / (\gamma^2 \Omega\, R_m) \qquad (12)$$

where b is a constant and R_m is the impedance characterizing the beam-induced losses in a sc rf module[4]. The impedance R_m is defined in section 4.7. Note that (11) and (12) are independent of the number of bunches k_b.

2.4 General observations

It can be seen from the formula (8) that the beam-beam effect diminishes at high energy while all other limits will become much more severe at higher energy, the cryogenic power limit having the steepest drop with energy. For our parameters, as will be seen later, it is this effect which will eventually be the strongest limit in energy; it would occur even well before the rf gradient limit if the actual cryoplants were not upgraded.

3. Optics, required voltage and gradient

The possible optics configurations for high energies are reviewed elsewhere [3] and the final choice for the run in 1999-2000 will emerge both from the forthcoming LEP Performance Workshops in Chamonix and further tests performed during the 1998 run. For our purpose, it is sufficient to consider two somewhat extreme cases, all others will either fall in between or be very similar to one of the two: the 90°/60° with J_x = 1.5 as the optics having the lowest energy reach and 102°/90° with J_x = 1 allowing for the highest energy. The 108°/90° optics may have a similar potential as the latter but needs further testing.

Table 1 gives the parameters used for the calculations. The emittance values are from WIGWAM[5], the beta values and the emittance ratios are the ones achieved in 1997 [6,7].

Table 1 - Optics parameters

$\Delta\mu_x$	$\Delta\mu_y$	J_x	ε_{xo}(100 GeV) nm	β_x m	β_y m	κ %	$\Omega.10^{20}$ m^2
90°	60°	1.5	37.9	1.5	0.050	1.0	2.71
102°	90°	1.0	44.4	2.0	0.050	1.0	3.65

$\Delta\mu_i$ are the phase advances per cell in the arcs in the horizontal (x) and vertical (y) plane. Table 2 gives the radiation loss per turn U_0 and the rf voltage required for 24 h quantum lifetime as a function of beam energy [5]. If the superconducting cavities all operate at their nominal field (6 MV/m for 272 Nb-film cavities, 5 MV/m for the 16 Nb-cavities), the maximum available voltage is 2.92 GV. Adding 0.13 GV for the remaining 52 Cu-cavities yields a maximum voltage of 3.05 GV. Although the likelihood for two klystrons tripping has become rather small [6] and further improvements are expected, we need an operational margin for the tripping of one klystron and for some cavities not fully operational. Hence, we assume that two klystrons feeding 16 Nb-film cavities are off, i.e. a loss of 0.16 GV, which leads to an effective nominal voltage of

$$V_{rfn} = 2.89 \text{ MV}$$

by 1999.

Table 2 shows that from a certain energy onwards the required voltage V_{rf} exceeds this nominal voltage. An increase of the accelerating gradient E_a in the Nb-film cavities to the values also given in Table 2 is the only means to cover the voltage deficit. As can be seen from the table, values close to 7 MV/m, about 15 % above the nominal value of E_a are required to reach 100 GeV. The values for 102 GeV are given for completeness.

Table 2 - Loss per turn, required rf voltage and Nb-film gradient

E (GeV)	96	98	100	102	Optics
U_0 (GV)	2.48	2.70	2.92	3.16	
V_{rf} (GV)	2.81	3.05	3.30	3.57	90°/60° J_x = 1.5
E_a (MV/m)	5.85	6.40	6.98	7.59	- " -
V_{rf} (GV)	2.73	2.96	3.21	3.47	102°/90° J_x = 1.0
E_a (MV/m)	5.65	6.19	6.75	7.36	- " -

Given all optics and rf parameters the individual limits in the L, E plane can now be evaluated.

4. Individual limits

4.1 Apart from the energy limit imposed by the maximum achievable accelerating gradient in the sc rf Nb-film cavities, the available dynamic aperture (A) can in principle also impose an upper limit on the energy. Since for good lifetime we require

$$A \geq n\sigma$$

and since the beam size grows with energy according to

$$\sigma \sim \gamma$$

a certain maximum energy cannot be exceeded for a given optics and J_x. As A, σ and, in collision, also n depend on the optics, the maximum energy is optics dependent.

The present experimental data indicate that the 90°/60° (J_x = 1.5) optics may not be limited by this effect up to 100 GeV. However, the 102°/90° (J_x = 1.0) optics might have a limit below 100 GeV, requiring then an appropriate emittance reduction by increasing J_x through a change in the rf frequency in order to avoid this limit [3)]. However, given the provisional character of these measurements and the doubts about the validity of extrapolation to higher energy, we ignore this possible energy limit by dynamic aperture for the 102°/90° (J_x = 1) optics. More measurements and simulations in 1998 should allow a better estimation of this effect. A more detailed discussion can be found elsewhere [3)].

4.2 The requirements on power supplies and water cooling of magnetic elements and issues related to deterioration of magnetic field quality by saturation are treated in the LEP2 Design Report [8)]. Other measures to be taken so that LEP components do not limit the beam energy to a value below 100 GeV are discussed in the companion paper [3)].

Next we turn to the effects which bring about a limit on the beam current as a function of energy. First we deal with effects limiting the total current $2I_0$.

4.3 Limit by rf power

To first order, neglecting mismatch by non-optimum coupling to the cavities, the maximum possible current is determined by the available rf power

$$P_{rf} \sim 2I_0U_0 \tag{13}$$

Since we have 34 klystrons available (2 are off) each providing at least 1 MW, and since the Cu rf system can add about 0.5 MW, a total of 34.5 MW is available. Using U_0 from table 2 with (13) yields the maximum possible beam currents shown in table 3.

Table 3, Approximate current limits for P_{rf} = 34.5 MW

E (GeV)	96	98	100	102
$2I_0$ (mA)	13.9	12.8	11.8	10.9

A more accurate result is obtained by using the approach of D. Boussard [9)] who considers the forward power per sc cavity which is limited to 125 kW as one 1 MW klystron feeds eight sc cavities

$$P_f = \frac{R}{16}\left(2I_0 \sin\phi_s + 2\,\frac{V_c}{R}\right)^2 \tag{14}$$

where $R = (R/Q).\ Q_{ext} = 464\ .\ 2 \times 10^6 = 9.2 \times 10^8\ \Omega$ [a)]
V_c - cavity voltage
$\sin\phi_s = U_0 / V_{rf}$

[a)] Note that the Linac definition of impedance is used.

Table 4 gives the maximum possible beam currents calculated from (14). Comparison with table 3 reveals that the differences are rather small.

Table 4 gives also the expected luminosity for the two optics under consideration calculated using (1) and the parameters of table 1. The luminosity decreases strongly with energy as expected from scaling law (7).

Table 4 - Maximum beam current and peak luminosity limited by P_{rf}.

E (GeV)	96	98	100	102	Optics
$2I_0$ (mA)	14.1	13.0	11.7	10.4	90° / 60° $J_x = 1.5$
$L / 10^{32} (cm^{-2}s^{-1})$	1.79	1.46	1.14	0.87	- " -
$2I_0$ (mA)	14.1	13.0	11.8	10.6	102° / 90° $J_x = 1.0$
$L / 10^{32} (cm^{-2}s^{-1})$	1.32	1.08	0.86	0.67	- " -

4.4 Effects of the synchrotron radiation power

i) If the heating of a particular set of components cannot be exceeded, also the total synchrotron radiation power

$$P_s = 2I_0U_0$$

has to be limited. We consider here the maximum cooling capacity of a major component, namely the vacuum chamber in the dipoles, which has a total length of about 20 km. The maximum design cooling capacity is 1.7 kW / m[8)] limiting the maximum total beam current to 11 mA at 100 GeV. Since this value not being a very hard limit is within a few percent of the P_{rf} limit shown in table 4 and since it is governed by the same scaling law in energy, the luminosity limits in table 4 apply also in this case.

ii) The synchrotron radiation induces gas desorption generating a dynamic pressure rise which increases the particle loss by bremsstrahlung in the rest gas. This effect is treated together with the expected lifetime from the beam-beam collisions (see section 4.6).

4.5 Robinson instability

The second Robinson instability in the multi-cavity rf system of LEP has been examined in detail and the threshold on $2I_0$ will be above 15 mA with the foreseen fast feed-back system [10)]. Hence, we can ignore this limitation on total beam current.

Next we turn to limitations involving both the total beam current I_0 and the bunch current I_b.

4.6 Beam lifetime

Three effects contribute to the particle loss:

- beam-beam bremsstrahlung in the interaction points
- beam-gas bremsstrahlung accentuated by the dynamic pressure rise
- Compton scattering on thermal photons.

i) the lifetime determined by beam-beam bremsstrahlung [11] depends on the luminosity L, the number of interaction points n_x, the bremsstrahlung cross-section σ_B and the number of particles per beam N. The decay rate is given by

$$\frac{1}{\tau_{bb}} = \frac{n_x \sigma_b L}{N} = n_x \sigma_B e f_o \left(\frac{L}{I_0}\right)$$

Expressing the luminosity by the beam-beam tune-shift ξ_y we get

$$\tau_{bb} = \frac{2 r_e \beta_y}{n_x \sigma_B f_0 \xi_y \gamma}$$

and after inserting the numerical values the lifetime becomes

$$\tau_{bb} = \frac{0.39}{\xi_y} \frac{100\,\text{GeV}}{E(\text{GeV})} \tag{17}$$

ii) from measurements of the single-beam lifetime at 91.5 GeV in 1997 [12] and substracting the Compton scattering on thermal photons we deduce a lifetime due to beam-gas interaction of

$$\tau_{bg}\,(h) = 0.75\,(A.h)\,/\,2I_0(A) \tag{15}$$

The dynamic pressure has been measured in LEP at all the energies LEP operated. A fit to the data taken at E $\geq$ 80 GeV yields an energy dependence [12]

$$\frac{dP}{dI_0} \sim \exp\,[0.045\,(E - 91.5\text{ GeV})]$$

implying an increase by 1.5 from 91.5 GeV to 100 GeV. Hence,

$$\tau_{bg}\,(h) = \frac{0.75(h.A)\exp[-0.045(E - 91.5\text{GeV})]}{2 I_0 (A)} \tag{16}$$

iii) the lifetime due to Compton scattering with thermal photons is given by [13)]

$$\tau_c \tilde{} \; 50 \text{ h}$$

for a 1% energy acceptance.

Table 5 summarizes the total beam lifetime resulting from the three effects for the energy range under consideration and for three values of the total beam current. For a given $2I_0$, the result depends on the optics and on the number of bunches k_b as ξ in (17) depends on both these parameters. Examining the individual terms shows that the beam-beam effect always dominates, which explains the short lifetime for k_b = 4. The beam-gas lifetime is never shorter than 45 h. Obviously, these figures are not very accurate, especially since the beam-gas lifetime is a rather rough estimate.

Table 5 - Expected beam lifetime in h

k_b	Optics	$2I_0$ (mA)	96 GeV	102 GeV
8	90/60 (J_x = 1.5)	6	10.6	11.0
		8	8.4	8.8
		10	7.0	7.3
8	102/90 (J_x = 1.0)	6	12.9	13.3
		8	10.4	10.7
		10	8.7	8.9
4	90/60 (J_x = 1.5)	6	6.3	6.8
		8	4.9	5.3
		10	4.1	4.3
4	102/90 (J_x = 1.0)	6	8.0	8.5
		8	6.3	6.7
		10	5.1	5.5

Inspection of the table shows that from lifetime point of view 10 mA is quite acceptable with k_b = 8.

For 4 bunches, 8 mA is an upper limit, since depending on the optics, it is either exceeding the beam-beam limit or the TMCI threshold (see section 4.10). The lifetime for 10 mA is only given for completeness. In 1997, an average turn-around-time (dead time for filling and ramping between two physics runs) of just below 2 h (minimum 0.75 h) has been achieved [6)]. Since this turn-around-time should also be achievable with a ramp going above 92 GeV, initial beam lifetimes down to 5 h as occurring with 8 mA seem to be acceptable, the more so as the lifetime will increase very quickly with the decaying beam current, provided that the luminosity also decays with current.

Table 6 gives the luminosity for k_b = 8 as function of energy for the two optics for 10 mA. Scaling with (6) yields the values for different $2I_0$ and k_b which will be needed in section 5.

Table 6 - Luminosity / 10^{32} ($cm^{-2}s^{-1}$) for $2I_0$ = 10 mA and k_b = 8

Optics	E (GeV)	96	98	100	102
90 / 60 (J_x = 1.5)		0.900	0.864	0.833	0.802
102 / 90 (J_x = 1.0)		0.668	0.639	0.619	0.594

4.7 Limit by cryogenic cooling power

As will be seen this is the most severe limit on the maximum energy apart from the accelerating gradient in the superconducting cavities. The scaling law (12) shows an extremely steep drop of luminosity as a function of energy with the luminosity vanishing at a certain energy.

First we examine the available cooling power at 4.5 K. The first line in Table 7 gives the number of sc rf modules in the respective interaction points at the start-up of 1999; the 20 modules in points 4 and 8 include the two modules which will be added in the shutdown 1998 / 1999 in each of these points. All modules contain four superconducting rf cavities.

The second line shows the cooling power P_c available for the dynamic load at 4.5 K if the cryoplants were not upgraded apart from the improvements by the tuning in 1997 [14)]. The dynamic load is the sum of the rf dissipation and the beam-induced dissipation in the cavities at 4.5 K.

The addition of 2 modules generates about 400 W additional static dissipation at 4.5 K for the power plant in points 4 and 8. These losses can be decomposed into:

- 54 W for 2 connections modules
- 180 W for additional liquefaction (90 W / mod.)
- 160 W for static losses in 2 modules.

This is taken into account when calculating P_c in the table in all cases.

The third line gives the available cooling power *per module* P_{cm}. It will become obvious that this cooling power is insufficient and would severely limit the potential of LEP2.

A relative straight-forward upgrade would lead to a dynamic cooling power of 10 kW at 4.5 K, which would give the power available for the dynamic load shown in the fourth and fifth line. As shown later, this power is marginal and would seriously compromise the chances for LEP to ever reach 100 GeV with a decent performance.

For this reason and after consultations of the cryoplant suppliers it has been decided to propose a more substantial upgrade to 12 kW using a configuration which maximizes the usefulness of this investment for LHC. This provides the cooling power required to reach 100 GeV with a reasonable margin as will be demonstrated later. The numerical values pertaining to this upgrade are given in the sixth and seventh line of table 7.

Table 7 - Cooling power at 4.5 K available for dynamic load

LEP Point	**2**	**6**	**4**	**8**	**Comment**
Number of modules	16	16	20	20	by 1999
P_c (kW)	6.70	6.90	6.43	6.23	After tuning 97, but no upgrade
P_{cm} (kW)	0.42	0.43	0.32	0.31	- “ -
P_c (kW)	10.00	10.00	9.61	9.61	Upgrade to 10 kW
P_{cm} (kW)	0.63	0.63	0.48	0.48	- “ -
P_c (kW)	12.00	12.00	11.61	11.61	Upgrade to 12 kW
P_{cm} (kW)	0.75	0.75	0.58	0.58	- “ -

Inspection of table 7 shows that the modules in points 4 and 8 have the lowest cooling power. Hence, they will limit the performance and are therefore considered below. With the upgrade to 12 kW the available cooling power for the dynamic load will reach 0.58 kW per module in these points. This is close to the cooling power the LHe circuit in these modules can cope with which is 0.60 kW. Hence, with 12 kW we use the modules at their design capacity.

The dynamic load at 4.5 K per module is given by

$$P_{cm} = 4\,\frac{V_c(E_a)^2}{(R/Q).Q(E_a)} + \frac{R_m(\sigma_s)}{n_b k_b}\,(2I_0)^2 \qquad (18)$$

where the first term is the rf dissipation in the four cavities with
V_c = 10.24 MV rf voltage per cavity at 6 MV/m
R/Q = 464 Ω normalized shunt impedance per cavity (linac definition) [15)]
Q = 3.2 x 10^9 at 6.0 MV/m [3)]
Q = 2.3 x 10^9 at 7.0 MV/m [3)]

Since the range in E_a which we have to consider is fairly small and since the real average Q-values of fully equipped and installed modules cannot be measured with precision and are therefore not very well known, a crude model of $Q(E_a)$ is used for the calculation:

- below and up to 6 MV/m Q is constant;
- above 6 MV/m Q is decreasing linearly having the above quoted value at 7 MV/m.

The second term of (18) is the beam-induced dynamic load measured in 1996 [15)] and 1997 [4)] which is characterized by

$$R_m \cong 16\ \mathrm{M}\Omega$$

the loss impedance for “long” bunches $\sigma_s \geq 9$ mm. The denominator contains:
n_b = 2 number of beams and
k_b = 8 number of bunches.

Using E_a from table 2, $V(E_a)$ can be calculated as a function of energy. Putting $P_{dm} = P_{cm}$ where the latter is taken from table 7 we can calculate the maximum admissible total current $2I_0 = f(E, \text{optics})$ shown in table 8 and in turn the peak luminosity given in table 9 for nominal 6.2 kW, 10 kW and 12 kW dynamic cooling power.

Table 8 shows a strong dependence of $(2I_0)$ on the optics which is due to the influence of the optics on the term V^2/Q in equation (18).

Table 8 - Maximum beam current $2I_0$(mA)

P_c (kW) / E (GeV)	96	98	100	Optics
6.2	6.8	-	-	90° / 60° J_x = 1.5
10	14.6	11.2	-	"
12	17.7	15.0	8.0	"
6.2	8.0	-	-	102° / 90° J_x = 1.0
10	15.2	12.9	5.8	"
12	18.2	16.3	11.6	"

Table 9 - Maximum luminosity $L/10^{32}$ ($cm^{-2}s^{-1}$) limited by the available dynamic cooling power P_c at 4.5 K

P_c (kW) / E (GeV)	96	98	100	Optics
6.2	0.41	-	-	90° / 60° J_x = 1.5
10	1.93	1.09	-	"
12	2.84	1.95	0.53	"
6.2	0.43	-	-	102° / 90° J_x = 1.0
10	1.55	1.05	0.21	"
12	2.23	1.71	0.83	"

If the energy, resp. the gradient is so high that all the available cooling power P_{cm} is used up by the rf power dissipation, the sustainable beam current will vanish and also the luminosity. Equation (18) becomes in this case

$$P_{cm} = 4\,\frac{V^2(E_a)}{(R/Q)Q(E_a)} \qquad (19)$$

It can be solved for E_a which in turn defines the energy where the current vanishes. The numerical results are given in table 10. Note that obviously this energy is independent of the parameters of the second term in (18).

Table 10 - Accelerating gradient and beam energy at vanishing beam current

P_c (kW)	E_a (MV / m)	E for 90° / 60° (GeV) J_x = 1.5	E for 102° / 90° (GeV) J_x = 1.0
6.2	6.16	97.2	98.0
10	6.84	99.6	100.6
12	7.12	100.6	101.5

It can be seen from the tables that the requirement of a good luminosity (~ 5 x 10^{31} $cm^{-2}s^{-1}$) at 100 GeV with a reasonable margin implies a dynamic cooling power of at least 12 kW.

4.8 Limit at higher-order mode power

The main issue is the higher-order mode power P_{hom} which can be absorbed by the hom couplers in the sc rf cavities. Experimental evidence [16] suggests that some coherent addition of the fields induced by the individual bunches takes place, depending on the azimuthal position of the modules in the LEP ring. Although k is strictly speaking related to the energy loss by the beam, the equation

$$P_{hom} \sim k(\sigma_s)(2I_0)I_b$$

still holds for hom dissipation in a component, provided the value of k derived from the measured dissipation is taken. Here k is defined by the highest hom power measured in any module.

The modules are being upgraded such that the couplers, cables or coaxial lines and loads can absorb more power than the original 600 W per cavity. Measurements have shown that the new system can cope with at least 1.7 kW/cavity [16]. Since the relevant maximum loss factor k is 1.0 V/pC[16] for σ_s close to 1 cm, the new system can deal with at least $2I_0$ = 17 mA with 8 bunches, resp. 12 mA with 4 bunches. Since this is above other thresholds (see section 4.10), we can safely ignore this effect.

At last, we examine two effects which impose a limit on the bunch current I_b: the maximum admissible beam-beam tuneshift and the Transverse Mode Coupling Instability (TMCI).

4.9 Beam-beam limit

We use as upper limits $\xi_x = 0.035$ and $\xi_y = 0.055$. The former limit is inferred from indications obtained during operation at the Z° peak. The latter is the vertical beam-beam tuneshift reached in 1997 with both optics [6]. Since

$$\xi_x = \xi_y (\beta_x \kappa / \beta_y)^{1/2}$$

holds, we get with the parameters of table 1

$$\xi_x = 0.55\, \xi_y \qquad \text{for } 90°/60°$$
$$\xi_x = 0.63\, \xi_y \qquad \text{for } 102°/90°$$

For $\xi_y = 0.055$, ξ_x reaches 0.030 or 0.035 respectively. Hence, if we respect the vertical beam-beam limit, the horizontal limit will not be exceeded either and it is justified to consider only the vertical beam-beam effect.

The numerical values for luminosity, obtained from (1) and (2) are given in table 11 showing that the upper limit on beam current and luminosity increases with beam energy as expected from scaling law (9).

Table 11 - Maximum beam currents and peak luminosity limited by $\xi_y = 0.055$

E_b(GeV)	96	98	100	102	Optics
$2I_0$ (mA)	12.7	13.4	14.3	15.2	90° / 60° $J_x = 1.5$
$L / 10^{32}(cm^{-2}\ s^{-1})$	1.44	1.57	1.69	1.84	- " -
$2I_0$ (mA)	17.3	18.4	19.5	20.7	102° / 90° $J_x = 1.0$
$L / 10^{32}(cm^{-2}\ s^{-1})$	1.97	2.15	2.32	2.52	- " -

4.10 Transverse Mode Coupling Instability

This effect limits the single-bunch current at injection as the threshold is approximated for a given transverse impedance by [8)]

$$I_b \sim Q_s E / f(\sigma_s)$$

For a synchrotron tune $Q_s = 0.15$, the numerical value is $I_b = 1.0$ mA [8)] rather independent of the optics, implying a limit on the total current of 16 mA for $k_b = 8$ which is comfortably above the P_{rf} limit. Even if Q_s would be lowered to the at present commonly used $Q_s = 0.12$, $2I_0$ would still be 13 mA which is about comparable with the P_{rf} limit. It is also known that the limits are somewhat lower with two beams in LEP and with eight bunches per beam configured in four bunch trains. However, we neglect these subtle effects.

5. Synopsis and discussion of the results

All these limits can be plotted in the L, E diagram for each of the optics and for $k_b = 4$ and 8. Fig. 1a and 1b give the results for the 90° / 60° ($J_x = 1.5$) optics with $k_b = 8$ and 4; Fig. 2a and 2b show the same plot for 102° / 90° ($J_x = 1.0$) $k_b = 8$ and 4. The numerical values for $k_b = 4$ are obtained from the figures for $k_b = 8$ worked out in section 4 using the scaling laws described under section 2. The total beam currents pertaining to the limits shown in the L, E diagrams are presented in Fig. 3a, b and 4a, b.

Fig. 1 and 2 show "hard" limits, forming the boundary of the accessible part of the L, E plane, strongly shaded. "Softer" limits have a lighter shading, as the 10 mA line for $k_b = 8$ where we may expect problems with the vacuum system but perhaps also with the control of the rf system.

Inspection of Fig. 1 and 2 shows that upgrading of the cryoplants to 12 kW for the dynamic load at 4.5 k is mandatory if 100 GeV should remain in our reach with a reasonable margin. The present capacity for the dynamic load of 6.2 kW would clearly preclude energies higher than 96 GeV.

The operation with 8 bunches has the highest "hard" luminosity limits but the required total current may become excessive for the vacuum system and/or rf. Then we have the option to operate with 4 bunches where respectable luminosities can be reached with achievable beam currents but the lifetime may be shorter and, therefore, the ratio of average to peak luminosity lower. The good energy reach of the 102°/90° optics is evident if $J_x = 1.0$ is allowed by the dynamic aperture. Clearly, this optics performs also quite well when used at a lower energy but with say $J_x = 1.5$.

Examining all these options leads to the conclusion that a luminosity of 5×10^{31} should be possible up to 100 GeV and even higher luminosities than this value are conceivable.

In order to estimate the integrated luminosity we scale from the run at 91.5 GeV in 1997 performed with the well-tried 90°/60° optics where 60 pb^{-1} were obtained in 71 days leading to an average of 0.85 pb^{-1}/d with a peak value of 1.7 pb^{-1}/d (1.9 pb^{-1} in optimally chosen 24 h)[17]. This corresponds to efficiencies η of 20 % and 39 % (44 %), where

$$\eta = \int L_t / LT$$

with the instantaneous luminosity L_t, $L = 5 \times 10^{31}\ cm^{-2}s^{-1}$ peak luminosity and T duration of run.

Table 11 gives the range of expected values for a peak luminosity of 5×10^{31} and $8 \times 10^{31}\ cm^{-2}s^{-1}$.

Table 11 - Expected average integrated luminosities per day (pb^{-1}/d)

$\eta / L(10^{31}\ cm^{-2}s^{-1})$	5×10^{31}	8×10^{31}
20 %	0.85	1.4
40 %	1.7	2.7

Thus, a value of $\geq$ 1 pb^{-1}/d seems to be reasonably within reach leading to an integrated luminosity of $\geq$ 200 pb^{-1} if one operates for 100 d in each of the years 1999 and 2000. This would meet the requirement of 200 pb^{-1} at highest energy [1,2].

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* * *

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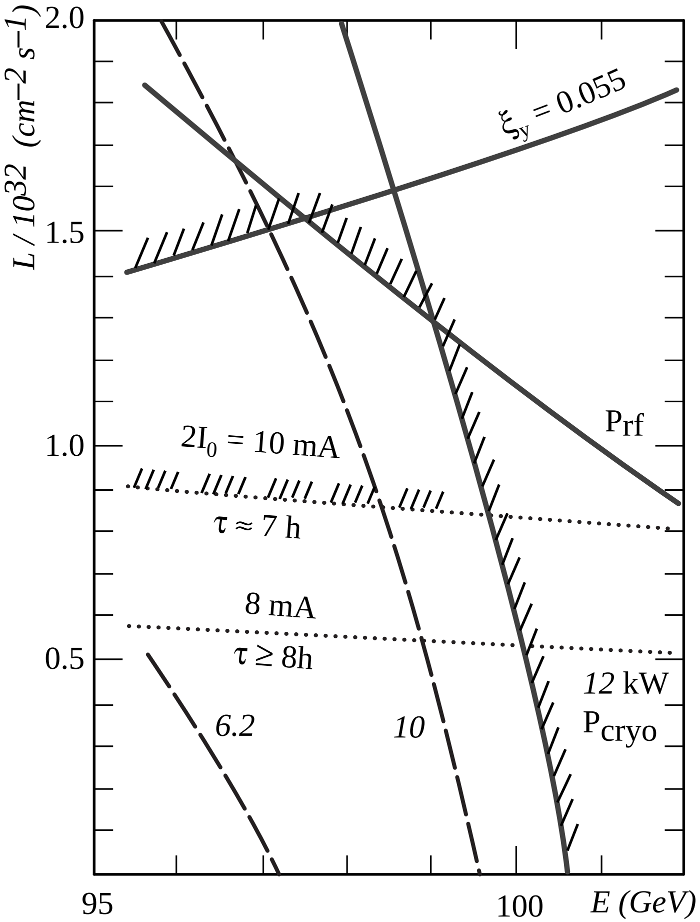

Fig. 1a - LEP luminosity limits

for $\Delta\varphi_x = 90°, \Delta\varphi_y = 60°, J_x = 1.5, k_b = 8$

$\beta_x^* = 1.5\text{m}, \beta_y^* = 5\text{cm}, \kappa = 1\%$

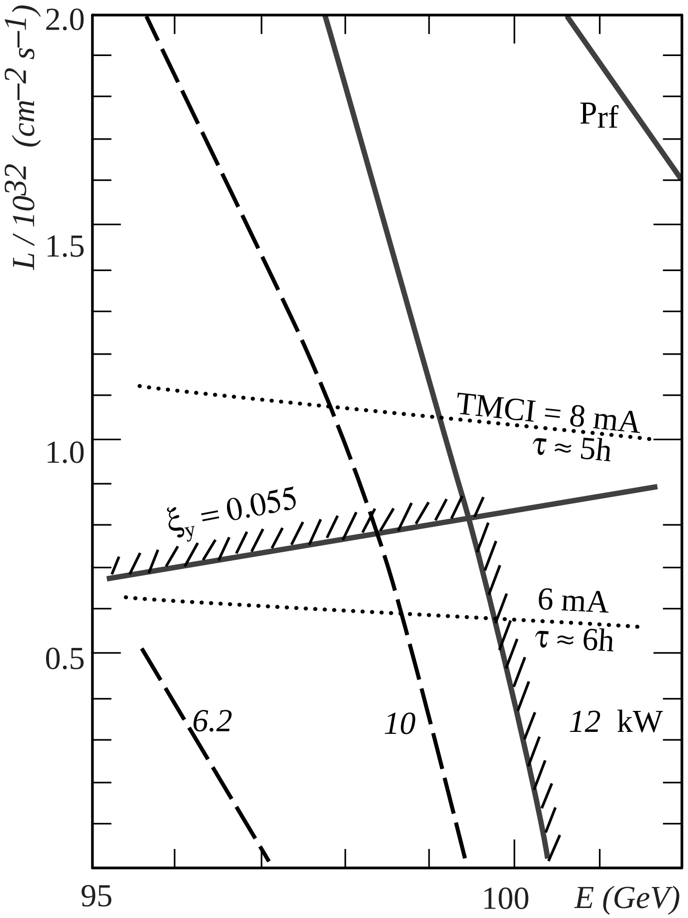

Fig. 1b - LEP Luminosity limits

for $\Delta\varphi_x = 90°, \Delta\varphi_y = 60°, J_x = 1.5, k_b = 4$

$\beta_x^* = 1.5\text{m}, \beta_y^* = 5\text{cm}, \kappa = 1\%$

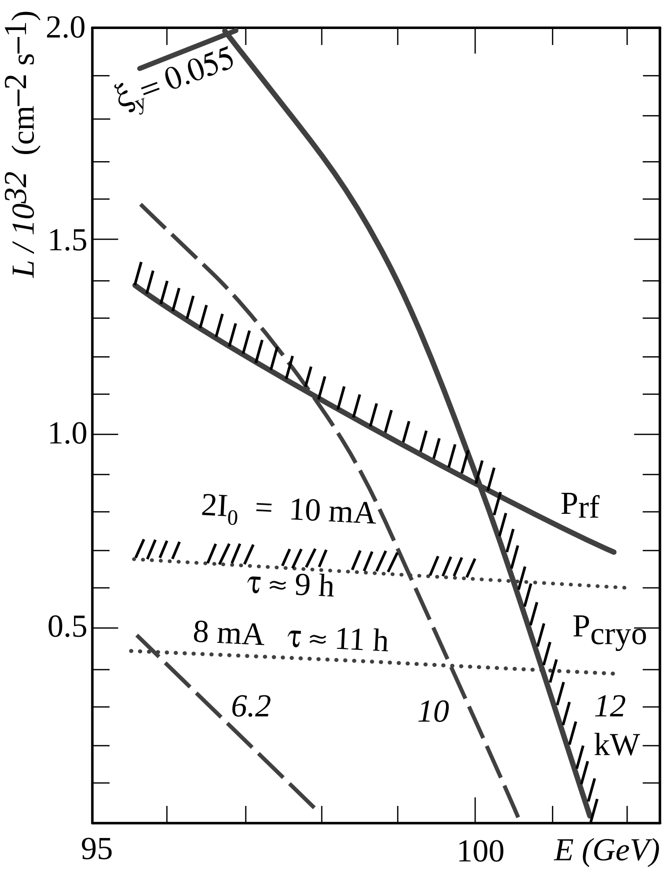

Fig. 2a - LEP luminosity limits

for $\Delta\varphi_x = 102°, \Delta\varphi_y = 90°, J_x = 1.0, k_b = 8$

$\beta_x^* = 2.0m, \beta_y^* = 5cm, \kappa = 1\%$

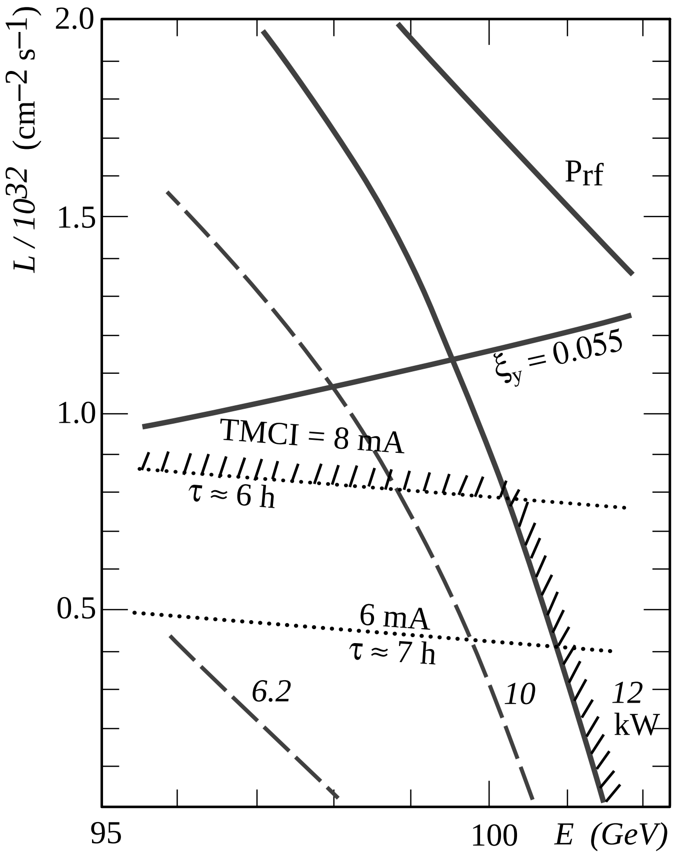

Fig. 2b - LEP Luminosity limits

for $\Delta\varphi_x = 102°, \Delta\varphi_y = 90°, J_x = 1.0, k_b = 4$

$\beta_x^* = 2.0m, \beta_y^* = 5cm, \kappa = 1\%$

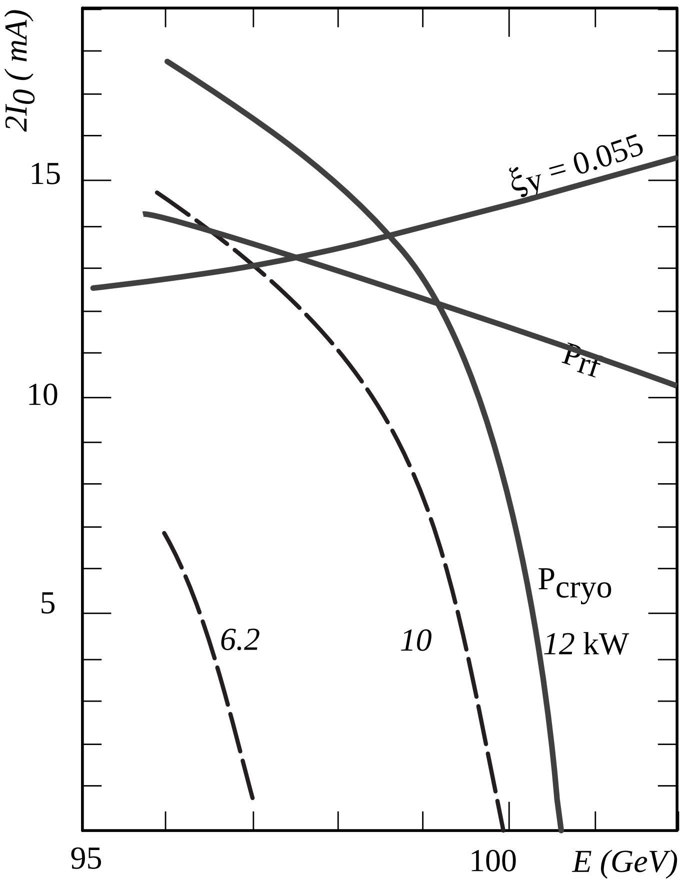

Fig. 3a - Limits on total beam current ($2I_0$)
for $\Delta\varphi_x = 90°, \Delta\varphi_y = 60°, J_x = 1.5, k_b = 8$
$\beta_x^* = 1.5m, \beta_y^* = 5cm, \kappa = 1\%$

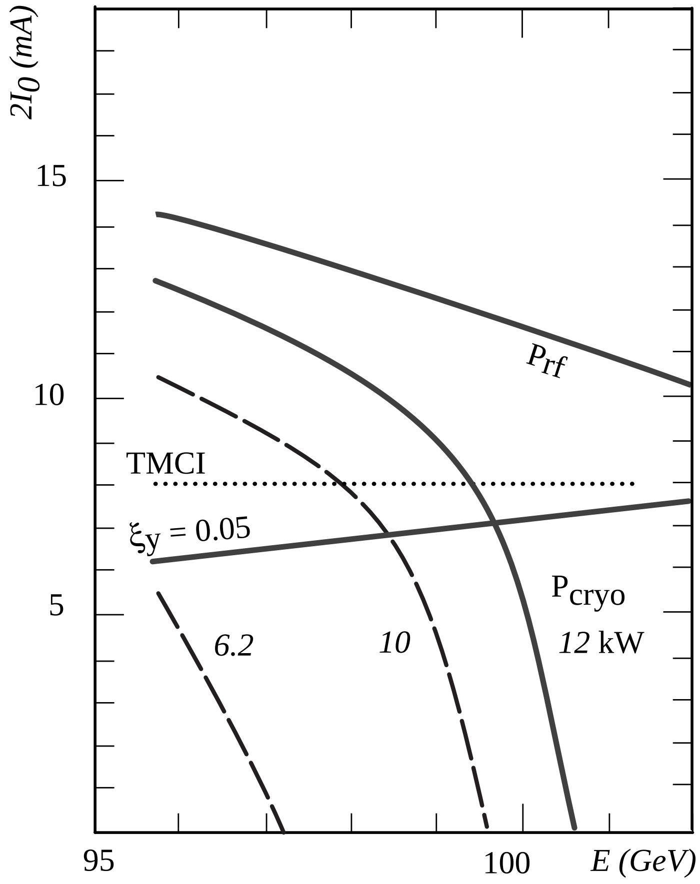

Fig. 3b - Limits on total beam current
for $\Delta\varphi_x = 90°, \Delta\varphi_y = 60°, J_x = 1.5, k_b = 4$
$\beta_x^* = 1.5m, \beta_y^* = 5cm, \kappa = 1\%$

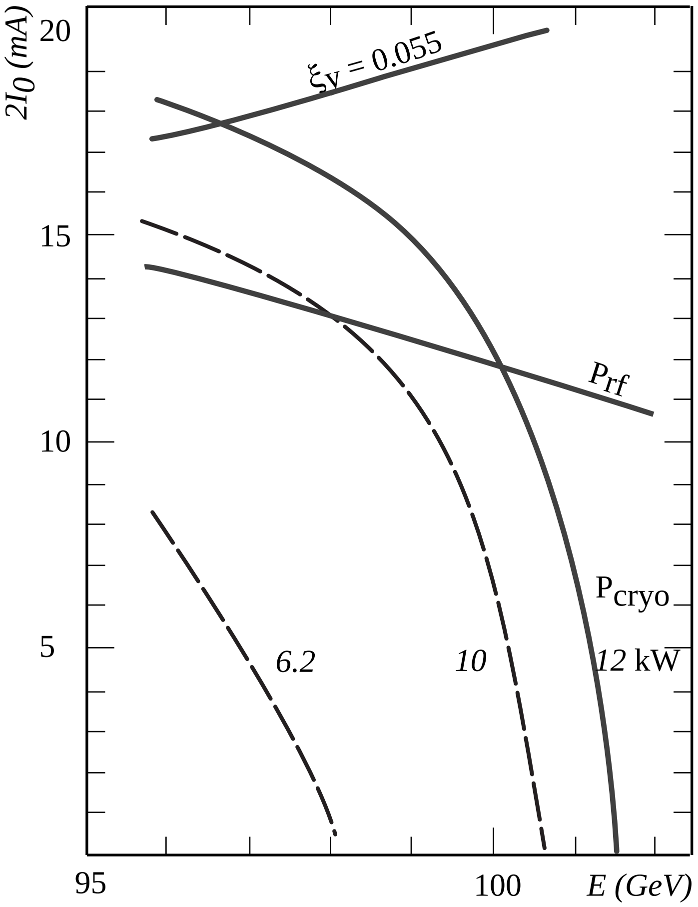

Fig. 4a - Limits on total beam current ($2I_0$)
for $\Delta\varphi_x = 102°, \Delta\varphi_y = 90°, J_x = 1.0, k_b = 8$
$\beta_x^* = 2.0\text{m}, \beta_y^* = 5\text{cm}, \kappa = 1\%$

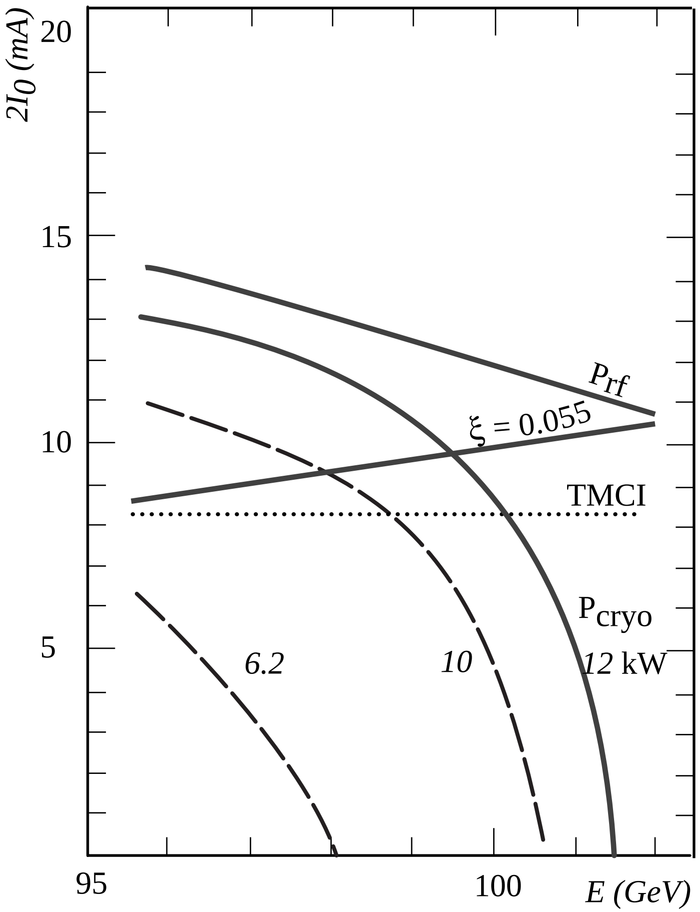

Fig. 4b - Limits on total beam current
for $\Delta\varphi_x = 102°, \Delta\varphi_y = 90°, J_x = 1.0, k_b = 4$
$\beta_x^* = 2.0\text{m}, \beta_y^* = 5\text{cm}, \kappa = 1\%$