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Dualities and Hidden Supersymmetry in String Quantum Cosmology

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Abstract

A supersymmetric approach to string quantum cosmology based on the non–compact, global duality symmetries of the effective action is developed. An $N = 2$ supersymmetric action is derived whose bosonic component is the Neveu–Schwarz/Neveu–Schwarz sector of the $(d+1)$–dimensional effective action compactified on a d–torus. A representation for the supercharges is found and the form of the zero–and one–fermion quantum states is determined. The purely bosonic component of the wavefunction is unique and manifestly invariant under the symmetry of the action. The formalism applies to a wide class of non–linear sigma–models.

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1 Introduction

Two approaches to the subject of quantum gravity that have received considerable attention in recent years are quantum cosmology [1] and the superstring theory [2]. In the canonical quantization of Einstein gravity, the classical Hamiltonian, $H = 0$, becomes a quantum operator. The physical state, Ψ, of the universe is then identified as the eigenstate of this operator with zero eigenvalue, $\hat{H}\Psi = 0$. This equation decouples into two components, $N^i \hat{H}_i \Psi = 0$ and $N\hat{H}_0\Psi = 0$, where N^i and N denote the shift and lapse functions, respectively. The first constraint implies the invariance of the wavefunction under spatial diffeomorphisms and the second is the Wheeler–DeWitt equation [1]. In the minisuperspace approximation, where inhomogeneous modes are frozen out before quantization, this is the only non–trivial constraint and can be solved, in principle, by imposing suitable boundary conditions [3].

String theory remains the most promising theory for a unification of the fundamental interactions. It is now widely thought that the five separate theories are non–perturbatively equivalent and are related by 'duality' symmetries, referred to as S–, T– and U–duality, respectively [4, 5, 6]. In general, these dualities are discrete subgroups of the non–compact, global symmetry groups of the low–energy effective supergravity actions. T–duality is a perturbative symmetry in the string coupling, but S–duality is non–perturbative. U–duality interchanges string and sigma–model coupling constants and, in this sense, represents a unification of S– and T–dualities.

Quantum gravitational effects would have played a key role in the very early universe and this represents one of the few environments where predictions of string theory may be quantitatively tested. A central paradigm of early universe cosmology is that of inflation, where the expansion briefly underwent a very rapid, acceleration. The above frameworks may be employed to study the very early universe and a question that naturally arises is whether they are compatible. At present, it is far from clear how such a question could be fully addressed. Consequently, a more pragmatic approach is to study how the unique features of string theory, such as its duality symmetries, may be employed to gain further insight into quantum cosmology, and vice–versa.

In string quantum cosmology, one solves the Wheeler–DeWitt equation derived from the tree–level string effective action [7, 8, 9]. The interpretive framework of quantum cosmology may then be employed to investigate whether string theory leads to realistic cosmologies and, in particular, whether inflation is probable. A quantum cosmological approach was recently advocated for solving the problem of how inflation ends in pre-big-bang string cosmology [10, 11, 12]. The well known factor ordering problem is also resolved in this approach because the symmetries of the action imply that the minisuperspace metric should be manifestly flat [10]. Moreover, these symmetries allow the Wheeler–DeWitt equation to be solved in general for a wide class of models [13]. For example, in the anisotropic Bianchi type IX model, the wavefunction becomes increasingly peaked around the isotropic limit at large spatial volumes [14].

When restricted to spatially flat, isotropic Friedmann–Robertson–Walker (FRW) cosmologies, the dilaton–graviton sector of the string effective action is invariant under an inversion of the scale factor and a shift in the dilaton field [15]. This 'scale factor duality' is a subgroup of T–duality and leads to a supersymmetric extension of the quantum cosmology, where the classical minisuperspace Hamiltonian may be viewed at the quantum level as the bosonic component of an $N = 2$ supersymmetric Hamiltonian [8, 16]. This is important because supersymmetric quantum cosmology may resolve the problems that arise in the standard approach in constructing a conserved, non–negative norm from the wavefunction. (For a recent review see, e.g., Refs. [17, 18]).

Thus, string quantum cosmology is well motivated. The purpose of the present paper is to develop a supersymmetric approach to quantum cosmology by employing the non–compact, global symmetries of the string effective action. All ten–dimensional string theories contain a dilaton, graviton and antisymmetric two–form potential in the Neveu–Schwarz/Neveu–Schwarz (NS–NS) sector of the theory. Furthermore, an interesting cosmology is the spatially flat and homogeneous, Bianchi type I universe admitting d compact Abelian isometries. We therefore consider the NS–NS sector of the effective action compactified on a d–torus. The reduced action is invariant under a global $\mathrm{O}(d,d)$ 'T–duality', where the scalar fields parametrize the coset $\mathrm{O}(d,d)/[\mathrm{O}(d)\times\mathrm{O}(d)]$. This leads to an $\mathrm{O}(d,d)$ invariant Wheeler–DeWitt equation [10].

The paper is organized as follows. After reviewing the derivation of the Wheeler–DeWitt equation in Section 2, we proceed in Section 3 to derive an $N = 2$ supersymmetric action whose bosonic component is $\mathrm{O}(d,d)$ invariant. The corresponding super–constraints on the wavefunction are then derived. These constraints are solved for the zero–fermion and one–fermion states in Section 4. We conclude in Section 5 with a discussion of how the analysis may be extended to a general class of non–linear sigma–models.

Unless otherwise stated, units are chosen such that $\hbar = c = 1$.

2 O(d,d) Invariant Wheeler–DeWitt Equation

2.1 Effective Action

The NS–NS sector of the $(d+1)$–dimensional, tree–level string effective action is given by [19]

$$S = \frac{1}{2\lambda_s^{d-1}} \int d^{d+1}x\sqrt{|g|}e^{-\Phi}\left[R + (\nabla\Phi)^2 - \frac{1}{12}H_{\alpha\beta\gamma}H^{\alpha\beta\gamma} + V\right], \qquad (2.1)$$

where the Yang–Mills fields are assumed to be trivial, Φ is the dilaton field, V is an interaction potential, R is the Ricci curvature scalar of the space–time with metric $\mathcal{G}$ and signature $(-,+,+,\ldots,+)$, $g \equiv \det\mathcal{G}$, $H_{\alpha\beta\gamma} \equiv \partial_{[\alpha}B_{\beta\gamma]}$ is the field strength of the antisymmetric two–form potential, $B_{\beta\gamma}$, and $\lambda_s \equiv (\alpha')^{1/2}$ is the fundamental string length scale.

We assume a spatially closed, flat, homogeneous (Bianchi type I) space–time, where the dilaton and two–form potential are constant on the surfaces of homogeneity, $t =$ constant. Without loss of generality, we may specify $\mathcal{G}_{00} = -1$ and $\mathcal{G}_{0i} = B_{0i} = 0$. Integrating over the spatial variables in Eq. (2.1) then implies that

$$S = \int d\tau \left[\bar{\Phi}'^2 + \frac{1}{8} \mathrm{Tr} \left(M'(M^{-1})' \right) + V e^{-2\bar{\Phi}} \right], \tag{2.2}$$

where

$$\bar{\Phi} \equiv \Phi - \frac{1}{2} \ln |g| \tag{2.3}$$

is the shifted dilaton field,

$$\tau \equiv \int^t dt_1 e^{\bar{\Phi}(t_1)} \tag{2.4}$$

is the 'dilaton' time parameter,

$$M \equiv \begin{pmatrix} G^{-1} & -G^{-1}B \\ BG^{-1} & G - BG^{-1}B \end{pmatrix} \tag{2.5}$$

is a symmetric $2d \times 2d$ matrix, G is the metric on the spatial hypersurfaces, a prime denotes differentiation with respect to τ and we have specified $\lambda_s \equiv 2$ [20]. The dilaton has also been shifted by the constant value $\Phi_0 = -\ln(\lambda_s^{-d} \int d^d x)$. The matrix M satisfies the conditions

$$M\eta M = \eta, \qquad M = M^T, \tag{2.6}$$

where

$$\eta \equiv \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \tag{2.7}$$

and I is the $d \times d$ unit matrix. It is therefore an element of the group $\mathrm{O}(d, d)$ and its inverse is given linearly by $M^{-1} = \eta M \eta$.

The kinetic sector of action (2.2) is invariant under a global $\mathrm{O}(d, d)$ transformation [20]:

$$\tilde{\bar{\Phi}} = \bar{\Phi}, \qquad \tilde{M} = \Omega^T M \Omega, \qquad \Omega^T \eta \Omega = \eta, \tag{2.8}$$

where Ω is a constant matrix. Since the shifted dilaton field transforms as a singlet under the action of Eq. (2.8), this symmetry is respected when V is an arbitrary function of $\bar{\Phi}$.

The classical Hamiltonian for this cosmological model is given by

$$H_{\mathrm{bos}} = \frac{1}{4} \Pi^2_{\bar{\Phi}} - 2\mathrm{Tr}(M \Pi_M M \Pi_M) - V e^{-2\bar{\Phi}}, \tag{2.9}$$

where

$$\Pi_{\bar{\Phi}} = 2\bar{\Phi}', \qquad \Pi_M = -\frac{1}{4}M^{-1}M'M^{-1} \tag{2.10}$$

are the momenta conjugate to $\bar{\Phi}$ and M, respectively. The equations of motion for the matrix M can be integrated directly to yield the first integral $M\eta M' = C$ [20], where C is a constant, $2d \times 2d$ matrix satisfying the conditions

$$C^T = -C, \qquad M\eta C = -C\eta M. \tag{2.11}$$

The first integral represents a conservation law and may also be written in terms of the conjugate momenta (2.10):

$$M\Pi_M = -\frac{1}{4}C\eta. \tag{2.12}$$

2.2 Quantum Cosmology

The cosmology is quantized by identifying the momenta (2.10) with the differential operators

$$\Pi_{\bar{\Phi}} = -i\frac{\delta}{\delta\bar{\Phi}}, \qquad \Pi_M = -i\frac{\delta}{\delta M}. \tag{2.13}$$

Substituting Eq. (2.13) into Eq. (2.9) then leads to the Wheeler–DeWitt equation [10]:

$$\left[\frac{\delta^2}{\delta\bar{\Phi}^2} + 8\mathrm{Tr}\left(\eta\frac{\delta}{\delta M}\eta\frac{\delta}{\delta M}\right) + 4Ve^{-2\bar{\Phi}}\right]\Psi(\bar{\Phi}, M) = 0. \tag{2.14}$$

However, a further constraint should also be imposed on the wavefunction because the matrix M belongs to the group $\mathrm{O}(d,d)$. This results in the conservation law (2.12) and is analogous to the 'rigid rotator' model for a particle moving in a spherically symmetric potential well. In this model, angular momentum is conserved due to a global O(3) symmetry. When states of definite angular momentum are considered, there arises a centrifugal barrier term in the effective action and we encounter a similar situation in the model considered above. The conservation law (2.12) is responsible for the analogue of the 'centrifugal barrier' term when the Wheeler-DeWitt equation is solved subject to the requirement that the wavefunction satisfies the necessary $\mathrm{O}(d,d)$ invariance properties. Thus, condition (2.12) should apply at the quantum cosmological level and this implies that the wavefunction should satisfy the first–order constraint [10]

$$iM\frac{\delta\Psi}{\delta M} = \frac{1}{4}C\eta\Psi. \tag{2.15}$$

In general, the constraint (2.15) can not be solved in closed form. On the other hand, it does imply that the wavefunction in Eq. (2.14) can be separated by specifying $\Psi(M,\bar{\Phi}) = X(M)Y(\bar{\Phi})$, where $X(M)$ and $Y(\bar{\Phi})$ are functions of M and $\bar{\Phi}$, respectively. The Wheeler–DeWitt equation then simplifies to an ordinary, differential equation in the shifted dilaton field:

$$\left[\frac{d^2}{d\bar{\Phi}^2} + B^2 + 4V(\bar{\Phi})e^{-2\bar{\Phi}}\right] Y(\bar{\Phi}) = 0, \tag{2.16}$$

where

$$B^2 \equiv \frac{1}{2}\mathrm{Tr}\,(C\eta)^2 \tag{2.17}$$

is a constant and represents the 'centrifugal barrier' term alluded to earlier. When $V(\bar{\Phi})$ is constant, Eq. (2.16) can be solved in full generality in terms of a linear superposition of Bessel functions [10].

3 Supersymmetric String Quantum Cosmology

3.1 N=2 Supersymmetry

In this Section we derive an $N = 2$ supersymmetric Lagrangian whose bosonic sector is invariant under global $\mathrm{O}(d, d)$ transformations. In supersymmetric point particle mechanics, the $N = 2$ case allows an interaction potential to be included. Homogeneous $N = 2$ supersymmetric quantum cosmologies coupled to a single scalar field have been studied previously in a different context [21, 22]. It has been further shown that the scale factor duality of isotropic FRW string cosmologies is associated with an $N = 2$ supersymmetry [8]. We extend previous analyses to the class of spatially flat, anisotropic (Bianchi type I) cosmologies including a non–trivial NS–NS two–form potential.

In formulating an $N = 2$ supersymmetric action, we consider superfields of the generic form

$$X^\mu(\tau, \theta, \bar{\theta}) \equiv x^\mu(\tau) + i\bar{\psi}^\mu(\tau)\theta + i\psi^\mu(\tau)\bar{\theta} + F^\mu(\tau)\theta\bar{\theta}, \tag{3.1}$$

where the bosonic functions, $\{x^\mu(\tau), F^\mu(\tau)\}$, and anticommuting complex spinor functions, $\{\psi^\mu(\tau), \bar{\psi}^\mu(\tau)\}$, are arbitrary functions of the dilaton time (2.4), $\{\theta, \bar{\theta}\}$ are constant, anticommuting, complex spinors and μ is a parameter labelling the degrees of freedom in minisuperspace.

The generators for the supersymmetry are defined by

$$\begin{aligned} \hat{Q}_1 &\equiv -\frac{\partial}{\partial\bar{\theta}} - i\theta\frac{\partial}{\partial\tau} \\ \hat{Q}_2 &\equiv \frac{\partial}{\partial\theta} + i\bar{\theta}\frac{\partial}{\partial\tau} \end{aligned} \tag{3.2}$$

and the supersymmetry transformation rule for the superfield (3.1) is

$$\delta X^\mu = -i\left(\xi_1\hat{Q}_1 + \xi_2\hat{Q}_2\right) X^\mu, \tag{3.3}$$

where ξ_i are arbitrary parameters that commute with the bosonic variables and anticommute with all fermionic variables.

We now define the superfields

$$m_{ij}(\tau,\theta,\bar{\theta}) \equiv M_{ij}(\tau) + i\bar{\psi}_{ij}(\tau)\theta + i\psi_{ij}(\tau)\bar{\theta} + F_{ij}(\tau)\theta\bar{\theta} \tag{3.4}$$

$$\mathcal{D}(\tau,\theta,\bar{\theta}) \equiv \sqrt{2}\bar{\Phi}(\tau) + i\bar{\chi}(\tau)\theta + i\chi(\tau)\bar{\theta} + f(\tau)\theta\bar{\theta}, \tag{3.5}$$

where $M_{ij}(\tau)$ is given by Eq. (2.5), $\{\psi_{ij},\bar{\psi}_{ij},\chi,\bar{\chi}\}$ are anticommuting, complex spinors and $(i,j) = (1,2,\ldots,2d)$. The spatial metric and antisymmetric, two–form potential determine the bosonic component of the superfield (3.4) and the shifted dilaton field (2.3) plays the equivalent role in Eq. (3.5).

We then define two further superfields in terms of Eqs. (3.4) and (3.5):

$$\Sigma(\tau,\theta,\bar{\theta}) \equiv \frac{1}{8}\hat{D}_1 m_{ij}\eta^{jk}\hat{D}_2 m_{kl}\eta^{li} \tag{3.6}$$

$$Y(\tau,\theta,\bar{\theta}) \equiv \frac{1}{2}\hat{D}_1\mathcal{D}\hat{D}_2\mathcal{D} - W(\mathcal{D}), \tag{3.7}$$

where the derivative operators are

$$\hat{D}_1 \equiv -\frac{\partial}{\partial\bar{\theta}} + i\theta\frac{\partial}{\partial\tau} \tag{3.8}$$

$$\hat{D}_2 \equiv \frac{\partial}{\partial\theta} - i\bar{\theta}\frac{\partial}{\partial\tau} \tag{3.9}$$

and the potential, $W(\mathcal{D})$, is an arbitrary function of $\mathcal{D}$.

The sum of Eqs. (3.6) and (3.7) is viewed as an effective Lagrangian in the action:

$$I_{\rm SUSY} \equiv \int d\tau \int d\theta d\bar{\theta}\,(\Sigma + Y)\,. \tag{3.10}$$

It may be verified by substituting Eqs. (3.4) and (3.5) into Eqs. (3.6) and (3.7), expanding the potential $W(\mathcal{D})$ around $\bar{\Phi}$, and collecting coefficients in the Grassmann variables θ and $\bar{\theta}$ that the action (3.10) is invariant under the supersymmetry transformations (3.3) up to a total time derivative in the $\theta\bar{\theta}$ coefficient. This coefficient is given by $L \equiv L_g + L_1$, where

$$L_g = \frac{1}{8}\left[i\psi_{ij}\eta^{jk}\bar{\psi}'_{kl}\eta^{li} - i\psi'_{ij}\eta^{jk}\bar{\psi}_{kl}\eta^{li} + M'_{ij}\eta^{jk}M'_{kl}\eta^{li}\right] \tag{3.11}$$

$$L_1 = \left(\bar{\Phi}'\right)^2 + \frac{i}{2}\left(\bar{\chi}\chi' - \bar{\chi}'\chi\right) + \frac{1}{2}f^2 - \frac{1}{\sqrt{2}}f\partial_{\bar{\Phi}}W - \frac{1}{4}\left(\partial^2_{\bar{\Phi}}W\right)[\bar{\chi},\chi]_- \tag{3.12}$$

and $W = W(\bar{\Phi})$. There is an additional term in Eq. (3.11) of the form $F_{ij}\eta^{jk}F_{kl}\eta^{li}$, but since there is no potential contribution in the action from the matrix (2.5), its equation of motion implies that we may specify $F_{ij} = 0$ without loss of generality. On the other hand, the equation of motion for the auxiliary field, f, is given by

$$f = \frac{1}{\sqrt{2}}\partial_{\bar{\Phi}}W \tag{3.13}$$

and substituting Eq. (3.13) into Eq. (3.12) eliminates this field from the action.

Integrating over the Grassmann variables in Eq. (3.10) therefore implies that the $N=2$ supersymmetric action is given by

$$
\begin{aligned}
I_{\rm SUSY} = \int d\tau \Big[& \frac{1}{8}\left(i\psi_{ij}\eta^{jk}\bar{\psi}'_{kl}\eta^{li} - i\psi'_{ij}\eta^{jk}\bar{\psi}_{kl}\eta^{li} + M'_{ij}\eta^{jk}M'_{kl}\eta^{li}\right) \\
& + \left(\bar{\Phi}'\right)^2 + \frac{i}{2}\left(\bar{\chi}\chi' - \bar{\chi}'\chi\right) - \frac{1}{4}\left(\partial_{\bar{\Phi}}W\right)^2 - \frac{1}{4}\left(\partial^2_{\bar{\Phi}}W\right)[\bar{\chi},\chi]_- \Big] . \qquad (3.14)
\end{aligned}
$$

The action (3.14) reduces to the bosonic action (2.2) in the limit where the Grassmann variables vanish if we identify the potential W:

$$
\left(\partial_{\bar{\Phi}}W\right)^2 = -4V(\bar{\Phi})e^{-2\bar{\Phi}}. \qquad (3.15)
$$

Thus, a necessary condition for an $N=2$ supersymmetric extension of the effective action (2.2) is that the interaction potential, V, must be semi–negative definite.

The classical momenta conjugate to the bosonic and fermionic degrees of freedom in action (3.14) are

$$
\begin{aligned}
\Pi_{M_{mn}} &= \frac{\partial L}{\partial M'_{mn}} = \frac{1}{4}\eta^{nk}M'_{kl}\eta^{lm} \\
K_{mn} &= \frac{\partial L}{\partial \psi'_{mn}} = -\frac{i}{8}\eta^{nk}\bar{\psi}_{kl}\eta^{lm} \\
\bar{K}_{mn} &= \frac{\partial L}{\partial \bar{\psi}'_{mn}} = -\frac{i}{8}\psi_{ij}\eta^{jm}\eta^{ni} \\
\Pi_{\bar{\Phi}} &= \frac{\partial L}{\partial \bar{\Phi}'} = 2\bar{\Phi}' \\
\Pi_{\chi} &= \frac{\partial L}{\partial \chi'} = -\frac{i}{2}\bar{\chi} \\
\Pi_{\bar{\chi}} &= \frac{\partial L}{\partial \bar{\chi}'} = -\frac{i}{2}\chi, \qquad (3.16)
\end{aligned}
$$

respectively, where the negative sign appears in the expressions for $\bar{\psi}_{ij}$ and χ because the left derivative of the Grassmann variables is taken. The classical Hamiltonian derived from the action (3.14) is given by

$$
H = M'_{ij}\eta^{jk}\Pi_{M_{kl}}\eta^{li} + \psi'_{ij}\eta^{jk}K_{kl}\eta^{li} + \bar{\psi}'_{ij}\eta^{jk}\bar{K}_{kl}\eta^{li} + \bar{\Phi}'\Pi_{\bar{\Phi}} + \chi\Pi_{\chi} + \bar{\chi}\Pi_{\bar{\chi}} - L \qquad (3.17)
$$

and substituting Eqs. (3.16) into Eq. (3.17) implies that it takes the form

$$
H = 2\Pi_{ij}\eta^{jk}\Pi_{kl}\eta^{li} + \frac{1}{4}\Pi^2_{\bar{\Phi}} + \frac{1}{4}\left(\partial_{\bar{\Phi}}W\right)^2 + \frac{1}{4}\left(\partial^2_{\bar{\Phi}}W\right)[\bar{\chi},\chi]_- , \qquad (3.18)
$$

where the anticommuting property of the Grassmann variables has been employed and $\Pi_{ij} \equiv \Pi_{M_{ij}}$. The bosonic component of the Hamiltonian (3.18) corresponds to Eq. (2.9). The first term in this expression describes the Hamiltonian for the matrix M_{ij}. The fermions do not appear in this component of the Hamiltonian, as is always the case in supersymmetric quantum mechanics when the fermions are free.

3.2 Quantum Constraints

The model is quantized by assuming the standard operator realizations for the bosonic variables in Eq. (2.13) and further imposing the spinor algebra

$$[\psi_{ij}, \psi_{kl}]_+ = \left[\bar{\psi}_{ij}, \bar{\psi}_{kl}\right]_+ = 0, \qquad \left[\psi_{ij}, \bar{\psi}_{kl}\right]_+ = \eta_{ik}\eta_{jl}$$
$$[\chi, \chi]_+ = [\bar{\chi}, \bar{\chi}]_+ = 0, \qquad [\chi, \bar{\chi}]_+ = 1$$
$$\left[\bar{\chi}, \bar{\psi}_{ij}\right]_+ = [\chi, \psi_{ij}]_+ = \left[\chi, \bar{\psi}_{ij}\right]_+ = [\bar{\chi}, \psi_{ij}]_+ = 0. \tag{3.19}$$

A representation satisfying Eq. (3.19) is given in terms of the set of Grassmann variables $\{\zeta_{ij}, \beta\}$:

$$\psi_{kl} = \eta_{kp}\frac{\partial}{\partial\zeta_{pr}}\eta_{rl}, \qquad \bar{\psi}_{ij} = \zeta_{ij}$$
$$\chi = \frac{\partial}{\partial\beta}, \qquad \bar{\chi} = \beta. \tag{3.20}$$

Notice that we have imposed $[\psi_{ij}, \bar{\psi}_{kl}]_+ = \eta_{ik}\eta_{jl}$ in Eq. (3.19). The canonically conjugate momenta for ψ_{ij} and $\bar{\psi}_{kl}$ are given by Eq. (3.16) and, if we had employed the canonical anticommutation relations between ψ_{ij} and K_{mn} and between $\bar{\psi}_{ij}$ and $\bar{K}_{mn}$, we would not have obtained the anticommutation relation presented in Eq. (3.19). We have implicitly employed $[\psi_{ij}, K_{kl}]_+ = 4\eta_{ik}\eta_{jl}$ and $[\bar{\psi}_{ij}, \bar{K}_{kl}]_+ = 4\eta_{ik}\eta_{jl}$ rather than 'one' on the right hand side. This is due to the fact that there is a factor of 1/8 in the kinetic energy terms of ψ_{ij} and $\bar{\psi}_{kl}$ rather than the conventional factor of 1/2 that appears in the Dirac Langrangian. (For example, the χ and $\bar{\chi}$ pieces in Eq. (3.14) have the standard factor). We adopted the standard anticommutation relation between ψ_{ij} and $\bar{\psi}_{kl}$ because the calculations can then be performed in a straightforward manner without keeping track of additional numerical constants. If the anticommutation relation in Eq. (3.19) had been modified with another numerical constant, our expressions for the supercharges Q and $\bar{Q}$ given below would also have been modified. However, an identical expression for the Hamiltonian would have been obtained from the anticommutator of Q and $\bar{Q}$.

We now define the supercharges

$$Q \equiv 2\Pi_{ij}\eta^{jk}\psi_{kl}\eta^{li} + \frac{1}{\sqrt{2}}\left(\Pi_{\Phi} + i\partial_{\Phi}W\right)\chi \tag{3.21}$$

$$\bar{Q} \equiv 2\Pi_{mn}\eta^{nr}\bar{\psi}_{rp}\eta^{pm} + \frac{1}{\sqrt{2}}\left(\Pi_{\Phi} - i\partial_{\Phi}W\right)\bar{\chi}, \tag{3.22}$$

where Q is a non–Hermitian, linear operator and $\bar{Q}$ is its adjoint. Substituting Eqs. (3.21) and (3.22) into Eq. (3.18), and employing the anticommutation relations (3.19), implies that the Hamiltonian operator may be written as

$$2H = \left[Q, \bar{Q}\right]_+, \qquad Q^2 = \bar{Q}^2 = 0, \tag{3.23}$$

where $[H, Q]_- = [H, \bar{Q}]_- = 0$. Thus, there exists an $N = 2$ supersymmetry in the quantum cosmology [16, 23]. This may be viewed as a direct extension of the $\mathrm{O}(d, d)$ 'T–duality' of the toroidally compactified NS–NS action (2.1).

Finally, supersymmetry implies that the wavefunction of the universe is annihilated by the supercharges, $Q\Psi = \bar{Q}\Psi = 0$. These reduce to a set of first–order differential equations and we derive solutions to these constraints in the following Section.

We conclude this Section by remarking that factor ordering problems in supersymmetric quantum mechanics in curved space have been addressed previously [24]. In general, four fermion terms with the curvature tensor appear, but we have not considered such problems here because we assumed a toroidal compactification and this implies that the curvature is zero. Moreover, as discussed by Gasperini *et al.* [10], the operator ordering issue is settled in the standard procedure by demanding the $\mathrm{O}(d, d)$ invariance of the Hamiltonian.

4 Quantum States

4.1 Zero–fermion State

The quantum constraints are solved by defining the conserved 'fermion number':

$$F \equiv \bar{\psi}_{ij}\eta^{jk}\psi_{kl}\eta^{li} + \bar{\chi}\chi, \tag{4.1}$$

where

$$[H, F]_- = 0, \qquad [Q, F]_- = Q, \qquad [\bar{Q}, F]_- = -\bar{Q}. \tag{4.2}$$

This implies that states with a fixed fermion number may be individually considered. The fermion vacuum, $|0\rangle$, is defined:

$$\psi_{ij}|0\rangle = \chi|0\rangle = 0 \qquad \forall \qquad i, j. \tag{4.3}$$

The state with zero fermion number, $|\psi_0\rangle$, is defined as $|\psi_0\rangle \equiv h(M_{ij}, \bar{\Phi})|0\rangle$, where h is an arbitrary function. This state is a function of the bosonic degrees of freedom only and is automatically annihilated by the supercharge Q. It is annihilated by its adjoint, $\bar{Q}$, if the conditions

$$\frac{\delta h}{\delta M_{ij}} = 0, \qquad \left[\frac{\partial}{\partial\bar{\Phi}} + \frac{dW}{d\bar{\Phi}}\right] h = 0 \tag{4.4}$$

are simultaneously satisfied. Modulo a constant of proportionality, the general solution to Eq. (4.4) is

$$|\psi_0\rangle = e^{-W(\bar{\Phi})}|0\rangle. \tag{4.5}$$

This solution is *uniquely* determined by the potential (3.15) and is a function of the shifted dilaton field only. It is therefore manifestly invariant under the global $\mathrm{O}(d, d)$ symmetry (2.8) of the bosonic action (2.2).

It is interesting to relate Eq. (4.5) to solutions of the Euclidean Hamilton–Jacobi equation. This is given by

$$\left(\frac{\delta I}{\delta\bar{\Phi}}\right)^2 - 8M_{ij}\frac{\delta I}{\delta M_{jk}}M_{kl}\frac{\delta I}{\delta M_{li}} = -4V(\bar{\Phi})e^{-2\bar{\Phi}}, \tag{4.6}$$

where $I = I(M_{ij}, \bar{\Phi})$ is a Euclidean action of the classical theory. The Euclidean analogue of the momentum constraint (2.12) implies that separable solutions to Eq. (4.6) can be found by substituting in the ansatz $I(M_{ij}, \bar{\Phi}) = F(M_{ij}) + G(\bar{\Phi})$, where F and G are functions of M_{ij} and $\bar{\Phi}$, respectively. Although a closed expression for $F(M_{ij})$ can not be determined in general, the form of $G(\bar{\Phi})$ can be found in terms of quadratures for an arbitrary dilaton potential $V(\bar{\Phi})$. It follows that

$$G(\bar{\Phi}) = \int^{\bar{\Phi}} d\bar{\Phi}_1 \left[B^2 - 4V(\bar{\Phi}_1)e^{-2\bar{\Phi}_1}\right]^{1/2}, \tag{4.7}$$

where B^2 is defined in Eq. (2.17).

Comparison with Eq. (3.15) therefore implies that the exponent, $W(\bar{\Phi})$, in Eq. (4.5) may be interpreted as a Euclidean action of the cosmology when $B^2 = 0$ and I is independent of M_{ij}. This corresponds to the case where the momenta conjugate to M_{ij} vanish. The bosonic wavefunction (4.5) represents an approximate WKB Euclidean solution to the Wheeler–DeWitt equation (2.14) when $B^2 = 0$ but it is an exact state in the supersymmetric quantization, and moreover, is uniquely selected by the symmetry.

To lowest–order, the potential in the (super–) string effective action (2.1) is a cosmological constant determined by the dimensionality of space–time, $V \equiv \Lambda = 2(d-9)/3\alpha'$. In this case the cosmological constant is related to the central charge deficit. We can put forward an argument that supersymmetric quantum cosmology preserving the $\mathrm{O}(d,d)$ T-duality implies an upper bound, $d \leq 9$, on the number of spatial dimensions in the universe. If one interprets $P = |\Psi|^2$ as an unrenormalized probability density, then $P \propto \exp(-4|\Lambda|^{1/2}\Omega_V g_s^{-2})$, where Ω_V is the proper spatial volume and $g_s \equiv e^{\Phi/2}$ is the string coupling. It is interesting that the probability density is peaked in the strong–coupling regime. Furthermore, for fixed coupling, the value of P increases as $\Lambda \to 0^-$. In this sense, therefore, smaller values of $|\Lambda|$ are favoured.

4.2 One–fermion State

We now proceed to find the form of the one–fermion state, $|\psi_1\rangle$. A general ansatz for this component of the wavefunction is given in terms of the fermion vacuum by

$$|\psi_1\rangle = \left(f_{ij}\eta^{jk}\bar{\psi}_{kl}\eta^{li} + f_\chi\bar{\chi}\right)e^{-W}|0\rangle, \tag{4.8}$$

where f_{ij} and f_χ are arbitrary functions of the bosonic variables over the configuration space. Operating on this state with $\bar{Q}$ and employing Eq. (3.19) implies that it is annihilated if $f_{ij} \equiv 2\Pi_{ij} f$ and $f_\chi \equiv (\Pi_{\bar\Phi} - i\partial_{\bar\Phi} W) f/\sqrt{2}$, respectively, where $f = f(M_{ij}, \bar\Phi)$ is an arbitrary function. Substituting these definitions into Eq. (4.8) then implies that

$$|\psi_1\rangle \equiv \bar{Q} f e^{-W(\bar\Phi)} |0\rangle. \tag{4.9}$$

Operating on Eq. (4.9) with Q and employing Eq. (3.23) implies that

$$Q|\psi_1\rangle = 2Hfe^{-W}|0\rangle = 0. \tag{4.10}$$

By employing Eqs. (3.19) and (4.3), it follows that $[\bar\chi, \chi]_-|0\rangle = -|0\rangle$. Eq. (4.10) is therefore satisfied if f is a solution to the differential equation:

$$\left[2\frac{\delta}{\delta M_{ij}} \eta_{jk} \frac{\delta}{\delta M_{kl}} \eta_{li} + \frac{1}{4}\frac{\partial^2}{\partial \bar\Phi^2} - \frac{1}{2}\frac{dW}{d\bar\Phi}\frac{\partial}{\partial\bar\Phi} \right] f(M_{ij}, \bar\Phi) = 0. \tag{4.11}$$

For a separable solution, $f \equiv X(M_{ij})Y(\bar\Phi)$, Eq. (4.11) implies that

$$\left[\frac{d^2}{d\bar\Phi^2} - 2\frac{dW}{d\bar\Phi}\frac{d}{d\bar\Phi} + c^2 \right] Y(\bar\Phi) = 0 \tag{4.12}$$

and

$$\left[\frac{\delta}{\delta M_{ij}} \eta_{jk} \frac{\delta}{\delta M_{kl}} \eta_{li} - \frac{c^2}{8} \right] X(M_{ij}) = 0, \tag{4.13}$$

where c is a separation constant. In the special case where $c = 0$, Eq. (4.12) may be solved exactly:

$$Y = \int^{\bar\Phi} d\bar\Phi_1 e^{2W(\bar\Phi_1)} \tag{4.14}$$

for an arbitrary potential $W(\bar\Phi)$.

In general, Eq. (4.12) can not be solved in closed form for arbitrary $W(\bar\Phi)$ and c. However, for a constant dilaton potential, $V = \Lambda$, we may define the new variable

$$z \equiv \frac{1}{4|\Lambda|^{1/2}} e^{\bar\Phi} \tag{4.15}$$

This implies that Eq. (4.12) takes the form

$$\left[z^2 \frac{d^2}{dz^2} + (z-1)\frac{d}{dz} + c^2 \right] Y = 0 \tag{4.16}$$

and Eq. (4.16) can be solved in the limit $z \gg 1$. A detailed study of this equation is beyond the scope of the present work, however.

4.3 An SL(2,R) Subgroup

Thus far, we have constructed the $\mathrm{O}(d,d)$ invariant supersymmetric Hamiltonian and derived the corresponding super–constraint equations in the general setting. Moreover, we have obtained the state in the zero fermion sector that is annihilated by the supercharges. This is uniquely determined by the potential given in Eq. (3.15). It is rather difficult to proceed further with the general form of the M–matrix (2.5), however. In view of this, we now consider a more simple scenario, where we deal with an $\mathrm{SL}(2,R)$ matrix corresponding to a subgroup of $\mathrm{O}(d,d)$.

The $\mathrm{O}(d,d)$ group may be written as a product of $\mathrm{SL}(2,R)$ subgroups when the components of the metric and two–form potential are identified in an appropriate fashion. For example, there are three $\mathrm{SL}(2,R)$ subgroups for $d=6$ [25]. Here we consider just one of them to illustrate our main points. It is then possible to make further progress in finding the solutions to the Wheeler–DeWitt equation. It is worth remarking at this stage that the group $\mathrm{SL}(2,R)$ also appears within the context of the S-duality group and thus, as we shall discuss later, this analysis is useful in the context of type IIB string cosmology. The $\mathrm{SL}(2,R)$ subgroup we consider here is part of the T–duality group.

It is well known that an $\mathrm{SL}(2,R)$ matrix can be introduced with unit determinant:

$$T \equiv \begin{pmatrix} e^{q\Delta} + e^{-q\Delta}\mathcal{P}^2 & e^{-q\Delta}\mathcal{P} \\ e^{-q\Delta}\mathcal{P} & e^{-q\Delta} \end{pmatrix}, \tag{4.17}$$

where q is a constant. The two scalar moduli fields, Δ and $\mathcal{P}$, parametrise the coset $\mathrm{SL}(2,R)/\mathrm{SO}(2)$. Let us consider another $\mathrm{SL}(2,R)$ matrix

$$V = \begin{pmatrix} e^{\frac{1}{2}q\Delta} & 0 \\ e^{-\frac{1}{2}q\Delta}\mathcal{P} & e^{-\frac{1}{2}q\Delta} \end{pmatrix}. \tag{4.18}$$

The matrix V is also of unit determinant and can be thought of intuitively as the 'vielbein' of the $\mathrm{SL}(2,R)$ metric, whereas T is like the metric itself, since $T = V^T V$. Notice that under a global $\mathrm{SL}(2,R)$ transformation, $\mathcal{G}$, and a local $\mathrm{SO}(2)$ transformation, O, $V \to OV\mathcal{G}$. Thus, for a given $\mathcal{G}$, an O can always be chosen that preserves the form of V. Thus, the symmetric matrix, T, transforms as $T \to \mathcal{G}^T T \mathcal{G}$.

The action (2.2) can be written in an $\mathrm{SL}(2,R)$ invariant form by replacing the kinetic term for the M–matrix by $\mathrm{Tr}[T'(T^{-1})']/4$. The inverse of T may be written linearly as $T^{-1} = -JTJ$, where

$$J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad J^2 = -I. \tag{4.19}$$

The classical equations of motion for the moduli fields then imply that momentum conjugate to $\mathcal{P}$, $\Pi_{\mathcal{P}} = -\mathcal{P}' \exp(-2q\Delta)$, is conserved. Thus, the quantum constraint (2.15) takes the simple form $i\partial\Psi/\partial\mathcal{P} = L_{\mathcal{P}}\Psi$, where $L_{\mathcal{P}}$ is an arbitrary constant. For separable solutions, $\Psi \equiv X(T)Y(\bar{\Phi})$, the $X(T)$ component of the wavefunction can then be evaluated by separating the Wheeler–DeWitt equation (2.14). The component form of Eq. (2.14) may be derived by identifying M_{ij} and η_{kl} with Eqs. (4.17) and (4.19), respectively. Alternatively, for this model it may be derived directly at the level of the Lagrangian. It follows that

$$\left[\frac{1}{2q^2}\frac{\partial^2}{\partial\Delta^2} + \frac{1}{2}e^{2q\Delta}\frac{\partial^2}{\partial\mathcal{P}^2} + B^2\right] X(T) = 0 \tag{4.20}$$

and the general solution to this equation that is consistent with the first–order momentum constraint (2.15) is given by

$$X = e^{-iL_{\mathcal{P}}\mathcal{P}} Z_{\sqrt{2}B}\left(L_{\mathcal{P}}e^{q\Delta}\right), \tag{4.21}$$

where $Z_{\sqrt{2}B}$ is a linear combination of modified Bessel functions of order $\sqrt{2}B$.

The supersymmetric extension may be performed for this model as outlined in Section 3 by defining a superfield, $N_{ij} \equiv T_{ij} + i\bar{A}_{ij}\theta + iA_{ij}\bar{\theta} + C_{ij}\theta\bar{\theta}$, analogous to Eq. (3.4). Similar conclusions therefore apply and, in particular, the structure of Eq. (4.13) for the one–fermion state is formally equivalent to that of Eq. (4.20). Thus, the solutions are again given in terms of modified Bessel functions.

5 Conclusion and Discussion

In this paper, we have derived an $N = 2$ supersymmetric quantum cosmology from the toroidally compactified string effective action. The supersymmetric Hamiltonian operator reduces to the $\mathrm{O}(d,d)$ invariant Hamiltonian in the classical limit. The existence of the supersymmetry imposes strong constraints on the wavefunction of the universe and implies that it should by annihilated by the supercharges. These are first–order constraints and correspond to the Dirac–type square root of the Wheeler–DeWitt equation (2.14). Solutions to these constraints were found for the zero– and one–fermion states. The general form of the bosonic component of the wavefunction was determined and found to be invariant under the non–compact, global $\mathrm{O}(d,d)$ symmetry (T–duality) of the classical action.

The supersymmetric approach we have employed may be applied to a wide class of non–linear sigma–models. The generalized sigma–model action in the 'Einstein' frame is given by [26]

$$S = \int d^4x\sqrt{-g}\left[R - \frac{1}{2}\gamma^{ij}(\phi)\nabla^\mu\phi_i\nabla_\mu\phi_j - 2\Lambda\right], \tag{5.1}$$

where the scalar fields $\{\phi_i\}$ may be viewed as coordinates on a target space with metric γ_{ij}, Λ is a cosmological constant and units are chosen such that $16\pi G \equiv 1$. We assume that the target space is a non–compact, Riemannian, symmetric space G/H, where G is a non–compact Lie group with a maximal compact subgroup H [27]. Eq. (5.1) represents the bosonic sector of many four–dimensional supergravity theories when the gauge fields are trivial and $\Lambda = 0$, including type II theories compactified to four dimensions and all those with $N \geq 4$ supersymmetry [5]. The metric γ_{ij} may be written as

$$dl^2 = \gamma_{ij} d\phi^i d\phi^j \equiv \mathrm{Tr}\left(dN dN^{-1}\right), \tag{5.2}$$

where the matrix N is an element of the group G. Substituting Eq. (5.2) into Eq. (5.1) then implies that

$$S = \int d^4x \sqrt{-g} \left[R - \frac{1}{2}\mathrm{Tr}\left(\nabla N \nabla N^{-1}\right) - 2\Lambda\right]. \tag{5.3}$$

We now consider the spatially homogeneous and isotropic FRW universes with a line element given by $ds^2 = -dt^2 + e^{2\alpha(t)} d\Omega_k^2$, where $d\Omega_k^2$ is the line element on the three–space with constant curvature, $k = \{-1, 0, +1\}$ for negatively curved, flat and positively curved models, respectively, and $a(t) \equiv e^{\alpha(t)}$ is the scale factor of the universe. Integrating over the spatial variables in action (5.3) implies that

$$S = \int dT \left[-6(\partial_T \alpha)^2 + \frac{1}{2}\mathrm{Tr}\left(\partial_T N \partial_T N^{-1}\right) - V_{\mathrm{eff}}(\alpha)\right], \tag{5.4}$$

where $\phi_i = \phi_i(t)$, ∂_T denotes differentiation with respect to the rescaled time variable

$$T \equiv \int^t dt_1 e^{-3\alpha(t_1)} \tag{5.5}$$

and

$$V_{\mathrm{eff}}(\alpha) \equiv 2\Lambda e^{6\alpha} - 6k e^{4\alpha}. \tag{5.6}$$

Eqs. (2.2) and (5.4) are formally very similar and, for constant Λ, the latter is invariant under the global action of the group G:

$$\bar{\alpha} = \alpha, \qquad \bar{N} = \Omega^T N \Omega, \tag{5.7}$$

where $\Omega \in G$ is a constant matrix. Since the logarithm of the scale factor transforms as a singlet, its role is equivalent to that of the shifted dilaton field (2.3) and Eq. (5.6) then represents a G–invariant effective potential for the scale factor. This implies that supersymmetric quantum cosmologies may be derived from the non–linear sigma–model (5.4) when the matrix N satisfies appropriate linearity conditions analogous to Eq. (2.6). In the $\mathrm{O}(d,d)$ model, the metric (2.7) satisfies $\eta^2 = I$ and, together with Eq. (2.6), this implies the important relation $M^{-1} = \eta M \eta$. Consequently, the target space metric may be written uniquely in terms of M and η. Thus, the analysis of Section 3 applies directly to *all* non–linear sigma–models where N is symmetric and its inverse is given linearly by

$$N^{-1} = \pm \theta N \theta, \qquad \theta^2 = \pm I, \qquad \theta \in G. \tag{5.8}$$

For example, it has recently been conjectured that the five string theories have a common origin in a new quantum theory, referred to as M–theory [6]. The low–energy limit of this theory is $N=1$, eleven–dimensional supergravity and this leads to $N=8$ supergravity after toroidal compactification to four dimensions [28]. The bosonic sector of this theory admits 28 Abelian vector gauge fields and 70 scalar fields that take values in the homogeneous coset space $\mathrm{E}_{7(7)}/[\mathrm{SU}(8)/\mathrm{Z}_2]$. The discrete subgroup $\mathrm{E}_7(Z)$ is the conjectured U–duality of type II string theory compactified on a six–torus [5]. When the gauge fields are frozen, the effective action has the form given by Eq. (5.3) with $\Lambda=0$. In this case, N is a symmetric matrix in $\mathrm{E}_{7(7)}$ that may be viewed as a positive metric in the internal space corresponding to the 56–dimensional fundamental representation of $\mathrm{E}_{7(7)}$ [28]. The symplectic invariant of $\mathrm{E}_{7(7)}$ is

$$\theta = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}, \qquad \theta^2 = \begin{pmatrix} -I & 0 \\ 0 & -I \end{pmatrix}, \tag{5.9}$$

where I is the 28×28 unit matrix and the linearity condition (5.8) is therefore satisfied for this model.

In view of various evidences that M-theory provides intricate relations among the five string theories, there have been attempts to explore the cosmological implications of M–theory and one is naturally led to study cosmological scenarios in type IIA and IIB theories with the effects of the Ramond-Ramond sector included [29]. It would be interesting to investigate quantum cosmologies for M-theory and string theories along the lines followed in this work.

When $d=3$, Eq. (2.1) also exhibits an $\mathrm{SL}(2,R)$ 'S–duality' [30]. In four dimensions, the three–form field strength of the NS–NS two–form potential is dual to a one–form corresponding to the field strength of a pseudo–scalar axion field, σ:

$$H^{\alpha\beta\gamma} \equiv e^{\Phi}\epsilon^{\alpha\beta\gamma\delta}\nabla_\delta\sigma, \tag{5.10}$$

where $\epsilon^{\alpha\beta\gamma\delta}$ is the covariantly constant antisymmetric four–form. Performing the conformal transformation

$$\tilde{g}_{\mu\nu} = \Theta^2 g_{\mu\nu}, \qquad \Theta^2 \equiv e^{-\Phi} \tag{5.11}$$

implies that the NS–NS action (2.1) transforms to

$$S = \int d^4x\sqrt{-\tilde{g}}\left[\tilde{R} - \frac{1}{2}\left(\tilde{\nabla}\Phi\right)^2 - \frac{1}{2}e^{2\Phi}\left(\tilde{\nabla}\sigma\right)^2\right] \tag{5.12}$$

and this may be written in the form of Eq. (5.3) with $\Lambda=0$ by defining

$$N \equiv \begin{pmatrix} e^{\Phi} & \sigma e^{\Phi} \\ \sigma e^{\Phi} & e^{-\Phi}+\sigma^2 e^{\Phi} \end{pmatrix}. \tag{5.13}$$

The dilaton and axion fields parametrize the $\mathrm{SL}(2,R)/\mathrm{U}(1)$ coset. Since the inverse of the symmetric matrix (5.13) is given by $N^{-1}=-JNJ$, where J is defined in Eq.

(4.19), a supersymmetric extension of the FRW cosmologies may also be developed for this model.

In conclusion, therefore, $N = 2$ supersymmetric quantum cosmologies may be derived from the non–linear sigma–models associated with S–, T– and U–dualities of string theory.

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