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Bound states and resonances in the scalar sector of the MSSM

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Abstract

The trilinear couplings of squarks and sleptons to the Higgs bosons can give rise to a spectrum of bound states with exotic quantum numbers, for example, those of a leptoquark.

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The scalar sector of the Minimal Supersymmetric Standard Model (MSSM) comprises the Higgs fields and the superpartners of quarks and leptons. The gauge interactions of these particles are well understood because the charges of the scalars, by virtue of supersymmetry, are the same as those of their fermionic partners. In addition, the squarks and sleptons can have some scalar interactions. The quartic coupling of the scalar fields are related to the Yukawa couplings and the gauge couplings by supersymmetry. However, the strength of the trilinear interactions is to a large extent unconstrained, except indirectly, from the requirement of vacuum stability. It is possible (and, in some models, desirable) that these dimensionful couplings be large in comparison to the masses of some scalar particles. The main focus of our analyses is on such trilinear scalar interactions because they are effectively "attractive" and can cause the appearance of bound states in the theory. These bound states can be observed as resonances in high-energy experiments and can, for example, cause an increase in the cross-section qualitatively similar to that reported last year at HERA [1].

We denote the SU(2)-doublet chiral superfields of quarks and leptons as $Q^\alpha_{_L}$ and $L^\alpha_{_L}$ respectively, and use the same notation for their scalar components. The corresponding right-handed SU(2)-singlets are $u_{_R}$, $d_{_R}$, and $l_{_R}$. Here and below the Greek letters stand for the SU(2) indices, the color SU(3) indices are suppressed. When relevant, the additional flavor indices, in Latin characters, will indicate the generations of (s)quarks and (s)leptons.

The superpotential of the MSSM includes the following terms

$$W = \epsilon^{\alpha\beta}[y_u Q^\alpha_{_L} H^\beta_2 u_{_R} + y_d Q^\alpha_{_L} H^\beta_1 d_{_R} + y_l L^\alpha_{_L} H^\beta_2 l_{_R} - \mu H^\alpha_1 H^\beta_2] + ..., \tag{1}$$

and generates, in particular, the trilinear interactions of the form

$$V_{3,\mu} = 2y\mu\epsilon^{\alpha\beta} H^\alpha_1 Q^\beta_{_L} u_{_R} + \text{h.c.} \tag{2}$$

that preserve supersymmetry. In addition, the trilinear terms

$$V_{3,_A} = A\epsilon^{\alpha\beta} H^\alpha_2 Q^\beta_{_L} u_{_R} + \text{h.c.}, \tag{3}$$

which couple the same squark bilinear $Q_{_L} u_{_R}$ to the "wrong" Higgs, appear in the scalar potential as a consequence of supersymmetry breaking. The potential can be written as follows

$$V = V_{2,H} + V_2 + V_{3,\mu} + V_{3,A} + V_4, \tag{4}$$

where $V_{2,H}$ comprises the terms quadratic in the Higgs fields, V_2 contains the mass terms of the squarks and sleptons, V_4 comprises the gauge D-terms and the terms of the form $y^2Q^2q^2$, $y^2Q^2H^2$, *etc.*

The trilinear coupling can play a crucial role in creating the bound states and resonances of squarks and sleptons through the exchange of the Higgs fields. For clarity, we will assume a particular form for the scalar potential that will retain all the relevant features of the general scalar interaction in the MSSM but will greatly simplify the discussion. First, we make use of a well-known MSSM prediction that there is a light neutral Higgs h^0. It is reasonable to neglect the propagation of heavier Higgs scalars in the ladder diagrams. Second, we will assume that the squarks and the sleptons have degenerate masses around m_0 (this assumption could be motivated by the constraints on the FCNC, although the latter can be satisfied without the squark degeneracy). In addition, we assume the equality of the trilinear couplings for the squarks and sleptons in question. One can easily generalize on all these assumptions, none of which is crucial to the main conclusions of our analysis. However, the algebraic entanglement involved in tracking a large number of parameters may unnecessarily complicate the discussion. We now want to examine the possibility of a bound state of two squarks, two sleptons, or a squark and a slepton, that exchange the lightest Higgs boson. The relevant interaction can be described by the approximate potential written in the flavor-diagonal basis of squarks and sleptons:

$$V_a = m_0^2(|Q^\alpha|^2 + |q|^2 + |L^\alpha|^2 + |l|^2) + \frac{1}{2} m_{_H}^2 |H^\alpha|^2 - A\epsilon^{\alpha\beta} H^\alpha Q^\beta q - A\epsilon^{\alpha\beta} H^\alpha L^\beta l + ..., \tag{5}$$

The quartic couplings V_4 can be neglected as long as the interaction relevant for the

Bethe-Salpeter equation is dominated by the large trilinear terms, the case in which we are mainly interested.

Theories of the kind described by the potential in equation (5) have historically been the testing ground for solving the Bethe–Salpeter equation [2],

$$\left[\left(\frac{1}{2}E+p\right)^2+m_0^2\right]\left[\left(\frac{1}{2}E-p\right)^2+m_0^2\right]\psi(p)=\frac{4iA^2}{(2\pi)^4}\int d^4k\frac{\psi(k)}{(p-k)^2+m_{_H}^2}, \quad (6)$$

where $\psi(p)$ is the wave function, E is the bound state energy, and $m_{_H}$ is the physical mass of the (lightest) Higgs. The potential in equation (5) can be re-written in the form that matches exactly the interaction studied in Ref. [4] if one diagonalizes the squark and slepton bilinears that enter into the cubic terms by a unitary transformation. We can, therefore, use the results of Refs. [2, 3, 4] for the energy spectrum of the bound states.

For fixed energy E, equation (6) is a Fredholm equation that has a discrete spectrum of eigenvalues $\lambda \equiv A^2$ that depend on E. In other words, for a given value of the coupling λ one looks for such energy E that makes λ an eigenvalue. Then the bound state energy E is characterized by a discrete spectrum. If $m_{_H}=0$ and α is small, the n'th bound state has energy [2, 3]

$$E_n = 2m_0\left(1-\frac{\alpha^2}{8n^2}\right), \quad (7)$$

where

$$\alpha=\frac{1}{16\pi}\frac{A^2}{m_0^2}. \quad (8)$$

The bound states exist for any value of α if the Higgs field H is massless. In the ladder approximation, the bound state energy approaches zero at $\alpha=\pi n(n+1)$, which can be used as a semi-quantitative reference point for the strength of the attractive interaction. The exchange of a scalar field with mass $m_{_H}$ creates a bound state only if $\alpha > \alpha_{min} \approx 1.68(m_{_H}/m_0)$ [3].

It is interesting to juxtapose these bound states and Q-balls [5]. Bosons can form a coherent state that allows a semiclassical description in terms of non-topological solitons if the number of particles is sufficiently large. The connection between sparticle bound states and Q-balls clarifies the role of the trilinear terms. The existence of small Q-balls [6] in the MSSM relies on the requisite trilinear couplings [7] that make it possible for the energy of the coherent state with a given baryon or lepton number to be less than the combined mass of the free particles that carry the same charge. The same trilinear term in the scalar potential can be viewed as an attractive interaction between the constituent squarks that exchange the Higgs fields. The latter description is more appropriate in the few-body limit, where the semiclassical description of Q-balls breaks down. Nevertheless, it is tempting to compare the expression for the ground state energy E_1 in equation (7) to the mass of a small Q-ball. One can suspect that a small Q-ball is merely an alternative description of some bound states. In some cases, such point of view is justified. For example, a bound state of two leptons can be thought of as a Q-ball with charge $Q = 2$ associated with the lepton number $\mathrm{U}(1)_{L}$ symmetry. A bound state of a slepton and a squark can be seen as a $\mathrm{U}(1)_{B-L}$ soliton, and so on. Of course, only a subset of sparticle bound states can be linked to Q-balls. The masses of small Q-balls (those, for which the thin-wall approximation is not valid) for a potential (5) with $m_{H} = m_0 = m$ were calculated in Ref. [6]:

$$E_{Q} = Qm \left[1 - \frac{Q^2}{54 S_{\psi}^2} \left(\frac{A^2}{m^2} \right)^2 \right], \tag{9}$$

where $S_{\psi} = 4.85$ is a quantity found numerically. The expression (9) was obtained in a semiclassical approximation that becomes unreliable when $Q \sim 1$. The energy in equation (7) was calculated [2, 3] in the ladder approximation from the Bethe-Salpeter equation. Although there is no reason to expect a good agreement between the two approximations, one of which (9) is pushed beyond its limit of validity, we notice that for $n = 1$ and $Q = 2$ they give the same dependency on the parameters A and m. In addition, formula (9) would give the same quantitative result if constant S_{ψ} assumed

a somewhat higher value.

We believe that taking into account the detailed structure of the mass terms in the MSSM and the inclusion of all degrees of freedom in the Higgs sector is unlikely to alter one's conclusions with respect to the existence of the bound states. Some generalizations of formula (7) to the case of the scalar fields with different masses and unequal trilinear couplings can be found in [4]. We conclude that the squarks and sleptons of the MSSM can form bound states. The binding energy is determined by the coupling α in equation (8). If the squarks and the sleptons have different masses m_1 and m_2, and couple to the light Higgs with the couplings A_1 and A_2 respectively, then the relevant coupling is $\tilde{\alpha} = (1/16\pi)A_1A_2/m_1m_2$ [4].

Phenomenologically, the trilinear coupling A in equation (3) can be as large as a few TeV. The upper limit on the value of A comes from considerations of vacuum stability with respect to tunneling into a possibly lower color and charge breaking (CCB) minimum in the scalar potential [8, 9]. The strongest limit of this kind exists for the third generation trilinear coupling, A_t, because the tunneling into a CCB minimum associated with a small Yukawa coupling is suppressed [10]. The empirical formula inferred from the numerical analyses [9] is

$$(A_t/y_t)^2 + 3\mu^2 < 7.5(m^2_{\tilde{t}_L} + m^2_{\tilde{t}_R}), \tag{10}$$

where $m_{\tilde{t}_L}$ and $m_{\tilde{t}_R}$ are the squark masses and $y_t \approx 1$ is the top Yukawa coupling[4]. Clearly, the ratio $A_t/m_{\tilde{t}_R}$ can be of order 10 if the right-handed stop is very light.

While the masses and widths of the bound states may vary, their quantum numbers are determined by the particle content of the MSSM. Many of them can produce resonances at present and future experiments. For example, let us consider a "leptoquark" bound state that can show up as a resonance in an electron-proton collider. The corresponding diagram is shown in Fig. 1. This resonance can be observed through an

[4]If one requires that the color and charge conserving vacuum be the global minimum of the potential, the coefficient 7.5 in equation (10) is replaced by 3 [8]. We note that A_t in Refs. [8, 9] differs from ours because we absorbed the Yukawa coupling into the definition of A.

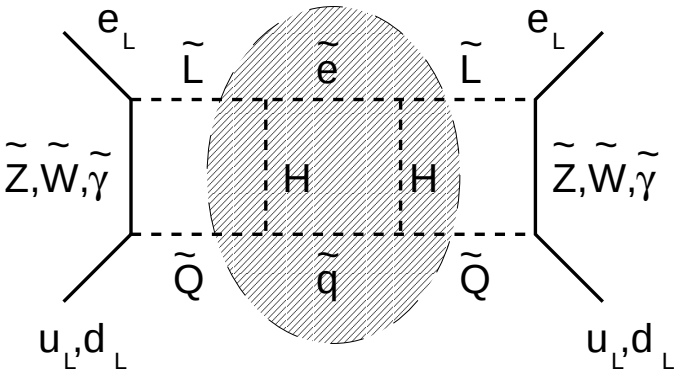

Figure 1: The scalar "leptoquark" bound state can create a resonance at an electron-proton collider. The SU(2)-doublets $\tilde{Q}$ and $\tilde{L}$ are the first-generation squark and slepton if the gaugino is $\tilde{Z}$ or $\tilde{\gamma}$. They can be of a different flavor (for example, stop and $\tilde{\tau}$) for the $\tilde{W}$ exchange. Only one of many possible diagrams is shown.

increase in the cross-section, much like that reported by HERA experiments [1] at high Q^2. The experimental status of these events remains uncertain but will undoubtedly be clarified in the near future. A squark with mass ~ 200 GeV and with R-parity violating couplings [11] has been proposed as an explanation of the HERA events. We emphasize that the leptoquark resonances can exist in the MSSM with conserved R-parity. They correspond to the bound states of squarks and sleptons of the type illustrated in Fig. 1. Perhaps, the resonances in the 200 GeV mass range can account for the HERA events if their coupling to the light fermions is sufficiently large. Of course, the parameters of such resonances are model-dependent.

We expect a variety of resonances to be detectable at a lepton collider, for instance, at LEP, NLC, or a muon collider (Fig. 2).

The two-particle bound states can have two squarks (including a "positronium" state that comprises a squark and an anti-squark), two sleptons, or a slepton and a squark. Depending on the trilinear terms, the leptoquark resonances can have different quantum numbers that correspond to $e^{\pm}d$, $e^{\pm}u$, etc. In addition, there can be colorless three-particle states bound by the exchange of the Higgs fields (*cf.* Ref. [4]) as well as gluons. The states of greatest interest are, presumably, those that correspond to the large trilinear couplings and have, therefore, a greater binding energy. Some of these

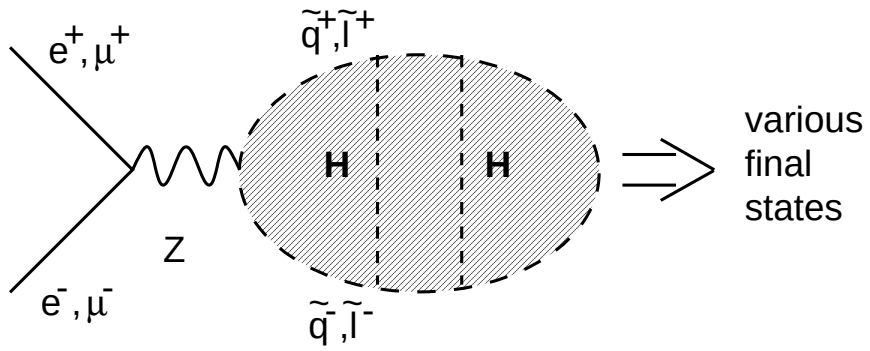

Figure 2: An example of a resonance at the e^+e^- or $\mu^+\mu^-$ collider.

multi-particle states may also show up as resonances. We leave the details for future publication.

In summary, we have shown that large trilinear couplings in the scalar sector of the MSSM can give rise to a new family of bound states and resonances, in particular those with the exotic quantum numbers, that may be observed in experiment.

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