

## INFLATION IN SOFTLY BROKEN SEIBERG-WITTEN MODELS

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In a recent paper we proposed a new model of inflation based on the soft-breaking of  $N=2$  supersymmetric  $SU(2)$  Yang-Mills theory. The advantage of such a model is the fact that we can write an *exact* expression for the effective scalar potential, including perturbative and non-perturbative effects. We find that the scalar condensate that plays the role of the inflaton can drive a long period of cosmological expansion in the weak coupling Higgs region, and end inflation in the strong coupling monopole region, where reheating takes place. The model predicts the right amount of temperature anisotropies in the microwave background, a precise spectral tilt,  $n = 0.91$ , and negligible gravitational wave perturbations.

In a recent paper<sup>1</sup> we proposed a new model of inflation where the role of the inflaton is played by the scalar condensate  $u = Tr \varphi^2$  that parametrizes inequivalent vacua in  $N=2$   $SU(2)$  super Yang-Mills. Duality and analyticity arguments allow one to write down an *exact* effective action for the light degrees of freedom in both the weak and in the strong coupling regions<sup>2</sup>. The soft breaking of  $N=2$  directly down to  $N=0$  via a spurion superfield preserves the analyticity properties of the Seiberg-Witten solution and produces a low energy effective scalar potential which includes all perturbative and non-perturbative effects<sup>3</sup>. This powerful result was studied in the context of low energy QCD. We simply realized that this *exact* scalar potential could be consistently used at much higher energies, of order the GUT scale, and be responsible for cosmological inflation<sup>1</sup>. The advantage with respect to other inflationary models based on supersymmetry is the complete control we have on the scalar potential, both along the quasi-flat direction and in the true vacuum of the theory, where reheating takes place.

In the Higgs region, along the positive real axis, it is possible to write in a compact way the Kähler metric and the scalar potential as<sup>1</sup>

$$\mathcal{K}(u) = \frac{2k^2}{\Lambda^2 \pi^2} K K', \quad (1)$$

$$V(u) = \frac{f_0^2 \Lambda^2}{\pi^2} \left[ 1 - 2 \left( \frac{K - E}{k^2 K} - \frac{1}{2} \right) \right]. \quad (2)$$

The functions  $K(k)$  and  $E(k)$  are complete elliptic functions of the first and second kind respectively, and  $E'(k) \equiv E(k')$ , where  $k'^2 + k^2 = 1$ . All these are functions of  $k^2 = 2/(1 + u/\Lambda^2)$  in the complex moduli plane  $u$ . The scalar

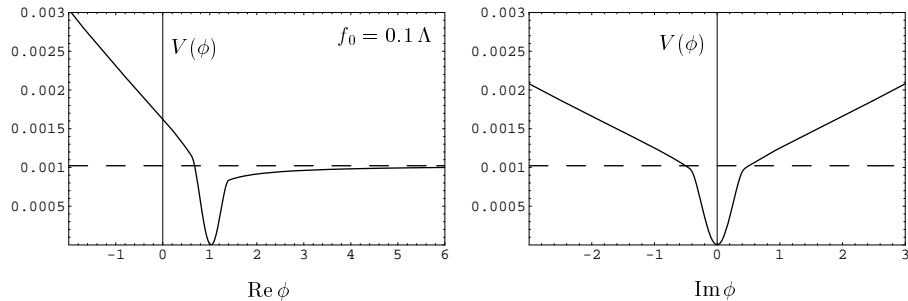


Figure 1: The exact scalar potential  $V(\phi)$  in moduli space, for  $f_0 = 0.1\Lambda$ . The left panel shows the potential as a function of  $\text{Re } \phi$  for  $\text{Im } \phi = 0$  and presents a minimum at  $\phi_{\text{min}} = \Lambda + f_0/\sqrt{8}$ . The right panel shows the potential as a function of  $\text{Im } \phi$  for  $\text{Re } \phi = \phi_{\text{min}}$ . The plateau (dashed line) corresponds to a vacuum energy density of  $V_0 = f_0^2 \Lambda^2 / \pi^2$ .

potential is thus a non-trivial function of the inflaton field, and depends on only two parameters, the dynamical scale  $\Lambda$  and the supersymmetry breaking scale  $f_0$ , satisfying  $f_0 < \Lambda < M \equiv M_{\text{P}}/\sqrt{8\pi}$  for the consistency of the theory<sup>3</sup>. In order to study the cosmological evolution of the inflaton under the potential (2) one should embed this model in supergravity. It is possible to show<sup>3</sup> that the only gravitational corrections to the potential are proportional to  $m_{3/2}^2 f_0^2$ , and therefore completely negligible in our case, since inflation in this model turns out to occur at the GUT scale, see below, and thus much above the phenomenological gravitino mass scale.

One also has to take into account the non-trivial Kähler metric for  $u$ <sup>3</sup>. The Lagrangian for the scalar field  $u$  in a curved background can then be written as

$$\mathcal{L} = \frac{1}{2} \mathcal{K}(u) g^{\mu\nu} \partial_\mu u \partial_\nu \bar{u} - V(u) = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \bar{\phi} - V(\phi), \quad (3)$$

where  $g_{\mu\nu}$  is the spacetime metric, and we have redefined the inflaton field through  $d\phi \equiv \mathcal{K}(u)^{1/2} du$ . We have plotted in Fig. 1 the scalar potential  $V(\phi)$  as a function of the real and imaginary parts of  $\phi$ . The vacuum energy,  $V_0 = f_0^2 \Lambda^2 / \pi^2$ , was added to the scalar potential in order to ensure that the absolute minimum is at zero cosmological constant.

The flatness of this potential at  $\text{Im } \phi = 0, \text{Re } \phi > \phi_{\text{min}}$  looks like an excellent candidate for inflation. It was not included by hand but arised naturally from the soft-breaking of supersymmetry<sup>3</sup>, albeit with a complicated functional form (2). There are two parameters in this model,  $\Lambda$  and  $f_0$ . Non of these have to be fine tuned to be small in order to have successful inflation. Moreover,

the trajectories away from the positive real axis do not give inflation<sup>1</sup>.

We will now consider the range of values of  $f_0$  and  $\Lambda$  that give a phenomenologically viable model, applying the usual machinery to study inflationary cosmology with a scalar field potential<sup>4,5</sup>. It turns out that for all values of the parameters  $\Lambda$  and  $f_0 < \Lambda$ , the extreme flatness of the potential at  $\phi > \phi_{\min}$  allows one to use the slow roll approximation<sup>5</sup> in the Higgs region all the way to the monopole region, where the slope of the potential is so large that inflation ends and reheating starts as the condensate oscillates around the minimum. As a consequence of the factorization of the symmetry breaking parameter  $f_0^2$  in the potential (2), the slow-roll parameters do not depend on  $f_0$ <sup>1</sup>. This further simplifies our analysis. For a given value of  $\Lambda/M$  it is easy to find the value of  $\phi_e$  at the end of inflation and from there compute the value  $\phi_{60}$  corresponding to  $N = 60$   $e$ -folds from the end of inflation.

Quantum fluctuations of the scalar condensate,  $\delta\phi$ , will create perturbations in the metric,  $\mathcal{R} = H\delta\phi/\dot{\phi}$ , which cross the Hubble scale during inflation and later re-enter during the matter era. Those fluctuations corresponding to the scale of the present horizon left 60  $e$ -folds before the end of inflation, and are responsible via the Sachs-Wolfe effect for the observed temperature anisotropies in the microwave background<sup>6</sup>. From the amplitude and spectral tilt of these temperature fluctuations we can constrain the values of the parameters  $\Lambda$  and  $f_0$ .

Present observations of the power spectrum of temperature anisotropies on various scales, from COBE DMR to Saskatoon and CAT experiments, impose the following constraints on the amplitude of the tenth multipole and the tilt of the spectrum<sup>7</sup>,

$$Q_{10} = 17.5 \pm 1.1 \mu\text{K}, \quad (4)$$

$$n = 0.91 \pm 0.10. \quad (5)$$

Assuming that the dominant contribution to the CMB anisotropies comes from the scalar metric perturbations, we can write<sup>7</sup>  $A_S = 5 \times 10^{-5} (Q_{10}/17.6 \mu\text{K})$ , where  $A_S^2 = (H/2\pi M)^2/2\epsilon$  and  $n = 1+2\eta-6\epsilon$ , in the slow-roll approximation<sup>5</sup>. Since during inflation at large values of  $\phi$ , corresponding to  $N = 60$ , the rate of expansion is dominated by the vacuum energy density  $V_0$ , we can write, to very good approximation,  $H^2 = f_0^2\Lambda^2/3\pi^2M^2$ , and thus the amplitude of scalar metric perturbations is

$$A_S^2 = \frac{1}{24\pi^4} \frac{f_0^2\Lambda^2}{M^4} \frac{1}{\epsilon_{60}}. \quad (6)$$

Let us consider, for example, a model with  $\Lambda = 0.1M$ . In that case the end of inflation occurs at  $\phi_e \simeq 1.5\Lambda$ , still in the Higgs region, and 60  $e$ -folds

correspond to a relatively large value,  $\phi_{60} = 14\Lambda$ , deep in the weak coupling region. The corresponding values of the slow roll parameters are  $\epsilon_{60} = 2 \times 10^{-5}$  and  $\eta_{60} = -0.04$ , which gives  $A_S^2 = 5f_0^2/24\pi^4\Lambda^2$  and  $n = 0.91$ . In order to satisfy the constraint on the amplitude of perturbations (4), we require  $f_0 = 10^{-3}\Lambda$ , which is a very natural value from the point of view of the consistency of the theory. In particular, these parameters correspond to a vacuum mass scale of order  $V_0^{1/4} \simeq 4 \times 10^{15}$  GeV, very close to the GUT scale. For other values of  $\Lambda$  we find numerically the relation  $\log_{10}(f_0/\Lambda) = -4.5 + 1.54 \log_{10}(M/\Lambda)$ , which is a very good fit in the range  $1 \leq M/\Lambda \leq 10^3$ . For  $M > 800\Lambda$ , the soft breaking parameter  $f_0$  becomes greater than  $\Lambda$ , where our approximations breaks down, and we can no longer trust our exact solution. Meanwhile the spectral tilt is essentially invariant,  $n = 0.913 - 0.003 \log_{10}(M/\Lambda)$ , in the whole range of  $\Lambda$ . It is therefore a concrete prediction of the model. Surprisingly enough it precisely corresponds to the observed value (5). This might change however when future satellite missions will determine the spectral index  $n$  with better than 1% accuracy<sup>8</sup>.

There are also tensor (gravitational waves) metric perturbations in this model, with amplitude  $A_T^2 = 2(H/\pi M)^2 = 2f_0^2\Lambda^2/3\pi^4M^4$  and tilt  $n_T = -2\epsilon$ . The relative contribution of tensor to scalar perturbations in the microwave background on large scales can be parametrized<sup>5</sup> by  $T/S \simeq 12.4\epsilon$ . A very good fit to the ratio  $T/S$  in this model is given by  $\log_{10}(T/S) = -2.6 - 0.91 \log_{10}(M/\Lambda)$ , in the same range as above. Since  $M \geq \Lambda$ , we can be sure that no significant contribution to the CMB temperature anisotropies will arise from gravitational waves.

We have therefore found a new model of inflation, based on *exact* expressions for the scalar potential of a softly broken N=2 supersymmetric SU(2) theory, to all orders in perturbations and with all non-perturbative effects included. Inflation occurs along the weak coupling Higgs region where the potential is essentially flat, and ends when the gauge invariant scalar condensate enters the strong coupling confining phase, where the monopole acquires a VEV, and starts to oscillate around the minimum of the potential, reheating the universe. A simple argument suggests that during reheating explosive production of particles will occur in this model. The evolution equation of a generic scalar (or vector) particle has the form of a Mathieu equation and presents parametric resonance for certain values of the parameters<sup>9</sup>. An efficient production of particles occurs for large values of the ratio  $q = g^2\Phi^2/4m^2$ , where  $g$  is the coupling between  $\phi$  and the corresponding scalar field,  $\Phi$  is the amplitude of oscillations of  $\phi$ , and  $m$  is its mass. As the inflaton field oscillates around  $\phi_{\min}$  it couples strongly,  $g \sim 1$ , to the other particles in the supermultiplet since the minimum is in the strong coupling region. The amplitude of

oscillations is of order the dynamical scale,  $\Phi \sim \Lambda$ , while the masses of all particles (scalars, fermions and vectors) are of order the supersymmetry breaking scale,  $m \sim f_0 \ll \Lambda$ <sup>10</sup>. This means that the  $q$ -parameter is large, thus inducing strong parametric resonance and explosive particle production<sup>9</sup>. These particles will eventually decay into ordinary particles, reheating the universe.

We are assuming throughout that we can embed this inflationary scenario in a more general theory that contains two sectors, the inflaton sector, which describes the soft breaking of N=2 supersymmetric SU(2) and is responsible for the observed flatness and homogeneity of our universe, and a matter sector with the particle content of the standard model, at a scale much below the inflaton sector. The construction of more realistic scenarios remains to be explored, in which the two sectors communicate via some messenger sector. For example, one could consider this SU(2) as a subgroup of the hidden  $E_8$  of the heterotic string and the visible sector as a subgroup of the other  $E_8$ . No-scale supergravity could then be used as mediator of supersymmetry breaking from the strong coupling inflaton sector to the weakly coupled visible sector. For the scales of susy breaking considered above,  $f_0 \sim 10^{-5} M_{\text{P}}$ , we can obtain a phenomenologically reasonable gravitino mass,  $m_{3/2} \sim f_0^3/M_{\text{P}}^2 \sim 10$  TeV. This gravity-mediated supersymmetry breaking scenario needs further study, but suggests that it is possible in principle to do phenomenology with this novel inflationary model.

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