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Theories with Gauge-Mediated Supersymmetry Breaking

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Abstract

Theories with gauge-mediated supersymmetry breaking provide an interesting alternative to the scenario in which the soft terms of the low-energy fields are induced by gravity. These theories allow for a natural suppression of flavour violations in the supersymmetric sector and have very distinctive phenomenological features. Here we review their basic structure, their experimental implications, and the attempts to embed them into models in which all mass scales are dynamically generated from a single fundamental scale.

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Contents

1 Introduction

The naturalness (or hierarchy) problem [276, 158] is considered to be the most serious theoretical argument against the validity of the Standard Model (SM) of elementary particle interactions beyond the TeV energy scale. In this respect, it can be viewed as the ultimate motivation for pushing the experimental research to higher energies. The naturalness problem arises from the difficulty, in field theory, in keeping fundamental scalar particles much lighter than the highest energy within the range of validity of the theory, Λ_{UV}. This difficulty is a consequence of the lack of a symmetry prohibiting a scalar mass term or, in more technical terms, of the presence of quadratic divergences in the quantum corrections to scalar masses.

The SM Lagrangian contains a single dimensionful parameter, in the mass term for the Higgs field, which determines the size of the electroweak scale. A reasonable criterion for "naturalness" limits the validity cut-off scale Λ_{UV} to be at most a loop factor larger than the mass scale of fundamental scalars. We are then led to the conclusion that the SM can be valid only up to the TeV scale, if the naturalness criterion is satisfied. New physics should appear at this scale, modifying the high-energy behaviour and divorcing the Higgs mass parameter from its ultraviolet sensitivity.

An elegant solution to the naturalness problem is provided by supersymmetry [142, 271, 274]. Since supersymmetry relates bosons to fermions, a scalar mass term is never, in a supersymmetric theory, generated by quantum corrections, if the corresponding fermion mass is forbidden by a chiral symmetry. Moreover, a supersymmetric theory is free from quadratic divergences [275, 163, 122, 146].

In the real world, supersymmetry must be broken. However, if supersymmetry provides a solution to the naturalness problem, it must then be an approximate symmetry of the theory above the TeV scale. This is possible when supersymmetry is broken only softly [138, 86, 249], *i.e.* by terms that do not introduce quadratic divergences. These terms always have dimensionful couplings, and the naturalness criterion implies that the corresponding mass scale cannot exceed the TeV region. The soft terms provide gauge-invariant masses to all supersymmetric partners of the known SM particles. These masses give a precise physical meaning to the SM ultraviolet cut-off Λ_{UV}.

As the soft terms determine the mass spectrum of the new particles, the mechanism of supersymmetry breaking is the key element for understanding the low-energy aspects of supersymmetric theories. However, the mechanism of communicating the original supersymmetry breaking to the ordinary particle supermultiplets plays an equally or even more important rôle. As an analogy, one can think of the case of the SM, in which the Higgs vacuum expectation value (VEV) determines the scale of electroweak breaking, but the detailed mass spectrum of bosons and fermions is dictated by the coupling constants of the forces that communicate the information of electroweak breaking, *i.e.* gauge and Yukawa interactions, respectively.

A major difference between the case of supersymmetry breaking and this SM analogy is that the supertrace theorem [123] essentially rules out the possibility of constructing simple models in which supersymmetry breaking is communicated to ordinary supermultiplets by tree-level renormalizable couplings. Indeed, in a globally supersymmetric theory with a gauge group free from gravitational anomalies [10], the sum of the particle tree-level squared masses, weighted by the corresponding number of degrees of freedom, is equal in the bosonic and fermionic sectors [123]:

$$\mathrm{STr}\mathcal{M}^2 = \sum_J (-1)^{2J}(2J+1)\mathcal{M}_J^2 = 0 \ . \tag{1.1}$$

Here $\mathcal{M}_J$ denotes the tree-level mass of a particle with spin J. Rather generically, this theorem implies, in cases of tree-level communication, the existence of a supersymmetric particle lighter than its ordinary partner.

As a consequence of this difficulty, the paradigm for constructing realistic supersymmetric theories is to assume that the sector responsible for supersymmetry breaking (the analogue of the Higgs sector) has no renormalizable tree-level couplings with the “observable sector”, which contains the ordinary particles and their supersymmetric partners. Moreover, the effective theory describing the observable sector, obtained by integrating out the heavy particles in the supersymmetry-breaking sector, should have a non-vanishing supertrace.

It is not too hard to satisfy these conditions. The supertrace theorem, after all, follows from the properties of renormalizability that force the kinetic terms to have the minimal form. Let us consider the effective theory describing the ordinary supermultiplets, the goldstino field, and possibly other associated light fields, but with the heavy fields of the supersymmetry-breaking sector integrated out. If this theory has non-canonical (and therefore non-renormalizable) kinetic terms for matter and gauge multiplets, involving interactions with the goldstino superfield, we generally induce scalar and gaugino masses, which break supersymmetry but violate the supertrace theorem. Therefore, to understand the question of supersymmetry-breaking communication is to identify the interactions that generate the non-renormalizable effective Lagrangian.

One possibility is to consider a theory that is altogether non-renormalizable, and such that the supertrace over the whole spectrum is non-vanishing. The best-motivated example is given by gravity. Indeed the most general supergravity Lagrangian, in the presence of supersymmetry breaking, leads to [60, 30, 205, 73, 153, 261] an effective theory for the low-energy modes containing the desired soft terms. This is the scenario most commonly considered in phenomenological applications (for reviews, see *e.g.* refs. [207, 150, 32]), and it is certainly a very attractive one as, for the first time, gravity ventures to play an active rôle in electroweak physics.

Another possibility is that the relevant dynamics at the microscopic level is described by a renormalizable Lagrangian, and, at tree level, the theory has a vanishing supertrace and no mass splittings inside the observable supermultiplets. However the low-energy modes are described by an effective Lagrangian, which has non-renormalizable kinetic terms (and non-vanishing supertrace) at the quantum level, induced by known gauge interactions. This is the case of theories with gauge-mediated supersymmetry breaking, which we are going to review here.

The fundamental difference between the two approaches is related to the problem of flavour. In the limit of vanishing Yukawa couplings, the SM Lagrangian is invariant under a global $U(3)^5$ symmetry, with each $U(3)$ acting on the generation indices of the five irreducible fermionic representations of the gauge group $(q_L, u^c_R, d^c_R, \ell_L, e^c_R)_i$. This symmetry, called flavour (or family) symmetry, follows from the property that gauge interactions do not distinguish between the three generations of quarks and leptons. We ignore the dynamical origin of the Yukawa couplings or, ultimately, of the flavour-symmetry breaking, but let us just define Λ_F to be the relevant energy scale of the corresponding new physics. Above Λ_F lie some unknown dynamics responsible for flavour breaking. Below Λ_F these dynamics are frozen, leaving their scars on the flavour-breaking structure of Yukawa couplings.

A possible realization of the flavour dynamics is given by models where a subgroup G_F

of $U(3)^5$ is a fundamental symmetry (local or global) which is broken spontaneously by the vacuum expectation values of a set of scalar fields $\phi_F = \{\phi_a\}$, the flavons. In these scenarios the hierarchical structure of the fermion spectrum can either be obtained via multiple stages of G_F breaking or by assuming that the flavon VEVs are somewhat smaller (one or two orders of magnitude) than the scale M_F which sets their coupling to the quarks and leptons. This possibility corresponds to the Froggatt-Nielsen mechanism [126] and the scale M_F could correspond to either the mass of some heavy states or simply to the Planck scale [183]. It should be clear that, for the purpose of our discussion, the flavour scale Λ_F can be indifferently identified with either $\langle\phi_F\rangle$ or M_F. Consider for instance the Froggatt-Nielsen case. By integrating out the flavour sector, the effective Yukawa couplings are functions of ϕ_a/M_F, whose form is restricteded by the G_F selection rules. In a similar way, if soft terms are already present at Λ_F, the effective squark and slepton masses and A-terms are functions of the flavons as well. Now these soft term matrices will necessarily contain new sources of flavour violation, in addition to those given by the Yukawa matrices.

In the gravity-mediated approach, the soft terms are generated at the Planck scale, and therefore necessarily at a scale larger than or equal to Λ_F. There is then no obvious reason why the supersymmetry-breaking masses for squarks and sleptons should be flavour-invariant. Even if at tree level, for some accidental reason, they are flavour-symmetric, loop corrections from the flavour-violating sector will still distort their structure. Even contributions from ordinary grand unified theories (GUTs) can lead to significant flavour-breaking effects in the soft terms [154, 34, 35].

These flavour-breaking contributions to the soft terms are very dangerous [86, 113, 29]. The mismatch between the diagonalization matrices for quarks and squarks (and analogously for leptons and sleptons) leads to flavour-violating gaugino vertices, and eventually to large contributions to flavour-changing neutral-current (FCNC) processes. Studies of the $\bar{K}^0$–K^0 mass difference, $\mu \to e\gamma$ and similar processes set very stringent bounds on the relative splittings among different generations of squarks and sleptons [152, 127].

Of course this does not mean that gravity-mediated scenarios cannot give a realistic theory. In models with a spontaneously broken flavour group G_F, like the Froggatt-Nielsen models, the selection rules of G_F can lead to approximate universality or to approximate alignment between particle and sparticle masses [103, 209, 184, 236, 36]. A similar result may also be obtained by a dynamical mechanism [89]. Indeed it may also be that at the level of quantum gravity soft terms are flavour-invariant. At any rate, it seems unavoidable [34, 35] that in these scenarios, flavour violations should be, at best, just at the edge of present bounds and should soon be visible.

On the other hand, in gauge-mediated theories, soft terms are generated at the messenger scale M, which is *a priori* unrelated to Λ_F. If $M \ll \Lambda_F$, the soft terms feel the breaking of flavour only through Yukawa interactions. Yukawa couplings are the only relevant sources of flavour violation, as in the SM. More precisely, all other sources of flavour violation at the messenger scale correspond to operators of dimension larger than 4, suppressed by the suitable powers of $1/\Lambda_F$. The contribution of these operators to soft masses is necessarily suppressed by powers of M/Λ_F. As a consequence, the GIM mechanism is fully operative and it can be generalized to a super-GIM mechanism, involving ordinary particles and their supersymmetric partners. Since it is reasonable to expect that Λ_F is as large as the GUT or the Planck scales, in gauge-mediated theories the flavour problem is naturally decoupled, in contrast to the case of supergravity or, in a different context, of technicolour theories [273, 264].

Moreover, in gauge-mediated theories, it is possible to describe the essential dynamics without dealing with gravity. This may be viewed as an aesthetic drawback, as it delays the complete unification of forces. However, from a more technical point of view, it is an advantage, because the model can be solved using only field-theoretical tools, without facing our present difficulties in treating quantum gravity. This is particularly interesting in view of the recent developments in the understanding of non-perturbative aspects of supersymmetric theories (for reviews, see *e.g.* refs. [165, 259, 224]).

Finally, as will be illustrated in this review, gauge-mediated theories are quite predictive in the supersymmetric mass spectrum and have distinctive phenomenological features. Future collider experiments can put these predictions fully to test.

This review is organized as follows. In sect. 2 we describe the general structure of models with gauge-mediated supersymmetry breaking. Their main phenomenological features are discussed in sect. 3. In sect. 4 we briefly outline some theoretical tools needed to study non-perturbative aspects of supersymmetry breaking. These techniques are used in sect. 5, where we discuss the present status of models with dynamical supersymmetry breaking and gauge mediation. Finally, in sect. 6 we consider mechanisms for generating the Higgs mixing parameters μ and $B\mu$, in the context of gauge mediation.

2 The Structure of Models with Gauge-Mediated Supersymmetry Breaking

In this section we will explain the main features of models with gauge-mediated supersymmetry breaking and describe the structure of the emerging soft-breaking terms.

2.1 The Building Blocks of Gauge Mediation

The first ingredient of these models is an *observable sector*, which contains the usual quarks, leptons, and two Higgs doublets, together with their supersymmetric partners. Then the theory contains a sector responsible for supersymmetry breaking. We will refer to it as the *secluded sector*, to distinguish it from the hidden sector of theories where supersymmetry breaking is mediated by gravity. For the moment we will leave the secluded sector unspecified, since it still lacks a standard description. For our purposes, all we need to know is that the goldstino field overlaps with a chiral superfield X, which acquires a VEV along the scalar and auxiliary components

$$\langle X \rangle = M + \theta^2 F \ . \tag{2.1}$$

As will be discussed in detail in the following, the parameters M and $\sqrt{F}$, which are the fundamental mass scales in the theory, can vary from several tens of TeV to almost the GUT scale. We will start by considering the simplest case, in which X coincides with the goldstino superfield. However in sect. 2.5 we will also discuss secluded sectors in which the goldstino is a linear combination of different fields.

Finally the theory has a *messenger sector*, formed by some new superfields that transform under the gauge group as a real non-trivial representation and couple at tree level with the goldstino superfield X. This coupling generates a supersymmetric mass of order M for the messenger fields and mass-squared splittings inside the messenger supermultiplets of order F. This sector is also unknown and it is the main source of model dependence. It is fairly reasonable to expect that the secluded and messenger sectors have a common origin, and models in which these two sectors are unified will be discussed in sect. 5.

The simplest messenger sector is described by N_f flavours of chiral superfields Φ_i and $\bar{\Phi}_i$ ($i = 1, ..., N_f$) transforming as the representation $\mathbf{r} + \bar{\mathbf{r}}$ under the gauge group. In order to preserve gauge coupling-constant unification, one usually requires that the messengers form complete GUT multiplets. If this is the case, the presence of messenger fields at an intermediate scale does not modify the value of M_{GUT}, but the inverse gauge coupling strength at the unification scale α_{GUT}^{-1} receives an extra contribution

$$\delta\alpha_{GUT}^{-1} = -\frac{N}{2\pi} \ln \frac{M_{GUT}}{M} \ , \tag{2.2}$$

$$N = \sum_{i=1}^{N_f} n_i \ . \tag{2.3}$$

Here n_i is twice the Dynkin index of the gauge representation $\mathbf{r}$ with flavour index i, *e.g.* $n = 1$ or 3 for an $SU(5)$ **5** or **10**, respectively. We will refer to N as the *messenger index*, a quantity that will play an important rôle in the phenomenology of gauge-mediated theories. From eq. (2.2) we infer that perturbativity of gauge interactions up to the scale M_{GUT} implies

$$N \lesssim 150/\ln \frac{M_{GUT}}{M} \ . \tag{2.4}$$

If M is as low as 100 TeV, then N can be at most equal to 5. However, this upper bound on N is relaxed for larger values of M. For instance, for $M = 10^{10}$ GeV, eq. (2.4) shows that N as large as 10 is allowed.

It should also be noticed that, in the minimal $SU(5)$ model, the presence of messenger states at scales of about 100 TeV is inconsistent with proton-decay limits and with b–τ unification, unless large GUT threshold corrections are added [55, 26]. Anyhow, these constraints critically depend on the GUT model considered and, after all, minimal $SU(5)$ is not a fully consistent model.

In the case under consideration the interaction between the chiral messenger superfields Φ and $\bar{\Phi}$ and the goldstino superfield X is given by the superpotential term

$$W = \lambda_{ij} \bar{\Phi}_i X \Phi_j \ . \tag{2.5}$$

After replacing in eq. (2.5) the X VEV, see eq. (2.1), we find that the spinor components of Φ and $\bar{\Phi}$ form Dirac fermions with masses λM, while the scalar components have a squared-mass matrix

$$\left(\Phi^\dagger \quad \bar{\Phi} \right) \begin{pmatrix} (\lambda M)^\dagger (\lambda M) & (\lambda F)^\dagger \\ (\lambda F) & (\lambda M)(\lambda M)^\dagger \end{pmatrix} \begin{pmatrix} \Phi \\ \bar{\Phi}^\dagger \end{pmatrix} . \tag{2.6}$$

Here we have dropped flavour indices and, with a standard abuse of notation, we have denoted the superfields and their scalar components by the same symbols. If there is a single field X, then the matrices λM and λF can be simultaneously made diagonal and real, and the scalar messenger mass eigenvectors are $(\Phi + \bar{\Phi}^\dagger)/\sqrt{2}$ and $(\bar{\Phi} - \Phi^\dagger)/\sqrt{2}$, with squared-mass eigenvalues $(\lambda M)^2 \pm (\lambda F)$. It is now convenient to absorb the coupling constant λ in the definition of M and F, $\lambda_{ii} M \to M_i$, $\lambda_{ii} F \to F_i$. In the following we will implicitly assume this redefinition.

2.2 Soft Terms in the Observable Sector

The mass scale $\sqrt{F}$ is the measure of supersymmetry breaking in the messenger sector. However, we are of course mainly interested in the amount of supersymmetry breaking in the observable sector. Ordinary particle supermultiplets are degenerate at the tree level, since they do not directly couple to X, but splittings arise at the quantum level because of gauge interactions between observable and messenger fields. While vector bosons and matter fermion masses are protected by gauge invariance, gauginos, squarks, and sleptons can acquire masses consistently with the gauge symmetry, once supersymmetry is broken. Gaugino masses are generated at one loop, but squark and slepton masses can only arise at two loops, since the exchange of both gauge and messenger particles is necessary. The corresponding Feynman diagrams are drawn in fig. 1. They were first evaluated in ref. [9], where it was shown that they lead to an acceptable particle spectrum with positive scalar squared masses.

Higher-loop calculations in theories with many particles are quite involved. Therefore it is often useful to employ a simple and systematic method [141] to extract the soft terms for observable fields in theories in which supersymmetry breaking is communicated by renormalizable and perturbative (but not necessarily gauge) interactions. Let us briefly discuss the method.

The positivity of the messenger squared masses requires $F < M^2$. Moreover, as we will see in the following, in most realistic cases it is appropriate to assume $F \ll M^2$. With this assumption, in the effective field theory below the messenger scale M, supersymmetry breaking can be treated as a small effect. This allows us to use a manifestly supersymmetric formalism to keep track of supersymmetry-breaking effects as well. First we define the effective theory,

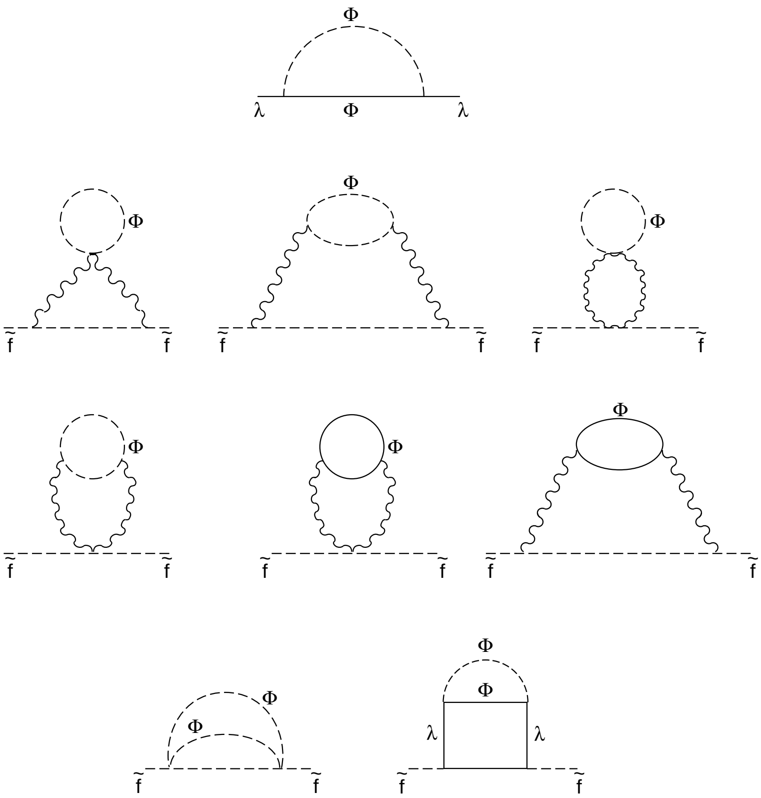

Figure 1: *Feynman diagrams contributing to supersymmetry-breaking gaugino (λ) and sfermion ($\tilde{f}$) masses. The scalar and fermionic components of the messenger fields Φ are denoted by dashed and solid lines, respectively; ordinary gauge bosons are denoted by wavy lines.*

valid below the mass scale M, by integrating out the heavy messenger fields. We are interested only in renormalizable terms, which are not suppressed by powers of M. Due to the non-renormalization of the superpotential, all the relevant M dependence of the low-energy effective theory is contained in the gauge and matter wave-function renormalizations S and Z_Q. In the presence of a single mass scale M, this dependence is logarithmic, and it can be calculated by solving the renormalization-group (RG) equations in the exact supersymmetric theory. At the end, the mass parameter M can be analytically continued in superspace into a chiral superfield X. Holomorphy dictates the correct analytic continuation $M \to X$ to be performed in S. On the other hand, the only substitution in Z_Q that is consistent with the chiral reparametrization $X \to e^{i\phi} X$ is given by $M \to \sqrt{XX^\dagger}$. By replacing X with its background value, given in eq. (2.1), we can derive all the relevant supersymmetry-breaking effects. We obtain that the supersymmetry-breaking gaugino masses, squark and slepton masses, and coefficients of the trilinear A-type terms, defined by

$$\mathcal{L}_{\rm soft} = -\frac{1}{2}\left(\tilde{M}_\lambda \lambda_g \lambda_g + \text{h.c.}\right) - m_{\tilde{Q}}^2 Q^\dagger Q - \left(\sum_i A_i Q_i \partial_{Q_i} W(Q) + \text{h.c.}\right) , \tag{2.7}$$

have the general expressions [141]

$$\tilde{M}_\lambda(t) = -\frac{1}{2}\frac{\partial \ln S(X,t)}{\partial \ln X}\bigg|_{X=M} \frac{F}{M} , \tag{2.8}$$

$$m_{\tilde{Q}}^2(t) = -\frac{\partial^2 \ln Z_Q(X,X^\dagger,t)}{\partial \ln X \; \partial \ln X^\dagger}\bigg|_{X=M} \frac{FF^\dagger}{MM^\dagger} , \tag{2.9}$$

$$A_i(t) = \frac{\partial \ln Z_{Q_i}(X,X^\dagger,t)}{\partial \ln X}\bigg|_{X=M} \frac{F}{M} . \tag{2.10}$$

Here $t = \ln M^2/Q^2$ and Q is the low-energy scale at which the soft terms are defined. The gauge and chiral wave-function renormalizations S and Z_Q are obtained by integrating the well-known RG differential equations in the supersymmetric limit.

Explicit formulae for the soft terms in gauge mediation will be given in the next section. Here we just want to remark that, by inserting the one-loop approximations for S and Z_Q in eqs. (2.8)–(2.10), we can directly obtain both the leading-loop messenger contribution to the soft terms and the resummation of the leading logarithms in the evolution from the messenger scale M to the low-energy scale Q. It is also interesting to notice that the gauge-mediated two-loop sfermion masses are obtained by integrating the one-loop RG equation. This is because, at the leading order, Z_Q is a function of $\alpha_s \log XX^\dagger$. The two derivatives in eq. (2.9) are responsible for a leading contribution to soft masses of order α_s^2. In the ordinary Feynman-diagram approach, soft scalar masses arise from finite two-loop graphs. Following the method of ref. [141], they can be reconstructed from the behaviour of the wave-function renormalization far away from threshold, derived from the RG equation. This method is computationally much simpler than the explicit evaluation of the relevant Feynman diagrams.

2.3 Physical Mass Spectrum

We present here the complete formulae, in the leading-log approximation, for the soft terms generated by chiral messengers communicating via gauge interactions. The formulae in the next-to-leading order approximation are given in the appendix. These results can be derived either by directly evaluating the relevant Feynman diagrams or by applying the method described in the previous section. The supersymmetry-breaking gaugino masses are

$$\tilde{M}_{\lambda_r}(t) = k_r \frac{\alpha_r(t)}{4\pi} \Lambda_G \qquad (r = 1, 2, 3) , \tag{2.11}$$

$$\Lambda_G = \sum_{i=1}^{N_f} n_i \frac{F_i}{M_i} \left[1 + \mathcal{O}(F_i^2/M_i^4) \right] , \tag{2.12}$$

where $k_1 = 5/3$, $k_2 = k_3 = 1$, and the gauge coupling constants are normalized such that $k_r \alpha_r$ $(r = 1, 2, 3)$ are all equal at the GUT scale. In the simple case in which there is a single X superfield, and the ratio F_i/M_i is therefore independent of the flavour index i, eq. (2.12) becomes

$$\Lambda_G = N \frac{F}{M} \left[1 + \mathcal{O}(F^2/M^4) \right] , \tag{2.13}$$

where the messenger index N is defined in eq. (2.3). Next-to-leading corrections in α_s can affect the gluino mass prediction by a significant amount, especially at small values of N and M [20, 225].

Neglecting Yukawa-coupling effects, the supersymmetry-breaking scalar masses at the scale Q $(t = \ln M^2/Q^2)$ are

$$m_{\tilde{f}}^2(t) = 2 \sum_{r=1}^{3} C_r^{\tilde{f}} k_r \frac{\alpha_r^2(0)}{(4\pi)^2} \left[\Lambda_S^2 + h_r \Lambda_G^2 \right] , \tag{2.14}$$

$$h_r = \frac{k_r}{b_r} \left[1 - \frac{\alpha_r^2(t)}{\alpha_r^2(0)} \right] , \tag{2.15}$$

$$\alpha_r(t) = \alpha_r(0)\left[1+\frac{\alpha_r(0)}{4\pi}b_r t\right]^{-1} \ . \tag{2.16}$$

Here $\alpha_r(0)$ are the gauge coupling constants at the messenger scale M and, in the case of a single X superfield,

$$\Lambda_S^2 = N\frac{F^2}{M^2}\ \left[1+\mathcal{O}(F^2/M^4)\right] \ . \tag{2.17}$$

In the general case of non-universal values of F_i and M_i (*i.e.* with several superfields X overlapping with the goldstino), eq. (2.17) is no longer valid, and the ratio Λ_G^2/Λ_S^2 is not just given by the messenger index N. However, in this case, we can treat Λ_S and Λ_G as free parameters, instead of N and F/M. In eq. (2.14) $C_r^{\tilde f}$ is the quadratic Casimir of the $\tilde f$ particle, $C=\frac{N^2-1}{2N}$ for the N-dimensional representation of $SU(N)$, and $C=Y^2=(Q-T_3)^2$ for the $U(1)$ factor. Here b_r are the β-function coefficients

$$b_3=-3, \quad b_2=1, \quad b_1=11 \ . \tag{2.18}$$

The physical scalar squared mass is obtained by adding to eq. (2.14) the D-term contribution $M_Z^2\cos 2\beta(T_3^{\tilde f}-Q^{\tilde f}\sin^2\theta_W)$.

In deriving these mass formulae, we have assumed $F \ll M^2$. Although this approximation is in most cases justified since, for $F > M^2$, a scalar messenger particle has negative squared mass, it is not appropriate whenever $F \simeq M^2$. Notice that in this case the method of ref. [141], discussed in sect. 2.2, fails because all higher covariant-derivative operators contribute to supersymmetry-breaking terms at the same order. An explicit Feynman diagram calculation is now necessary. This has been performed in refs. [93, 193], with the result

$$\Lambda_G = \sum_{i=1}^{N_f} n_i \frac{F_i}{M_i} g(F_i/M_i^2) \ , \tag{2.19}$$

$$\begin{aligned} g(x) &= \frac{1}{x^2}\left[(1+x)\ln(1+x)\right] + (x\to -x) \\ &= 1+\frac{x^2}{6}+\frac{x^4}{15}+\frac{x^6}{28}+\mathcal{O}(x^8) \ , \end{aligned} \tag{2.20}$$

$$\Lambda_S^2 = \sum_{i=1}^{N_f} n_i \frac{F_i^2}{M_i^2} f(F_i/M_i^2) \ , \tag{2.21}$$

$$\begin{aligned} f(x) &= \frac{1+x}{x^2}\left[\ln(1+x) - 2\mathrm{Li}_2\left(\frac{x}{1+x}\right)+\frac{1}{2}\mathrm{Li}_2\left(\frac{2x}{1+x}\right)\right] + (x\to -x) \\ &= 1+\frac{x^2}{36}-\frac{11}{450}x^4-\frac{319}{11760}x^6+\mathcal{O}(x^8) \ . \end{aligned} \tag{2.22}$$

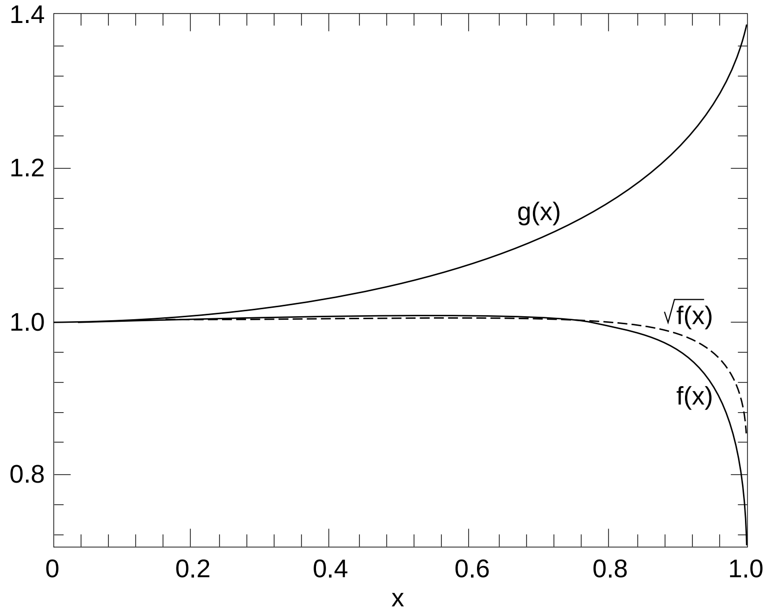

Figure 2: *The functions* $f(x)$, $g(x)$, *and* $\sqrt{f(x)}$.

The functions $g(x)$ and $f(x)$, which represent the corrections with respect to the $F \ll M^2$ case, are shown in fig. 2; $g(x)$ is always larger than 1, reaching the maximum value $g(1) = 2\ln 2 = 1.4$; $f(x)$ is approximately equal to 1, within 1%, for $x < 0.8$, and reaches the minimum value $f(1) = (2\ln 2)(1 + \ln 2) - \pi^2/6 = 0.7$.

The leading contributions to $\Lambda_{G,S}$, in an expansion in F/M^2, are universal for the different GUT components of messengers. This is true because these leading contributions are proportional to the ratio F/M, which is a universal quantity, independent of the coupling constant λ between the messenger superfields and the goldstino superfield X, see eq. (2.5). On the other hand, the argument of the functions g and f in eqs. (2.19) and (2.21) is F/M^2, a quantity which is not equal for messengers with different SM quantum numbers. Therefore, depending on the particular choice of λ, the inclusion of the correction functions g and f can be relevant only for part of the messenger multiplet. This effect gives an uncertainty on the mass prediction which is typically small for squarks and sleptons, but could be up to 40% for gauginos. Notice that, if the messengers form degenerate multiplets at the GUT scale, then F/M^2 is larger for a messenger weak doublet than for a messenger colour triplet. Therefore, the enhancement of the gaugino mass due to the correction function g is larger for the W-ino than for the gluino. This will result in an apparent violation of gaugino-mass unification.

The scalar masses have been computed neglecting Yukawa-coupling effects. This is not a good approximation for the stop system, which is described by the 2×2 mass matrix

$$m^2_{\tilde{t}} = \begin{pmatrix} m^2_{\tilde{Q}_L} + m_t^2 + (\frac{1}{2} - \frac{2}{3}\sin^2\theta_W)\cos 2\beta M_Z^2 & m_t(A_t - \mu\cot\beta) \\ m_t(A_t - \mu\cot\beta) & m^2_{\tilde{t}_R} + m_t^2 + \frac{2}{3}\sin^2\theta_W \cos 2\beta M_Z^2 \end{pmatrix} . \quad (2.23)$$

The third-generation supersymmetry-breaking masses $m^2_{\tilde{Q}_L}$ and $m^2_{\tilde{t}_R}$ at the scale Q ($t = \ln M^2/Q^2$) are given by

$$\begin{aligned} m^2_{\tilde{Q}_L}(t) = \quad & 2\sum_{r=1}^{3} k_r \, \frac{\alpha_r^2(0)}{(4\pi)^2} \, \left[(C_r^{\tilde{Q}_L} - \frac{K_t}{12}a_r)\Lambda_S^2 + h_r C_r^{\tilde{Q}_L}\Lambda_G^2\right] \\ & -\frac{K_t}{6}(H_1 - K_t H_2^2)\Lambda_G^2 \ , \end{aligned} \quad (2.24)$$

$$\begin{aligned} m^2_{\tilde{t}_R}(t) = \quad & 2\sum_{r=1}^{3} k_r \, \frac{\alpha_r^2(0)}{(4\pi)^2} \, \left[(C_r^{\tilde{t}_R} - \frac{K_t}{6}a_r)\Lambda_S^2 + h_r C_r^{\tilde{t}_R}\Lambda_G^2\right] \\ & -\frac{K_t}{3}(H_1 - K_t H_2^2)\Lambda_G^2 \ , \end{aligned} \quad (2.25)$$

Here $a_r = 2(C_r^{\tilde{Q}_L} + C_r^{\tilde{t}_R} + C_r^{H_2}) = (13/9, 3, 16/3)$, and

$$E = \prod_{r=1}^{3}\left[\frac{\alpha_r(0)}{\alpha_r(t)}\right]^{\frac{a_r}{b_r}} , \qquad F = \int_0^t dt \ E \ , \quad (2.26)$$

$$\begin{aligned} H_1 = & \frac{\alpha_X t_X}{4\pi} H_3 \frac{E}{F}\left(\frac{t}{t_X} - 1\right) + \left(\frac{\alpha_X t_X}{4\pi}\right)^2 \left\{\left[\frac{E}{F}\left(\frac{t}{t_X} - 1\right) + \frac{1}{F}\right]\sum_{r=1}^{3} a_r \frac{\alpha_r(0)}{4\pi} \right. \\ & \left. + \sum_{r=1}^{3} a_r b_r \frac{\alpha_r^2(0)}{(4\pi)^2} + \left[\sum_{r=1}^{3} a_r \frac{\alpha_r(0)}{4\pi}\right]^2\right\} , \end{aligned} \quad (2.27)$$

$$H_2 = \frac{\alpha_X t_X}{4\pi}\left[\frac{E}{F}\left(\frac{t}{t_X} - 1\right) + \frac{1}{F} - \frac{1}{t_X} + \sum_{r=1}^{3} a_r \frac{\alpha_r(0)}{4\pi}\right] , \quad (2.28)$$

$$H_3 = \sum_{r=1}^{3} \frac{a_r}{b_r} k_r \frac{\alpha_r(0) - \alpha_r(t)}{4\pi} , \quad (2.29)$$

$$\alpha_X = \frac{k_s b_r - k_r b_s}{b_r \alpha_s^{-1}(0) - b_s \alpha_r^{-1}(0)} \qquad \forall r \neq s \qquad r, s = 1, 2, 3 , \quad (2.30)$$

$$t_X = \frac{4\pi\left[k_r \alpha_s^{-1}(0) - k_s \alpha_r^{-1}(0)\right]}{k_s b_r - k_r b_s} \qquad \forall r \neq s \qquad r, s = 1, 2, 3 , \quad (2.31)$$

$$K_t = \frac{6F}{E}\frac{h_t^2(t)}{(4\pi)^2} . \quad (2.32)$$

Here h_t is the running top-quark Yukawa coupling, related to the top-quark mass by $h_t = (2\sqrt{2}G_F)^{1/2} m_t / \sin\beta$. Since K_t is equal to the squared top-Yukawa coupling in units of the infrared fixed-point value, the condition that h_t does not reach the Landau pole before the messenger scale M implies $K_t < 1$. The definitions of α_X and t_X are independent of the specific indices r and s, because we are assuming gauge-coupling unification. They correspond to the unification coupling constant and the unification mass scale ($t_X = \ln M^2/M_X^2$) in a fictitious theory with no messengers. The assumption of gauge-coupling unification allows us to solve integrals which, in general, cannot be expressed by elementary functions.

There is no one-loop messenger contribution to the supersymmetry-breaking trilinear terms. However A terms are generated in the leading-log approximation by the RG evolution proportional to gaugino masses. In particular, the stop A term appearing in eq. (2.23) is given by

$$A_t(t) = (H_3 - K_t H_2)\Lambda_G \; . \tag{2.33}$$

The A terms corresponding to the other Yukawa couplings can be obtained from eq. (2.33) by setting $K_t = 0$ and replacing a_r with the coefficients of the corresponding interaction.

The above formulae assume that Yukawa couplings other than the one for the top quark are negligible. Therefore, they are not valid for very large $\tan\beta$ because, as $\tan\beta$ approaches m_t/m_b, bottom-quark Yukawa effects become significant.

The parameter μ, which appears in eq. (2.23), is the Higgs mixing mass, defined by the superpotential interaction $\mu H_1 H_2$. This mass term breaks the Peccei–Quinn symmetry, and therefore it cannot be generated by gauge interactions alone. The origin of the μ parameter is still one of the main problematic issues [111] in theories with gauge mediation, as we will discuss in sect. 6. Here we will simply assume that a hard mass parameter μ exists at the messenger scale M. Its value at the low-energy scale is then given by

$$\mu(t) = \mu(0)\ (1 - K_t)^{1/4} \prod_{r=1}^{3} \left[\frac{\alpha_r(0)}{\alpha_r(t)}\right]^{\frac{a_r^\mu}{2b_r}} \; , \tag{2.34}$$

where $a_r^\mu = 2(C_r^{H_1} + C_r^{H_2}) = (1, 3, 0)$.

Analogously, a supersymmetry-breaking Higgs mixing mass $B\mu$ will appear in the scalar potential. The running value of the coefficient B at low energy is

$$B(t) = B(0) + (H_4 - \frac{K_t}{2} H_2)\Lambda_G + \delta B^{(NLO)}(t) \; , \tag{2.35}$$

$$H_4 = \sum_{r=1}^{3} \frac{a_r^\mu}{b_r} k_r \frac{\alpha_r(0) - \alpha_r(t)}{4\pi} \ , \tag{2.36}$$

$$\delta B^{(NLO)}(t) = -\frac{\alpha_s^2(t) h_t^2(t)}{8\pi^4} t \ \Lambda_G + \sum_{r=1}^{3} a_r^\mu k_r \frac{\alpha_r^2(t)}{(4\pi)^2} \Lambda_G \ . \tag{2.37}$$

Here $\delta B^{(NLO)}$ contains terms suppressed by $1/t$ with respect to the leading radiative contributions. The total contribution from this class of terms comes from ultraviolet and infrared theshold effects and some RG evolution, and it is scheme-independent. The result in eq. (2.37) contains only ultraviolet thresholds and running in $\overline{\rm DR}$, and the scheme dependence is cancelled by the effective-potential contribution [245, 141]. The term in eq. (2.37) becomes important for relatively low values of M, where t is small.

Notice that the low-energy value of B is non-vanishing even if $B(0) = 0$. This has motivated [22] several phenomenological studies [95, 27, 245, 46, 128] of the very predictive case $B(0) = 0$, in which the low-energy value of B is determined in terms of the gaugino mass. This case is also interesting because the theory is automatically free from any dangerous new CP-violating parameter, aside from the SM ones, *i.e.* the usual Kobayashi–Maskawa phase and Θ_{QCD}.

The Higgs soft mass parameters $m^2_{H_{1,2}}$ are given by

$$m^2_{H_1}(t) = 2 \sum_{r=1}^{3} C_r^{H_1} k_r \ \frac{\alpha^2(0)}{(4\pi)^2} \ \left[\Lambda_S^2 + h_r \Lambda_G^2\right] + \delta m_{H_1}^{(1-loop)2}(t) \ , \tag{2.38}$$

$$\begin{aligned} m^2_{H_2}(t) = {} & 2 \sum_{r=1}^{3} k_r \ \frac{\alpha^2(0)}{(4\pi)^2} \ \left[(C_r^{H_2} - \frac{K_t}{4} a_r)\Lambda_S^2 + k_r h_r C_r^{H_2} \Lambda_G^2\right] \\ & -\frac{K_t}{2}(H_1 - K_t H_2^2)\Lambda_G^2 + \delta m_{H_2}^{(NLO)2}(t) + \delta m_{H_2}^{(1-loop)2}(t) \ . \end{aligned} \tag{2.39}$$

Here, as it is customary, we added to the running $\overline{\rm DR}$ masses the contributions $\delta m_{H_{1,2}}^{(1-loop)2}$ from infrared thresholds. These effects can be computed using the one-loop effective potential V_{1-loop} in $\overline{\rm DR}$ [129, 38]:

$$\delta m_{H_{1,2}}^{(1-loop)2} = \frac{1}{2H_{1,2}} \frac{\partial V_{1-loop}}{\partial H_{1,2}}\bigg|_{H_{1,2}=\langle H_{1,2}\rangle} \ . \tag{2.40}$$

Finally $\delta m_{H_2}^{(NLO)2}$ contains the constant (*i.e.* not log-enhanced) term in the $\alpha_s^2 h_t^2$ three-loop correction to the value of $m^2_{H_2}$ at the messenger scale M. This has been calculated in ref. [141]

in $\overline{\rm DR}$ (see also the appendix)

$$\delta m_{H_2}^{(NLO)2}(t) = -\frac{\alpha_s^2(t) h_t^2(t)}{8\pi^4} \Lambda_S^2. \tag{2.41}$$

Although the parameter μ is not determined by the underlying theory, we can compute it from the condition of correct electroweak breaking:

$$\mu^2 = -\frac{M_Z^2}{2} + \frac{1}{\tan^2\beta - 1} \left(m_{H_1}^2 - \tan^2\beta \; m_{H_2}^2 \right) \ , \tag{2.42}$$

$$B = \frac{\sin 2\beta}{2\mu} \left(m_{H_1}^2 + m_{H_2}^2 + 2\mu^2 \right) \ . \tag{2.43}$$

The second equation determines the low-energy value of B in terms of $\tan\beta$. If the condition $B(0) = 0$ is imposed, eq. (2.43) predicts the value of $\tan\beta$, which turns out to be rather large. In eqs. (2.42) and (2.43), the parameters μ and B are calculated at the scale Q^2. For physical applications, we can choose $Q^2 = m_{\tilde{t}}^2$. This minimizes most low-energy threshold effects, because below the stop mass scale the RG running is negligible. Therefore the term in eq. (2.41) is significant only when the logarithm of $M/\tilde{m}_t$ is not very large. The full next-to-leading expressions of the Higgs mass parameters can be found in the appendix.

It is important to stress that, if μ and B are generated radiatively through some new interactions, it is very plausible [111] that also $m_{H_{1,2}}^2$ receive extra corrections. These corrections then modify the values of μ and B extracted from the electroweak-breaking conditions.

2.4 Properties of the Mass Spectrum

The most important feature of the gauge-mediated mass spectrum is of course flavour universality, which is guaranteed by the symmetry of gauge interactions. This property is maintained if gravity-mediated contributions do not reintroduce large flavour violations, *e.g.* in the K^0–$\bar{K}^0$ system or in $\mu \to e\gamma$ transitions. We therefore require that gravity-mediated contributions do not account for more than, say, one per mille of the soft squared masses. Since gravity generates soft terms with typical size F/M_P, the flavour criterion gives a rough upper bound on the messenger mass scale

$$M \lesssim \frac{1}{10^{\frac{3}{2}}} \frac{\alpha}{4\pi} M_P \sim 10^{15} \text{ GeV} \ , \tag{2.44}$$

where $M_P = (8\pi G_N)^{-1/2} = 2.4 \times 10^{18}$ GeV is the reduced Planck mass.

An attractive feature of the gauge-mediated mass formulae is that gaugino mass terms are generated at one loop, and (positive) squark mass terms are generated at two loops. Since fermion and boson bilinears have different canonical dimensions, all supersymmetry-breaking mass parameters have the same scaling property $\tilde{m} \sim (\alpha/\pi)F/M$. However, as we will see in sect. 6, it is not automatic for the μ and B parameters to satisfy this property.

Another important property of the mass formulae derived here is that they allow a high degree of predictivity. The whole supersymmetric spectrum is determined by the effective supersymmetry-breaking scale $\Lambda = F/M$, the messenger index N, the messenger mass M, and $\tan\beta$. The parameter μ can be fitted from the electroweak-breaking condition (up to a phase ambiguity), if we assume the absence of any non-minimal contribution to $m^2_{H_{1,2}}$.

Comprehensive analyses of the supersymmetric particle masses in gauge-mediated models have been presented in refs. [95, 27]. Examples of the mass spectra are shown in fig. 3. We have plotted all sparticle masses in units of the B-ino mass $M_1(t) = 5\alpha_1(t)N\Lambda/(12\pi)$ evaluated at the energy scale $Q = M_1$. Since M_1 is essentially independent of M and $\tan\beta$, this choice is equivalent to normalizing the spectrum to Λ. Moreover, the mass ratios plotted in fig. 3 (apart from $m_{\tilde{t}}/M_1$) are fairly independent of Λ and $\tan\beta$, unless Λ (or, equivalently M_1) is rather small. In this case, D-term contributions can affect slepton masses, although they are never very important for squarks. Figure 4 illustrates this regime, showing how D-terms modify the slepton spectrum, and how they can even drive $m_{\tilde{\nu}_L}$ lighter than $m_{\tilde{e}_R}$, for large values of N and small values of Λ and M. This happens, however, only in a very marginal region of parameters, at the border with the experimental limit on slepton masses. The stop-mass eigenvalues are quite sensitive to $\tan\beta$ because of left–right mixing effects, and the lightest stop mass decreases for smaller $\tan\beta$. However, the stop squared masses always remain positive, unless one chooses values of $\tan\beta$ so small that the top-quark Yukawa Landau pole is reached around the scale M. The case $\mu < 0$ gives a larger mixing than the case $\mu > 0$, because the μ contribution in eq. (2.23) adds up to the A_t term, which is always positive, in our notation.

Notice the large hierarchy between the strongly interacting and weakly interacting particles. For small values of M, this hierarchy is determined by the ratio $\alpha_3(0)/\alpha_2(0)$. For larger values of M, where the RG running is important, the value of N plays a crucial rôle. If N is small,

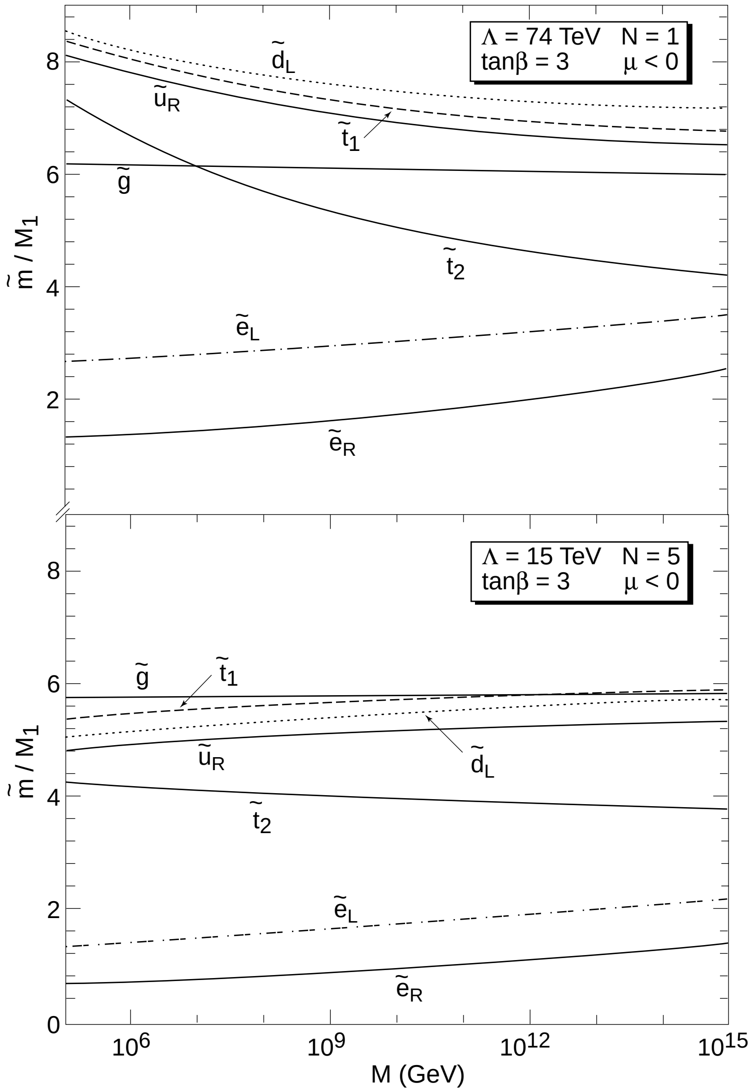

Figure 3: *Different supersymmetric particle masses in units of the B-ino mass M_1, as a function of the messenger mass M. The choice of parameters is indicated, and in both cases it corresponds to a B-ino mass of 100 GeV.*

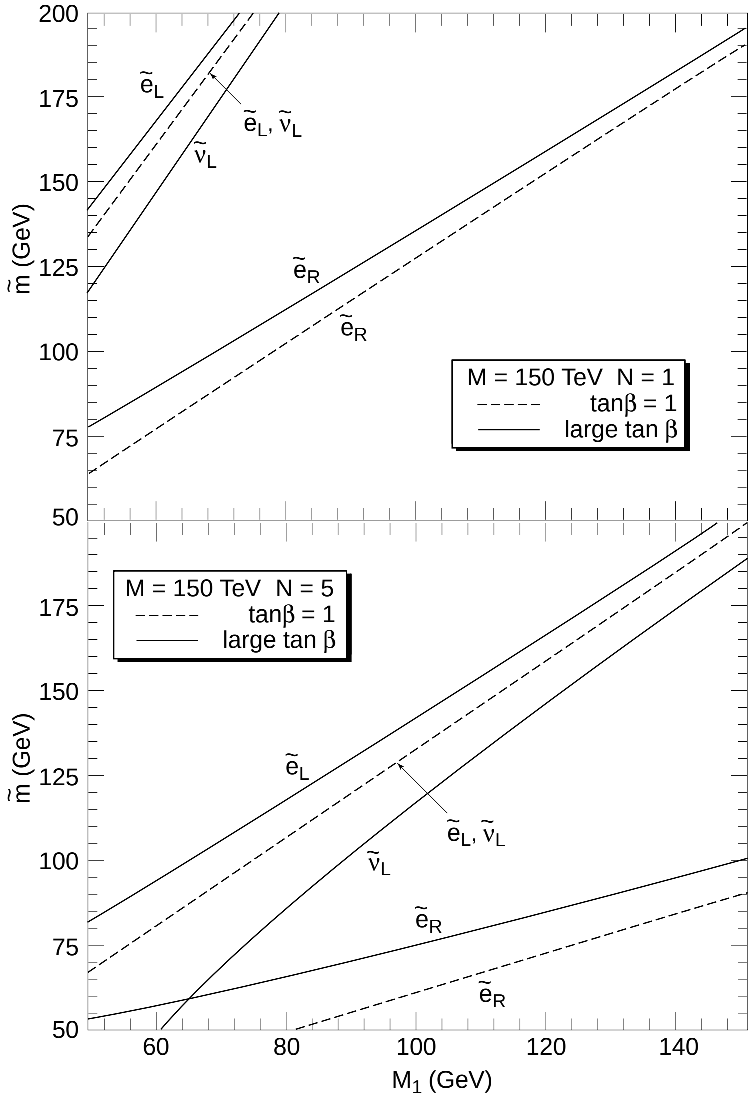

Figure 4: *Slepton masses as a function of the B-ino mass M_1, for the indicated choice of parameters. Dashed lines correspond to the case of vanishing D terms* ($\tan\beta = 1$)*, and the solid lines to the case of maximal D terms* ($\cos 2\beta = -1$)*.*

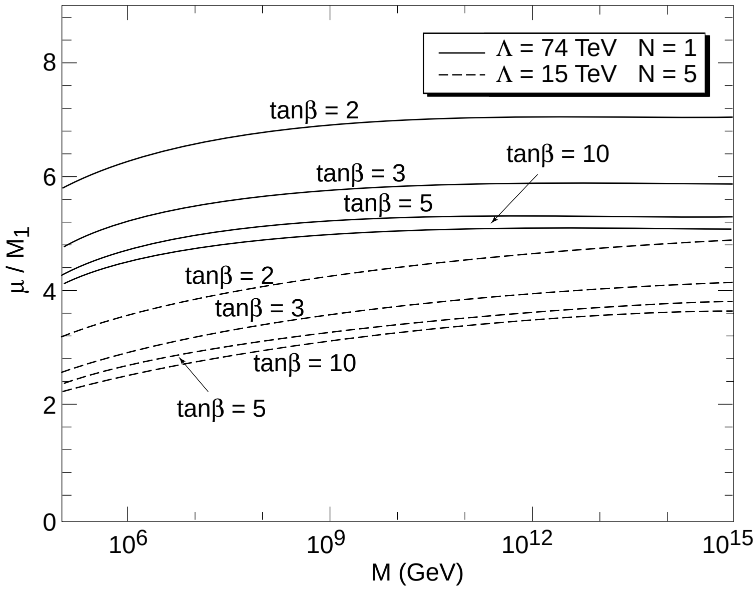

Figure 5: *The ratio between μ and the B-ino mass M_1, as a function of the messenger mass M. The choice of parameters is indicated, and in both cases it corresponds to a B-ino mass of 100 GeV.*

the most important effect is the decrease of the ratio $\alpha_3(0)/\alpha_2(0)$ as M increases, and squarks become lighter and closer in mass to sleptons (see upper frame of fig. 3). On the other hand, if N is large, gaugino masses increase and their effects in the RG evolution dominate the squark and slepton masses. In this case, squarks become heavier as M grows (see lower frame of fig. 3). The ratio between gaugino and scalar masses is determined by the messenger index N. Increasing N, scalars become lighter and the ratio $m_{\tilde{e}_R}/M_1$ can be smaller than 1.

Another remarkable success of the gauge-mediated mass spectrum is that eqs. (2.42) and (2.43) have an acceptable solution [102], and therefore radiative electroweak-symmetry breaking [162] can be achieved. This happens because the negative contribution to $m^2_{H_2}$ is proportional to the large stop mass. Indeed for small M, eq. (2.39), with the addition of the low-energy threshold corrections, can be approximated by

$$m^2_{H_2} = m^2_{\tilde{e}_L} - \frac{3h_t^2}{4\pi^2} m^2_{\tilde{t}} \left(\ln \frac{M}{m_{\tilde{t}}} + \frac{3}{2} \right) . \qquad (2.45)$$

Therefore, even for moderate values of M, the coefficient in front of the logarithm is large enough to drive $m^2_{H_2}$ negative and trigger electroweak-symmetry breaking.

If the condition of electroweak breaking is imposed, then the value of μ is determined by eq. (2.42), with the result shown in fig. 5. Rather large values of μ are required to compensate the stop contribution in eq. (2.42). The ratio μ/M_1 decreases for large N, but in this regime M_1 is no longer the smallest supersymmetric particle mass in the spectrum. This rather large value of μ, originating from the hierarchy between strongly and weakly interacting particles is at the basis of fairly stringent upper bounds on the supersymmetric particle masses obtained from the naturalness criterion. For instance, using the criterion [31] that no independent parameter should be correlated by more than 10%, it is found [65, 44, 11] that the right-handed selectron has to be lighter than 100 GeV or less, depending on the parameters. This limit does not significantly change for large N, since the ratio $\mu/m_{\tilde{e}_R}$ only slowly increases as N grows. It was also suggested in ref. [11] that messengers belonging to split GUT multiplets may alleviate the fine-tuning problem present in the minimal model.

With the value of μ extracted from the electroweak-breaking condition, we can now compute the spectrum of neutralinos and charginos. Since μ typically turns out to be larger than M_Z, with a good approximation we can treat electroweak breaking as a perturbation and obtain [191]

$$m_{\chi_1^0} = M_1 - \frac{M_Z^2 \sin^2\theta_W (M_1 + \mu \sin 2\beta)}{\mu^2 - M_1^2} , \tag{2.46}$$

$$m_{\chi_1^\pm} = m_{\chi_2^0} = M_2 - \frac{M_W^2 (M_2 + \mu \sin 2\beta)}{\mu^2 - M_2^2} , \tag{2.47}$$

$$m_{\chi_2^\pm} = \mu + \frac{M_W^2 (\mu + M_2 \sin 2\beta)}{\mu^2 - M_2^2} , \tag{2.48}$$

$$m_{\chi_3^0} = \mu + \frac{M_Z^2 (1 + \sin 2\beta)(\mu - M_1 \cos^2\theta_W - M_2 \sin^2\theta_W)}{2(\mu - M_1)(\mu - M_2)} , \tag{2.49}$$

$$m_{\chi_4^0} = -\mu - \frac{M_Z^2 (1 - \sin 2\beta)(\mu + M_1 \cos^2\theta_W + M_2 \sin^2\theta_W)}{2(\mu + M_1)(\mu + M_2)} . \tag{2.50}$$

The lightest neutralino is mainly B-ino, $\chi_1^\pm$ and χ_2^0 form a degenerate W-ino weak triplet, and the nearly higgsinos $\chi_2^\pm$ and $\chi_{3,4}^0$ have masses roughly equal to μ. Figures 6 and 7 show the light neutralino and chargino masses in the regime where the approximation in eqs. (2.46)–(2.50) is not reliable, namely small Λ. Actually, for $N = 1$ this approximation is still good in almost all the range of Λ, since μ is still large enough to decouple higgsinos from gauginos. Significant deviations appear when $N = 5$, as shown in figs. 6 and 7, because both μ and M_2 can both become comparable to M_Z.

The CP-odd neutral Higgs mass is given by

$$m_A^2 = \frac{2B\mu}{\sin 2\beta} \simeq \frac{\mu^2}{\sin^2\beta} \tag{2.51}$$

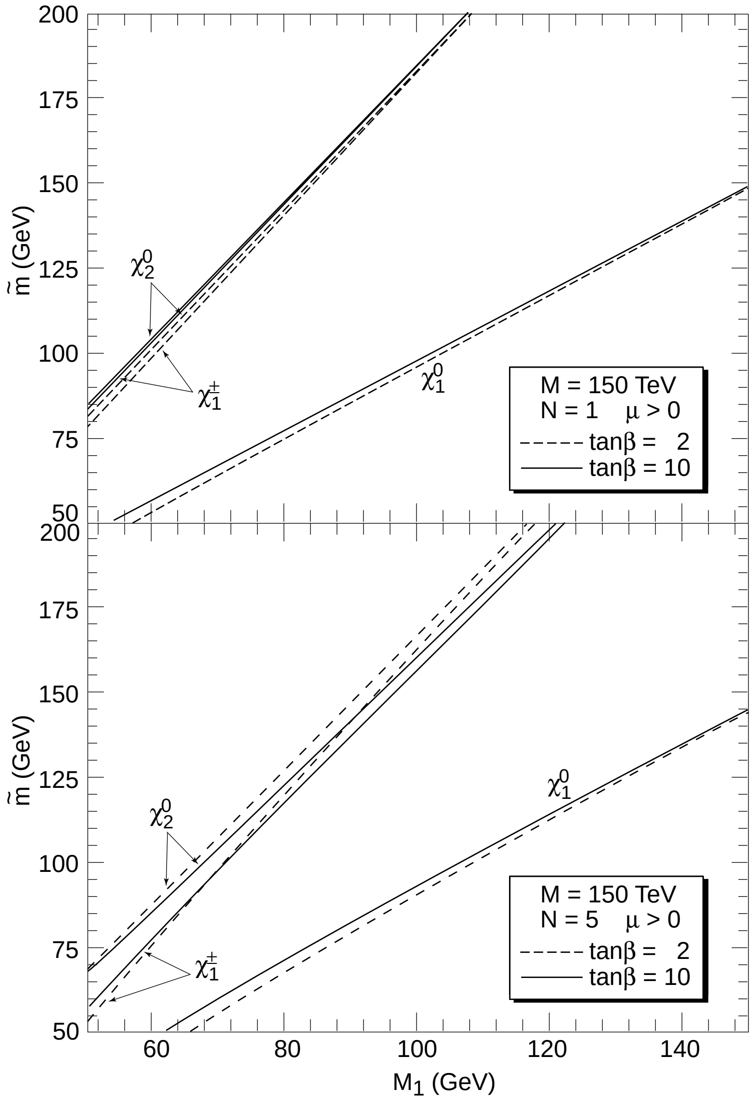

Figure 6: *The masses of the lightest neutralino* (χ_1^0) *and chargino* ($\chi_1^\pm$) *and the next-to-lightest neutralino* (χ_2^0) *as a function of the B-ino mass* M_1*, for* $\mu > 0$ *and the indicated choice of parameters.*

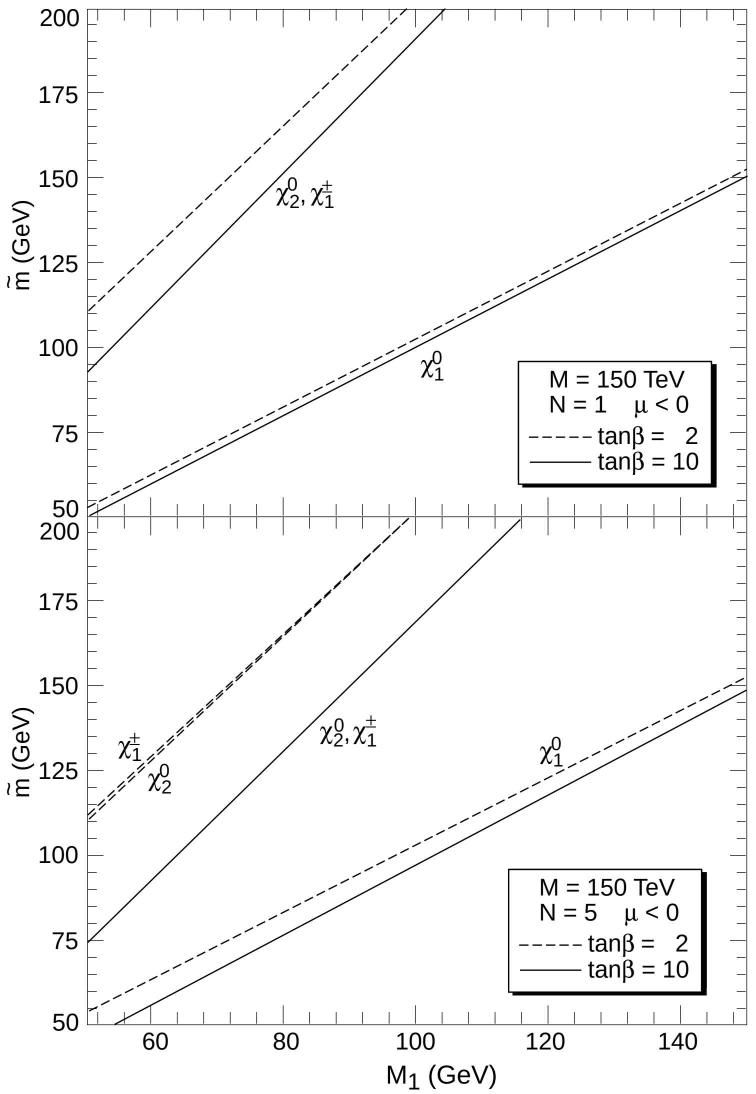

Figure 7: *The masses of the lightest neutralino (χ_1^0) and chargino ($\chi_1^\pm$) and the next-to-lightest neutralino (χ_2^0) as a function of the B-ino mass M_1, for $\mu < 0$ and the indicated choice of parameters.*

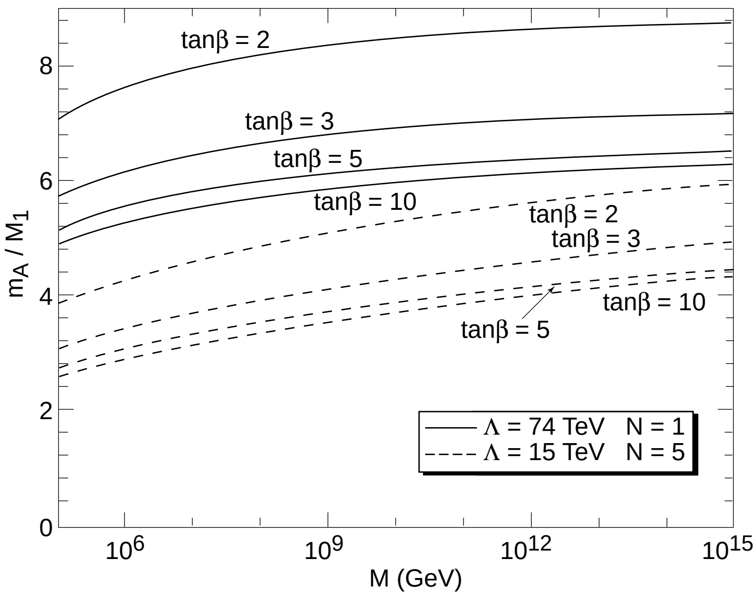

Figure 8: *The ratio between CP-odd Higgs boson mass m_A and the B-ino mass M_1, as a function of the messenger mass M. The choice of parameters is indicated, and in both cases it corresponds to a B-ino mass of 100 GeV.*

and shown in fig. 8. Since m_A is rather large, the theory at low energies is approximately described by a single Higgs doublet, and the Higgs phenomenology at LEP2 should resemble the SM case with a light Higgs. The charged Higgs boson mass is given by $m^2_{H^\pm} = M^2_W + m^2_A$.

The lightest CP-even Higgs boson mass m_h receives important radiative corrections proportional to the top quark Yukawa coupling [216, 115, 151], and it can be predicted in terms of the fundamental parameters Λ, M, N, and $\tan\beta$ [95, 27, 247]. Including the leading two-loop effects, in the limit of large m_A, the Higgs boson mass can be approximated as [51]

$$m_h^2 = M_Z^2 \cos^2 2\beta \left(1 - \frac{3\sqrt{2}}{4\pi^2} G_F m_t^2 \, t_S\right) +$$

$$\frac{3\sqrt{2}}{2\pi^2} G_F m_t^4 \left\{ \frac{\tilde{X}_t}{2} + t_S + \frac{1}{16\pi^2}\left[3\sqrt{2} G_F m_t^2 - 32\pi\alpha_s(M_t)\right](\tilde{X}_t + t_S)t_S \right\} \ , \tag{2.52}$$

$$t_S = \ln\left(\frac{M_S^2}{M_t^2}\right) \qquad M_S^2 = m_{\tilde{t}_1} m_{\tilde{t}_2} \ , \tag{2.53}$$

$$\tilde{X}_t = \frac{2(A_t - \mu\cot\beta)^2}{M_S^2}\left[1 - \frac{(A_t - \mu\cot\beta)^2}{12 M_S^2}\right] \ , \tag{2.54}$$

$$m_t = \frac{M_t}{1 + \frac{4}{3\pi}\alpha_s(M_t)} \ . \tag{2.55}$$

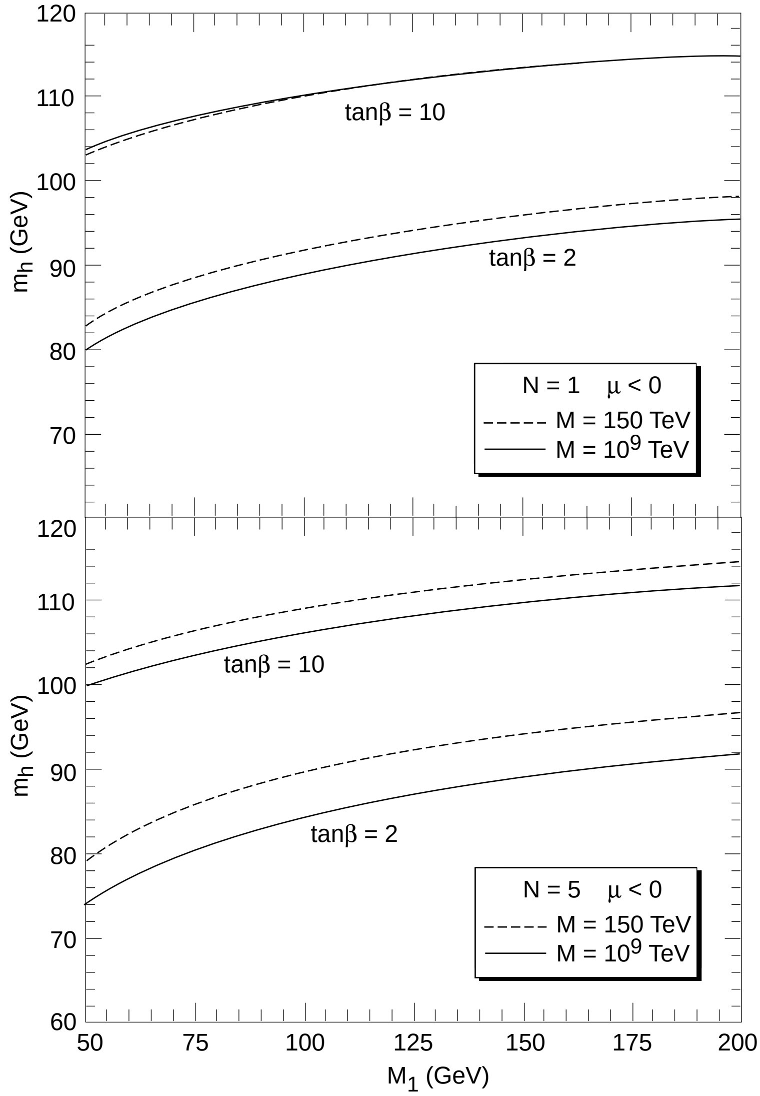

Figure 9: *The mass of the lightest CP-even Higgs boson as a function of the B-ino mass M_1 for the indicated choice of parameters.*

Here m_t, M_t are the running and $\overline{\rm MS}$ pole top-quark masses, respectively. The Higgs boson mass is shown in fig. 9 in the case $\mu < 0$, where stop mixing effects are maximized. The largest values of m_h are obtained at large $\tan\beta$. For large $\tan\beta$ and small N, the stop mixing is negligible and the value of m_h shown in fig. 9 corresponds to the value obtained in the usual supersymmetric model with vanishing stop mixing [51]. As we increase M, A_t grows and the typical squark mass decreases; the two effects roughly compensate each other and the value of m_h is not modified. At larger N, the squarks are lighter and the value of m_h is smaller. The effect of increasing M is now more important since A_t receives a large renormalization proportional to the gaugino mass. In any case, a considerable fraction of the parameter space is within the reach of LEP2. If the Higgs boson is not discovered, the LEP2 search will provide an extremely severe bound on the model.

In conclusion, gauge-mediated models have a very predictive mass spectrum. If supersymmetry is discovered, it seems very likely that these predictions can be used to distinguish these models from the generic spectrum of gravity mediation. In the presence of unification relations, gaugino masses are the same in both scenarios, but scalar masses are different. Even if M is close to the GUT scale, the initial condition of scalar masses is not unified. This is because the gauge bosons are already split and do not form a complete GUT multiplet. From eq. (2.14), we find

$$\frac{2}{7}\frac{m^2_{\tilde{Q}_L}(0)}{m^2_{\tilde{E}_R}(0)} = \frac{3}{8}\frac{m^2_{\tilde{U}_R}(0)}{m^2_{\tilde{E}_R}(0)} = \frac{3}{7}\frac{m^2_{\tilde{D}_R}(0)}{m^2_{\tilde{E}_R}(0)} = \frac{2}{3}\frac{m^2_{\tilde{L}_L}(0)}{m^2_{\tilde{E}_R}(0)} = 1 \; . \tag{2.56}$$

However, for large N and large M the mass spectrum is "gaugino-dominated" and the signal of gauge mediation is more difficult to be distinguished. Studies of the comparison between the mass spectrum of gauge-mediated and gravity-mediated models can be found in refs. [95, 27, 53, 262].

2.5 Variations of the Minimal Model

We have seen that the mass formulae obtained in gauge mediation are very predictive. It is then important to assess how much these predictions depend on the assumptions of the minimal model. Moreover, some of the variations we consider below have important virtues and they could be the natural outcome of simple fundamental models.

As mentioned before, the prediction for the μ parameter extracted from the electroweak-breaking conditions is modified by possible new contributions to $m^2_{H_{1,2}}(0)$, which arise in theories where all mass parameters are generated radiatively [111], see sect. 6. However, the parameter μ is not significantly modified as long as the new contributions to the Higgs mass parameters are smaller than the usual gauge-mediated contributions, since the two effects add in quadratures. The effects of arbitrary new contributions to $m^2_{H_{1,2}}(0)$ in the prediction of μ and of the mass spectrum have been discussed in ref. [95].

In our study of the minimal gauge-mediated model, we have assumed that the goldstino resides in a single chiral superfield, and therefore the matrices F and M are proportional to each other. If this is not the case, in the basis in which M is diagonal and real, F is still a generic matrix in flavour space. Now supersymmetry-breaking scalar masses can receive a new contribution from an induced Fayet–Iliopoulos term [117]. The one-loop contribution, proportional to the sfermion ($\tilde{f}$) and messenger (Φ) hypercharge Y, is [94]

$$\Delta m^2_{\tilde{f}} = \frac{\alpha_1}{4\pi} Y_{\tilde{f}} \ \mathrm{Tr} Y_\Phi \ \Lambda^2_D \ , \tag{2.57}$$

where the trace is taken over the complete GUT messenger representation. At leading order in F/M^2, Λ^2_D is independent of the specific component of the GUT multiplet Φ:

$$\Lambda^2_D = \frac{1}{2} \sum_{i,j=1}^{N_f} \frac{|F_{ji}|^2 - |F_{ij}|^2}{M_i^2} f_D\left(\frac{M_j^2}{M_i^2}\right) \ , \tag{2.58}$$

$$f_D(x) = \frac{2}{(1-x)} + \frac{(1+x)}{(1-x)^2} \ln x \ . \tag{2.59}$$

Equation (2.57) vanishes either if the messengers form complete GUT multiplets ($\mathrm{Tr}Y_\Phi$=0) or if the messenger sector is invariant under a "messenger parity", defined in ref. [94], which guarantees $\Lambda_D = 0$, since one can choose a basis in which M is diagonal and real and F Hermitian. This cancellation is welcome, since the one-loop contribution to scalar masses in eq. (2.57) is proportional to the sfermion hypercharge and therefore is not positive-definite. Were it to dominate over the ordinary two-loop gauge contribution, it would lead to an unacceptable mass spectrum. However one can consider theories with messengers embedded in a GUT, but with no "messenger parity". Now, even if $\mathrm{Tr}Y_\Phi = 0$, a hypercharge D-term contribution to scalar masses is generated either at higher order in F/M^2 ($m^2_{\tilde{f}} \sim [\alpha_1/\pi]F^4/M^6$) or at two loops ($m^2_{\tilde{f}} \sim [\alpha_1\alpha_3/\pi^2]F^2/M^2$). Complete expressions for these contributions can be found in ref. [94]. Numerically, these effects can be important for slepton masses and for the Higgs mass parameters, but are insignificant for squark masses.

Another possible variation is described by renormalizable superpotential couplings between messenger and matter superfields. These couplings are usually assumed to vanish, because generically they break flavour invariance, reintroducing the flavour problem, whose solution was the primary motivation for gauge mediation. However they are allowed by gauge invariance and their consequences have been analysed in ref. [106]. It has been found [111, 106] that messenger-matter couplings do not induce soft masses at one loop, at leading order in F/M^2, if the goldstino is contained in a single superfield X. However they contribute to scalar masses at two loops, and (in contrast with gauge mediation) give a non-vanishing A-type trilinear interaction at the one-loop level. Complete formulae for the supersymmetry-breaking terms induced by messenger-matter couplings can be found in ref. [141].

So far we have considered the case in which the messenger particles belong to chiral superfields. However, it is also possible that gauge supermultiplets behave as messengers. We are envisaging a situation in which the supersymmetry-breaking VEV is also responsible for spontaneous breaking of some gauge symmetry containing the SM as an unbroken subgroup. The vector bosons corresponding to the broken generators, together with their supersymmetric partners, receive masses proportional to M. However supersymmetry-breaking effects proportional to F split the gauge supermultiplets at tree level and consequently soft terms for observable fields are generated by quantum effects. These terms have been computed in ref. [141]. The gaugino masses have the same expressions as in eq. (2.11), once the messenger index N, see eq. (2.3) is identified with

$$N = nN_f - 2(C_G - C_H) \ . \tag{2.60}$$

Here nN_f is the usual chiral messenger contribution. The second term in eq. (2.60) describes the new contribution from gauge messengers and it is defined as follows. We have assumed that the scalar component VEV of the goldstino superfield X spontaneously breaks the gauge group $G \to H$, in such a way that the gauge coupling constant is continuous at the threshold. C_G and C_H are the quadratic Casimirs of the adjoint representations of G and H (equal to N for an $SU(N)$ group). Equation (2.60) shows that the pure vector-messenger effect is to reduce the value of N, and also to allow negative values of the total N. Complete formulae for the gauge-messenger contribution to soft terms can be found in ref. [141]. It turns out that scalar squared masses receive new negative contributions, which can destabilize the ordinary gauge group and which pose a serious constraint to construct acceptable models. It is also interesting to notice that gauge messengers lead to one-loop contributions to A terms, in contrast to the case of chiral messengers.

A conceivable possibility, although not realized in any complete model built so far, is that not only the secluded sector, but the messengers themselves feel some strongly interacting gauge force. In this case, the perturbative calculations presented in sect. 2.3 are no longer valid. Exchange of many new "gluons" along the messenger lines give unsuppressed effects. Using naive dimensional analysis to count the factors of 4π, it has been shown [186, 70] that the soft terms induced by strongly interacting messengers have the same form as in the case of weakly interacting messengers, up to (not calculable) factors of order 1. Furthermore, the leading-order expressions for the gaugino masses are not corrected by the new strong interactions, since they are protected by a "screening theorem" [20].

The authors of ref. [112] have investigated the interesting possibility of identifying the messengers with the GUT multiplets containing ordinary Higgs bosons. Unfortunately, this case requires a considerable amount of fine-tuning to maintain the weak scale much smaller than the messenger mass scale. The case in which the messengers do not form complete GUT multiplets has been studied in ref. [193]. Finally, the papers in refs. [107, 195, 59] discuss models in which the soft terms are generated by new gauge interactions beyond those of the ordinary $SU(3) \times SU(2) \times U(1)$.

3 Phenomenology of Models with Gauge-Mediated Supersymmetry Breaking

3.1 The Lightest Supersymmetric Particle: the Gravitino

As a result of the spontaneous breakdown of supersymmetry, the physical spectrum contains a massless spin-1/2 fermion, the goldstino. When the globally supersymmetric theory is coupled to gravity and promoted to a locally supersymmetric theory, the goldstino provides the longitudinal modes of the spin-3/2 partner of the graviton, the gravitino. As a result of this super-Higgs mechanism, the gravitino acquires a supersymmetry-breaking mass which, under the condition of vanishing cosmological constant, is given by [272, 81]

$$m_{3/2} = \frac{F_0}{\sqrt{3}M_P} \ . \tag{3.1}$$

Here $M_P = (8\pi G_N)^{-1/2} = 2.4\times10^{18}$ GeV is the reduced Planck mass. We denote by F_0 the total contribution of the supersymmetry-breaking VEV of the auxiliary fields, normalized in such a way that the vacuum energy of the globally supersymmetric theory is $V = F_0^2$. Thus F_0 does not coincide with the definition of F, which appears in the sparticle masses through $\Lambda = F/M$. While F_0 is the fundamental scale of supersymmetry breaking, F is the scale of supersymmetry breaking felt by the messenger particles, *i.e.* the mass splitting inside their supermultiplets. The ratio $k \equiv F/F_0$ depends on how supersymmetry breaking is communicated to the messengers. If the communication occurs via a direct interaction, this ratio is just given by a coupling constant, like the parameter λ in the case described by eq. (2.5). It can be argued that this coupling should be smaller than 1, by requiring perturbativity up to the GUT scale [16]. If the communication occurs radiatively, then k is given by some loop factor, and therefore it is much smaller than 1. We thus rewrite the gravitino mass as

$$m_{3/2} = \frac{F}{k\sqrt{3}M_P} = \frac{1}{k}\left(\frac{\sqrt{F}}{100 \text{ TeV}}\right)^2 2.4 \text{ eV} \ , \tag{3.2}$$

where the model-dependent coefficient k is such that $k < 1$, and possibly $k \ll 1$.

In gauge-mediated models, the gravitino is the lightest supersymmetric particle (LSP) for any relevant value of F. Indeed, as argued in sect. 2.4, a safe solution to the flavour problem requires that gravity-mediated contributions to the sparticle spectrum should be much smaller than gauge-mediated contributions. Since $m_{3/2}$ is exactly the measure of gravity-mediated effects, it is indeed the solution of the flavour problem in gauge mediation, see eq. (2.44), which implies that the gravitino is the LSP.

If R parity is conserved, all supersymmetric particles follow decay chains that lead to gravitinos. In order to compute the decay rate we need to know the interaction Lagrangian at lowest order in the gravitino field. Since, for $\sqrt{F} \ll M_P$, the dominant gravitino interactions come from its spin-1/2 component, the interaction Lagrangian can be computed in the limit of global supersymmetry. In the presence of spontaneous supersymmetry breaking, the supercurrent J_Q^μ satisfies the equation

$$\partial_\mu J_Q^\mu = -F_0\gamma^\mu\partial_\mu\tilde{G} \ , \tag{3.3}$$

which is the equivalent of the usual current algebra relation for soft pions. This can be viewed as the goldstino equation of motion. The corresponding interaction Lagrangian is

$$\mathcal{L} = -\frac{1}{F_0} J_Q^\mu \partial_\mu \tilde{G} \ . \tag{3.4}$$

This shows how the goldstino interacts with derivative couplings suppressed by $1/F_0$, which are typically more important than the gravitational couplings suppressed by powers of $1/M_P$. Since we are interested in the goldstino couplings to field bilinears, we can replace J_Q^μ in eq. (3.4) by its expression for free fields and obtain

$$\mathcal{L} = -\frac{k}{F} \left(\bar{\psi}_L \gamma^\mu \gamma^\nu \partial_\nu \phi - \frac{i}{4\sqrt{2}} \bar{\lambda}^a \gamma^\mu \sigma^{\nu\rho} F^a_{\nu\rho} \right) \partial_\mu \tilde{G} + \text{h.c.} \tag{3.5}$$

Here ϕ and ψ are the scalar and fermionic components of a generic chiral supermultiplet and λ^a and $F^a_{\mu\nu}$ are the Majorana spinor and gauge field strength belonging to a vector supermultiplet.

The Lagrangian in eq. (3.5) can also be derived by using the supersymmetric analogue of the equivalence theorem [118, 120, 56, 57]. This theorem allows the replacement, in high-energy processes, of an external-state gravitino field with $\sqrt{2/3}\ \partial_\mu \tilde{G}/m_{3/2}$. If this substitution is done in the relevant supergravity Lagrangian, one indeed recovers eq. (3.5). For this reason, it is perfectly adequate for our purposes to describe the LSP in terms of the goldstino properties. The only rôle played by gravity is to generate the LSP mass given in eq. (3.2).

For on-shell particles, by using the equations of motion, the goldstino Lagrangian in eq. (3.5) can be written as a Yukawa interaction with chiral fields and a magnetic moment-like interaction with gauge particles,

$$\mathcal{L} = \frac{k}{F} \left[(m_\psi^2 - m_\phi^2) \bar{\psi}_L \phi + \frac{M_\lambda}{4\sqrt{2}} \bar{\lambda}^a \sigma^{\nu\rho} F^a_{\nu\rho} \right] \tilde{G} + \text{h.c.} \tag{3.6}$$

Notice that both goldstino interactions are proportional to the mass splitting inside the supermultiplet and inversely proportional to the scale of supersymmetry breaking [118, 120].

For our purposes the interaction Lagrangians in eqs. (3.5) and (3.6) are sufficient to describe the relevant processes involving the goldstino. Derivations of the complete effective Lagrangian involving multi-goldstino interactions can be found in refs. [68, 134, 47, 187, 48, 69].

3.2 The Next-to-Lightest Supersymmetric Particle

The next-to-lightest supersymmetric particle (NLSP) plays an important rôle in the phenomenology of gauge mediation. Assuming R-parity conservation, we expect that all supersymmetric particles will promptly decay into cascades leading to the NLSP, with the NLSP then decaying into the gravitino via $1/F$ interactions. Therefore the nature of the NLSP determines the signatures in collider experiments and some cosmological properties of gauge mediation. From the study in sect. 2.4, we have seen that the NLSP can be, depending on the parameter choice, the neutralino, the stau, or, in a very restricted region of parameters, the sneutrino. Let us review these possibilities.

The NLSP neutralino has, in most cases, a dominant B-ino component, since the ratio μ/M_1 is typically larger than 1. An exception occurs for large N and small M and Λ, as the NLSP neutralino is a non-trivial superposition of different states. Another exception is given by models where the physics generating the parameter μ also contributes to supersymmetry-breaking Higgs masses.

From eq. (3.6) we find the following NLSP χ_1^0 decay rates [50, 15, 92, 27]:

$$\Gamma(\chi_1^0 \to \gamma \tilde{G}) = \frac{k^2\ \kappa_\gamma m_{\chi_1^0}^5}{16\pi F^2} = k^2\ \kappa_\gamma \left(\frac{m_{\chi_1^0}}{100\ \text{GeV}}\right)^5 \left(\frac{100\ \text{TeV}}{\sqrt{F}}\right)^4\ 2\times 10^{-3}\ \text{eV}\ , \tag{3.7}$$

$$\frac{\Gamma(\chi_1^0 \to Z^0 \tilde{G})}{\Gamma(\chi_1^0 \to \gamma \tilde{G})} = \frac{\kappa_Z}{\kappa_\gamma}\left(1-\frac{M_Z^2}{m_{\chi_1^0}^2}\right)^4\ , \tag{3.8}$$

$$\frac{\Gamma(\chi_1^0 \to h^0 \tilde{G})}{\Gamma(\chi_1^0 \to \gamma \tilde{G})} = \frac{\kappa_h}{\kappa_\gamma}\left(1-\frac{m_h^2}{m_{\chi_1^0}^2}\right)^4\ , \tag{3.9}$$

$$\kappa_\gamma = |N_{11}\cos\theta_W + N_{12}\sin\theta_W|^2\ , \tag{3.10}$$

$$\kappa_Z = |N_{11}\sin\theta_W - N_{12}\cos\theta_W|^2 + \frac{1}{2}|N_{13}\cos\beta - N_{14}\sin\beta|^2\ , \tag{3.11}$$

$$\kappa_h = |N_{13}\sin\alpha - N_{14}\cos\alpha|^2\ . \tag{3.12}$$

Here N_{1i} are the χ_1^0 components in standard notation (see, *e.g.* ref. [150]) and $\tan 2\alpha = \tan 2\beta (m_A^2 + M_Z^2)/(m_A^2 - M_Z^2)$. The decay mode into photon and goldstino is very likely to dominate. Even if the decay modes into the Z^0 boson or the neutral Higgs boson are kinematically allowed, they are quite suppressed by the β^8 phase-space factor. Moreover, if χ_1^0 is mainly B-ino, $\kappa_Z/\kappa_\gamma = 0.3$ and κ_h/κ_γ is negligible. Complete expressions for the neutralino decay rates into three-body final states can be found in ref. [27].

For roughly

$$N > \frac{66}{5(13\xi_1 - 2)} \ , \quad \xi_1 \equiv \frac{\alpha_1^2(M_1)}{\alpha_1^2(M)} = \left[1 + \frac{11}{4\pi}\alpha_1(M_1)\ln\frac{M_1^2}{M^2}\right]^2 \ , \tag{3.13}$$

the right-handed slepton is lighter than χ_1^0. The transition occurs at moderate values of N for small M (*e.g.* $N = 1.7$ for $M = 10^5$ GeV), but requires large values of N for large M (*e.g.* $N > 5$ for $M = 10^{12}$ GeV). This is the result of the significant renormalization of $m_{\tilde{E}_R}$ proportional to the gaugino mass, in the regime of large N and M.

Among the three generations of right-handed sleptons, $\tilde{\tau}_R$ is the lightest because of mixing effects proportional to m_τ in the stau mass matrix:

$$m^2_{\tilde{\tau}} = \begin{pmatrix} m^2_{\tilde{L}_L} + m^2_\tau - (\frac{1}{2} - \sin^2\theta_W)\cos 2\beta M_Z^2 & m_\tau(A_\tau - \mu\tan\beta) \\ m_\tau(A_\tau - \mu\tan\beta) & m^2_{\tilde{E}_R} + m^2_\tau - \sin^2\theta_W \cos 2\beta M_Z^2 \end{pmatrix} \ . \tag{3.14}$$

As the mixing grows with $\tan\beta$, see eq. (3.14), the stau can become the NLSP for values of N quite smaller than shown in eq. (3.13). In particular, for extremely large values of $\tan\beta$, the determinant of the matrix in eq. (3.14) can become negative and destabilize the electromagnetically neutral vacuum [22, 245].

The NLSP stau decay rate is

$$\Gamma(\tilde{\tau} \to \tau\tilde{G}) = \frac{k^2 m^5_{\tilde{\tau}}}{16\pi F^2} = k^2 \left(\frac{m_{\tilde{\tau}}}{100 \text{ GeV}}\right)^5 \left(\frac{100 \text{ TeV}}{\sqrt{F}}\right)^4 \ 2 \times 10^{-3} \text{ eV} \ . \tag{3.15}$$

If the mixing is small, the lightest stau is mainly right-handed. In this case, $\tilde{e}_R$ and $\tilde{\mu}_R$ are so close in mass to $\tilde{\tau}$ that their three-body decays into the NLSP are very much suppressed by phase space. Under these conditions, all three right-handed sleptons decay directly into the corresponding charged lepton and goldstino. Particles which, in spite of not being the NLSP, have a dominant two-body decay into their supersymmetric partner and a $\tilde{G}$ have been called [16] "co-NLSPs". For larger values of $\tan\beta$, typically $\tan\beta > 4$–8 depending on the parameter choice, the first two generations of sleptons decay as $\tilde{\ell}_R \to \ell\tau\tilde{\tau}$ and the stau is the "only" NLSP. Another possibility is that χ_1^0, although not the NLSP, is nearly mass degenerate with $\tilde{\tau}$, and therefore decays dominantly into a photon and a goldstino. In this case, $\tilde{\tau}$ and χ_1^0 are again "co-NLSPs".

The possibility that a sneutrino is the NLSP is very marginal. As seen in sect. 2.4, it requires large values of N and values of Λ so low that the discovery of supersymmetry should take place quite soon. In this case, the sneutrino decays into a neutrino and a goldstino.

These various cases correspond to a quite different phenomenology in high-energy experiments, as we will discuss in sect. 3.3.

Finally, a completely different option is that the gluino is a stable LSP. This occurs in models in which the coloured messengers are heavier than their weak GUT partners, and in which the mass scale M is close to the GUT scale [241, 242]. This could be a natural possibility in theories with Higgs–messengers mixings, as a consequence of a doublet–triplet splitting mechanism.

3.3 Signals in Collider Experiments

The phenomenology of supersymmetric-particle production in collider experiments is a well-studied subject (see *e.g.* refs. [23, 108, 223, 265] and references therein). Here we will not try to give a comprehensive review of these studies, but merely discuss the main peculiarities of gauge-mediated models with respect to the ordinary searches for supersymmetry.

Depending on the nature of the NLSP, its decay modes, and its decay rate, the signals in collider experiments predicted by gauge-mediated models can be quite different. As we have seen in sect. 2.4, the NLSP can be the lightest neutralino χ_1^0, the lightest stau $\tilde{\tau}$ or, in a very marginal corner of parameter space, the lightest sneutrino. Moreover, there is the possibility of having co-NLSPs, *i.e.* particles other than the NLSP that have a direct two-body decay into the gravitino. This occurs when the mass difference between NLSP and co-NLSP is small enough to suppress the ordinary supersymmetric decay and when F_0 is adequately low to allow for a sizeable decay rate into gravitinos. Candidates for co-NLSP are χ_1^0 (with $\tilde{\tau}$ NLSP), $\tilde{\tau}$ (with χ_1^0 NLSP), and $\tilde{e}_R$ and $\tilde{\mu}_R$ (with $\tilde{\tau}$ NLSP or co-NLSP). All supersymmetric particles decay in cascade processes leading to an NLSP or a co-NLSP which, depending on the case, decays into a photon or a charged lepton and a gravitino. An NLSP sneutrino decays into neutrino and gravitino and therefore, from the detector point of view, it behaves like a stable invisible particle.

From the NLSP decay rate, see eqs. (3.7) and (3.15), we obtain that the average distance travelled by an NLSP with mass m and produced with energy E is

$$L = \frac{1}{\kappa_\gamma} \left(\frac{100 \text{ GeV}}{m} \right)^5 \left(\frac{\sqrt{F/k}}{100 \text{ TeV}} \right)^4 \sqrt{\frac{E^2}{m^2} - 1} \times 10^{-2} \text{ cm} . \tag{3.16}$$

Here κ_γ is given in eq. (3.10) for χ_1^0 and it is equal to 1 for the stau. Mainly depending on the unknown value of $\sqrt{F/k}$, the NLSP can either decay within microscopic distances or decay well outside the solar system. Therefore from the collider experiments' point of view, there are two relevant regimes, which correspond to different search strategies.

If $\sqrt{F/k}$ is large (roughly larger than 10^6 GeV), the NLSP decays outside the detector and therefore behaves like a stable particle. If χ_1^0 is the NLSP, the collider signatures closely resemble those of the ordinary supersymmetric scenarios with a stable neutralino. The only handle to distinguish a gauge-mediated origin is given by the properties of the mass spectrum, described in sect. 2.4. On the other hand, for a $\tilde{\tau}$ NLSP, the signature is quite novel, with a stable charged massive particle going through the detector, leaving an anomalous ionization track.

For small $\sqrt{F/k}$ (typically $\sqrt{F/k} \lesssim 10^6$ GeV), the NLSP promptly decays and the experimental signature is given by events with missing transverse energy, imbalance in the final-state momenta and a pair of photons or charged leptons, possibly accompanied by other particles. This is a very characteristic signal, which typically allows better detection efficiency than the usual missing-energy signal of ordinary supersymmetry. Moreover, in this case, looking for NLSP pair production, it is possible to extend the search to a portion of the parameter space not accessible in the corresponding gravity-mediated scenario, where the LSP is invisible.

The intermediate region between the two regimes is particularly interesting. In this case the NLSP decay length could be measurable as a vertex displacement of the final-state photon (or tau). It is interesting experimentally, because it allows a better background rejection, but also theoretically, because a measurement of the decay length gives direct information on the value of $\sqrt{F/k}$. This is a unique opportunity, since other measurements are mainly sensitive only to the mass scale $\Lambda = F/M$, which roughly determines the mass spectrum. A study of how hadron collider sensitivity to long-lived NLSP can be maximized can be found in ref. [62]. There has also been a proposal [190] for extending the search to NLSP lifetimes much longer than allowed by present detectors. The suggestion is to design dedicated collider experiments

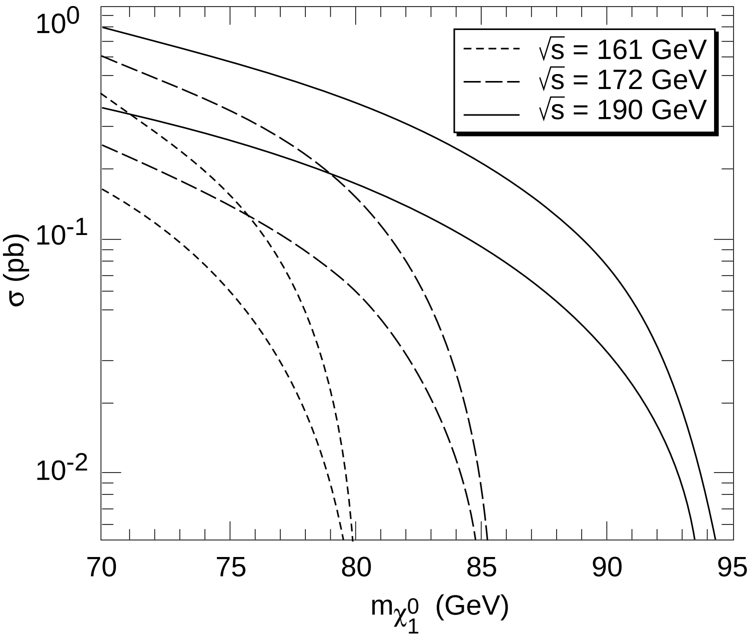

Figure 10: *The minimum and maximum cross section for* $e^+e^- \to \chi_1^0\chi_1^0$ *at the indicated centre-of-mass energies, as calculated in ref. [16]. These bounds hold only if the dynamics generating* μ *and* B *does not influence the supersymmetry-breaking Higgs scalar masses. Here it is also assumed that* χ_1^0 *is the NLSP. If this is not the case, the minimum of the cross section still holds, but cross sections larger than the maximum value can be obtained. (Courtesy S. Ambrosanio, G. Kribs, and S. Martin.)*

where the interaction point is shielded and the detector measuring the NLSP decay is located in a tunnel some distance away.

Let us first discuss the phenomenology of gauge mediation at e^+e^- colliders, and more specifically at LEP. If χ_1^0 is the NLSP with a prompt decay mode $\chi_1^0 \to \gamma\tilde{G}$, NLSP pair production leads to events with two photons and missing energy [263, 90, 14]. As we have seen in sect. 2.4, if we assume that the supersymmetry-breaking Higgs mass parameters are not influenced by the dynamics generating μ, then χ_1^0 is mainly B-ino and its production cross section is therefore determined by the slepton mass. In gauge-mediated models, the ratios $m_{\tilde{e}_{L,R}}/m_{\chi_1^0}$ are bounded from above. This guarantees that, for a given $m_{\chi_1^0}$, the cross section for $e^+e^- \to \chi_1^0\chi_1^0$ has a minimum and a maximum, which were calculated in ref. [16] and are reproduced in fig. 10, for values of $\sqrt{s}$ relevant for LEP.

The two photons coming from χ_1^0 pair production have a flat energy distribution in the range $E_{min} < E_\gamma < E_{max}$, with

$$E_{max,min} = \frac{1}{4}\left(\sqrt{s} \pm \sqrt{s - 4m^2_{\chi^0_1}}\right) \ . \qquad (3.17)$$

At LEP, the main source of background comes from $e^+e^- \rightarrow \gamma\gamma\nu\bar{\nu}$, which has an invariant missing mass peaked around M_Z, although the distribution has a non-negligible tail due to the contribution from t-channel $W^\pm$ exchange. The background from $e^+e^- \rightarrow \gamma\gamma\gamma$, with one photon unobserved, contributes only to events with small missing mass. With some acceptance cuts on the photon energy and angle and a cut on the invariant missing mass 5 GeV $< M_{inv} <$ 80 GeV, one obtains a good background rejection, with a signal efficiency between 50 and 80% [15, 17] (see also refs. [136, 80, 137, 198]).

All LEP collaborations have searched for diphoton plus missing energy events. Their combined 95% CL limit [171, 78] on the cross section for producing χ^0_1 with $\chi^0_1 \rightarrow \gamma\tilde{G}$ at $\sqrt{s} = 172$ GeV is about $\sigma_{\chi^0_1} < 0.3$ pb for $m_{\chi^0_1} < 70$ GeV, and $\sigma_{\chi^0_1} < 0.15$ pb for 70 GeV $< m_{\chi^0_1} < 85$ GeV. From fig. 10 we see that, in a fairly parameter-independent way, neutralino masses below 73 GeV are excluded, as long as the neutralino decay occurs inside the detector.

Slepton pair production, in the scenario with χ^0_1 as NLSP, leads to a final state with two leptons, two photons, and missing energy. The two leptons are expected to be soft at LEP, because of the limited available phase space. However, if the mass difference between $\tilde{\ell}$ and χ^0_1 is large enough for the final-state lepton to be observed, the process of slepton pair production is essentially free from SM background and it represents a clean signal for the χ^0_1 NLSP scenario.

Let us now turn to the case of $\tilde{\tau}$ NLSP, with moderate $\tan\beta$ such that $\tilde{e}_R$ and $\tilde{\mu}_R$ are almost degenerate with $\tilde{\tau}$ and behave as co-NLSPs. The production cross sections for $\tilde{\mu}$ and $\tilde{\tau}$ are model-independent (see *e.g.* ref. [140]) while the cross section for $\tilde{e}$ depends on the neutralino masses and often suffers a destructive interference between s-channel and t-channel exchange in the parameter region relevant for gauge mediation. Therefore one can expect an excess of $\tilde{\mu}$ events over $\tilde{e}$ events. If all sleptons decay inside the detector, the signature of slepton pair production is given by two leptons and missing energy. The signal is analogous to the one in gravity mediation, where the slepton decays into the LSP neutralino and the corresponding lepton. Above the W threshold, the main SM background comes from W^+W^- production. Since the gravitino is nearly massless, the slepton-production kinematics is more similar to the SM background than in the gravity-mediated case, where the LSP neutralino has a non-zero mass. The best handle to identify the signal is then given by the angular distribution of the charged leptons [16, 84]. For the background, the initial- and final-state leptons with same charge are preferentially in the same direction, while the angular distribution of the signal is flatter.

For large values of $\tan\beta$, the mass differences $m_{\tilde{\tau}} - m_{\tilde{e}}$ and $m_{\tilde{\tau}} - m_{\tilde{\mu}}$ become significant. Now $\tilde{e}$ and $\tilde{\mu}$ are no longer co-NLSPs and new decay channels are open. The three-body decays $\tilde{\ell} \to \tilde{\tau}\nu_\ell\bar{\nu}_\tau$ $(\ell = e, \mu)$ are always negligible because of the small couplings between charginos and right-handed sleptons. However, the three-body decays $\tilde{\ell}^- \to \tilde{\tau}^\pm\tau^\mp\ell^-$ $(\ell = e, \mu)$, mediated by virtual neutralinos, can have significant rates [17]. Depending on the value of $\tan\beta$ (which determines $m_{\tilde{\tau}} - m_{\tilde{\ell}}$) and the value of F_0 (which determines the gravitino coupling), the three-body decay may or may not dominate over the two-body mode $\tilde{\ell} \to \ell\tilde{G}$. If it does, slepton production leads to a striking signal with $\ell^+\ell^-\tau^\pm\tau^\pm\tilde{\tau}^\mp\tilde{\tau}^\mp$ or $\ell^+\ell^-\tau^\pm\tau^\mp\tilde{\tau}^\mp\tilde{\tau}^\pm$ in the final state, with possible vertex displacements [17, 63]. As the neutralino mass (which determines the $\tilde{\ell}$ decay widths) is increased, the final state with same-sign $\tilde{\tau}$'s is suppressed with respect to the case of opposite-sign $\tilde{\tau}$'s.

The case in which the $\tilde{\tau}$ NLSP decays outside the detector is also of great experimental interest, since it leads to observable anomalous ionization tracks. LEP experiments have studied stable slepton production, and their combined unsuccessful searches at $\sqrt{s} = 172$ GeV [171, 58] set a 95% CL limit of 76 GeV on the mass of an NLSP $\tilde{\tau}$ decaying outside the detector.

In the $\tilde{\tau}$ NLSP scenario, χ_1^0 pair production, if kinematically allowed, plays an important rôle and can become the discovery mode. This is because the production cross section for χ_1^0, not suppressed by the β^3 factor, can be larger than for $\tilde{\ell}$, even if $m_{\chi_1^0} > m_{\tilde{\ell}}$. The signal from χ_1^0 pair production is given by four leptons and missing energy, because of the decay chain $\chi_1^0 \to \ell\tilde{\ell} \to \ell^+\ell^-\tilde{G}$. In the large $\tan\beta$ region, where $m_{\tilde{\tau}} < m_{\tilde{e}}, m_{\tilde{\mu}}$, the final-state leptons are predominantly τ's. Two out of the four leptons in the final state can be very soft because of the limited phase space available in the χ_1^0 decay. However, because of the Majorana nature of χ_1^0, there is equal probability that the two hard leptons have the same or opposite charge [83, 17]. This provides a clean discovery signal.

Let us now consider the search at hadron colliders. Because of the large mass hierarchy between strongly and weakly interacting particles, production of (mainly right-handed) sleptons dominates over squark production and, similarly, chargino–chargino and chargino–neutralino productions have larger cross sections than gluino production. This is in contrast to ordinary searches for supersymmetry, which have focused on gluino and squarks as the discovery modes.

If χ_1^0 is the NLSP with a prompt decay, the typical signal is given by photons and missing energy, accompanied by charged leptons and/or jets (for a detailed study of the case with a single messenger flavour $N = 1$ and $M \simeq \sqrt{F}$, see ref. [24]). Diphoton events with large missing transverse energy have been searched for by the CDF [268] and the D0 [1] experiments at the Tevatron. The measured $\not{E}_T$ distributions were found to be in agreement with SM background by both collaborations. D0 [1] has set a bound on the cross section $\sigma(p\bar{p} \to \gamma\gamma\not{E}_T + X) < 185$ fb at 95 % CL, imposing cuts on the photon transverse energies, pseudorapidities, and the transverse missing energy of $E_T^\gamma > 12$ GeV, $|\eta^\gamma| < 1.1$, $\not{E}_T > 25$ GeV, respectively. This bound roughly corresponds [15] to excluding neutralinos lighter than 70 GeV and charginos lighter than 120 GeV, in scenarios with χ_1^0 decaying into a photon and a gravitino.

Actually CDF has detected one controversial event [221] with two energetic electrons, two energetic photons, and missing energy that cannot be attributed to the SM. This event has been interpreted [90, 14, 92, 15] as a possible signal coming from production of sleptons or charginos, decaying into unstable neutralinos. This possibility is now excluded [24] in the uni-messenger case $N = 1$, because of the absence of large anomalous rates for jets + γ's + $\not{E}_T$ events. No detailed study has been presented for $N > 1$, although this case appears more promising since, for a given slepton mass, it predicts heavier charginos and neutralinos, and therefore a reduction in anomalous events containing hadronic jets.

The phenomenology of long-lived $\tilde{\tau}$ NLSP at hadron colliders has been considered in ref. [121]. Long-lived sleptons are penetrating particles that deposit little energy in the hadron calorimeter, but appear in the tracking and muon chambers. If they are highly relativistic, they are misidentified as muons. At the Tevatron, in the mass range most relevant for discovery, they are often only moderately relativistic and therefore have a fast rate of energy loss through ionization. Their signal is then given by anomalous highly ionizing tracks.

CDF has searched for highly ionizing tracks with pseudorapidity $|\eta| < 0.6$ and energy loss corresponding to a relativistic factor $0.4 < \beta\gamma < 0.85$ [157]. By computing slepton production in Drell–Yan processes, the authors of ref. [121] conclude that these cuts reduce the signal by 25, 44, and 65% for $m_{\tilde{\tau}} = 100$, 200, and 300 GeV, and eliminate the background. The resulting lower limit on $m_{\tilde{\tau}}$ is about 50 GeV, which is weaker than the above-mentioned LEP limit of 76 GeV. However, searches at the upgraded Tevatron with $\sqrt{s} = 2$ TeV and large integrated luminosity are highly competitive with the future LEP results, since the limit on $m_{\tilde{\tau}}$ can be improved to 110 GeV for $\int \mathcal{L} = 2$ fb^{-1}, and 230 GeV for $\int \mathcal{L} = 30$ fb^{-1}. Of course, these considerations apply not only to the $\tilde{\tau}$, but also to $\tilde{e}$ and $\tilde{\mu}$, if they behave as co-NLSPs. More relativistic sleptons can be searched by studying anomalous dimuon events and apparent violations of universality in the ratio $\sigma(\mu^+\mu^-)/\sigma(e^+e^-)$. Because of the significant SM background, these searches are less powerful and can at most be used as an independent confirmation of a previously discovered signal.

For large $\tan\beta$, $\tilde{e}$ and $\tilde{\mu}$ are no longer co-NLSPs and have a three-body decay into $\tilde{\tau}$, as previously discussed. Their pair production then leads to very characteristic multilepton events with little hadronic activity. The Tevatron reach for $m_{\tilde{e}}$ and $m_{\tilde{\mu}}$ in this mode is 140 GeV for $\int \mathcal{L} = 2$ fb^{-1} and 230 GeV for $\int \mathcal{L} = 30$ fb^{-1} [121]. This is comparable to the discovery reach of the highly ionizing track analysis.

Because of the β^3 suppression of scalar-production cross sections, searches for neutralinos and charginos at the Tevatron can often probe a larger region of parameter space than slepton searches, even in models where $\tilde{\tau}$ is the NLSP. Under the conditions specified in sect. 2.4, the Higgs mixing parameter μ is large, and the two dominant processes of gaugino production are $p\bar{p} \to W^* \to \chi_1^\pm \chi_2^0$ and $p\bar{p} \to \gamma^*, Z^* \to \chi_1^\pm \chi_1^\pm$, where $(\chi_1^\pm, \chi_2^0)$ is the W-ino $SU(2)$ triplet. These states decay into left-handed sleptons, which then decay into right-handed sleptons along the chains

$$\chi_2^0 \to \ell\tilde{\ell}_L \to \ell\ell\ell\tilde{\ell}_R \,, \tag{3.18}$$

$$\chi_2^0 \to \nu\tilde{\nu}_L \to \ell\nu\nu\tilde{\ell}_R \,, \tag{3.19}$$

$$\chi_1^\pm \to \nu\tilde{\ell}_L \to \ell\ell\nu\tilde{\ell}_R \,, \tag{3.20}$$

$$\chi_1^\pm \to \ell\tilde{\nu}_L \to \ell\ell\nu\tilde{\ell}_R \,. \tag{3.21}$$

The signature is given by multilepton production with little associated hadronic activity and possibly highly ionizing tracks. Present searches are sensitive to W-ino masses up to about 190 GeV, but in the future they can be extended to 310 GeV for $\int \mathcal{L} = 2$ fb^{-1} and 410 GeV for $\int \mathcal{L} = 30$ fb^{-1} [121].

In conclusion, although gauge-mediated models are more predictive than gravity-mediated models in the determination of the new particle spectrum, they allow for a complicated taxonomy of collider signals, which depend on the specific parameter choice. Typically, the newly predicted signals are cleaner than the usual ones and lead to a better background rejection. This is particularly interesting for the Tevatron because any luminosity increase will allow for considerable improvements in the discovery reach, as most of the relevant processes are almost background-free. On the other hand, searches at e^+e^- colliders are limited by $\sqrt{s}$. It should also be recalled that the Higgs search plays a significant rôle in probing gauge-mediated models, as emphasized in sect. 2.4. Certainly the LHC has the capability of exhausting the search for supersymmetric particles. However, much more work on event simulations has to be done, as the study of the experimental signals for gauge mediation has just begun.

3.4 Searches in Low-Energy Experiments

The search for new physics can be pursued either by directly producing new particles in high-energy colliders or by investigating effects caused by virtual-particle exchange in processes at low energies. The study of FCNC processes is a typical example of the latter experimental strategy. The major theoretical success of gauge-mediated models is to predict a nearly exact flavour invariance of the supersymmetry-breaking terms. Therefore it may seem hopeless to look for new effects in FCNC processes. Exceptions are the cases in which the process is sensitive to new particle exchange, even for flavour violations coming from ordinary Cabibbo–Kobayashi–Maskawa effects. The inclusive B-meson decay $B \to X_s\gamma$ is known to be an example of such a case [43, 33, 219, 217, 131].

The contributions to $BR(B \to X_s\gamma)$ from supersymmetric particles in gauge-mediated models can be discussed in two different regimes. For moderate $\tan\beta$, all squarks are heavy and the dominant effect comes from loop diagrams involving top quark and charged Higgs boson. They constructively interfere with the SM diagrams and increase the value of $BR(B \to X_s\gamma)$. Using the present 95% CL upper limit on $BR(B \to X_s\gamma)$ from CLEO [8], the complete next-to-leading theoretical calculation leads to the limit $m_{H^\pm} > 340$ GeV [67]. As we have seen in sect. 2.4, if the Higgs mass parameters are not affected by the dynamics generating μ, the electroweak-breaking condition typically requires large μ and therefore a rather heavy charged Higgs boson (see fig. 8 and recall that $m^2_{H^\pm} = M^2_W + m^2_A$). In this respect, although the bound from $BR(B \to X_s\gamma)$ for moderate $\tan\beta$ is important, it is not too restrictive on the parameter space. On the other hand, independently of any consideration of electroweak-breaking conditions, one can view the constraint from $BR(B \to X_s\gamma)$ as a requirement for a large value of μ, in gauge-mediated models with moderate $\tan\beta$.

In the region of very large $\tan\beta$, the loop diagrams with stop and chargino exchange become important and, depending on the sign of μ, they can constructively or destructively interfere with the SM contribution. In the first case (which occurs for positive μ), one finds very stringent constraints: *e.g.* $\mu > 700$ GeV, for $\tan\beta = 42$, $N = 1$, and $M = 1.1\ \Lambda$ [82]. In the second case, when μ is negative, there is an approximate cancellation between different contributions and no useful bound can be derived. On the other hand, $BR(B \to X_s\gamma)$ can now be reduced with respect to the SM value. This could be interesting, since the present CLEO measurement [8] is about 2 σ below the SM prediction, although preliminary results from ALEPH [222] indicate larger rates.

The case in which $B = 0$ at the messenger scale reproduces a situation with large $\tan\beta$ and negative μ. For small values of the messenger mass M, the contributions from charged Higgs and charginos approximately cancel each other, and deviations from the SM value of $BR(B \to X_s\gamma)$ are predicted to be small. However, for larger values of M ($M \gtrsim 10^3$–$10^4\Lambda$), the chargino contribution is more important and $BR(B \to X_s\gamma)$ can be significantly lowered [128].

Another potentially interesting source of experimental information for gauge-mediated models comes from measurements of the muon anomalous magnetic moment a_μ. In the limit of large $\tan\beta$, the supersymmetric contribution is approximately given by (see refs. [61, 197, 52] for complete formulae)

$$\delta a_\mu \simeq \frac{\alpha}{8\pi \sin^2\theta_W} \frac{m_\mu^2}{\tilde{m}^2} \tan\beta \simeq 15 \times 10^{-10} \left(\frac{100 \text{ GeV}}{\tilde{m}}\right)^2 \tan\beta \ , \tag{3.22}$$

where $\tilde{m}$ is the typical mass scale of weakly interacting supersymmetric particles. From the SM result and the experimental measurement [39], one obtains a bound on new contributions, $|\delta a_\mu| < 200 \times 10^{-10}$. This already constrains gauge-mediated models with light sleptons and very large $\tan\beta$ [52], but the significance of this constraint may be much more important in the future. The experimental precision in the measurement of a_μ is expected to be improved by the E821 experiment at the Brookhaven National Laboratory to the level of 4×10^{-10}. If the theoretical prediction in the SM, unfortunately affected by hadronic uncertainties, could also be calculated at the same level of accuracy, the measurement of a_μ could compete with high-energy experiments.

Although gauge-mediated models have the same source of flavour violation as the SM, *i.e.* the Yukawa couplings, they can have additional sources of CP violation. In the minimal case with a single X field, the situation is particularly simple. The phases of the parameters M and F can be reabsorbed in the definition of the messenger fields through a global-symmetry and an R-symmetry transformation. However, once the Higgs mixing mass parameters μ and B are introduced, we are left with an irremovable phase; this, for instance, can be chosen to be $\arg(B^* M_3)$, the relative phase between B and the gluino mass. This phase is constrained by limits on the electric dipole moments of the electron and the neutron. The corresponding bounds [125, 147] are $\arg(B^* M_3) \lesssim 0.1(m_{\tilde{\ell}}/100\ \mathrm{GeV})^2$ and $\arg(B^* M_3) \lesssim (m_{\tilde{q}}/\mathrm{TeV})^2$, respectively.

If the dynamical mechanism that generates μ and B correlates their phases with the phase of the original supersymmetry-breaking scale F/M, then all soft terms are CP-conserving. A simple example of this possibility is the condition $B(M) = 0$ [22, 106]. In this case, all relative phases (and, in particular, all relative signs) of the different parameters are determined.

Theories with gauge-mediated supersymmetry breaking also offer a possible scenario for implementing a solution of the strong CP problem through the Nelson–Barr mechanism [202, 40]. If the theory has a (spontaneously broken) CP invariance and the determinant of the mass matrix for all coloured fermions is real, then θ_{QCD} vanishes at tree level and it is computable in perturbation theory. Since, in the supersymmetric limit, θ_{QCD} is not renormalized [114], all corrections must be proportional to supersymmetry-breaking effects. In a general gravity-mediated scenario, θ_{QCD} receives dangerously large corrections from GUT or Planck mass particles that invariably exist in models attempting to solve the strong CP problem [103]. In gauge-mediated models, supersymmetry shields θ_{QCD} from these effects, since the observable sector feels supersymmetry breaking only at much lower scales [41]. The computable corrections to θ_{QCD} from CP violation in the Yukawa sector are too small to give a significant effect to the neutron electric dipole moment [110], and the strong CP problem can be solved.

Finally a more unusual (and more speculative) way of searching for low-scale supersymmetry-breaking effects was suggested in ref. [91]. In superstring theories we expect the occurrence of gravitationally coupled massless scalars, called moduli. It is possible that these states acquire masses only after supersymmetry breaking. If the mass scale $\sqrt{F}$ is low, the moduli are very light and their Compton wavelengths can lie in the range between a micron and a millimeter. Although moduli may cause cosmological difficulties (see sect. 3.5), it is interesting that their couplings to matter can induce new forces, stronger than gravity, in this range of distances. New experiments have been proposed [239, 172] to search for non-gravitational forces at distances down to 100 or even 10 microns.

3.5 Gravitino Cosmology

In models in which supersymmetry breaking is mediated by gravity, the gravitino mass sets the scale for the soft terms, and therefore it is expected to lie in the range between 100 GeV and 1 TeV. Its lifetime is dictated by gravity to be $\tau \sim 10^6$ sec $(\mathrm{TeV}/m_{3/2})^3$. This late decay leads to an enormous entropy production after nucleosynthesis, unless the gravitino number density has not been diluted after the original thermalization. If the gravitino is lighter than few TeV, a successful nucleosynthesis sets an upper bound of about 10^{10} GeV [88, 116] to T_{max}, the temperature at which the ordinary radiation-dominated Universe starts. Here T_{max} could correspond to the reheating temperature after an inflationary epoch, or to the temperature of a significant entropy production. This bound is rather uncomfortable for many inflation scenarios and necessarily requires some low-temperature mechanism for baryogenesis.

In gauge-mediated models the gravitino effects on cosmology are quite different. The gravitino is stable, and therefore an important bound comes from its contribution to the energy density and not from lethal effects of its decay products. If gravitinos are in thermal equilibrium at early times and freeze out at the temperature T_f, their contribution to the present energy density is [220]

$$\Omega_{3/2}h^2 = \frac{m_{3/2}}{\mathrm{keV}} \left[\frac{100}{g_*(T_f)}\right] . \tag{3.23}$$

Here h is the Hubble constant in units of 100 km sec^{-1} Mpc^{-1} and $g_*(T_f)$ is the effective number of degrees of freedom at T_f, typically between 100 and 200 in a supersymmetric model. Therefore, if $m_{3/2} <$ keV, or equivalently

$$\sqrt{F} < \sqrt{k}\ 2\times 10^6\ \mathrm{GeV} , \tag{3.24}$$

gravitinos do not lead to overclosure of the Universe and, in contrast with the gravity-mediated case, there is no need for mechanisms of late entropy production.

The situation is less satisfactory if $m_{3/2} >$ keV, since some means of gravitino dilution are now necessary, and stringent bounds on T_{max} are found. At temperatures below T_{max}, gravitinos are produced by sparticle decay and scattering processes. For keV $< m_{3/2} <$ 100 keV, or equivalently 2×10^6 GeV $< \sqrt{F}/\sqrt{k} < 2 \times 10^7$ GeV, the decays of thermalized sparticles dominate the gravitino production mechanisms [196]. Then the bound on the Universe energy density implies $T_{max} < \tilde{m}$, where $\tilde{m}$ is the typical mass of the supersymmetric particles. For $m_{3/2} >$ 100 keV, the scattering processes dominate and the upper bound on T_{max} is given by [196]

$$T_{max} < 10 \text{ TeV} \times h^2 \left(\frac{m_{3/2}}{100 \text{ keV}}\right)\left(\frac{\text{TeV}}{M_3}\right)^2 , \tag{3.25}$$

where M_3 is the gluino mass. These constraints are quite stringent and they require an inflation with very low reheating temperature or a mechanism of late entropy production. Moreover, in this case, we have to rely on a baryogenesis scenario occurring at temperatures not much higher than the weak scale.

Although the gravitino is stable, a danger for ordinary nucleosynthesis predictions comes from the NLSP decay. From eqs. (3.7) and (3.15), we find that the NLSP lifetime is roughly

$$\tau_{NLSP} = \frac{1}{k^2}\left(\frac{100 \text{ GeV}}{m_{NLSP}}\right)^5 \left(\frac{\sqrt{F}}{100 \text{ TeV}}\right)^4 \; 3 \times 10^{-13} \text{ sec} . \tag{3.26}$$

The damage that the NLSP decay can make depends on the final-state particles. Decays into photons are safe as long as $\tau_{NLSP} < 10^7$ sec. This condition is satisfied as soon as eq. (2.44) is verified. However, much stronger constraints arise for hadronic decays of the NLSP [88, 116, 96]. These require that the messenger mass should be lower than values between 10^{12}–10^{14} GeV, depending on the parameter choice and the nature of the NLSP [135]. When combined with the requirement of no gravitino overabundance, this bound also implies that the reheating temperature after inflation must be less than 10^7 GeV [135]. Notice that these limits can be evaded in theories with R-parity breaking, in which the NLSP lifetime is decoupled from the supersymmetry-breaking scale.

Another potential cosmological difficulty is the "moduli problem" [54, 28]. This problem, also present in gravity-mediated theories, is actually exacerbated in models with low-energy supersymmetry breaking. It arises because in string theory there are flat directions in field space, called moduli, which may acquire masses only after supersymmetry breaking. They correspond to excitations that are only gravitationally coupled and have masses of the order of $m_{3/2}$. As the Universe cools down, the moduli oscillate around their minima, storing an enormous amount of energy density. In gravity-mediated scenarios, they decay at the same time as gravitinos. If their mass is below about 10 TeV, their decay products erase the nucleosynthesis predictions. Even if they are heavier than 10 TeV, they are cosmologically worrysome since the entropy production at the time of their decay dilutes any pre-existing baryon asymmetry. One again has to invoke a low-temperature baryogenesis scenario.

In gauge-mediated theories with low F, the moduli are practically stable, at least when cosmological times are concerned. The contribution to the present energy density of the Universe from the kinetic energy stored in their oscillations is

$$\Omega h^2 = 4 \times 10^{13} \left(\frac{m_{3/2}}{\mathrm{keV}} \right)^{\frac{1}{2}} . \tag{3.27}$$

A very efficient way of entropy production should be found to dilute this unacceptably large energy density. In order to avoid the regeneration of moduli, the entropy production should occur at temperatures below the supersymmetry-breaking scale $\sqrt{F}$. Strong constraints on the moduli masses in gauge mediation also come from the limits on the observed fluxes of gamma rays [174, 156, 21]

Finally, we mention that limits on light gravitinos have been obtained from various astrophysical considerations of star cooling. These limits only apply to models with other ultralight particles or for values of $\sqrt{F}$ much below the range of interest to gauge mediation.

3.6 Dark Matter

Theories with gravity-mediated supersymmetry breaking have the appealing feature that the lightest neutralino is a viable dark matter candidate, as long as R parity is (almost) exactly conserved. In gauge-mediated theories the neutralino decays with a lifetime shorter than the age of the Universe, and therefore cannot constitue the galactic halos. On the other hand, gravitinos could be the dark matter and give the dominant contribution to the present energy density if $m_{3/2}$ is about a keV, see eq. (3.23), or equivalently, if $\sqrt{F}$ saturates the bound in eq. (3.24). Gravitinos would behave as "warm" dark matter, since they just become non-relativistic at the time at which the growing horizon encompasses a perturbation of a typical galactic size. Gravitinos much heavier than a keV could also form the dark matter but, in this case, the temperature T_{max} defined in sect. 3.5 should be just right to give the correct gravitino abundance. This may be regarded as an unappealing scenario, since it requires a correlation between two quantities, F and T_{max}, which *a priori* originate from different physics. A bleak feature of dark matter gravitinos is that terrestrial experiments looking for galactic halo particles have no hope to detect any signal.

It is interesting to observe that, in contrast with the case of gravity-mediated theories, the gravitino could form the dark matter even in the presence of considerable R-parity breaking. Indeed, if the R-breaking interaction violates baryon (but not lepton) number, then the gravitino has no kinematically accessible channels as long as $\sqrt{F} < \sqrt{k}\ 2 \times 10^9$ GeV. However, certain baryon-number violating couplings are, in this case, tightly constrained by proton decay into gravitinos [64]. On the other hand, if the R-breaking interaction violates lepton number, then the gravitino can decay $\tilde{G} \to \nu\gamma$. For instance with a superpotential interaction $\lambda_{ijk} L_L^i L_L^j E_R^k$, the gravitino decay occurs via a one-loop diagram involving lepton–slepton exchange. The largest contribution comes from the coupling λ_{i33} which, in the limit of degenerate $\tilde{\tau}_L$ and $\tilde{\tau}_R$ gives [45]

$$\Gamma(\tilde{G} \to \nu_i \gamma) = \frac{\alpha \lambda_{i33}^2}{128\pi^4} \frac{m_{3/2} m_\tau^2}{M_P^2} \ln^2 \frac{m_{\tilde{\tau}}}{m_\tau} \ . \tag{3.28}$$

The strongest constraint on this decay comes from limits on the diffuse photon background; for a keV gravitino mass, this implies [246] that the lifetime should exceed 10^{26} sec, a time much longer than the age of the Universe. In turn, this requires $\lambda_{i33} \lesssim 10^{-2}$. This bound for $i = 2, 3$ can be more stringent than present bounds [37, 109] from low-energy processes, but still allows for significant R-parity violating effects in high-energy experiments.

It has also been suggested [45] that there could be two components of relic gravitinos playing a cosmological rôle. One is given by the gravitinos, which were thermalized in the early Universe, the other is given by the gravitinos from the NLSP decay, which act as a hot component in a mixed warm/hot dark matter scenario. However, in order to have a significant component of gravitinos from NLSP decay, one would need a rather extreme mass hierarchy, with sleptons heavier than 7 TeV for a (purely B-ino) neutralino of 30 GeV.

Another option for a dark matter candidate is given by the lightest messenger particle [93, 155]. Indeed, the superpotential in eq. (2.5) satisfies a "messenger number" invariance and therefore justifies the presence of a stable messenger particle. In the minimal case of messengers belonging to a $\mathbf{5}$ and $\bar{\mathbf{5}}$ of $SU(5)$, the lightest messenger particle has the same quantum numbers as the scalar left-handed neutrino. In order not to overclose the Universe, this particle should be lighter than about 3 TeV. Direct observations [42, 130] have already ruled out at 90% CL a 3 TeV weakly interacting scalar particle accounting for more than 30% of a galactic halo with local density of 0.3 GeV/cm^3. However, the authors of ref. [155] have suggested a model in which the messenger dark matter particle would evade detection.

The presence of a "messenger number" invariance is indeed in most cases quite disturbing. The relic abundance of the lightest messengers overcloses the Universe for typical values of the messenger mass. Breaking this invariance with renormalizable interactions between the messenger and observable sectors generally reintroduces the flavour problem. Therefore one should maybe rely on dimension-five Planck-mass suppressed interactions, which allow the messenger to decay without inducing significant flavour violation in the observable sector [105, 93].

Another conceivable option for a dark matter particle is to have a global symmetry in the strongly interacting supersymmetry-breaking sector, which forces the stability of the lightest "secluded baryon" B_φ [93]. In this case a lower bound on the B_φ relic abundance can be derived from unitarity of the annihilation cross section [145]:

$$\Omega_{B_\varphi} h^2 > (m_{B_\varphi}/300 \text{ TeV})^2 \ . \tag{3.29}$$

Therefore a stable particle with mass in the 100 TeV range, strongly interacting under a new gauge force of the secluded sector, could be a good dark matter candidate.

It has also been suggested that, in theories with low-energy supersymmetry breaking, flat directions along some field configurations can support stable non-topological solitons. These states can be abundantly produced in the early Universe and form the dark matter [178].

4 Basic Tools for Studying Dynamical Supersymmetry Breaking

Before describing the attempts to construct realistic models with dynamical supersymmetry breaking (DSB), we want to review in this section, for the ease of the reader, the basic tools necessary to analyse the non-perturbative dynamical properties of supersymmetric theories. We will just recall here the basic ingredients we need for our analysis, and refer the reader to refs. [165, 259, 224] for recent reviews on non-perturbative aspects of supersymmetric theories. For reviews on DSB, see also refs. [260, 267, 235].

4.1 The Witten Index

The Hamiltonian H of a supersymmetric system equals the square of the supercharge Q ($H = Q^2$), and Q transforms a bosonic state (b) into a fermionic state (f):

$$Q|b\rangle = \sqrt{E}|f\rangle \qquad Q|f\rangle = \sqrt{E}|b\rangle \ . \tag{4.1}$$

Together, these properties imply that bosonic and fermionic states of non-zero energy E always come in pairs. On the other hand, states with vanishing energy are annihilated by Q, so that they are not necessarily paired. The Witten index [279] is defined as

$$\mathrm{Tr}(-1)^F \equiv \sum_E n_B(E) - n_F(E) = n_B(0) - n_F(0) \ , \tag{4.2}$$

where $n_B(E)$ and $n_F(E)$ are respectively the number of bosonic and fermionic states with energy E. As $n_B(E) = n_F(E)$ for $E \neq 0$, only the supersymmetry-preserving vacua ($E = 0$) can contribute to a non-zero index. Thus, a sufficient criterion for unbroken supersymmetry is $\mathrm{Tr}(-)^F \neq 0$. Conversely, a necessary requirement for broken supersymmetry is that $\mathrm{Tr}(-1)^F$ be zero (or ill-defined). The crucial property of the index is that it is a discrete quantity, and is thus constant under "continuous" deformations of the parameters of the theory. If the index is found to be non-zero at weak coupling, supersymmetry is certainly unbroken, even at strong coupling. We should add that by "continuous" deformations we mean those that do not modify the asymptotic behaviour of the action in field space. In particular, a change in the large field behaviour of the superpotential can affect the index. The picture in that case is that vacua "come in from" (or "go to") infinity. In sect. 4.4 we will discuss a specific example in which this pathology is explicitly realized. Therefore when $\mathrm{Tr}(-1)^F \neq 0$ over a set of non-zero measure of parameters, it will stay so almost everywhere, and supersymmetry may be broken only at special points.

On the other hand, when $\mathrm{Tr}(-1)^F = 0$, we can only conclude that supersymmetry is *probably* broken, but counter-examples with $n_B(0) = n_F(0) \neq 0$ are known [279]. In this case, more information may be obtained with the use of holomorphy [250]. If the vacuum energy E_{vac} is described by a holomorphic function W of fields and couplings via $E_{vac} = K^{-1}_{ij} \partial_i W \partial^*_j W^*$ then, barring singularities in the Kähler metric K_{ij}, phase transitions are precluded. In other words, if E_{vac} is non-zero over a finite range of parameters, it will remain non-zero for all values, except at special isolated points [160]. In particular if $E_{vac} \neq 0$ at weak coupling, supersymmetry will also be broken at strong coupling.

In ref. [279] the index of supersymmetric pure gauge theories was calculated in a finite volume and found to be non-zero. For example, in $SU(N)$ gauge theories, it is $\mathrm{Tr}(-1)^F = N$. The Witten index does not vary if we add massive vector matter as, with a continuous change of parameters, it can be calculated in the limit of very large mass, where the effective theory is just pure Yang–Mills. Thus $E_{vac} = 0$ for any finite value of the mass m. In particular, the theory with massless vector matter, when it exists, preserves supersymmetry. This conclusion provides a useful criterion to search for dynamical supersymmetry breaking. Gauge theories that break supersymmetry dynamically must either be chiral or have massless fermions at every point of their parameter space (we will discuss an example of such a case in sect. 4.4).

4.2 Global Symmetries, R Symmetry, and Supersymmetry Breaking

A *sufficient* condition for the occurrence of DSB has been suggested by Affleck, Dine, and Seiberg (ADS) in refs. [4, 6, 7]. It relies on two basic requirements. The first one is that there be no non-compact flat directions in the classical scalar potential; the second one is that there exist a spontaneously broken global symmetry. Under these circumstances, supersymmetry must be broken. Indeed, a spontaneously broken global symmetry implies the existence of a Goldstone boson, and unbroken supersymmetry leads to an additional massless scalar to complete the supermultiplet. This extra massless mode corresponds to a non-compact flat direction[2], in contradiction with the first requirement. Whence the conclusion that supersymmetry is broken. We will refer to this condition for DSB as to the ADS criterion.

The ADS criterion is useful in strongly coupled theories, where the breakdown of a global symmetry can be established by the complexity (or absence) of solutions to 't Hooft's anomaly matching [4], or directly by use of instanton calculations [194]. Even though this argument applies to any global symmetry, in all known models satisfying these criteria, the spontaneously broken symmetry is actually an R symmetry. As pointed out in ref. [203], there is indeed a deep connection between R symmetry and supersymmetry breaking. Consider theories that, after the strong gauge dynamics have been integrated out, have a low-energy description in terms of a Wess–Zumino model, although typically with a complicated Kähler potential. For these theories, the presence of an R symmetry is a *necessary* condition for supersymmetry breaking, if the superpotential is a *generic* function of the fields, *i.e.* if it contains enough terms. Theories with non-generic superpotentials, which typically have some interaction or mass terms set to zero or fine-tuned, are unstable under small variations in the couplings.

Let us consider the effective theory described by a Wess–Zumino model with n chiral superfields ϕ_i, $i = 1, \ldots, n$ and no gauge fields. The condition for unbroken supersymmetry is given by the n equations $\partial_{\phi_i} W = 0$. These are just n complex equations in n complex unknowns, so that for a generic superpotential, with no symmetry requirements, there is always a solution. This simple fact reflects all the difficulty in achieving DSB. Consider, on the other hand, a theory with an R symmetry which is spontaneously broken, say, by the VEV of the field ϕ_1 [203]. One can parametrize the fields by ϕ_1 and by the R-invariants $\psi_i = \phi_i^{1/r_i}/\phi_1^{1/r_1}$, for $i = 2, \ldots, n$, where r_i is the charge of ϕ_i. Since W has charge 2, it will be of the form $W = \phi_1^{2/r_1}\mathcal{W}(\psi)$. By using $\phi_1 \neq 0$ the zero energy condition is

$$\partial_{\phi_1} W = \frac{2}{r_1}\phi_1^{-1+2/r_1}\mathcal{W}(\psi) = 0 \quad \Rightarrow \mathcal{W} = 0 \tag{4.3}$$

$$\partial_{\psi_i} W = \phi_1^{2/r_1}\partial_{\psi_i}\mathcal{W}(\psi) = 0 \quad \Rightarrow \partial_{\psi_i}\mathcal{W} = 0 \quad i = 2, \ldots, n. \tag{4.4}$$

Equations (4.3) and (4.4) correspond to n equations in $n-1$ variables ψ_i, which cannot be solved for a *generic* $\mathcal{W}$. Apart from singular points in the Kähler metric, which need a specific study, we conclude that supersymmetry is broken. Notice that the case of a non-R spontaneously

[2] In special cases, the extra massless scalar could be a Goldstone boson itself, thus evading the conclusion of non-compact flat directions and, ultimately, of supersymmetry breaking.

broken global symmetry does not lead to the same conclusion. Now W is just a function of the invariants I_A, $A = 1, \ldots, k$, where $k < n$. So, when written in terms of I_A, the stationary conditions for W are just k equations in k variables. In general these equations admit a solution, precisely as in the absence of a symmetry. Conversely, this proves that, for a generic superpotential with no R symmetries, a spontaneously broken global symmetry leads to a flat direction with zero energy.

These arguments show that the presence of an R symmetry is a *necessary* condition for supersymmetry breaking, while a spontaneously broken R symmetry provides a *sufficient* condition [203]. These conclusions hold only for theories that allow a calculable low-energy description in terms of a Wess–Zumino model with a generic superpotential. There are interesting cases in which these conditions are not satisfied. For instance, in some cases, the effective superpotential generated by the non-perturbative dynamics is non-generic and DSB can occur without an R symmetry.

Notice that the familiar perturbative O'Raifertaigh model [218] corresponds to the case of a generic superpotential with an R symmetry and with a non-compact pseudoflat direction, *i.e.* with constant positive energy. Everywhere along the pseudoflat direction, away from the origin, the R symmetry is spontaneously broken, and the *sufficient* condition for supersymmetry breaking is satisfied. At the origin, the R symmetry is preserved, but supersymmetry remains broken. The *necessary* condition for supersymmetry breaking is verified.

The connection between spontaneously broken R symmetry and DSB may be the cause of a phenomenological problem. Since the gluino mass has R charge 2, the R Goldstone boson, usually referred to as the R-axion, couples to the QCD anomaly. In the absence of explicit R breaking, the R-axion decay constant is very constrained by astrophysical considerations [269, 243]. However, as we will discuss at the end of sect. 5.1, there are several ways to overcome this problem.

4.3 Flat Directions and Supersymmetric QCD

At tree level, in the absence of a superpotential, a supersymmetric gauge theory typically has a large set of vacua. These are the points with vanishing D-terms:

$$D_A \equiv \sum_i \phi_i^\dagger T_A \phi_i = 0 \ . \tag{4.5}$$

Here T_A are the gauge generators in the representation under which the chiral superfields ϕ_i transform. Understanding the space of flat directions (usually referred to as classical moduli space) is crucial to study a model. It is often a non-trivial problem to find explicitly all the solutions to eq. (4.5). In some cases the techniques of refs. [3, 5, 7] can be useful. Fortunately there is a general theorem [49, 240, 132, 133, 185] stating that the space of solutions to eq. (4.5) is in a one-to-one correspondence with the VEVs of the complete set of gauge-invariant functions of the chiral fields ϕ_i. In other words, the moduli space is the space of independent chiral invariants. The coordinates on the moduli space correspond to massless chiral supermultiplets. In general, the global description of this space is given in terms of a set of invariants satisfying certain constraints. These constraints, which select the independent invariants, are determined by Fierz identities in the gauge contractions and Bose symmetry of the scalar fields. Although finding all the invariants and the constraints can sometimes be difficult, this theorem greatly simplifies the search for the solutions of eq. (4.5), and in practice it is very useful.

After adding a superpotential W, some flat directions are lifted, *i.e.* the F terms are non-vanishing along the D-flat direction. In particular, if one can show that every invariant is fixed by the condition $F_i = -\partial_{\phi_i} W = 0$, then all flat directions have been lifted, and one of the requirements of the ADS criterion discussed in sect. 4.2 is satisfied. In terms of the invariants the vacua are described by the zeros of holomorphic functions, and this simplifies things considerably.

As an explicit example consider an $SU(3) \times SU(2)$ model [7] with matter content $Q(\mathbf{3}, \mathbf{2})$, $\bar{U}(\bar{\mathbf{3}}, \mathbf{1})$, $\bar{D}(\bar{\mathbf{3}}, \mathbf{1})$ $L(\mathbf{1}, \mathbf{2})$. A complete set of gauge invariants is

$$X = Q\bar{U}L, \qquad Y = Q\bar{D}L, \qquad Z = Q\bar{U}Q\bar{D}. \tag{4.6}$$

As we will show in sect. 4.4, by adding a tree superpotential

$$W = Q\bar{U}L, \tag{4.7}$$

this model breaks supersymmetry dynamically. It is easy to see that the superpotential in eq. (4.7) lifts all D-flat directions. Multiplying the equation $0 = \partial_{\bar{U}} W$ by $\bar{U}$ and $\bar{D}$, we respectively get $X = 0$ and $Y = 0$. In the same way, by contracting $\partial_L W = 0$ with $Q\bar{D}$ we get $Z = 0$.

This example actually illustrates a general technique which is very useful to verify if a certain flat direction is lifted by the superpotential. One should first construct gauge invariants by contracting in all possible ways the equations $\partial_{\phi_i} W = 0$ with chiral superfields. If this procedure determines all independent chiral invariants, then all D-flat directions have been lifted by the superpotential.

Another important example is given by supersymmetric quantum chromodynamics (SQCD) [266, 2, 5] with gauge group $SU(N_c)$ and N_f flavours of chiral multiplets in the fundamental and antifundamental representations $Q_i, \bar{Q}^i$, $i = 1, \ldots, N_f$. The moduli space is parametrized by the mesons $M^i_j = \bar{Q}^i Q_j$, the baryons $B_{i_1 \ldots i_{N_c}} = \epsilon_{\alpha_1 \ldots \alpha_{N_c}} Q^{\alpha_1}_{i_1} \ldots Q^{\alpha_{N_c}}_{i_{N_c}}$ (where α_i are gauge indices) and the antibaryons $\bar{B}^{i_1 \cdots 1_{N_c}}$. Notice that the baryons exist only for $N_f \geq N_c$ (since Q_i are bosonic operators) and that for $N_f < N_c$ the mesons provide a complete, non-redundant parametrization of the moduli space. On the contrary, for $N_f \geq N_c$ mesons and baryons are redundant and satisfy certain classical constraints. For instance for $N_f = N_c$ there are just one baryon B and one antibaryon $\bar{B}$, and they satisfy the constraint $\mathrm{Det} M - B\bar{B} = 0$.

A last ingredient of great use in determining the low-energy dynamics of supersymmetric gauge theories is holomorphy [256, 13, 258, 250]. In these theories the gauge kinetic term and the superpotential are determined by chiral operators integrated over $\int d^2\theta$. The corresponding coupling constants can therefore be treated themselves as spurionic chiral superfields, with non-vanishing VEVs only in their scalar components. The way these couplings appear in the effective action must respect the background supersymmetry under which also the couplings are treated as chiral superfields. For instance if a term $\lambda\phi^3$ appears in the tree-level W, the appearance of λ^* in the effective superpotential is forbidden.

As far as the gauge coupling constant is concerned, various different definitions (schemes) can be given. Not for all of them can we extend the coupling to a chiral superfield, and the "physical" coupling defined by the behaviour of scattering amplitudes is not one of these. As discussed in refs. [256, 258] (for additional discussions, see also refs. [173, 19]), the holomorphic quantity is the coefficient of the gauge kinetic term in the Wilsonian effective action, the so-called Wilsonian coupling g_W. This and the topological angle Θ define a chiral superfield S_W, whose scalar component is just $1/g_W^2 - i\Theta/8\pi^2$. As physical quantities should not change when $\Theta \to \Theta + 2\pi$, it is easily proved that holomorphy implies that S_W is renormalized only at one loop, to all orders in perturbation theory[3]. This property of S_W is nothing but a special case of the non-renormalization theorem that applies to chiral operators. The Wilsonian coupling is very useful: since it runs only at one loop, the RG-invariant strong interaction scale is just

$$\Lambda^b = \mu^b e^{-\frac{8\pi^2}{g_W^2}+i\Theta} = \mu^b e^{-8\pi^2 S_W} \ . \tag{4.8}$$

Here b is the one-loop β-function coefficient, equal to $3N_c - N_f$ in massless SQCD, and μ is the renormalization scale. Notice that Λ is also a chiral superfield, with definite quantum numbers under all anomalous symmetries, under which Θ is shifted. The quantum numbers of Λ constrain the form of non-perturbative renormalizations of the superpotential [250].

Since S_W runs only at one loop, its matching at thresholds at which heavy states are integrated out is simply done at tree level, for instance by requiring continuity of S_W through the threshold[4]. This makes S_W very useful to describe effective theories. As an example, if we integrate out one flavour of quark superfields with mass m in SQCD, the effective scale of the low-energy $N_f - 1$ theory is simply

$$\Lambda_{eff}^{3N_c-N_f+1} = m \ \Lambda^{3N_c-N_f} \ . \tag{4.9}$$

As another example, consider the theory along a D-flat direction in which some flavours get VEVs. As previously discussed, this flat direction can be described by the VEVs of some gauge invariants, in this case the mesons or the baryons. For instance, by giving a VEV to just one meson $M_{N_f}^{N_f}$ in massless SQCD, the gauge group is broken down to $SU(N_c-1)$ with $N_f - 1$ flavours, as one flavour disappears because it is eaten by the Higgs mechanism. In this case the low-energy scale is $\Lambda_{eff}^{3N_c-N_f-2} = \Lambda^{3N_c-N_f}/M_{N_f}^{N_f}$. For $N_f < N_c$, when all mesons acquire VEVs, the low-energy theory is pure $SU(N_c - N_f)$ with a scale

$$\Lambda_{eff}^{3(N_c-N_f)} = \frac{\Lambda^{3N_c-N_f}}{\mathrm{Det} M} \ . \tag{4.10}$$

[3]When there is an anomalous symmetry under which $\Theta \to \Theta + \alpha$, with α a real number, one can also exclude non-perturbative renormalizations of S_W (see *e.g.* ref. [19]).

[4]For a discussion of the various scheme choices for g_W^2, see ref. [124].

Let us now briefly describe the properties of SQCD with different numbers of flavours N_f. Consider first the case $N_f = N_c - 1$. The classical moduli space is described by the $(N_c - 1) \times (N_c - 1)$ meson matrix M. At a generic point $\mathrm{Det} M \neq 0$, all flavours get VEVs, and the gauge group is completely broken. The global symmetries of the system and holomorphy [5, 7, 250] constrain W_{eff} to have the form

$$W_{eff} = c \, \frac{\Lambda^{2N_c+1}}{\mathrm{Det} M}, \tag{4.11}$$

where c is a constant. Notice that $\Lambda^{2N_c+1} \propto \exp(-8\pi^2/g_W^2)$, which is precisely the suppression factor of a one-instanton amplitude. The calculation of instanton effects in the broken phase is reliable, as large instantons are suppressed by the gauge-boson mass, and this calculation explicitly shows that $c \neq 0$ [5, 257, 124].

The result in eq. (4.11) can be extended to the massive case where $W_{tree} = \mathrm{Tr}\; mM$, and $\mathrm{Det}(m) \neq 0$. Using the arguments of ref. [250], based on holomorphy, one can conclude that the effective superpotential is just the sum of W_{tree} and eq. (4.11), for any m and Λ. This is easily seen in the simplest case of $SU(2)$ with one flavour, where there is just one meson $M = \bar{Q}Q$ and a mass term $W_{tree} = mM$. By using the global symmetries, the general form of the effective superpotential is $W_{eff} = mM \, f(\Lambda^5/mM^2)$. The function $f(z)$ is holomorphic (analytic with no singularities at finite z), and its asymptotic behaviour is known. The limit $z \to 0$ corresponds to the free field theory, so that $f(0) = 1$. The limit $z \to \infty$ corresponds to the massless theory, where, by eq. (4.11), $W_{eff} = c\Lambda^5/M$, so that $\lim_{z\to\infty} f(z) = cz$. With the help of holomorphy, we can reconstruct $f(z) = 1 + cz$.

In the case of SQCD with $N_f < N_c - 1$ massless flavours, the global symmetries and holomorphy constrain the superpotential to be

$$W_{eff} = c' \left(\frac{\Lambda_{N_c,N_f}^{3N_c-N_f}}{\mathrm{Det} M} \right)^{1/(N_c-N_f)}, \tag{4.12}$$

where Λ_{N_c,N_f} is the corresponding dynamical scale. Unlike the case $N_f = N_c - 1$, the power of Λ_{N_c,N_f} does not coincide with the one-instanton effect, so we cannot perform a direct calculation of the constant c'. Indeed, at a generic point on the classical moduli space, there is now an unbroken $SU(N_c - N_f)$ gauge group. So one expects additional non-perturbative effects other than instantons. Equation (4.12) can however be obtained from the theory with $N_c - 1$ flavours by adding a mass m to $N_c - 1 - N_f$ flavours. We have argued above that the exact superpotential for this theory is just eq. (4.11) plus the mass term [250]. By integrating out the mesons containing a massive quark, we obtain W_{eff} for the theory with N_f flavours. This has the form of eq. (4.12) with $\Lambda_{N_c,N_f}^{3N_c-N_f} = m^{N_c-N_f-1}\Lambda^{2N_c+1}$, which is precisely the scale determined by matching the theory with N_f to the theory with $N_c - 1$ flavours, see eq. (4.9). The constant c' can then be computed in terms of the constant c, which appears in eq. (4.11), and one concludes that $c' \neq 0$ [72, 257, 124].

The superpotential in eq. (4.12) has no supersymmetric minima at finite M, but vanishes at infinity as an inverse power of M. Thus massless SQCD with $0 < N_f \leq N_c - 1$ is not a well-defined theory, and it has no stable vacuum. On the other hand by adding a mass term $m\text{Tr}M$ to eq. (4.12), one finds N_c supersymmetric vacua characterized by

$$\langle M_i^j \rangle = \delta_i^j \Lambda_{N_c,N_f}^2 \left(\frac{m}{\Lambda_{N_c,N_f}} \right)^{-(N_c-N_f)/N_c} e^{2\pi i k/N_c}, \quad k = 1, \ldots, N_c. \tag{4.13}$$

Notice that N_c is precisely the number of vacua suggested by the Witten index $\text{Tr}(-1)^F = N_c$, calculated in supersymmetric pure $SU(N_c)$ theories at finite volume (see sect. 4.1).

As we said, for $N_f = N_c - 1$, the effective superpotential in eq. (4.12) is generated by the one-instanton amplitude. What is then its origin for smaller N_f? In order to answer this question we must consider the effective gauge theory at a generic point where $\text{Det}M \neq 0$. Here the VEVs of the N_f flavours break $SU(N_c)$ to pure $SU(N_c - N_f)$ with no charged matter. Recalling the expression of the dynamical scale for the effective theory, see eq. (4.10), we observe that the superpotential in eq. (4.12) is $W_{eff} = \Lambda_{eff}^3$. This is precisely the term that would be generated by the gauge kinetic term $\int d^2\theta W^\alpha W_\alpha$ in $SU(N_c - N_f)$, if the glueball field $W^\alpha W_\alpha$ were to receive a VEV $\sim \Lambda_{eff}^3$. Therefore the interpretation of eq. (4.12) is just that gauginos $\lambda^\alpha \lambda_\alpha = W^\alpha W_\alpha|_{\theta=\bar{\theta}=0}$ condense in the vacuum of the low-energy pure $SU(N_c - N_f)$ theory[5]. This result confirms other approaches where $\langle \lambda\lambda \rangle = \Lambda_{eff}^3$ was derived by direct instanton calculus [213, 214, 215, 13] or by an effective Lagrangian for the glueball field [270]. Notice that the $SU(N_c)$ theory has a discrete Z_{2N_c} R symmetry under which $\lambda \to e^{2\pi i k/2N_c}\lambda$, $k = 1, \ldots, 2N_c$, broken down to Z_2 by $\langle \lambda\lambda \rangle \neq 0$. Again, the resulting N_c-equivalent vacua are in agreement with the index $\text{Tr}(-1)^F = N_c$.

For $N_f = N_c$, SQCD confines [251] with the light bound states given by the meson matrix

[5]The same conclusion can be reached [13], more rigorously, by using eq. (4.13) and the Konishi anomaly [176, 177].

M_i^j and the baryons B, $\bar{B}$. Quantum effects (instantons) modify the classical constraint $\mathrm{Det}M - B\bar{B} = 0$ to

$$\mathrm{Det}M - B\bar{B} = \Lambda^{2N_c}. \tag{4.14}$$

This field equation, defining the so-called quantum moduli space (QMS), can be obtained by introducing a Lagrange-multiplier superfield A with superpotential

$$W_{quantum} = A\left(\mathrm{Det}M - B\bar{B} - \Lambda^{2N_c}\right). \tag{4.15}$$

Notice that on any point of the QMS the field A pairs up with a linear combination of $M, B, \bar{B}$ and becomes massive. The above picture was derived inductively in ref. [251], as it satisfies a series of non-trivial consistency checks. In particular the massless spectrum from eq. (4.15) satisfies at any point 't Hooft's anomaly matching conditions, for flavour and R symmetries. Thus for $N_f = N_c$ massless SQCD not only exists but has an infinite degeneracy of vacua.

For $N_f = N_c + 1$ there is also confinement, but the classical moduli space is not modified [251][6]. For $N_f > N_c + 1$ the low-energy description can be done in terms of a dual gauge theory whose gauge bosons and matter fields are composites of the original ones [252]. These last cases, although of great theoretical interest, do not enter directly the model-building discussion of sect. 5. We refer the interested reader to the original papers and to the reviews in refs. [165, 223, 259].

4.4 Mechanisms for Dynamical Supersymmetry Breaking

In this section we review a few simple models that illustrate the known mechanisms by which supersymmetry is dynamically broken.

The minimal model with calculable DSB is the 3-2 model [7], based on gauge group $SU(3) \times SU(2)$. This model has been introduced in the previous section, where we proved that the superpotential in eq. (4.7) lifts all flat directions. Let us consider the limit in which $SU(3)$ is much stronger than $SU(2)$ ($\Lambda_3 \gg \Lambda_2$) so that, in first approximation, the $SU(2)$ non-perturbative dynamics can be neglected. The $SU(3)$ factor has two flavours, so according to the discussion in sect. 4.3 a dynamical superpotential is generated by instantons. With the

[6]For a general classification of theories with this property, see also ref. [76].

help of holomorphy, we obtain that the exact superpotential is just

$$W_{eff} = \frac{\Lambda_3^7}{(Q\bar{D})(Q\bar{U})} + \lambda(Q\bar{U})L \ , \tag{4.16}$$

where the fields inside the brackets form a $(\mathbf{1},\mathbf{2})$ under $SU(3) \times SU(2)$. The L equation of motion, $(Q\bar{U}) = 0$, is inconsistent with the equation $0 = \bar{D}\partial_{\bar{D}}W = \Lambda_3^7/[(Q\bar{D})(Q\bar{U})]$, because we know that there are no flat directions and $(Q\bar{D})$ cannot escape to infinity. There can thus be no supersymmetric vacuum. Notice the special rôle played by the fields L and $\bar{D}$, one of which appears *only* in the tree-level operator and the other one *only* in the denominator of the dynamical term. Their equations of motion are mutually inconsistent, because they want the same combination of fields (in this case $Q\bar{U}$) to be zero in one case and infinity in the other. This is indeed a general feature of a large class of models in which DSB is triggered by a non-perturbative superpotential.

Supersymmetry breaking in the 3-2 model can also be established using the ADS criterion. The superpotential in eq. (4.7), besides lifting all flat directions, also preserves an anomaly-free global $U(1) \times U(1)_R$ (where the second factor is an R symmetry) with charges $Q(1,0)$, $\bar{U}(2,2)$, $\bar{D}(-4,-4)$, $L(-3,0)$. The non-perturbative superpotential forces the fields away from the origin (in jargon "lifts the origin"), thereby spontaneously breaking the R symmetry. The conditions of the ADS criterion are met and supersymmetry is broken.

Some additional comments on the 3-2 model are in order. By simple scaling arguments the VEVs of all scalar fields scale like $v \sim \Lambda_3/\lambda^{1/7}$, see eq. (4.16). For $\lambda^{1/7} \ll 1$ the VEVs are much larger than Λ_3, so that the group $SU(3)$ is broken in the weak regime. Here the vacuum of the model can be studied in perturbation theory. More precisely, non-perturbative corrections to the Kähler potential are negligible and typically the tree-level approximation is sufficient to characterize the spectrum around the minimum of the potential. Indeed, in the limit $\lambda^{1/7} \ll 1$, the three original moduli X, Y, Z describe the light degrees of freedom. The vacuum can be studied by considering the non-linear σ-model for the light states X, Y, Z, obtained by integrating out the heavy modes in a theory with an originally flat tree-level Kähler metric [7, 229].

Let us assume the addition of the operator $\delta W = (Q\bar{U})(Q\bar{D})/M$ to the tree-level superpotential in eq. (4.7), where M is some heavy-mass scale [250, 203]. By holomorphy, the full non-perturbative superpotential now is

$$W_{eff} = \frac{\Lambda_3^7}{(Q\bar{D})(Q\bar{U})} + \lambda(Q\bar{U})L + \frac{(Q\bar{U})(Q\bar{D})}{M} \ . \tag{4.17}$$

It is easily checked that the new superpotential still lifts all flat directions. The perturbation δW, however, breaks the R symmetry explicitly. However, supersymmetry remains broken since the equation of motion for L still requires $(Q\bar{U}) = 0$, while the origin is lifted by the instanton term. The necessary condition for DSB discussed in sect. 4.2 (*i.e.* presence of an R symmetry) is not respected, because the superpotential in eq. (4.17) is a *non-generic* function of the fields. The lack of genericity is a direct consequence of holomorphy: not every possible R-breaking term appears in eq. (4.17). This illustrates an example of a model with DSB, but no harmful R-axion. For instance, assuming $M = M_P$ and $v > \sqrt{F} \gtrsim 10^5$ GeV, we find that the R-axion is heavier than 10 MeV, large enough to suppress fast emission in stellar cooling.

In ref. [166] the exact superpotential for the 3-2 model was derived for any value of Λ_3/Λ_2. As expected from holomorphy, supersymmetry was found to be always broken. It is interesting to consider the limit $\Lambda_2 \gg \Lambda_3$, in which only the $SU(2)$ dynamics is important. Now DSB is achieved by a different mechanism. The $SU(2)$ theory has 4 doublets (2 flavours), and confines with a QMS. All global symmetries are preserved only at the origin, but the origin is removed from the moduli space by the quantum deformation.

To give a simple characterization of the DSB mechanism on the QMS, consider an $SU(2)$ gauge theory with 4 fundamentals Q_i, $i = 1, \ldots, 4$, coupled to 6 singlets $R^{ij} = -R^{ji}$ [167, 166]

$$W_{tree} = \lambda R^{ij} Q_i Q_j. \tag{4.18}$$

This model has an $SU(4)$ flavour symmetry under which R is a **6** antisymmetric tensor. The superpotential lifts all mesonic D-flat directions $M_{ij} = Q_i Q_j$, while R^{ij} remain flat. The $SU(2)$ dynamics is a special case of the $N_f = N_c$ theories discussed in the previous section. For $N_c = N_f = 2$ the baryons and the mesons fit into the same antisymmetric tensor M_{ij} with classical constraint $\mathrm{Pf}M = 0$. The quarks Q_i are confined in the mesons M_{ij} and at the quantum level the exact effective superpotential is given by

$$W_{eff} = \lambda R^{ij} M_{ij} + A(\mathrm{Pf}M - \Lambda^4), \tag{4.19}$$

where A is a Lagrange-multiplier superfield and Λ is the scale of the $SU(2)$ dynamics. Equation (4.19) just describes the usual O'Raifertaigh model. Supersymmetry is broken as $F_{R^{ij}}$ and

F_A cannot both vanish at the same time. Notice also that eq. (4.19) satisfies an R symmetry, and that the rôles played by R_{ij} and A are similar to those of L and $\bar{D}$ in the 3-2 model. It is now the quantum deformation that lifts the origin, the only point satisfying $\partial_{R_{ij}} W = 0$.

As R_{ij} describe flat directions at tree level, this model does not satisfy the ADS criterion. However, at the quantum level, these flat directions are lifted with a vacuum energy $E_{vac} \sim \Lambda^4$. The quantum lifting of classical flat directions, which occurs in other DSB models [254], shows how the first requirement of the ADS criterion is not strictly necessary.

The model under consideration is a vector-like gauge theory, so one may wonder how DSB is reconciled with the Witten index. Let us assume that we give mass to all quarks by adding $W_{mass} = m^{ij} Q_i Q_j$ to eq. (4.18). Evidently, as R^{ij} are free to slide, there is always a point in R space where the effective quark mass $R^{ij} + m^{ij}$ is zero. Actually by the field redefinition $R^{ij} + m^{ij} \to R^{ij}$, the renormalizable Lagrangian is the same as that of the original theory. So quarks do not decouple in the limit $m \to \infty$, and the index is not that of a pure $SU(2)$ Yang–Mills theory. On the other hand, a mass term $m_R \mathrm{Pf} R$ added to eq. (4.19) explicitly breaks the R symmetry and restores supersymmetry. For small m_R the VEV of R_{ij} is proportional to Λ^2/m_R. Thus in the limit $m_R \to 0$ the supersymmetric vacuum escapes to infinity, and the index has a discontinuous change at $m_R = 0$ [166].

There are also chiral gauge theories that are believed to break supersymmetry in the strongly coupled regime, *i.e.* for values of the VEVs smaller than Λ, where the Kähler potential in general receives uncontrollable corrections. One such simple case is an $SU(5)$ gauge theory with matter in a single $\bar{F} = \bar{\mathbf{5}}$ and $A = \mathbf{10}$. This theory is easily analysed classically. Since we cannot construct chiral gauge invariants involving only $\bar{F}$ and A, there are no D-flat directions and, also, no superpotential can be added. At the tree level the only ground state is at the origin, where the gauge group is unbroken. At the origin the theory is strongly coupled, and we cannot make a direct and reliable calculation of the vacuum. There is, however, plentiful indirect evidence that supersymmetry is broken [4, 194]. The theory has a non-anomalous $U(1) \times U(1)_R$ global symmetry with charges $\bar{F}(3, 0)$ and $A(-1, -2)$. In ref. [4] the solutions to 't Hooft anomaly matching conditions for the low-energy effective theory were studied under the assumption of unbroken supersymmetry. The solutions were found to be fairly complicated, as the "simplest" solution found in ref. [4] involves five light fermions. Therefore it seems more plausible that the global Abelian symmetry is broken. If that is the case, the ADS conditions are satisfied and supersymmetry is expected to be broken.

A different argument for evidence of supersymmetry breaking in the $SU(5)$ model is based [194, 3] on the Konishi anomaly equation [176, 177]:

$$\frac{1}{2\sqrt{2}}\left\{\bar{Q}_{\dot{\alpha}}, \psi_a^{\dot{\alpha}} \varphi_a\right\} = \varphi_a \frac{\partial W}{\partial \varphi_a} + C_a \frac{g^2}{32\pi^2} \lambda\lambda \ . \quad (4.20)$$

Here ψ_a and φ_a are respectively the fermion and scalar components of a chiral superfield with Casimir C_a and the index a in eq. (4.20) is not summed. In the present model $W = 0$, so that the first term on the r.h.s. is absent. Then by taking the VEV of eq. (4.20), one finds that $\langle\lambda\lambda\rangle$ is an order parameter for supersymmetry breaking. Notice that $\langle\lambda\lambda\rangle \neq 0$ breaks the R symmetry, so that the Konishi anomaly establishes a rigorous connection between the spontaneous breaking of R and that of supersymmetry. By applying the techniques of refs. [213, 248, 12, 13], the authors of ref. [194] concluded that the product $\langle\lambda\lambda\rangle^2\langle\lambda\lambda\bar{F}AAA\rangle$ is non-zero. Here $\bar{F}$ and A indicate the scalar components of the corresponding fields and an $SU(5)$ invariant contraction inside the VEVs is understood. As there are no flat directions, the solution $\langle\lambda\lambda\rangle = 0$, $\langle\lambda\lambda\bar{F}AAA\rangle = \infty$ should be discarded. Then $\langle\lambda\lambda\rangle \neq 0$ and supersymmetry is broken[7].

More recently further circumstantial evidence for broken supersymmetry in this model (and in similar ones) has been gathered. One interesting approach [199, 230] is to add one flavour $\phi = \mathbf{5}$, $\bar{\phi} = \bar{\mathbf{5}}$ to make it a calculable theory. With the added matter fields there are D-flat directions along which the theory becomes weak and can be studied. At a generic point on the classical moduli space, $SU(5)$ is broken to pure $SU(2)$, with no charged matter. Gaugino condensation in $SU(2)$ generates a dynamical superpotential $W_{dyn} = \Lambda_5^6/[(AA\phi)(A\bar{F}\bar{\phi})]^{1/2}$. The runaway behaviour of W_{dyn} is avoided by adding a tree-level mass term $W_{tree} = m\phi\bar{\phi}$, which removes all flat directions[8]. The total superpotential $W_{eff} = W_{dyn} + W_{tree}$ is studied with the same technique as we described for the 3-2 model. Supersymmetry is found to be broken for small m, where a perturbative control of the Kähler potential around the minimum is available. One can, however, conclude by continuity that the Witten index is zero for any finite m and also[9] for $m = \infty$, which corresponds to the original theory. Holomorphy also

[7] A possible loophole of this argument is that other non-perturbative effects cancel the one-instanton contribution to the relevant Green's functions. While this cancellation seems very unlikely it is impossible to exclude it rigorously.

[8] All flat directions are removed since, after integrating out ϕ and $\bar{\phi}$, we get the original $SU(5)$ model back.

[9] The possibility $\lim_{m\to\infty} E_{vac} \to 0$ cannot be rigorously excluded [160], although it seems unlikely.

implies a non-zero vacuum energy E_{vac} for any finite m. Finally we should add that another "proof" of broken supersymmetry for this model has been given in ref. [237], by adding enough flavours for a weakly coupled dual description to exist. By giving mass to the additional flavours, supersymmetry is broken in the dual theory just by an O'Raifertaigh-type tree-level superpotential.

The model just discussed is the simplest of a class of models based on gauge group $SU(N)$, N odd, with matter consisting of $N-4$ antifundamentals $\bar{F}_i$ and one antisymmetric tensor A [194, 7]. These theories, with the inclusion of the most general tree-level cubic superpotential, have no flat directions and preserve a non-anomalous R symmetry. In all models, supersymmetry is expected to be broken in the strongly coupled regime. Indeed, without tree-level superpotential, these models have flat directions described by the invariants $A\bar{F}_i\bar{F}_j$. On a generic point along a flat direction, the gauge symmetry is broken and the effective theory is given exactly by the $SU(5)$ model previously discussed, which has been shown to break supersymmetry.

More models can be constructed from this class, as suggested in ref. [105]. The strategy is to take the $SU(N)$ group and remove some "off-diagonal" generators to reduce it to $SU(k) \times SU(N-k) \times U(1)$, while keeping the original matter content. The smaller gauge symmetry allows more D-flat directions, but also more superpotential terms. Indeed a generic cubic superpotential lifts all flat directions in these "daughter" theories [182]. Notice that the 3-2 model is obtained through this decomposition from the $SU(5)$ with $\bar{\mathbf{5}} \oplus \mathbf{10}$. In ref. [105] new models in this class, such as $SU(4)\times U(1)$ or $SU(6)\times U(1)$, were shown to break supersymmetry and to lead to interesting applications (see sect. 5.2). In refs. [74, 75] also more complicated cases such as $SU(2n)\times SU(3)\times U(1)$, $SU(2n{+}1)\times SU(4)\times U(1)$ and $SU(2n)\times SU(5)\times U(1)$ were shown to break supersymmetry dynamically. In the latter cases one of the group factors has a dual description, in which supersymmetry breaking is manifest. In particular, in the dual theory, some Yukawa couplings flow to mass terms, giving mass to a number of flavours. By integrating them out the resulting effective theory generates a superpotential, and supersymmetry is broken in the standard way. Finally, in ref. [182], an argument for broken supersymmetry was given in the general case $SU(k) \times SU(N-k) \times U(1)$. The approach was to consider the original $SU(N)$ theory with the addition of an adjoint representation Σ and a superpotential containing the terms

$$W_\Sigma = M\Sigma^2 + \lambda\Sigma^3. \tag{4.21}$$

The classical vacua determined by W_Σ have gauge group $SU(k) \times SU(N-k) \times U(1)$ and correspond to the "daughter" theories of interest. By considering a dual [179, 180] of the full theory with Σ one finds [182] that it has no supersymmetric minima. Thus one expects no such state in any of the "daughter" theories.

To conclude this review of DSB mechanisms we mention an example [164] where confinement triggers supersymmetry breaking. Consider an $SU(2)$ gauge theory with matter ψ in an isospin-3/2 representation. At the classical level there is one D-flat direction parametrized by $u = \psi^4$. In ref. [164] it is argued that the theory confines with one massless chiral multiplet parametrized by the modulus u. This hypothesis is consistent with 't Hooft's matching of the R-symmetry anomalies $\mathrm{Tr}R$ and $\mathrm{Tr}R^3$. Consider now the addition of a perturbation

$$W_{tree} = \frac{\psi^4}{M} \ , \qquad (4.22)$$

where M is some large mass. Classically, there is a unique supersymmetric minimum at the origin $\psi = 0$. Notice that, when written in terms of $u = \psi^4$, the superpotential is linear. With this parametrization, however, the classical Kähler potential $K^c = (uu^\dagger)^{1/4}$ is singular at the origin. Therefore the classical potential $V(u) = |\partial_u W|^2 / K^c_{uu^\dagger}$ is zero at $u = 0$. On the other hand, if the theory confines, u is the correct degree of freedom to describe the low-energy effective theory, so the quantum Kähler metric should be smooth (and positive) when expressed in terms of u. For instance at the origin we expect $K_{uu^\dagger} = \mathrm{const} + uu^\dagger + \ldots$ Thus

$$V = K^{-1}_{uu^\dagger} |\partial_u W|^2 > 0 \qquad (4.23)$$

at every point and supersymmetry is broken. This phenomenon, DSB from smoothing a singularity in the Kähler metric, takes place in other models too. One of these is the 4-3-1 model of refs. [74, 182], where the confined field is a cubic (rather then quartic) monomial of the elementary fields. Therefore supersymmetry can be broken with a renormalizable superpotential.

5 Models for Dynamical Supersymmetry Breaking and Gauge Mediation

5.1 Early Attempts

When the mass splittings inside supermultiplets arise at tree level, the supertrace sum rule $\mathrm{STr}\mathcal{M}^2 = 0$ holds [123]. This theorem prevents the construction of simple and realistic models in which supersymmetry is broken at tree level in the SM sector. A generic implication is the existence of sparticles below the mass range of quarks and leptons [86]. The supertrace mass formula, however, does not hold beyond tree level. It was soon realized that when the splittings inside supermultiplets arise from radiative corrections, the sparticles can all be made consistently heavier than the SM particles. This was a motivation of the first gauge-mediated supersymmetry-breaking models. Indeed the aim of refs. [85, 277, 98, 100] was to build a "supersymmetric technicolour" theory, in which the breakdown of supersymmetry is due to some strong gauge dynamics, while its mediation to the SM particles is just due to the usual SM gauge interactions. Those models came early in the understanding of strongly coupled supersymmetric theories, and some of the dynamical assumptions on which they were based turned out to be false in the light of deeper tools of analysis, such as the Witten index [279]. The idea of gauge mediation emerged, however, as an independent, and general, aspect of those models. Indeed the paradigm discussed in refs. [99, 101, 9, 201, 87], consists of an O'Raifertaigh [218] sector coupled to the messengers at the tree level.

As an example let us consider the messenger sector described by [99]

$$W = X(\lambda_1 \Phi_1^d \bar{\Phi}_1^d + \lambda_2 \Phi_1^t \bar{\Phi}_1^t - \mu^2) + m_1 \Phi_1^d \bar{\Phi}_1^d + m_2 \Phi_1^t \bar{\Phi}_1^t + M_3 \Phi_1 \bar{\Phi}_2 + M_4 \Phi_2 \bar{\Phi}_1 \ , \tag{5.1}$$

where $\Phi_{1,2} \oplus \bar{\Phi}_{1,2}$ collectively denote $\mathbf{5} \oplus \bar{\mathbf{5}}$ of messenger fields, while Φ^d and Φ^t respectively denote their doublet and triplet components. The field X is a singlet. In a certain range of parameters the absolute minimum at tree level is at $\Phi_{1,2} = \bar{\Phi}_{1,2} = 0$, with $F_X = \mu^2$ and X undetermined. For $\lambda_1/\lambda_2 \neq m_1/m_2$, the one-loop effective potential generically fixes the VEV of X such that $m'_{1,2} = m_{1,2} + X\lambda_{1,2}$ are both non-zero. In such a way the messenger supermultiplets are split and non-zero soft masses arise for both gauginos and sfermions, as discussed in sect. 2. Notice that the above superpotential is not the most general one that is consistent with the symmetries of the model, and that in particular $m_{1,2}$ explicitly break the R

symmetry, which one could have used to "naturally" enforce linearity in X. The weak principle of naturalness can however be invoked in supersymmetric model building, as the superpotential is not renormalized. Notice also that it is crucial to have $\lambda_1/\lambda_2 \neq m_1/m_2$, or else the masses $m_{1,2}$ can be eliminated by a shift in X, after which the mimimum is at $X = 0$. This preserves an R symmetry and gauginos remain massless.

In theories with tree-level supersymmetry breaking, like the one above, the fundamental mass scales are just inputs, whose origin is left unspecified. This implies, in particular, that the small size of the weak scale itself, albeit stable against large quantum corrections, is left unexplained. Undoubtedly, a more fundamental theory would be one where the relevant mass scales arise dynamically. The scale of supersymmetry breaking *can* indeed have this nice property [278]. This is because when supersymmetry is unbroken at tree level, it will stay so to all orders in perturbation theory. The reason for that is the non-renormalization theorem, whose range of applicability however does not extend beyond perturbation theory. Indeed there are theories where supersymmetry does break non-perturbatively. Consequently the ratio between the scale of breaking and the fundamental mass scale goes like e^{-1/g^2}, and it is exponentially small at weak coupling. This can explain in a very natural way the smallness of the supersymmetry breaking (or weak) scale with respect to the Planck mass. Notice, indeed, the analogy with the case of technicolour theories, in which a *chiral* symmetry, unbroken in perturbation theory, is used to generate the weak-scale hierarchy. A great advantage of supersymmetry, however, is that in many cases the non-perturbative effects that determine its breaking can be studied exactly.

The explanation of the gauge hierarchy is one general motivation for being interested in field theories with DSB. As a matter of fact, in gauge-mediated models the motivation is even stronger that in the gravity-mediated case. This is because in gauge-mediated models, the whole dynamics of breaking and mediation takes place at very low scales, where we should be able to describe it field-theoretically. In principle this may not be the case in the gravity-mediated scenario, where string theoretic effects may play a crucial rôle[10].

The first attempts to build realistic models of DSB with gauge mediation were discussed in ref. [7]. Many of the gauge theories that are known to break supersymmetry have flavour

[10]Of course we must always plead ignorance of what makes the cosmological constant vanish, in gauge and in gravity mediation.

symmetries, which can remain unbroken in the vacuum. A simple strategy [7] is then to consider models with an anomaly-free flavour group $G_F \supset SU(3) \times SU(2) \times U(1)$ (or better $G_F \supset SU(5)$, and consider only complete $SU(5)$ multiplets in order to preserve the unification of gauge couplings) and weakly gauge it. The resulting model still breaks supersymmetry as long as the SM gauge force is weak, since the vacuum energy must be continuous in the gauge coupling constant. Moreover the SM sector "knows" about supersymmetry breaking just by its gauge interactions. In principle this idea seems very promising. In practice, though, there are problems to implement it, some of which are largely present today. The main problem is the loss of asymptotic freedom in the SM gauge factors. Models that both break supersymmetry and have $G_F \supset SU(3) \times SU(2) \times U(1)$ have a large gauge group G_S. From the point of view of the SM group (or G_F) the different "colours" of G_S are just different flavours. This means that, G_S being large, there are in general many flavours of SM matter in the supersymmetry-breaking (messenger) sector. In practice this makes the gauge couplings of the SM blow up a few decades above the messenger scale. This is often well below the GUT scale. For instance, consider the class of DSB models of refs. [4, 194], based on $G_S = SU(N)$ (N odd) with an antisymmetric tensor A and $N-4$ antifundamentals $\bar{F}_i$ in the matter sector. The smallest model where an anomaly-free $SU(5)$ gauge group can be embedded is based on $SU(15)$, for which there are 15 families of messengers above the supersymmetry-breaking scale. A way out could be to embed only $SU(2)_W \times U(1)$ among the SM group factors in the messenger sector. But this would be problematic: on one side gauge unification would be typically lost and on the other the gluino mass would be tiny as it arises at a high order in perturbation theory.

Another problem of the early attempts was the spontaneous breakdown of the R symmetry, which is rather generic in models with spontaneous supersymmetry breaking, as discussed in sect. 4.2. The resulting axion couples to the QCD anomaly and is phenomenologically problematic as its scale f_a is typically low $\sim$ 10–100 TeV. This can however be considered a smaller problem. Indeed there may exist different sources of explicit R breaking, which are small enough not to restore supersymmetry (at least in a nearby vacuum) and large enough to give the axion a mass that renders it phenomenologically harmless. In ref. [203] it was shown that in some models this rôle can be played by $1/M_P$ suppressed dimension-five operators in W. Moreover, in ref. [25] it was pointed out that, when R is broken at the same scale as supersymmetry, the cancellation of the cosmological constant, by adding a constant term to W in supergravity, im-

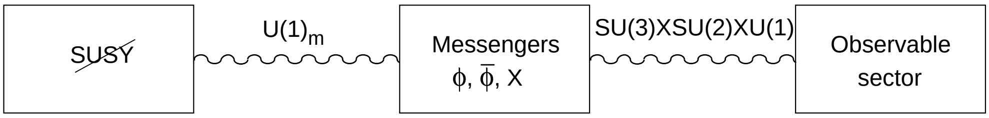

Figure 11: *The modular structure of the messenger $U(1)$ models.*

plies an amount of explicit R breaking that gives the axion an acceptably large mass. Another way out is to consider special R-breaking deformations of the DSB model [203, 250, 231, 182]. In some cases (see the example of the 3-2 model in sect. 4.4) the stationary equations are not affected and supersymmetry remains broken, although R-breaking effects give mass to the R-axion, sometimes just from the Kähler potential (see the 4-3-1 model of ref. [182]). Alternatively, one can introduce a new strongly coupled gauge group under which the R symmetry carries an anomaly. Non-perturbative effects give a large mass to the axion and, in most cases, do not restore supersymmetry [203]. Finally, there are now models where R is broken at a higher scale than supersymmetry. This scale can consistently be in the astrophysically allowed window 10^9–10^{12} GeV [269, 243], so that the R-axion could indeed solve the strong-CP problem.

5.2 Messenger $U(1)$ Models

The revival of DSB and gauge mediation, which was started by Dine, Nelson and collaborators [102, 104, 105] in 1993, had the main goal of tackling the difficulties due to the loss of asymptotic freedom. In order to build realistic models, the approach was to assume that the DSB sector is completely neutral under the gauge forces of the SM. The basic structure of this class of models is shown in fig. 11. There are basically three sectors, one to break supersymmetry, one to mediate it and one containing the ordinary particles. These sectors only communicate with one another by weak gauge forces. Supersymmetry is first broken in sector I, and the news of it is transferred to sector II (the messengers) via an Abelian gauge group $U(1)_m$, denoted as messenger hypercharge. Sector III, the observable sector, is neutral under $U(1)_m$, so that the breakdown of supersymmetry is felt there only at the next step, through the usual gauge interactions.

This structure may seem complicated, but models in this class provided the first examples of a realistic theory with calculable, dynamical, and universal soft terms. Moreover, this structure is quite generic and adaptive, as there is a vast class of DSB models that can play the rôle of sector I. Let us briefly outline a prototypical example discussed in ref. [105]. The DSB sector is based on an $SU(6)\times U(1)\times U(1)_m$ gauge group, which comes from the reduction procedure illustrated in sect. 4.4, with the following matter content

$$A(\mathbf{15},+2,0) \quad F(\mathbf{6},-5,0) \quad \bar F^{\pm}(\bar{\mathbf{6}},-1,\pm1) \quad \bar F^{0}(\bar{\mathbf{6}},-1,0) \quad S^{\pm}(\mathbf{1},+6,\pm1) \quad S^{0}(\mathbf{1},+6,0). \tag{5.2}$$

The tree-level superpotential is

$$W_I = \lambda_1 A\bar F^+\bar F^- + \lambda_2 F(\bar F^+ S^- + \bar F^- S^+) + \lambda_3 F\bar F^0 S^0. \tag{5.3}$$

This superpotential lifts all flat directions and respects a non-anomalous R symmetry. Moreover gaugino condensation generates a superpotential

$$W_{dyn} = \frac{\Lambda^7}{\sqrt{A^4\bar F^+\bar F^-\bar F^0\ F}}\ . \tag{5.4}$$

In eq. (5.4) gauge indices have been neglected, and there is only one gauge-invariant contraction, which respects the flavour $SU(3)$ symmetry of the $SU(6)$ strong interactions under which $(\bar F^{\pm},\bar F^0)$ form a triplet. Equation (5.4) lifts the origin, forcing the R symmetry to be spontaneously broken. Then, according to the arguments discussed in sect. 4, supersymmetry is also broken by non-zero F-terms. Balancing the terms in W_I+W_{dyn} leads to a typical VEV for the scalars $v\sim\Lambda/\lambda^{1/7}$, where λ generically denotes the size of the Yukawa couplings in eq. (5.3). For small enough Yukawas, $v\gg\Lambda$ and the theory can be analysed at weak coupling along the D-flat directions. Here the Kähler potential can be reliably approximated by its tree-level value and the vacuum can be studied in perturbation theory. We refer for the details to ref. [105]. The main points are the following. *i)* At the vacuum a $U(1)'_m\subset SU(6)\times U(1)\times U(1)_m$ is unbroken. *ii)* There are two light chiral superfields $\chi^{\pm}\subset\bar F^0$ with charge ±1 under $U(1)'_m$. The scalar components of $\chi^{\pm}$ have a positive supersymmetry-breaking mass m^2_χ. *iii)* No D term for $U(1)'_m$ is generated, as there is an unbroken charge parity under which the $U(1)_m$ vector is odd. This last property is very useful for practical model building [102, 104, 105].

In the end, *i)–iii)* are the only properties of sector I, which are needed to build a realistic messenger sector II. This sector is described by the following superpotential

$$W_{II} = k_1 X\varphi^+\varphi^- + k_2 X^3 + k_3 X\Phi\bar\Phi\ , \tag{5.5}$$

where X is a singlet, Φ, $\bar\Phi$ are the messengers, and $\varphi^{\pm}$ have charge ±1 under $U(1)_m$ and are singlets under the rest of the gauge group. The fields $\varphi^{\pm}$ communicate with $\chi^{\pm}$ in module I via the $U(1)'_m$ gauge bosons. At two loops the scalar components of $\varphi^\pm$ receive soft masses $m_\varphi^2 = -m_\chi^2(\alpha_m/\pi)^2 \ln(v/m_\chi)$, where v is a scalar VEV in sector I, determining the scale where m_χ is effectively generated. In the calculable regime (see above) $v \gg \Lambda > m_\chi$ holds, and one obtains $m_\varphi^2 < 0$. This negative mass is crucial to obtain the desired vacuum. The scalar potential in sector II is the sum of the F^2-terms generated from W_{II}, the $D_m^2 \propto (|\varphi^+|^2 - |\varphi^-|^2)^2$ term, and the negative soft masses for $\varphi^\pm$. In a certain range of parameters, the minimum of the potential has a non-vanishing VEV for $\varphi^\pm$, X and F_X, while the messenger Φ, $\bar{\Phi}$ VEVs are zero. This determines a one-scale messenger sector with $X \sim \sqrt{F_X} \sim m_\varphi \sim$ 10–100 TeV.

We stress that models in this class were the first explicit examples of calculable DSB with gauge mediation. The price that is being payed is the modular structure: the three boxes in fig. 11. The supersymmetry-breaking sector is far removed from the observable sector. A measure of this fact is that the superfield X has only a small overlap with the goldstino: $F_X \sim (\alpha_m/\pi)^2 m_\chi^2 \ll m_\chi^2 \lesssim F_0$, where F_0 is the goldstino decay constant. In these models, for $\alpha_m \sim 10^{-2}$ one has $\sqrt{F_0} \gtrsim 10^7$ GeV. This scale is indeed in the most dangerous range for the gravitino problem, as it requires a reheating temperature below the TeV scale, see sect. 3.5.

Moreover, the desired vacuum in sector II turns out to be only local. This is easily seen by considering first the $m_\varphi^2 = 0$ limit. Now we find a flat direction, see eq. (5.5), with $X = 0$ and $F_X = k_1 \varphi^+ \varphi^- + k_3 \Phi \bar{\Phi} = 0$, along which both $U(1)_m$ and $SU(3) \times SU(2) \times U(1)$ are broken. When the soft mass $m_\varphi^2 < 0$ is turned on, the potential along this direction becomes increasingly negative until $\varphi^\pm \sim m_\chi$, above which m_φ^2 goes to zero. At $\varphi^\pm \sim m_\chi$, however, the potential is $V \sim -|m_\varphi^2 m_\chi^2|$ which is much deeper than in the desired vacuum $V \sim -|m_\varphi^4|$. Notice that in the deeper minimum colour is broken.

The question of whether the local minimum at $F_X \neq 0$ is cosmologically acceptable was discussed in detail in ref. [79]. There are two aspects to this problem. One concerns the "likelihood" of being placed in such a vacuum in the early Universe without being pushed out by thermal fluctuations. The answer to this question is both difficult and dependent on details of the early history of the Universe, but it is not unreasonable to assume that the answer is positive. The other issue is the lifetime of the false vacuum in the cold Universe. For the model to be acceptable, this lifetime should necessarily be larger than the age of the Universe. This requirement puts restrictions on the parameter space. By studying a numerical approximation to the bounce action, one finds [79] that the coupling k_1, see eq. (5.5), must be smaller than

0.1. This bound is necessary but probably not sufficient. However, one can conclude that with some adjustment of parameters this model can be easily made acceptable. Another possibility is to enlarge the spectrum of sector II in order to make the desired vacuum the global one. By adding [79, 18] another singlet X' with the general couplings

$$\Delta W = k_1' X' \varphi^+ \varphi^- + k_2' X'^3 + k_3' X' \Phi \bar{\Phi} + (X, X' \text{ terms}) \tag{5.6}$$

the dangerous flat direction is lifted and the desired vacuum is global for a range of parameters.

5.3 $SU(N) \times SU(N-k)$ Models with Direct Gauge Mediation

A first interesting attempt to build models of direct gauge mediation, in which the messengers themselves belong to the DSB sector, has been performed by Poppitz and Trivedi [233] and by Arkani-Hamed, March-Russell, and Murayama [18]. The goal is still to find a DSB model with flavour group $G_F \supset SU(5)$, but now the problem of Landau poles is circumvented by considering theories for which $X \gg \sqrt{F_X}$ in a natural way. The larger value of the messenger mass X can displace the Landau pole above the GUT scale M_G, even for a "large" number of messenger families $N_f \sim$ 5–10. The idea is to consider DSB models like those introduced in refs. [231, 232], which have classically flat directions that are only lifted by non-renormalizable operators $W_{nr} = \phi^n / M_P^{n-3}$. In the models of interest a non-perturbative superpotential $W_{dyn} \sim \Lambda^{3-p}\phi^p$ is generated by the gauge dynamics, where we collectively denote all the fields by ϕ. Thus for $p < 1$, in the limit $M_P \to \infty$, the potential slopes to zero at infinity and the theory has no vacuum. For finite M_P, the vacuum is stabilized at a geometric average $\phi^{n-p} \sim M_P^{n-3}\Lambda^{3-p}$ and supersymmetry can be broken with generic F terms $F_\phi \sim \phi^{n-1}/M_P^{n-3}$. With the messenger masses and splittings given by ϕ and F_ϕ, one naturally obtains a two-scale model with F_ϕ/ϕ^2 suppressed by a positive power of Λ/M_P.

Notice that ϕ is fixed by $\phi^{n-2}/M_P^{n-3} = F_\phi/\phi \sim 10^4$ GeV (we consider large N_f). On the other hand, an acceptable suppression of the gravity-mediated soft masses requires $\phi \lesssim 10^{15}$ GeV, see eq. (2.44), which translates[11] into $n \lesssim 7$. In the $SU(N) \times SU(N-1)$ and $SU(N) \times SU(N-2)$ models of refs. [231, 232], the flat direction is lifted by a baryon, so that $n = N-1$ and $N-2$ respectively, which puts a generic upper bound $N \lesssim 9$. This is already a powerful

[11]A weaker bound on n can be obtained if we assume that the mass scale of the non-renormalizable interaction is lower than M_P [233].

constraint. The first attempt in ref. [233] focused on $SU(N)\times SU(N-2)$. In these theories the flat directions can be lifted [232] by preserving a flavour group $G_F = Sp(N-3)$, which is also unbroken at the vacuum. The minimal N for which an anomaly-free $SU(5)\subset Sp(N-3)$ can be gauged is $N = 13$. Thus the gravity contribution to soft terms is not suppressed but at best comparable to the one induced by gauge effects. Models like these are denoted as "hybrid".

In ref. [18], the problem of large N was tackled by allowing an anomalous embedding of $SU(5)$ and by adding the suitable spectator fields to cancel anomalies. On one side, this allows smaller N but, on the other, the final theory may not break supersymmetry, as the spectators can enter the superpotential. A working example was, however, found in ref. [18], based on a known [231, 232] $SU(7)\times SU(6)$ model. The particle content is given by $Q(\mathbf{7},\mathbf{6})$, $L^I(\bar{\mathbf{7}},\mathbf{1})$, $I = 1,\ldots,6$, $R_I(\mathbf{1},\bar{\mathbf{6}})$, $I = 1,\ldots,7$. A group $SU(5)_W$ acting on the indices $I = 1,\ldots,5$ is weakly gauged and we denote the corresponding elements in R_I and L^I just by $\mathcal{R}$ and $\mathcal{L}$, dropping the indices. Then a spectator field $\phi(\mathbf{1},\mathbf{1},\mathbf{5})$ under $SU(7)\times SU(6)\times SU(5)_W$ is added to cancel the $SU(5)^3_W$ anomaly. Imposing an R symmetry and a global $U(1)$ the most general superpotential is

$$W = \lambda_1 \mathcal{L}Q\mathcal{R} + \lambda_2 L^6 Q R_6 + \frac{\lambda_3}{M_P}\mathcal{L}\phi Q R_6 + \frac{\lambda_4}{M_P^3} B_7 + \frac{\lambda_5}{M_P^4}\mathcal{B}\phi \ , \tag{5.7}$$

where the last two terms involve the $SU(6)$ baryons $B_7 = \mathcal{R}^5 R_7$ and $\mathcal{B}_I = (\mathcal{R}^4)_I R_6 R_7$ ($I = 1,\ldots,5$), with the obvious contractions. Equation (5.7) lifts all flat directions involving the fields in this sector. According to the discussion in ref. [18], the vacuum lies along a two-dimensional D-flat direction where the diagonal $SU(5)\subset SU(6)\times SU(5)_W$ is unbroken. This $SU(5)$ is identified with the SM gauge interactions. The VEVs are $\langle\mathcal{R}\rangle = v_{\mathcal{R}}\times \mathbb{1}_{5\times5}$ and $\langle R^j_{6,7}\rangle = \delta^{6j} v_{6,7}$, where j is the SU(6) gauge index, with $|v_{\mathcal{R}}|^2 = |v_6|^2 + |v_7|^2$, as required by $SU(6)$ flatness. This direction is parametrized by the two baryons $B_{6,7} = \mathcal{R}^5 R_{6,7}$. The R VEVs, through $\lambda_{1,2}$ in eq. (5.7), give mass to all $SU(7)$ matter. Moreover, under the unbroken $SU(7)\times SU(5)_W$, the massive $Q\oplus\mathcal{L}\oplus L^6$ decompose as $(\mathbf{7},\mathbf{5})\oplus(\bar{\mathbf{7}},\mathbf{5})\oplus(\mathbf{7},\mathbf{1})\oplus(\bar{\mathbf{7}},\mathbf{1})$. Then the supersymmetry-breaking dynamics is very simple. The low-energy $SU(7)$ pure gauge theory generates a superpotential via gaugino condensation $W_{dyn}\propto \Lambda_7^{\frac{15}{7}} B_6^{\frac{1}{7}}$. The competition of this term with B_7 in eq. (5.7) gives rise to a stable vacuum, where supersymmetry is broken: $F_{B_{6,7}}\neq 0$.

It is useful to work in components and define $R_{\tilde{6},\tilde{7}}$ as the orthogonal combinations of $R_{6,7}$

such that $R_{\tilde{7}}$ has no VEV in the scalar component. Then, with an appropriate gauge choice, we can parametrize $\mathcal{R} = X \times 1\!\!1_{5\times5}$, $R_{\tilde{6}}^j = \delta^{6j} X$, $R_{\tilde{7}}^6 = Y$, $R_{\tilde{7}}^j = \bar{\Phi}^j$ $(j \le 5)$, where $X, Y, \bar{\Phi}$ are light fields and $\langle X \rangle = v_{\mathcal{R}}$, $\langle Y \rangle = \langle \bar{\Phi} \rangle = 0$. Notice that all $SU(5)_W$ charged fields in $\mathcal{R}$ and $R_{\tilde{6}}$ are eaten by the super-Higgs mechanism.

Parametrically the scalar and auxiliary field VEVs are given by $\mathcal{R} \sim X \sim \Lambda_7^{\frac{5}{12}} M_P^{\frac{7}{12}}$ and $F_{\mathcal{R}} \sim F_X \sim \Lambda_7^{\frac{25}{12}} M_P^{-\frac{1}{12}}$. Thus there are 7 flavours of $SU(5)_W$ that get both supersymmetric masses and splittings from $\mathcal{R}, F_{\mathcal{R}} \neq 0$ and act just as conventional messengers. Notice, in addition, that they are a crucial part of the DSB sector: the $SU(7)$ gaugino condensate that drives DSB is proportional to B_6, *i.e.* the messenger mass determinant.

This particular model suffers from a problem: the dominant contribution to the ordinary sfermion mass squared is negative. This a generic problem of the existing models of direct gauge mediation. We will discuss here one source of negative contributions to the mass squared, while another contribution will be discussed in sect. 5.4.

The most important effect is related to $\phi(\mathbf{1}, \mathbf{5})$. This field is needed to cancel the $SU(5)_W^3$ anomaly, *i.e.* to match the composite $\mathcal{B}^j = (\mathcal{R}^4 R_6 R_7)^j = X^5 \bar{\Phi}^j = (\mathbf{1}, \bar{\mathbf{5}})$, which is massless in the theory without ϕ. Its mass, see eq. (5.7), is roughly $\lambda_5 X^5 / M_P^4$, which is below the weak scale. The field $\mathcal{B}$ is however directly involved in DSB. Its scalars get positive soft masses $\mathcal{O}(F_X/X)^2 \sim (100 \text{ TeV})^2$, leading to a positive supertrace in the $\phi \oplus \mathcal{B}$ sector. The supertrace contributes, via two-loop RG evolution, to the masses of squarks and sleptons

$$\delta m_{\tilde{Q},\tilde{L}}^2 \sim -\left(\frac{\alpha}{4\pi} \frac{F_X}{X}\right)^2 \ln \frac{X}{m_\phi} < 0 \,. \tag{5.8}$$

This negative contribution is parametrically $\ln(X/m_\phi) \sim 30$ times bigger that the standard messenger result in gauge mediation.

This problem [18, 234] is common to models based on $SU(N) \times SU(N-k)$, which have k such light states. A positive mass supertrace in the messenger sector arises at tree level by decoupling the heavy vector superfields. It is easily illustrated by using our parametrization in components, where $R_{\tilde{7}}$ contains a light $SU(5)_W$ antifundamental $\bar{\phi}$ (which we earlier parametrized by the baryon $\mathcal{B}$) and the singlet Y. At the vacuum, Y has zero scalar VEV, but $F_Y \sim F_X \neq 0$. Then, see fig. 12, exchange of massive vectors in the coset $SU(6) \times SU(5)_W / SU(5)$, gives rise

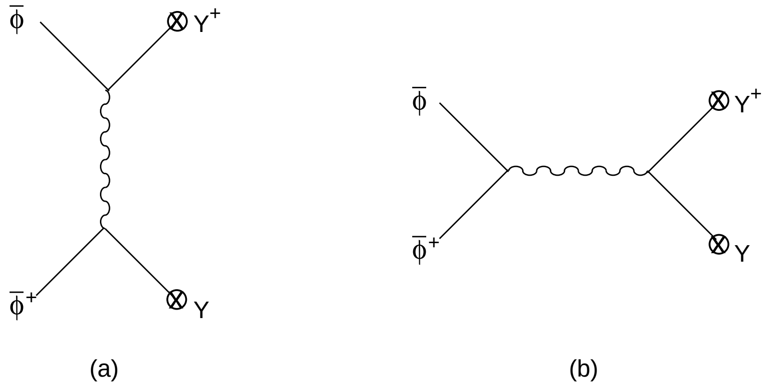

Figure 12: *Tree-level contributions to the $\bar\phi$ soft masses from the Y VEV.*

to positive soft masses for $\bar\phi$

$$m^2_{\bar\phi} = \frac{|F_Y|^2}{2M_V^2}\left(1 - \frac{1}{N}\right) > 0 \ , \tag{5.9}$$

where M_V is the vector mass, and where $N = 6$ corresponds to the present model. The negative $-1/N$ contribution, which arises from the diagram of fig. 12b is always subdominant.

5.4 Models with Quantum Moduli Spaces and Plateau Models

Another pathway for constructing simpler DSB theories with gauge mediation was opened by Izawa and Yanagida [167] and by Intriligator and Thomas [166]. These authors considered gauge theories with quantum moduli spaces (QMS) [250], which do not break supersymmetry, and coupled them to singlets to obtain effective O'Raifertaigh models. Now all field configurations where supersymmetry is unbroken belong only to the classical moduli space, and are removed by quantum deformation. The prototype example has been discussed in sect. 4.4, and it is based on an $SU(2)$ gauge theory with 4 fundamentals Q_i, $i = 1, \ldots, 4$ coupled to 6 singlets $R^{ij} = -R^{ji}$, with the tree-level superpotential given in eq. (4.18). At the quantum level, the superpotential is given by eq. (4.19) and supersymmetry is broken as $F_{R^{ij}}$ and F_A cannot both vanish at the same time. However, the determination of the vacuum state requires control of the Kähler potential. For the model in question, the only range where this can be done perturbatively is $|\lambda\sqrt{\mathrm{Pf}R}| \gg \Lambda$. The dynamics in this region will be discussed in the next section. The bottom line is that, at large $X \equiv \sqrt{\mathrm{Pf}R}$, the effective potential is well approximated by $V(X) \simeq |\lambda\Lambda^2|^2/Z_R(XX^\dagger)$, where Z_R is the R wave function. By solving the one-loop RG equation for

Z_R, one finds that V increases monotonically with R in the perturbative region[12]. On the one hand this is good, as the potential is stabilized at large R, on the other it is problematic, as R is driven to the region where we cannot control the Kähler potential.

Thus it is natural to expect that the vacuum is either at $R = \mathcal{O}(\Lambda)$ or at $R = 0$. In the former case, the field R could be directly used to give mass to the messengers by adding a term $R\Phi\bar{\Phi}$ to eq. (4.18). However, the modified model has supersymmetry-preserving vacua where $R = 0$ and $F_R = \lambda\Lambda^2 + \Phi\bar{\Phi} = 0$. These are similar to those existing in sector II of the messenger $U(1)$ models, discussed in sect. 5.2. This is a problem, since there is no obvious parametric suppression of the rate of decay of the false vacuum and the theory is probably cosmologically unacceptable.

A more realistic model requires an intermediate step in the coupling between R and the messengers [159, 168, 169, 211, 212]. In particular, this was done in the model of ref. [159] by adding a singlet Y with superpotential

$$W_Y = \lambda_Y Y^3 + \lambda_\Phi Y\Phi\bar{\Phi}. \tag{5.10}$$

In this model, the superpotential in eq. (4.18) preserves only an $SO(5)$ subgroup of the $SU(4)$ flavour group, under which the fields R transform as a $\mathbf{1} + \bar{\mathbf{5}}$: $R = (R_0, R_a)$, $a = 1, \ldots, 5$. In a range of parameters, the vacuum has $F_{R_0} \neq 0$ and $F_{R_a} = 0$. Furthermore the field Y is assumed to mix with R_0 in the Kähler potential $K \supset \beta R_0 Y^\dagger + \text{h.c.}$, where β is a constant of order unity[13]. The basic dynamical assumption of ref. [159] is that the vacuum of the original model ($\beta = 0$) is at $R_0 = R_a = 0$. At $\beta \neq 0$ the interactions with Y and $\Phi, \bar{\Phi}$, modify the vacuum in a perturbative manner. In a range of parameters one has a minimum with $\langle Y^2 \rangle \sim \beta\Lambda^2$, $F_Y \sim \Lambda^2/(4\pi)^2$ and $\Phi = \bar{\Phi} = 0$. Notice that F_Y is generated by radiative corrections to the Kähler potential from the interaction in eq. (5.10). Then one has $F_Y/Y \sim \Lambda/(4\pi)^2$, so that the usual gauge-mediated contributions to gauginos and sfermion masses squared correspond respectively to two- and four-loop effects. In this model, however, there are additional and generically larger contributions to the sfermions. This is because the second term in eq. (5.10) induces, at one

[12] Heuristically, V behaves like the running coupling of a pure Yukawa theory since Z_R, at one loop, is only affected by λ. As Z_R removes the flatness of the potential, the first condition of the ADS criterion is verified only after including one-loop effects.

[13] Notice that we use a field parametrization different from that in ref. [159]. There, Y is chosen to couple to a combination of the mesons M_{ij} in the superpotential, while the kinetic terms are diagonal. The two formulations are equivalent.

loop, a non-vanishing supertrace in the messenger sector. Inserting this supertrace in the usual two-loop diagrams for sfermion masses, we realize that there is an effective three-loop contribution to these masses. In order to avoid sfermions that are a factor $\mathcal{O}(4\pi)$ heavier than gauginos, the choice $\lambda_\Phi \lesssim \lambda_Y/(4\pi)$ must be made. In spite of this adjustment of couplings, we still consider the above model as an instructive example towards a realistic gauge-mediated model with X not much larger than $\sqrt{F_X}$.

A characterization of a class of realistic and calculable models based on QMS was given in refs. [200, 96]. The basic idea is to gauge part of the flavour group of models like those based on the superpotential in eq. (4.19), so that the fields R, which couple to the mesons on the QMS, are no longer singlets. This allows the creation of a stable minimum far away from the origin $R \gg \sqrt{F_R}$. The theories are based on a gauge group $G = SU(5)_W \times G_B \times G_S$, where $SU(5)_W$ again stands for $SU(3) \times SU(2) \times U(1)$, and G_S is the strong factor with a QMS. The messenger-sector superpotential is

$$W = \lambda_1 R\Phi\bar{\Phi} + \lambda_2 RQ\bar{Q} \ , \tag{5.11}$$

where R transforms non-trivially only under G_B, $\Phi \oplus \bar{\Phi}$ are $SU(5)_W$ fundamentals and singlets of G_S. Finally, $Q \oplus \bar{Q}$ form a vectorial representation of G_S, but are singlets under $SU(5)_W$. All the fields transform non-trivially under G_B. The following requirements must be satisfied. *i)* G_S has a QMS, *i.e.* the Dynkin index μ_Q of $Q \oplus \bar{Q}$ equals that of the adjoint μ_{G_S}; $G_S = SU(2)$ with four fundamentals is an example. *ii)* R contains just one D-flat direction parametrized by one invariant $u = u(R) \sim R^k$. *iii)* Along u, all the $\Phi, \bar{\Phi}$ and $Q, \bar{Q}$ get masses $\sim \lambda_1 R$ and $\sim \lambda_2 R$, respectively.

Far along $u \neq 0$, the low-energy theory has gauge group $G = SU(5)_W \times H_B \times G_S$ (with $H_B \subset G_B$). The only massless matter is given by the observable-sector fields plus the gauge singlet u. The additional group factors are just pure gauge. It is also assumed that H_B is weak, so that only the strong dynamics of G_S is relevant.

The effective theory picture is simple. Calling Λ the strong scale of the original G_S ($X \ll \Lambda$), the scale Λ_{eff} of the effective theory below X is determined by one-loop matching, $\Lambda_{eff} = \Lambda(\lambda_2 X/\Lambda)^{\mu_Q/3\mu_{G_S}}$. Above the G_S scale Λ_{eff}, the effective theory has a vanishing superpotential, $W = 0$. Below the scale Λ_{eff}, G_S confines and generates a superpotential for X via gaugino condensation. The symmetries, holomorphy, and the QMS relation $\mu_Q = \mu_{G_S}$, constrain W_{eff}

to be linear in $X = u^{1/k}$ (where in the previous example $X = \sqrt{\mathrm{Pf} R}$)

$$W_{eff} = \Lambda_{eff}^3 = \Lambda^{3-\mu_Q/\mu_{G_S}} (\lambda_2 X)^{\mu_Q/\mu_{G_S}} = \lambda_2 X \Lambda^2 \ . \tag{5.12}$$

Now supersymmetry is broken at any point on the X complex line. For $\mu_Q \neq \mu_{G_S}$ the scalar potential $|\partial_X W_{eff}|^2$ would push X either to the origin or to infinity, where supersymmetry is in general restored. Models with the simple microscopic superpotential in eq. (5.11) have in general other flat directions at $X = 0$ (involving the massless Φ and Q) where supersymmetry can be restored. Unlike the models discussed at the beginning of this section, however, it is now possible to stabilize $X \sim R$ far away from these points, *i.e.* $\langle X \rangle \gg \Lambda$.

In the region $\lambda_2 X \gg \Lambda$, the Kähler potential is perturbatively calculable. It is given by loops involving the heavy superfields $Q, \bar{Q}$, the messengers $\Phi, \bar{\Phi}$ and the heavy vector superfields in the coset G_B/H_B. All these particles get a mass from the VEV of X. At lowest order the result corresponds to the one-loop anomalous dimension of R

$$K(X, X^\dagger) = XX^\dagger \left[1 + \frac{1}{16\pi^2} (C_B g_B^2 - C_1 \lambda_1^2 - C_2 \lambda_2^2) \ln(XX^\dagger/M_P^2) + \dots \right] \ , \tag{5.13}$$

where the C's are positive coefficients and M_P is the cut-off scale. By resumming the logs via the RG, one gets $K(X, X^\dagger) = XX^\dagger Z_R \left(XX^\dagger/M_P^2 \right)$, where Z_R is the running R wave function. The effective potential is then

$$V_{eff} = \frac{|\partial_X W|^2}{\partial_X \partial_{X^\dagger} K} \simeq \frac{\lambda_2^2 \Lambda^4}{Z_R \left(XX^\dagger/M_P^2 \right)}. \tag{5.14}$$

The logarithmic evolution of Z_R with $|X|$ can generate local minima in the effective potential. Unlike the pure Yukawa case, $d \ln Z_R^{-1}/d \ln X$ is not necessarily negative at every X, as the gauge and Yukawa contributions can balance each other at some scale. It is crucial that X be part of a field R, which is charged under G_B. This provides the gauge contribution g_B^2 without which X is always pushed towards the origin, where we lose control of our approximation and typically restore supersymmetry. The stationary points arise via the Coleman–Weinberg mechanism and are determined by the zeros of the anomalous dimension for R

$$8\pi^2 \frac{d \ln Z_R}{d \ln |X|} = C_B \bar{g}_B^2 - C_1 \bar{\lambda}_1^2 - C_2 \bar{\lambda}_2^2 = 0 \ . \tag{5.15}$$

Here $\bar{g}_B, \bar{\lambda}_1, \bar{\lambda}_2$ are the running couplings evaluated at the scale $|X|$. Moreover, in order to ensure that the stationary point be a minimum, one should verify that $d^2 Z_R/d(\ln X)^2$ is negative. This mechanism for stabilizing a tree-level flat potential is just Witten's inverse hierarchy [278]. It is now natural to expect $X \gg \sqrt{F_X} \sim \Lambda$. This hierarchy ensures that the minimum on the plateau, even if it is not the true ground state, is nonetheless stable on cosmological time scales. In ref. [96] the rate of tunnelling into the true supersymmetric vacuum at $X = 0$ was estimated by using the semiclassical approximation [71], and was found to be larger than the lifetime of the Universe as long as $X \gtrsim 10\ \Lambda$. Again, here there is the open question of why this particular local minimum is chosen by the cosmological evolution.

A simple way to implement this idea is to consider $G_S = SU(2)$ and $G_B = SU(2) \times SU(2) \sim SO(4) \subset SU(4)$, where $SU(4)$ is the flavour group of the prototype model based on the superpotential in eq. (4.19). The spectrum under $SU(2)_S \times SU(2)_{B1} \times SU(2)_{B2} \times SU(5)$ is

$$Q = (\mathbf{2}, \mathbf{2}, \mathbf{1}, \mathbf{1}), \quad \bar{Q} = (\mathbf{2}, \mathbf{1}, \mathbf{2}, \mathbf{1}), \quad R = (\mathbf{1}, \mathbf{2}, \mathbf{2}, \mathbf{1}),$$
$$\Phi_3 \oplus \Phi_2 \oplus \Phi_1 = (\mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{3} \oplus \mathbf{2} \oplus \mathbf{1}) \quad \bar{\Phi}_3 \oplus \bar{\Phi}_2 \oplus \bar{\Phi}_1 = (\mathbf{1}, \mathbf{2}, \mathbf{1}, \bar{\mathbf{3}} \oplus \bar{\mathbf{2}} \oplus \bar{\mathbf{1}})\ , \tag{5.16}$$

where the Φ_i's have been split into their $SU(3) \times SU(2) \times U(1)$ irreps. Notice that a pair of SM singlets $\Phi_1, \bar{\Phi}_1$ is added in order to cancel the $SU(2)_{B1} \times SU(2)_{B2}$ global anomaly. The tree-level superpotential is given by

$$W = R\left(\lambda Q\bar{Q} + \lambda_3 \Phi_3 \bar{\Phi}_3 + \lambda_2 \Phi_2 \bar{\Phi}_2 + \lambda_1 \Phi_1 \bar{\Phi}_1\right). \tag{5.17}$$

The classically flat direction of interest is $X = (\mathrm{Det}R)^{\frac{1}{2}}$. Along X, we obtain $\langle R \rangle \propto 1\!\!1_2$, $H_B = SU(2)$ and conditions *i)–iii)* are satisfied. By studying the RG equations one finds [96] a significant region of parameter space where this model has a local mimimum on the plateau at large X. There it behaves as a conventional gauge-mediated supersymmetry-breaking model with two messenger families. The messenger mass X can be anywhere between 10^9 and 10^{14} GeV, where the lower bound is determined by asking perturbative control over the Kähler potential. Notice also that at the upper edge $X \sim 10^{13}$–10^{14} nucleosynthesis is problematic, see sect. 3.5. Around the minimum the scalar part of the goldstino superfield X gets a mass $m_X^2 \sim (\alpha/4\pi)^2 (F_X/X)^2$, which is of the order of the sfermion masses, $\sim 10^2$–10^3 GeV. At this stage, the pseudoscalar part is the R-axion. Notice that $\langle X \rangle$, the axion scale, as far as gauge mediation is concerned, can consistently be in the 10^9–10^{12} GeV axion window. So $\mathrm{Im}X$ provides an interesting QCD axion candidate.

A special and more ambitious possibility is to try to identify $Q, \bar{Q}$ with $\Phi, \bar{\Phi}$, so that the messengers themselves trigger supersymmetry breaking. This was done in ref. [200], in a theory based on $Sp(4)\times SU(5)\times SU(5)$, and in ref. [96], in a theory based on $SU(5)_1\times SU(5)_2\times SU(5)_W$. Let us consider, for illustrative purposes, the $SU(5)_1\times SU(5)_2\times SU(5)_W$ model, which is indeed quite simple. The matter content is given by $R(\mathbf{1},\mathbf{5},\bar{\mathbf{5}})$, $\Phi(\bar{\mathbf{5}},\mathbf{1},\mathbf{5})$ and $\bar{\Phi}(\mathbf{5},\bar{\mathbf{5}},\mathbf{1})$. Again $SU(5)_W$ is already broken to $SU(3)\times SU(2)\times U(1)$ and the standard matter transforms as $\mathbf{5}\oplus\bar{\mathbf{10}}$ of $SU(5)_W$. The most general renormalizable superpotential is

$$W=\lambda R\Phi\bar{\Phi}\ , \tag{5.18}$$

which lifts most flat directions. Along $u\equiv \mathrm{Det}R=X^5$ this model satisfies conditions *i)–iii)* and $W_{eff}=\lambda X\Lambda_1^2$. Notice that $\langle R\rangle = X\ \mathbb{1}_5$ breaks $SU(5)_2\times SU(3)\times SU(2)\times U(1)\rightarrow SU(3)\times SU(2)\times U(1)$, the SM gauge group. A detailed numerical study [96] shows that a stable minimum can be consistently found for any $X\gtrsim 10^{14}$ GeV. This is low enough to suppress gravity contributions to soft terms. Minima at smaller X turn out to imply a Landau pole in λ below the Planck scale. The fields Φ, $\bar{\Phi}$ with mass λX and splittings λF_X, correspond to five flavours of conventional messengers. Notice that this model is a special case of the $SU(N)\times SU(N-k)$ models discussed in sect. 5.3 ($k=0$), in which we do not need non-renormalizable interactions to stabilize the potential. The problem of the positive supertrace is solved since there are no light messengers.

The actual problem is that, in both models in which $Q,\bar{Q}$ are identified with the messengers, there are fatal negative contributions to the sfermion masses coming from the massive vector superfields. Indeed the SM gauge bosons live in the diagonal subgroup of $SU(5)\times SU(5)$. The vector multiplets corresponding to the coset space interact with the SM matter and feel supersymmetry breaking, since their masses are determined by the superfield X. Their contribution to the sfermion masses has been calculated in ref. [141]. It turns out that although squark squared masses are stabilized at low energy by the positive gluino contribution, slepton squared masses are negative. It is not clear at the moment if there exist simple and viable models with gauge messengers. Notice that this problem is fatal for the models in which $Q,\bar{Q}$ are identified with the messengers, but of course it is not present in models with no gauge messengers, as in the example presented above, based on $G_S=SU(2)$ and $G_B=SU(2)\times SU(2)$. The negative contributions from heavy vector loops are present also in the $SU(N)\times SU(N-k)$ models. But in that case there is an even larger negative contribution from the light messengers, as discussed in sect. 5.3.

An interesting and viable $SU(5)^3$ theory has been discussed in ref. [255]. The embedding of the SM gauge group is changed in order to avoid gauge messengers. Under $SU(5)_1 \times SU(5)_2 \times SU(5)_W$ the spectrum contains $R(\bar{\mathbf{5}}, \mathbf{5}, \mathbf{1})$, $\Phi(\mathbf{1}, \bar{\mathbf{5}}, \mathbf{5})$ $\bar{\Phi}(\mathbf{5}, \mathbf{1}, \bar{\mathbf{5}})$, $A(\mathbf{10}, \mathbf{1}, \mathbf{1})$ and $\bar{F}(\bar{\mathbf{5}}, \mathbf{1}, \mathbf{1})$. Now, along the flat direction Det$R$, the weak factor $SU(5)_W$ is unbroken, unlike the previous case, since the quantum number assignment is now different. The analysis in ref. [255] shows that the superpotential in eq. (5.18) and the strong dynamics lift all flat directions but Det$R$. Along Det$R$, $SU(5)_1 \times SU(5)_2$ is broken to the diagonal $SU(5)_D$, whose matter content is just $A \oplus \bar{F} = \mathbf{10} \oplus \bar{\mathbf{5}}$. Thus the low-energy $SU(5)_D$ dynamics break supersymmetry in the strong coupling regime [4, 194]. Although this happens in an uncalculable way, one can estimate the vacuum energy to be [255]

$$V \sim \Lambda_D^4 \sim \left(\frac{\Lambda_1^8 \Lambda_2^{10}}{\mathrm{Det}R}\right)^{4/13}, \tag{5.19}$$

where Λ_D and $\Lambda_{1,2}$ are the effective scales for $SU(5)_D$ and $SU(5)_{1,2}$. Equation (5.19) leads to a runaway behaviour of the field R. In general this slope can be compensated at large R by Planck-suppressed corrections to the tree-level superpotential. The lowest-dimension operator of this kind is $W_1 = \mathrm{Det}R/M_P^2$. By adding W_1 to eq. (5.18), the vacuum relaxes at $\langle R \rangle \sim (M_P^{13}\Lambda_1^8\Lambda_2^{10})^{1/31}$. Around this minimum, Φ and $\bar{\Phi}$ act as 5 flavours of messengers, while the heavy gauge fields are neutral under $SU(5)_W$. By fixing $F_R/R \sim 10^4$ GeV the messenger scale $\langle R \rangle$ is then $\sim 10^{13}$ GeV. This scale could be problematic for nucleosynthesis, see sect. 3.5.

Planck-suppressed effects in the Kähler potential play a central rôle in the mechanism proposed in ref. [208]. This scenario requires a simple messenger sector superpotential $W = kX^3 + \lambda X \Phi \bar{\Phi}$. The coupling of X to a bilinear in the supersymmetry-breaking sector,

$$\int d^4\theta \frac{QQ^\dagger X}{M_P}, \tag{5.20}$$

generates a tadpole for X when $(QQ^\dagger) \to |F_Q|^2 \theta^2 \bar{\theta}^2$. Over a wide range of k and λ this tadpole leads to an absolute minimum in the messenger sector at $\Phi, \bar{\Phi} = 0$ and at

$$X \sim m_{3/2}^{\frac{2}{3}} M_P^{\frac{1}{3}} \qquad \frac{F_X}{X} \sim X, \tag{5.21}$$

where $m_{3/2} \sim F_Q/M_P$ has been used. Notice that, because of the well-known "destabilizing" effects of singlets [228, 206, 181], the visible scale X of supersymmetry breaking is parametrically larger that $m_{3/2}$, a property crucial to solve the flavour problem. In the vacuum defined by eq. 5.21 gauge mediation proceeds in the usual way. This scenario gives automatically $F_X \sim X^2$. In order to have $F_X/X \sim 10^5$ GeV it must be $F_Q \sim 10^8$ GeV so that $m_{3/2}$ is in the MeV range.

This scenario shares an obvious similarity with that discussed below eq. 5.10, as the VEV of X is triggered by the Kähler potential. It also shares the overall structure of messenger $U(1)$ models, but now the connection between the first two boxes in Fig. 11 is determined by $1/M_P$ effects. An advantage of the scenario of ref. [208] is that an absolute minimum is obtained rather naturally.

A different class of models, based on a superpotential such as eq. (5.11), has been discussed in ref. [188]. The field R is now a gauge singlet. The corresponding flat direction is lifted by two competing quantum effects: gaugino condensate in one group factor lifts the origin $R = 0$ and DSB in another factor stabilizes the potential. A subclass is based on the gauge group $SU(N)_S \times SU(5)_B \times SU(5)_W$. The fields in eq. (5.11) tranform as $\Phi(\mathbf{1}, \bar{\mathbf{5}}, \mathbf{5})$, $\bar{\Phi}(\mathbf{1}, \mathbf{5}, \bar{\mathbf{5}})$, $Q^i = (\mathbf{N}, \mathbf{1}, \mathbf{1})$ and $\bar{Q}_i(\bar{\mathbf{N}}, \mathbf{1}, \mathbf{1})$, where i is a flavour index $i = 1, \ldots, N_f$ and $Q\bar{Q}$ in eq. (5.11) is identified with $\sum_i Q^i \bar{Q}_i$. The matter content is completed by $A(\mathbf{1}, \mathbf{10}, \mathbf{1})$ and $\bar{F}(\mathbf{1}, \bar{\mathbf{5}}, \mathbf{1})$, which do not appear in the superpotential. Along $R \neq 0$, $SU(N)_S$ is a pure supersymmetric Yang–Mills theory, and gaugino condensation generates a superpotential $W_S \propto R^{N_f/N}$. Then for $N_f < N$, the potential $V_S = |\partial_R W_S|^2$ will "push" R away from the origin. However, at $R \neq 0$, the gauge group $SU(5)_B$, whose only matter is contained in $\mathbf{10} \oplus \bar{\mathbf{5}}$, breaks supersymmetry with a vacuum energy $V_B \sim \Lambda_B^{32/13} R^{20/13}$, which stabilizes R. The bottom line is that, for $\Lambda_B \ll \Lambda_S$, R is stabilized in a supersymmetry-breaking ($F_R \neq 0$) minimum far away from the origin. This is typically a false vacuum but, as was the case for plateau models, it is cosmologically acceptable, since the true vacuum is "far" away in field space.

An interesting mechanism to stabilize a flat direction, different than the one presented in this section, can be obtained from confinement, as discussed in ref. [189]. This, however, can only happen in the region of field values of the order of the confinement scale Λ. The resulting phenomenological models have interesting features, such as composite messengers, but require assumptions about uncalculable terms in the Kähler potential.

5.5 Models with Composite X

In refs. [244, 148, 149, 253, 77] it was assumed that the field X corresponds to a gauge-invariant composite of the supersymmetry-breaking sector. This assumption was made to avoid intermediate sectors involving apparently *ad hoc* singlet fields. Indicating by K the invariant playing the rôle of X, the coupling to the messenger will have the form

$$W = \frac{K\Phi\bar{\Phi}}{M^{d-1}} \,, \tag{5.22}$$

where $d \geq 2$ is the dimension of K, and M is a fundamental mass scale. The basic feature of eq. (5.22) is that while X^2 scales like $1/M^{2(d-1)}$, F_X scales only like $1/M^{d-1}$. By treating $1/M$ as a small parameter, one expects a qualitative difficulty in getting $F_X < X^2$ to avoid a tachyon. In ref. [253] it was found that this generically requires a new mass scale $M \ll M_P$ to suppress these operators, since $M = M_P$ is too large. Alternatively [148, 253], one must assume some small (less than 10^{-6}) coupling in the supersymmetry-breaking sector, or try to stabilize the superpotential by some higher-dimensional operator [149]. A possibility is that this new mass scale arises from some confining gauge dynamics involving fields in both the observable and supersymmetry-breaking sectors. But this seems difficult to realize without creating new and dangerous terms in the effective Lagrangian.

The scenarios characterized in ref. [244], and constructed in ref. [77], realize the new mass scale as the mass of two composite fields S and $\bar{S}$. These are coupled to both sectors and by integrating them out, they mediate the interaction in eq. (5.22). This intermediate step in the communication of supersymmetry breaking avoids the appearance of dangerous interactions. Thus the effective scale in eq. (5.22) is $M = M_P^k \Lambda_c^{1-k}$, with $0 < k < 1$, where Λ_c is a combination of strong-dynamics scales smaller than M_P. One of the examples of ref. [77] is based on a gauge group $SU(3) \times SU(2) \times [SU(2) \times SU(4)]_c \times SU(5)_W$, where the factor in brackets provides the confining dynamics. It is rather non-trivial that the superpotential from confinement, together with a limited set of tree-level operators, gives rise to the needed interactions. It is also conceivable that realizations simpler than those presented in ref. [77] exist. Nevertheless, albeit without gauge singlets, the overall modular structure of fig. 11 seems to be present here, with the confining sector playing the rôle of module II. More generally one could measure the "remoteness" of the supersymmetry-breaking sector by observing that F_X is much smaller than F_0, the gravitino decay constant, as is always the case when X is composite. Nonetheless, the composite scenario, together with the ones discussed in the previous sections, illustrates the many different ways gauge mediation can be realized in the context of dynamical supersymmetry breaking. It is noteworthy that, in essentially all these models, the phenomenologically relevant parameters correspond to the quantities M, F, and N introduced in sect. 2.

6 The Origin of μ and $B\mu$

6.1 The μ Problem

Usually, the μ-problem [175] is referred to as the difficulty in generating the correct mass scale for the Higgs bilinear term in the superpotential

$$W = \mu H_1 H_2 \ , \tag{6.1}$$

which, for phenomenological reasons, has to be of the order of the weak scale. If this term were present in the limit of exact supersymmetry, then it would have to be of the order of the Planck scale or some other fundamental large scale (as the GUT scale). In the gravity-mediated scenario, however, if this term is absent in the supersymmetric limit and the Kähler potential is general enough, the correct μ is generated after supersymmetry is broken [139]. In particular the two terms

$$K = H_1 H_2 \left(\frac{X^\dagger}{M_P} + \frac{X X^\dagger}{M_P^2} + \dots \right) \ , \tag{6.2}$$

where X is the supersymmetry-breaking chiral spurion, generate respectively $\mu \sim F_X/M_P$ and $B\mu \sim (F_X/M_P)^2$. Therefore the sizes of both μ and $B\mu$ are of the order of the weak scale. Here $B\mu$ is the supersymmetry-breaking counterpart of μ, defined by the following term in the scalar potential

$$V = B\mu H_1 H_2 + \text{h.c.} \tag{6.3}$$

In gauge mediation the problem is somehow more acute. Now μ must be related to the scales X and F_X in the messenger sector, and the natural value is $\mu \sim (1/16\pi^2) F_X/X$. Indeed it is not hard to accomplish that, as we will show below. The real difficulty is to generate *both* μ and B of the same order. Consider the simplest possibility of a direct coupling $W = \lambda X H_1 H_2$. Even putting $\lambda \sim 1/16\pi^2$ by hand does not accomplish the job since $\mu = \lambda X$ while $B\mu = \lambda F_X$, so that $B = F_X/X \sim$ 10–100 TeV. The source of this problem is not the tree-level origin of μ, but the fact that μ and $B\mu$ are generated at the same order in the "small" parameter λ.

To illustrate this, let us consider the more realistic situation of a radiatively generated μ. A simple option to do that is to couple H_1 and H_2 to a pair of messengers [111]

$$W = \lambda H_1 \Phi_1 \Phi_2 + \bar{\lambda} H_2 \bar{\Phi}_1 \bar{\Phi}_2 \, . \tag{6.4}$$

Assuming that a single X determines the messenger masses, $W = X(\lambda_1\Phi_1\bar{\Phi}_1 + \lambda_2\Phi_2\bar{\Phi}_2)$, then the one-loop correction to the Kähler potential is

$$\int d^4\theta \left[\frac{\lambda\bar{\lambda}}{16\pi^2} f(\lambda_1/\lambda_2) H_1 H_2 \frac{X^\dagger}{X} + \text{h.c.}\right] . \tag{6.5}$$

Here $f(x) = (x \ln x^2)/(1 - x^2)$ and we have neglected higher-derivative terms, which only give $\mathcal{O}(F_X^2/X^4)$ corrections. Both μ and $B\mu$ are generated from eq. (6.5) and we again obtain

$$B = \frac{B\mu}{\mu} = \frac{F_X}{X} . \tag{6.6}$$

A similar result is expected also in the general case where $X^\dagger/X$ in eq. (6.5) is replaced by some function $G(X, X^\dagger)$. Only for specific, *tuned*, functions[14] G can one get a small B.

Equation (6.6) is the expression of the μ-problem in gauge mediation. The electroweak-symmetry breaking condition can be satisfied only at the price of a quite unreasonable fine-tuning. In order to solve this problem and avoid the relation in eq. (6.6), μ should neither be generated from a direct coupling to X at tree level nor via loop corrections to the Kähler potential. Let us now discuss possible solutions that have been proposed in the literature.

6.2 Dynamical Relaxation Mechanism

The basic point of the solution outlined in ref. [111] is that μ arises only from higher-derivative operators of the form

$$\int d^4\theta H_1 H_2 D^2 G(X^\dagger, X) . \tag{6.7}$$

Here D_α is the supersymmetric covariant derivative and G a function determined by loop corrections. The crucial point is that a D^2 acting on any function of X and $X^\dagger$ always produces an antichiral superfield. Then, independently of the form of G, no B term is generated along with μ from eq. (6.7). Different operators coming from higher-order corrections can be responsible of generating B of the correct size.

A possible implementation of this mechanism is given by the superpotential

$$W = S\left(\lambda_1 H_1 H_2 + \frac{\lambda_2}{2} N^2 + \lambda\Phi\bar{\Phi} - M_N^2\right) + X\Phi\bar{\Phi} , \tag{6.8}$$

[14] For instance, $G = \ln(X/X^\dagger)$ gives $B = 0$, but we do not know of any explicit example giving this result.

where $\Phi \oplus \bar{\Phi}$ are the messengers, $\mathbf{5} \oplus \bar{\mathbf{5}}$ of $SU(5)$, and S, N are gauge singlets. It is assumed that the DSB dynamics provides a VEV for X, F_X and also the mass $M_N^2 \sim F_X$. For a given model of DSB, one should also check that the interactions in eq. (6.8) do not destabilize the original DSB vacuum. We will later illustrate an explict DSB model, which generates all the appropriate mass terms.

The dynamics of this model is easily described. In the limit in which the source F_X of supersymmetry breaking is turned off, there is a minimum of the energy at $S, H_1, H_2 = 0$ with $F_S = 0$ saturated by a non-zero VEV $\lambda_2 \langle N \rangle^2 = 2M_N^2$. Around this vacuum S and N pair up and get a mass $\sqrt{2\lambda_2} M_N$. At this stage $\mu \propto \langle S \rangle = 0$. Notice now that messenger exchange in one-loop diagrams produces a wave-function mixing between S and X in the Kähler potential

$$\int d^4\theta \frac{5\lambda}{16\pi^2} S X^\dagger \ln(XX^\dagger) + \text{h.c.} \tag{6.9}$$

Thus, when F_X is turned on, a one-loop tadpole $\delta V \sim S F_X^2 / X$ appears in the scalar potential and a non-zero μ is generated

$$\mu = \lambda_1 \langle S \rangle = -\frac{5}{32\pi^2} \frac{\lambda \lambda_1}{\lambda_2} \frac{F_X^2}{X M_N^2} \ . \tag{6.10}$$

At the same order no $B\mu$ is generated or, equivalently, $F_S = 0$. This is understood by noticing that *i)* $F_S = F_S(N)$ is a function of N; *ii)* at one loop the effective potential depends on N only via $|F_S(N)|^2$, as N does not interact directly with $\Phi, \bar{\Phi}$. Then, at one loop, the minimization of the energy term F_S^2 ensures that $B \propto F_S$ relaxes to zero. At two loops, soft masses for N are generated and F_S becomes non-vanishing:

$$B\mu = \lambda_1 \langle F_S \rangle = -\frac{10\lambda^2 \lambda_1 \lambda_2}{(16\pi^2)^2} \left(1 + \frac{5F^2}{8\lambda_2^2 M_N^4} \right) \frac{F_X^2}{X^2} \ . \tag{6.11}$$

In addition to these terms, new contributions to the soft masses for H_1, H_2 arise at two loops

$$\delta m_{H_1}^2 = \delta m_{H_2}^2 = 10 \left(\frac{\lambda \lambda_1}{16\pi^2} \right)^2 \frac{F_X^2}{X^2} \ . \tag{6.12}$$

In an operator language, it is easy to verify that μ in this model is generated by eq. (6.7) after decoupling N and S. Notice that the fields N and S have large masses of order $\sqrt{F_X}$, and therefore belong to the messenger sector and no singlet fields are contained in the low-energy theory.

This mechanism can be implemented in a DSB theory, where M_N^2 in eq. (6.8) is not just an input. Models with QMS are well suited for this: the meson condensate $\langle Q\bar{Q} \rangle = \Lambda^2 \sim F_X$

is the natural source of M_N^2 via the microscopic coupling $SQ\bar{Q}$. This was explicitly realized in ref. [97] in a theory based on the gauge group $SU(5)_S \times SU(5)_W \times SU(2)_B$. In that model the messengers and the confining fields coincide $\Phi \oplus \bar{\Phi} = (\mathbf{5}, \bar{\mathbf{5}}, \mathbf{1}) \oplus (\bar{\mathbf{5}}, \mathbf{5}, \mathbf{1})$ and the superpotential is

$$W = S(\lambda_1 H_1 H_2 + \lambda_2 N^2/2 - \lambda_3 \Phi\bar{\Phi}) + \lambda_4 Y \Phi\bar{\Phi} + I(\lambda_5 Y^2 - \lambda_6 \psi\bar{\psi}), \tag{6.13}$$

where I, Y are additional singlets, ψ, $\bar{\psi}$ are $SU(2)_B$ doublets and, finally, all λ_i are $\mathcal{O}(1)$ Yukawa couplings. Equation (6.13) has a classical flat direction where $\lambda_5 Y^2 = \lambda_6 \psi\bar{\psi} \equiv X^2 \neq 0$ and all the other fields are at the origin. Along X the messengers $\Phi\bar{\Phi}$ are massive, so that the $SU(5)_S$ gaugino condensate generates $W_{eff} \sim X\Lambda^2$, as in the models described in sect. 5.4. Notice also that X is an admixture of the singlet Y and the $SU(2)_B$ doublets ψ, $\bar{\psi}$, so that this second component can lead to a stable minimum on the X plateau via Witten's inverse hierarchy. Thus, below the scale X, W_{eff} has the form of eq. (6.8), with $\lambda_3 \Phi\bar{\Phi} \to \Lambda^2 \equiv M_N^2$. Moreover messenger loops lead to the Kähler-potential terms needed to generate VEVs for S and F_S.

6.3 Extensions with Light Singlets

A possible way to dynamically generate μ and $B\mu$ is to add a singlet chiral superfield S to the observable sector and to include the most general superpotential free from mass parameters,

$$W = \lambda S H_1 H_2 + \frac{k}{3} S^3. \tag{6.14}$$

In this case the generation of μ corresponds to the spontaneous breakdown of a Z_3 symmetry under which each superfield in eq. (6.14) is rotated by $e^{i2\pi/3}$. If the soft terms are general enough, the VEVs of S, H_1, H_2 are all non-zero. We stress that the singlet S here has electroweak scale mass, while the one in the previous section had a large mass, of order $\sqrt{F_X}$, and decoupled from the observable sector.

We also recall that in models with low-energy supersymmetry breaking, in contrast with the gravity-mediated case [228], a light singlet offers less (or none at all) danger to the stability of the gauge hierarchy [204] (see also refs. [105, 66]). Consider, for instance, a GUT embedding of the Higgs sector with a gauge singlet S. Even if the Z_3 symmetry that protects $\langle S \rangle$ is broken, for instance, by the Higgs triplets mass terms, the induced tadpole is at most $\delta V \sim (\alpha/4\pi)^3 S F_X^2 / M_G$. Here three loops are needed to couple S to the messengers, and $1/M_G$ is the price paied to propagate GUT-scale particles. The resulting $\langle S \rangle$ is below the weak scale, as long as[15] $\sqrt{F_X} \lesssim 10^8$ GeV. Conversely, in DSB theories with $\sqrt{F_X} \sim 10^7$–10^8 GeV the correct μ term is provided by this tadpole; but this seems quite fortuitous.

The problem with the light-singlet approach is that in gauge mediation the soft terms are not general enough to give the desired vacuum. In particular a VEV for S requires either a sizeable negative soft mass m_S^2 or large A_λ and A_k, the coefficients of the A-terms associated with eq. (6.14). As S is not directly interacting with the messengers, the leading contributions to all these terms arise only through RG evolution. For a low messenger scale both A-terms are so small that they are irrelevant in the minimization. On the other hand m_S^2 gets a small contribution, which is negative for light messengers:

$$m_S^2 \simeq -2\lambda^2 \left(2m_{H_1}^2 - 3{\lambda_t}^2 m_{\tilde{t}}^2 \frac{\ln(X/m_{H_1})}{8\pi^2} \right) \frac{\ln(X/m_{H_1})}{8\pi^2} < 0 \ . \tag{6.15}$$

Here m_{H_1} is the soft mass of the Higgs coupled to down-type quarks. The smallness of this term is problematic for a correct implementation of the electroweak-breaking conditions. Moreover one always finds [102] a light scalar Higgs boson and an almost massless pseudoscalar, and the LEP bound on their associated production rules out this scenario. The presence of the light pseudoscalar is caused by the spontaneously broken R symmetry of the superpotential in eq. (6.14), which is explicitly violated only by the small A terms. In a detailed numerical study [144] it has been shown that this unsatisfactory situation persists for any value of the messenger mass M. In the model with a singlet described by the superpotential in eq. (6.14), the soft-breaking terms induced by gauge mediation either do not give the correct minimum or predict unacceptably light new particles.

A more promising direction is to couple directly S to some sizeable source of supersymmetry breaking. One possibility [102] is to add one light $SU(5)$ flavour $f, \bar{f}$ coupled to S via $W = \lambda_f S f \bar{f}$. If $\lambda_f \sim 1$ then m_S^2 receives a large negative contribution from the coloured triplets soft masses, which is the analogue of the stop contribution to the Higgs mass. Thus a VEV for S of the order of the weak scale can be generated, together with a consistent mass spectrum. However, this scenario implies the existence of exotic and stable (or very long-lived) matter $f, \bar{f}$ with weak-scale mass.

[15] This also allows, in the context of gauge mediation, to solve the Higgs doublet–triplet splitting problem by means of the sliding singlet [278, 161].

Another possibility is to couple S directly to the messengers [141]. This can be done in a theory with at least two messenger flavours and a superpotential

$$W = X(\bar{\Phi}_1\Phi_1 + \bar{\Phi}_2\Phi_2) + \lambda_S S\bar{\Phi}_1\Phi_2 \ , \tag{6.16}$$

which avoids the radiative generation of the dangerous tadpole mixing $SX^\dagger$. The form in eq. (6.16) is fixed by a discrete Z_3 symmetry under which $\bar{\Phi}_1, \Phi_2$, and S have charge 1/3, Φ_1 and $\bar{\Phi}_2$ have charges $-1/3$ and X is neutral. This symmetry, broken only at the weak scale, distinguishes between X and S. In the leading F_X/X^2 approximation, the messengers contribute to $A_{k,\lambda}$ and m_S^2 at one and two loops, respectively:

$$\frac{A_k}{3} = A_\lambda = -\frac{5}{4\pi}\alpha_{\lambda_S}\frac{F_X}{X} \ , \tag{6.17}$$

$$m_S^2 = \frac{5}{8\pi^2}\alpha_{\lambda_S}\left[\frac{7}{2}\alpha_{\lambda_S} - \frac{8}{5}\alpha_3 - \frac{3}{5}\alpha_2 - \frac{13}{3}\alpha_1\right]\left(\frac{F_X}{X}\right)^2 \ . \tag{6.18}$$

Here $\alpha_{\lambda_S} = {\lambda_S}^2/4\pi$ and all coupling constants are evaluated at the messenger scale X. Notice that the mixed gauge–Yukawa contribution to m_S^2 is negative. Indeed there is a range of λ_S where m_S^2 is negative and of the order of the Higgs soft term $m_{H_2}^2$ induced by stop loops. This allows a correct electroweak-symmetry breaking. Moreover, the A terms are now sizeable, and there is no approximate R-Goldstone boson.

In ref. [66] the superpotential of eq. (6.14) was considered in the limit $k = 0$. In this case, S could indeed be the sliding singlet which explains the doublet-triplet splitting. Let us not worry for the moment about the Peccei–Quinn symmetry of this limiting case and about its potentially dangerous axion. The basic point of ref. [66] is that a large enough A_λ-term for λSH_1H_2, generated by the messenger sector, can trigger the correct electroweak breaking. Notice that in the limit $\lambda \ll 1$ the term $\lambda^2|H_1H_2|^2$ can be neglected and the potential depends on λ and S only via the combination $\mu = \lambda S$. For a range of parameters where $A_\lambda^2 \sim m_{H_1}^2, m_{H_2}^2 \sim m_{weak}^2$ at the weak scale, the minimum of the potential gives the correct electroweak breaking and $\mu \sim m_{weak}$. This cannot be achieved within ordinary gauge mediation, but it is not excluded that soft terms of the correct size may be obtained with additional Higgs-messenger Yukawa couplings. In ref. [66] it is also pointed out that a Peccei–Quinn-breaking tadpole $\delta V = \rho S$ can give the axion a sufficiently large mass without destabilizing the weak scale. The tadpole could originate from Planck or GUT scale suppressed operators. For instance, the term $SXX^\dagger/M_P$ in the Kähler potential would give $\rho \sim F_X^2/M_P$. Notice that the hierarchy is preserved for

$\rho/\lambda \lesssim 10^6$ GeV^3, that is for $\sqrt{F_X} \lesssim \lambda^{1/4}10^6$ GeV. Now, for $\lambda < 10^{-7}$ the axion decay constant is $\langle S\rangle \gtrsim 10^9$ GeV, so that the cooling of red giants does not pose a problem. For larger λ, the axion mass $m_a^2 \simeq \rho/S = \lambda\rho/\mu$ should be larger than 1 MeV. This bound is compatible with the stability of the weak scale for $\lambda > 10^{-5}$, and collider bounds on axion-like scalars require $\lambda < 10^{-2}$. In conclusion, there is a significant range for λ where the axion does not cause a problem. Notice that F_X is constrained to be in its lower range of validity.

Another possibility, suggested in ref. [105], is to invoke a field S with non-renormalizable superpotential couplings $W = S^n H_1 H_2 + S^m$, in units of $1/M_P$. Soft terms, like those from a Kähler-potential interaction $SS^\dagger XX^\dagger/M_P^2$, induce a tree-level $\mu = \langle S\rangle^n \sim F_X^{n/(m-2)}$, so that for $m = 2n+2$ one has $\mu \sim \sqrt{F_X}$. This requires further adjustment of the relevant coupling constants to get the right μ. Anyhow, an interesting implication of this mechanism is that B is not generated at tree level along with μ. As previously discussed, this solves the supersymmetric CP problem, since the only source of A and B terms is the gaugino mass via RG evolution. Moreover the size of the radiatively induced B is small, so that $\tan\beta$ is predicted to be naturally large [22, 106, 245].

Finally, a different mechanism for generating the μ term was suggested in ref. [280]. It was argued that the cancellation of the cosmological constant in gauge mediation necessarily requires a new mass scale. Indeed the cosmological constant receives two different contributions

$$\langle V\rangle \sim \langle F\rangle^2 - \frac{\langle W\rangle^2}{3M_P^2} \, . \tag{6.19}$$

If supersymmetry is broken at low scales, both the auxiliary fields F and the superpotential W are determined by the same typical energy scale Λ. The cancellation (of course always at the price of a fine-tuning) between the two terms can only be achieved if we add to the superpotential a constant term of order $\bar\Lambda^3 \sim \Lambda^2 M_P$. It is possible that the scale $\bar\Lambda$ is dynamically generated by some new strongly interacting sector, which breaks the continuous R symmetry. If this new sector couples to the ordinary Higgs via non-renormalizable interactions, the μ term is generated by the condensate of a bilinear of new fields,

$$\mu \sim \frac{\bar\Lambda^2}{M_P} \sim \left(\frac{\Lambda}{M_P}\right)^{1/3} \Lambda \, . \tag{6.20}$$

For interesting values of Λ, the ratio $(\Lambda/M_P)^{1/3}$ can mimic the loop factor and give the correct size of the μ term.

7 Conclusions

Theories with gauge-mediated supersymmetry breaking provide an interesting alternative to more conventional theories, in which the information of supersymmetry breaking is communicated to the observable sector by gravity. They offer the possibility of solving the flavour problem, since the soft terms are generated at a low mass scale and are not sensitive to the high-energy physics, which is presumably responsible for the breaking of the flavour symmetry.

Moreover, since the relevant dynamics occur at an energy scale much smaller than the Planck mass, gauge-mediated models can be solved in the context of field theory, with no knowledge of quantum gravity aspects. Although this means that these models cannot describe the ultimate unified theory, they nevertheless allow us to calculate and predict important aspects of the low-energy physics. In particular, because of the recent developments in the understanding of non-perturbative aspects of supersymmetric theories (for a review, see *e.g.* refs. [165, 259, 224]), the question of dynamical supersymmetry breaking can be addressed in a quantitative way. This opens the possibility of constructing a theory in which the electroweak scale can be computed in terms of a single fundamental mass scale, like the Planck mass.

The last two years have seen a flourishing in the construction of models with gauge-mediated dynamical supersymmetry breaking. This activity follows the line of the seminal work in refs. [102, 104, 105]. Clearly this field is still evolving; however, progress has been made. Models have become simpler in structure, which is to say more believable candidates to describe reality. For instance, some realistic models have been found where the messengers themselves play an essential rôle in breaking supersymmetry. Also, with the variety of new models, all of the allowed values of the phenomenologically relevant parameters M and N can be reproduced, with the resulting diversity of cosmological and phenomenological implications.

Also, gauge-mediated theories have quite distinct phenomenological features. The supersymmetric mass spectrum is determined in terms of relatively few parameters. The most important parameter is $\Lambda = F/M$, which sets the scale of supersymmetry breaking in the observable sector. The supersymmetric particle masses are typically a one-loop factor smaller than Λ, with coefficients completely determined by the particle gauge quantum numbers.

The second parameter is the messenger mass scale M. Supersymmetric particle masses depend only logarithmically on M, from their RG evolution. This scale can vary roughly between several tens of TeV and 10^{15} GeV. The lower bound on M is determined by present experimental limits on supersymmetric particle masses; the upper bound from the argument that gravity contributions should be small enough not to reintroduce the flavour problem, see eq. (2.44).

The third important parameter is the messenger index N. If the goldstino is contained in a single chiral field, then N is the number of messenger flavours weighted by the corresponding Dynkin index, see eq. (2.3). Perturbativity of gauge couplings gives an upper bound on N, which significantly depends on the messenger mass M, see eq. (2.4). In theories in which F/M is not the same for all messenger fields, N is no longer an integer, but it retains its physical meaning of measuring the ratio between the typical gaugino and scalar squared mass at the scale M.

Other unknown parameters are connected with the solution to the μ problem. As discussed in sect. 6, the origin of the Higgs mixing parameters μ and B still seems problematic, although definite progress has been made towards a more direct generation of μ and B from the sector that breaks supersymmetry. From the phenomenological point of view, a simple option is to include μ and B terms by hand and impose the constraint of electroweak breaking. We are left with a single free parameter, usually chosen to be $\tan\beta$, apart from an ambiguity in the phase of μ. A more restrictive possibility is to assume $B = 0$ at the messenger scale M. In this case, there are no new free parameters and $\tan\beta$ is determined to be very large. However, one should keep it in mind that dynamical solutions to the μ problem often induce new contributions to the supersymmetry-breaking Higgs mass terms. We then find two (or one, if the two-Higgs-doublet sector has an isospin invariance) extra free parameters.

As we have discussed in this review, the phenomenological aspects of gauge-mediated theories are quite diverse, depending on the parameter choice. However, they can roughly be divided into two regimes. Models with low values of F_0, and therefore comparable values of M, have the characteristic that the NLSP decays promptly (for $\sqrt{F_0}$ roughly less than 10^6 GeV) and that the gravitino relic density is less than the critical density (for $\sqrt{F_0}$ roughly less than 10^7 GeV). Depending on whether the NLSP is a neutralino or a stau, the characteristic collider signals are anomalous events characterized by missing energy and γ or τ, respectively. For larger $\sqrt{F_0}$, the NLSP lifetime is longer and the collider phenomenology can resemble the well-known missing-energy supersymmetric signatures (for a neutralino NLSP) or can lead to a long-lived heavy charged particle penetrating the detector (for a stau NLSP). In this regime, the gravitino or the NLSP decay can cause cosmological difficulties.

Theories with gauge-mediated supersymmetry breaking have very appealing theoretical features and quite distinctive experimental signatures. We expect that the joint theoretical and experimental research will soon demostrate whether these theories are relevant for the description of nature.

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Appendix

In this appendix we give the complete matching conditions at the messenger scale for the gauge-mediation mass spectrum, including next-to-leading order radiative corrections in the gauge coupling constants and the top-quark Yukawa, in the limit $F \ll M^2$. Details of the calculation are presented in refs. [141, 20]. The results are given in the so-called $\overline{\rm DR}'$ scheme for soft terms, which is defined in ref. [170]. The formulae given below can be implemented in numerical codes that compute the next-to-leading order RG evolution [192] and the low-energy threshold corrections [226, 227], and then can be used in phenomenological studies.

We choose to define the matching scale as

$$M \equiv \lambda_G \langle X \rangle, \tag{A.1}$$

where $\langle X \rangle$ is the scalar component VEV of the chiral superfield containing the Goldstino, and λ_G is the coupling constant in eq. (2.5) evaluated at the GUT scale. We also define $F \equiv \lambda_G \langle F_X \rangle$, where F_X is the auxiliary component of the superfield X. Notice that the physical messenger thresholds are instead at

$$M_i = \frac{M}{Z_i(M)} \tag{A.2}$$

where Z_i is the wave-function renormalization of the i-th messenger flavour. We are also assuming that the wave functions for Φ_i and $\bar{\Phi}_i$ are equal. This is correct in most cases, and the expressions we give here can be easily generalized to the case in which this assumption does not hold.

The next-to-leading order expression of the gaugino mass at the scale M is

$$\tilde{M}_{\lambda_r}(M) = k_r \frac{\alpha_r(M)}{4\pi} N \frac{F}{M} \left[1 + \frac{\alpha_r(M)}{2\pi} T_{G_r} \right]. \tag{A.3}$$

Here we use the same notation as in sect. 2.3 and $T_{G_r} = N$ for $SU(N)$ groups and $T_{G_r} = 0$ for $U(1)$ groups. Because of the "gaugino screening" theorem discussed in ref. [20], the result in eq. (A.3) is independent of new messenger interactions and, in particular, of the assumption that the couplings λ_i unify at the GUT scale into a single coupling λ_G. The QCD corrections to gluino mass are important especially at small N and M and can exceed 10% [20, 225].

The next-to-leading expression of the supersymmetry-breaking scalar masses at the matching scale M is

$$m^2_{\tilde{f}}(M) = 2 \sum_{r=1}^{3} k_r \frac{\alpha_r^2(M)}{(4\pi)^2} N \frac{F^2}{M^2} \left(C_r^{\tilde{f}} + \frac{\Delta_r^{\tilde{f}}}{4\pi} \right). \tag{A.4}$$

Here the notation is analogous to the one followed in sect. 2.3 and $\Delta_r^{\tilde{f}}$, which represents the next-to-leading correction, is given by

$$\begin{aligned} \Delta_r^{\tilde{f}} = \quad & 2C_r^{\tilde{f}} \left[b_r + T_{G_r} + (Nk_r - b_r)(1 - \mathcal{Z}_r) \right] \alpha_r(M) - 4C_r^{\tilde{f}} \sum_{s=1}^{3} C_s^{\tilde{f}} \alpha_s(M) + C_r^{\tilde{f}} \gamma_r \\ & - \left[P_r^{\tilde{f}} + \frac{d_{\tilde{f}}}{2} a_r (1 - \mathcal{Z}_r) \right] \alpha_t(M). \end{aligned} \tag{A.5}$$

Here $\alpha_t = h_t^2/(4\pi)^2$ measures the top-quark Yukawa coupling, $a_r = 2(C_r^{\tilde{Q}_L} + C_r^{\tilde{U}_R} + C_r^{H_2}) = (13/9, 3, 16/3)$ and the only non-vanishing coefficients $P_r^{\tilde{f}}$ and $d_{\tilde{f}}$ are

$$P_3^{H_2} = 8, \quad P_2^{\tilde{U}_R^3} = 3, \quad P_1^{\tilde{Q}_L^3} = \frac{2}{3}, \quad P_1^{\tilde{U}_R^3} = -\frac{1}{3}, \quad P_1^{H_2} = \frac{2}{3}, \tag{A.6}$$

$$d_{\tilde{Q}_L^3} = 1, \quad d_{\tilde{U}_R^3} = 2, \quad d_{H_2} = 3. \tag{A.7}$$

We have also defined

$$\mathcal{Z}_r = \frac{1}{N} \sum_{i=1}^{N_f} n_i^r \ln Z_i^2 \tag{A.8}$$

$$\gamma_r = \frac{1}{N} \sum_{i=1}^{N_f} n_i^r \gamma_i. \tag{A.9}$$

The sum is extended over the messenger fields, n_i^r is twice the corresponding Dynkin index (*e.g.* $n_i^r = 1$ if the messengers belong to the fundamental of the group G_r, and $n_i^r = (6/5)Y^2$ for hypercharge), and the messenger index is $N = \sum_{i=1}^{N_f} n_i^r$. The γ_i's (see eq. (A.11) below) are determined by the messengers' anomalous dimensions: $\gamma_i = -4\pi d \ln Z_i(M)/d \ln M$.

The "screening theorem" [20] does not hold for scalar masses, and therefore eq. (A.4) depends on the messenger wave-functions Z_i. Here we assume unification of the couplings λ_i and therefore the messenger wave-functions can be calculated in terms of the unification scale M_G and the unified coupling λ_G,

$$Z_i = \left[\frac{\lambda_i^2(M)}{\lambda_G^2}\right]^{-1/D_i} \prod_{r=1}^{3} \left[\frac{\alpha_r(M)}{\alpha_r(M_G)}\right]^{\frac{2C_r^i(2-D_i)}{D_i(b_r-Nk_r)}}, \tag{A.10}$$

$$\gamma_i = \frac{\lambda_i^2(M)}{2\pi} - 4\sum_{r=1}^{3} C_r^i \alpha_r(M). \tag{A.11}$$

We have defined $D_i = 2 + \dim(R_{\Phi_i})$, where $\dim(R_{\Phi_i})$ is the dimension of the messenger representation (*e.g.* $\dim(R_{\Phi_i}) = N_G$ for a fundamental of an $SU(N_G)$ gauge group). The relations between the coupling constants at the scale M and the GUT scale M_G are

$$\alpha_r^{-1}(M) = \alpha_r^{-1}(M_G) + \frac{(b_r - Nk_r)}{2\pi} \ln \frac{M_G}{M}, \tag{A.12}$$

$$\lambda_i^2(M) = \frac{\lambda_G^2 E^i(M)}{1 - \frac{D_i}{8\pi^2}\lambda_G^2 F^i(M)} \tag{A.13}$$

$$E^i(\mu) = \prod_{r=1}^{3} \left[\frac{\alpha_r(M_G)}{\alpha_r(\mu)}\right]^{\frac{4C_r^i}{(b_r-Nk_r)}}, \qquad F^i(M) = \int_0^{\ln M/M_G} d\ln\mu \; E_i(\mu). \tag{A.14}$$

Finally, defining the trilinear A terms as in eq. (2.7), their next-to-leading order matching condition at the scale M is

$$A_{\tilde{f}_i}(M) = -2\sum_{r=1}^{3} k_r C_r^{\tilde{f}_i} \frac{\alpha_r^2(M)}{(4\pi)^2} N \frac{F}{M}(1 - \mathcal{Z}_r). \tag{A.15}$$

For phenomenological applications, it is useful to rewrite the next-to-leading corrections in the particular case in which the messengers form N fundamental and antifundamental multiplets of $SU(5)$. To further simplify eq. (A.5), we keep only QCD and top-Yukawa corrections and neglect the weak coupling (which is justified only for $M \ll M_G$).

For squarks different than the stop, eq. (A.5) becomes

$$\Delta_3^{\tilde q} = \frac{8}{3}\alpha_3(M)\left[N - \frac{7}{3} - 2(N+3)\ln Z_3\right] + \frac{2\lambda_3^2(M)}{3\pi}, \tag{A.16}$$

$$Z_3 = \left[\frac{\lambda_3^2(M)}{\lambda_G^2}\right]^{-\frac{1}{5}} \left[\frac{\alpha_3(M)}{\alpha_G}\right]^{\frac{8}{5(N+3)}}. \tag{A.17}$$

For left and right sleptons, we find

$$\Delta_2^{\tilde\ell_L} = \frac{3\lambda_2^2(M)}{8\pi} \tag{A.18}$$

$$\Delta_1^{\tilde\ell_L} = -\frac{8\alpha_3(M)}{15} + \frac{3\lambda_2^2(M)}{40\pi} + \frac{\lambda_3^2(M)}{20\pi}, \tag{A.19}$$

$$\Delta_1^{\tilde e_R} = 4\Delta_1^{\tilde\ell_L}. \tag{A.20}$$

Here λ_2 and λ_3 refer to the couplings which determine the masses of the $SU(2)$ doublet and $SU(3)$ triplet messengers contained in the fundamental representation of $SU(5)$. They are related to the unified coupling λ_G by eq. (A.13).

The corrections to the left and right stop masses are

$$\Delta_3^{\tilde Q_L^{(3)}} = \Delta_3^{\tilde q} - \frac{8}{3}(1 - \ln Z_3^2)\alpha_t(M), \tag{A.21}$$

$$\Delta_3^{\tilde U_R^{(3)}} = \Delta_3^{\tilde q} - \frac{16}{3}(1 - \ln Z_3^2)\alpha_t(M). \tag{A.22}$$

The correction to the soft mass of the Higgs H_2 is

$$\Delta_3^{H_2} = -8(2 - \ln Z_3^2)\alpha_t(M). \tag{A.23}$$

Notice that, neglecting weak next-to-leading corrections, Z_3 is the only messenger wave-function appearing in the mass formulae. Indeed, in this case, it is more convenient to use in eq. (A.5) the triplet messenger mass $M_3 = M/Z_3$ as a matching scale, instead of M. The expressions for the corrections $\Delta_r^{\tilde f}$ are then given by eqs. (A.16)–(A.23), setting $\ln Z_3 = 0$. This can be explicitly verified by using the one-loop RG equations for the soft masses to relate $m_{\tilde f}^2(M)$ to $m_{\tilde f}^2(M_3)$. The result in eq. (A.23) plays an important rôle in the analysis of electroweak breaking and it has been used in eq. (2.41).

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