# New constraints on the tau neutrino mass and fourth generation mixing

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We present new constraints on the mass $m_{\nu_\tau}$ of the tau neutrino and its mixing with a fourth generation neutrino. From an analysis of the partial widths of tau lepton decays we obtain the following bounds at the 90% confidence level: $m_{\nu_\tau} < 32\,\mathrm{MeV}$ and $\sin^2\theta < 0.007$, where $\theta$ describes the Cabibbo-like mixing of the third and fourth generation neutrinos.

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## I. INTRODUCTION

In a previous paper we derived constraints on the mass of the third generation neutrino $\nu_3$ and its mixing with a heavy fourth generation neutrino $\nu_4$ [1]. In this paper we update this analysis using recent experimental measurements. We determine significantly more stringent constraints on the mass $m_{\nu_3}$ and the Cabibbo-like mixing angle $\theta$, where the tau neutrino weak eigenstate is given by the superposition of two mass eigenstates $|\nu_\tau\rangle = \cos\theta|\nu_3\rangle + \sin\theta|\nu_4\rangle$. We compare the precise measurements of the $\tau$ partial widths for the following decays[1] : $\tau^- \to e^-\bar{\nu}_e\nu_\tau$, $\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau$, $\tau^- \to \pi^-\nu_\tau$, and $\tau^- \to K^-\nu_\tau$, with our theoretical predictions, as functions of $m_{\nu_3}$ and $\sin^2\theta$ to obtain upper limits on both these quantities.

## II. THEORETICAL PREDICTIONS

The theoretical predictions for the branching fractions $\mathcal{B}_\ell$ for the decay $\tau^- \to \ell^-\bar{\nu}_\ell\nu_\tau(X_{\mathrm{EM}})$, with $\ell^- = e^-, \mu^-$ and $X_{\mathrm{EM}} = \gamma,\ \gamma\gamma,\ e^+e^-, \ldots$, are given by:

$$\begin{aligned}\mathcal{B}_\ell^{\mathrm{theory}} &= \left(\frac{G_F^2 m_\tau^5}{192\pi^3}\right)\tau_\tau\left(1 - 8x - 12x^2\ln x + 8x^3 - x^4\right)\\ &\times\left[\left(1 - \frac{\alpha(m_\tau)}{2\pi}\left(\pi^2 - \frac{25}{4}\right)\right)\left(1 + \frac{3}{5}\frac{m_\tau^2}{m_W^2}\right)\right]\\ &\times\left[1 - \sin^2\theta\right]\left[1 - 8y(1-x)^3 + \cdots\right] \qquad (1)\end{aligned}$$

where $G_F = (1.16639 \pm 0.00002) \times 10^{-5}\,\mathrm{GeV}^{-2}$ is the Fermi constant [2]; $\tau_\tau = (290.55 \pm 1.06)\,\mathrm{fs}$ is the tau lifetime [3]; $m_\tau = (1776.96^{+0.18+0.25}_{-0.21-0.17})\mathrm{MeV}$ [4] is the tau mass, determined by BES from the $\tau^+\tau^-$ production rate near threshold which has no dependence on the tau neutrino mass; and $x = m_\ell^2/m_\tau^2$. The first term in brackets allows for radiative corrections [5–8], where $\alpha(m_\tau) \simeq 1/133.3$ is the QED coupling constant [8] and $m_W = 80.400 \pm 0.075\,\mathrm{GeV}$ is the W mass [9]. The second term in brackets describes mixing with a fourth generation neutrino which, being kinematically forbidden, causes a suppression of the decay rate. The third term in brackets parametrises the suppression due to a non-zero mass of $\nu_3$, where $y = m_{\nu_3}^2/m_\tau^2$ and the ellipsis denotes negligible higher order terms [1].

The branching fractions for the decays $\tau^- \to h^-\nu_\tau$, with $h = \pi/K$, are given by

$$\begin{aligned}\mathcal{B}_h^{\mathrm{theory}} &= \left(\frac{G_F^2 m_\tau^3}{16\pi}\right)\tau_\tau f_h^2|V_{\alpha\beta}|^2\left(1 - x\right)^2\\ &\times\left[1 + \frac{2\alpha}{\pi}\ln\left(\frac{m_Z}{m_\tau}\right) + \cdots\right]\left[1 - \sin^2\theta\right]\\ &\times\left[1 - y\left(\frac{2 + x - y}{1 - x}\right)\sqrt{1 - y\frac{(2 + 2x - y)}{(1-x)^2}}\right] \qquad (2)\end{aligned}$$

where $x = m_h^2/m_\tau^2$, $m_h$ is the hadron mass, $f_h$ are the hadronic form factors, and $V_{\alpha\beta}$ are the CKM matrix elements, $V_{ud}$ and $V_{us}$, for $\pi^-$ and $K^-$ respectively. From an analysis of $\pi^- \to \mu^-\bar{\nu}_\mu$ and $K^- \to \mu^-\bar{\nu}_\mu$ decays, one obtains $f_\pi|V_{ud}| = (127.4 \pm 0.1)\mathrm{MeV}$ and $f_K|V_{us}| = (35.18 \pm 0.05)\mathrm{MeV}$ [10, and references therein]. The ellipsis represents terms, estimated to be $\mathcal{O}(\pm 0.01)$ [10], which are neither explicitly treated nor implicitly absorbed into $G_F$, $f_\pi|V_{ud}|$, or $f_K|V_{us}|$. The second term in brackets describes mixing with a fourth generation neutrino while the third parametrises the effects of a non-zero $m_{\nu_3}$ [1].

The fourth generation neutrino mixing affects all the tau branching fractions with a common factor whereas a non-zero tau neutrino mass affects all channels with different kinematic factors. Therefore, given sufficient experimental precision, these two effects could in principle be separated.

## III. RESULTS

We use the recently updated world average values for the measured tau branching fractions [3]:

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[1] Throughout this paper the charge-conjugate decays are also implied. We denote the branching ratios for these processes as $\mathcal{B}_e, \mathcal{B}_\mu, \mathcal{B}_\pi, \mathcal{B}_K$ respectively; $\mathcal{B}_\ell$ denotes either $\mathcal{B}_e$ or $\mathcal{B}_\mu$ while $\mathcal{B}_h$ denotes either $\mathcal{B}_\pi$ or $\mathcal{B}_K$.

$$\mathcal{B}_e = (17.786 \pm 0.072)\,\%; \tag{3}$$
$$\mathcal{B}_\mu = (17.356 \pm 0.064)\,\%; \tag{4}$$
$$\mathcal{B}_\pi = (11.01 \pm 0.11)\,\%; \tag{5}$$
$$\mathcal{B}_K = (0.692 \pm 0.028)\,\%. \tag{6}$$

Substituting in equations 1 and 2 for the measured quantities we find that both $m_{\nu_\tau}$ and $\sin^2\theta$ are consistent with zero.

We therefore derive constraints on $m_{\nu_\tau}$ and $\sin^2\theta$ from a combined likelihood fit to the four tau decay channels. The likelihood is constructed numerically following the procedure of Ref. [11] by randomly sampling all the quantities used according to their errors. The CLEO measurement of the $\tau$ mass was used to further constrain $m_{\nu_3}$. From an analysis of $\tau^+\tau^- \rightarrow (\pi^+ n\pi^0\bar{\nu}_\tau)$ $(\pi^- m\pi^0\nu_\tau)$ events (with $n \leq 2, m \leq 2, 1 \leq n+m \leq 3$), CLEO determined the $\tau$ mass to be $m_\tau = (1777.8 \pm 0.7 \pm 1.7) + [m_{\nu_3}(\mathrm{MeV})]^2/1400\,\mathrm{MeV}$ [12]. The likelihood for the CLEO and BES measurements to agree, as a function of $m_{\nu_3}$ is included in the global likelihood.

The fit yields upper limits of

$$m_{\nu_3} < 38\,\mathrm{MeV} \tag{7}$$
$$\sin^2\theta < 0.008 \tag{8}$$

at the 95% C.L. or

$$m_{\nu_3} < 32\,\mathrm{MeV} \tag{9}$$
$$\sin^2\theta < 0.007 \tag{10}$$

at the 90% C.L. These results improve on our previous determinations of $m_{\nu_3} < 42\,\mathrm{MeV}$ and $\sin^2\theta < 0.014$ at 90% C.L. [1]

## IV. DISCUSSION

The limit on $m_{\nu_3}$ can be reasonably interpreted as a limit on $m_{\nu_\tau}$, since the mixing of $m_{\nu_3}$ with lighter neutrinos is also small [2]. The best direct experimental constraint on the tau neutrino mass is $m_{\nu_\tau} < 18.2\,\mathrm{MeV}$ at the 95% confidence level [13] which was obtained using many-body hadronic decays of the $\tau$. While our constraint is less stringent, it is statistically independent. Moreover, it is insensitive to fortuitous or pathological events close to the kinematic limits, details of the resonant structure of multi-hadron $\tau$ decays, and the absolute energy scale of the detectors. Since LEP has completed running on the Z it is unlikely that significantly improved constraints on $m_{\nu_\tau}$, using multi-hadron final states, will be forthcoming in the foreseeable future.

Future improved measurements of the tau branching fractions, lifetime, and the tau mass from direct reconstruction would enable significant improvements to be made in the determinations of both $m_{\nu_\tau}$ and $\sin^2\theta$. If CLEO and the b-factory experiments were to reduce the uncertainties on the experimental quantities by a factor of approximately 2, then the constraints on $m_{\nu_\tau}$ from the technique we have described would become the most competitive. Were a tau-charm factory to be built, then the determination of $m_{\nu_\tau}$ by direct reconstruction would again become the most sensitive technique.

Our upper limit on $\sin^2\theta$ is already the most stringent experimental constraint on mixing of the third and fourth neutrino generations.

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