Radiative Tau Lepton Pair Production as a Probe of Anomalous Electromagnetic Couplings of the Tau

S. S. Gau, T. Paul [1], J. Swain [2], L. Taylor [3]

Department of Physics, Northeastern University, Boston, MA 02115, USA

We calculate the squared matrix element for the process $e^+e^- \to \tau^+\tau^-\gamma$ allowing for anomalous magnetic and electric dipole moments at the $\tau\tau\gamma$ vertex. No interferences are neglected and no approximations of light fermion masses are made. We show that anomalous moments affect not only the cross section, but also the shape of the photon energy and angular distributions. We also demonstrate that in the case of the anomalous magnetic dipole moment, the contribution from interference involving Standard Model and anomalous amplitudes is significant compared to the contribution from anomalous amplitudes alone. A program to perform the calculation is available and it may be employed as a Monte Carlo generator.

Key words: tau, anomalous-magnetic-dipole-moment, electric-dipole-moment

1 Introduction

In the Standard Model, the charged leptons are identical in all respects except for their masses and their distinct and conserved lepton numbers. There is, however, no experimentally verified explanation for why there are three generations of leptons nor why their masses are so different. New insight might be forthcoming if the leptons were observed to have substructure, which could be manifest as an anomalous magnetic or electric dipole moment. The anomalous moments for the electron and muon are already measured to be in extremely good agreement with the predictions of QED[1–3]. Anomalous moments of the tau are relatively poorly measured and are of particular interest since, as the heaviest lepton, the tau may exhibit the most sensitivity to new physics.

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[1] E-mail: tom.paul@cern.ch

[2] E-mail: lucas.taylor@cern.ch

[3] E-mail: swain@neuhp1.physics.neu.edu

In general a photon may couple to a tau through its electric charge, magnetic dipole moment, or electric dipole moment (neglecting possible anapole moments). We may parametrise this coupling with the following matrix element:

$$\langle \tau(p')|J_\mu|\tau(p)\rangle = \bar{u}(p')\Gamma_\mu u(p). \tag{1}$$

Γ_μ represents the most general Lorentz-invariant form of the coupling of a tau to a photon:

$$\Gamma_\mu = F_1(q^2)\gamma_\mu + i\frac{F_2(q^2)}{2m_\tau}\sigma_{\mu\nu}q^\nu - F_3(q^2)\sigma_{\mu\nu}q^\nu\gamma_5, \tag{2}$$

where m_τ is the mass of the τ lepton, and $q = p' - p$ is the momentum transfer. As can be verified using the Gordon decomposition, the q^2-dependent form-factors, $F_i(q^2)$, have familiar interpretations for $q^2 = 0$ and with the τ on mass-shell: $F_1(0) \equiv q_\tau$ is the electric charge of the tau, $F_2(0) \equiv a_\tau = (g-2)/2$ is the static anomalous magnetic moment of the tau (where g is the gyromagnetic ratio), and $F_3(0) \equiv d_\tau/q_\tau$, where d_τ is the static electric dipole moment of the tau and q_τ is its charge. Hermiticity of the electromagnetic current forces all the F_i to be real.

In the Standard Model, a_τ is non-zero due to loop effects and has been calculated to be[4]

$$a_\tau = \alpha/2\pi + \ldots = 0.001\,177\,3(3). \tag{3}$$

A non-zero value of d_τ is forbidden by both P invariance and T invariance. Assuming CPT invariance, observation of a non-zero value of d_τ would imply CP violation in the τ system.

Several limits on $F_2(q^2)$ and $F_3(q^2)$ at various values of q^2 have been reported [5–10]. The q^2 dependence of these form factors depends strongly on the hypothetical mechanism giving rise to anomalous values of F_2 and F_3. It is commonly assumed that $F_2(0)$ corresponds to the static anomalous magnetic moment and $F_3(0)$ the static electric dipole moment, since the radiated photon has zero q^2. This condition is necessary but not sufficient; the tau must also be on-shell on both sides of the $\tau\tau\gamma$ vertex which is only valid in the limit of vanishing photon energy. This condition is satisfied for the precession measurements of electrons and muons [1]. None of the measurements of tau anomalous moments published thus far satisfies both of these conditions, so we urge caution in the interpretation of the limits obtained on the form factors F_2 and F_3 in terms of the static tau properties, a_τ and d_τ.

Table 1 shows published theoretical predictions and experimental limits on F_2 and F_3 for electrons, muons and taus. For the case of taus, the experimental

limits of reference [5] are derived from the $e^+e^- \to \tau^+\tau^-$ total cross section at PETRA at q^2 up to $(37\text{GeV})^2$, those of reference [6] are calculated from $\Gamma(Z \to \tau^+\tau^-)$, and those of [9] are found using the total $e^+e^- \to \tau^+\tau^-\gamma$ cross section from early LEP data. We note that this last result is the only one which corresponds to a direct measurement at $q^2 = 0$.

This paper describes a tree-level calculation of the process $e^+e^- \to \tau^+\tau^-\gamma$ which accounts for the effects of anomalous magnetic and electric dipole couplings. We present total cross sections as well as energy and angular distributions of the radiated photon which will be useful for measuring $F_2(0)$ and $F_3(0)$. We also demonstrate the importance of including interference involving Standard Model and anomalous amplitudes, which has not been considered experimentally up until now. The calculation described here may be employed as an event generator, making it possible to simulate detector effects and properly account for acceptance, as is necessary for a meaningful interpretation of the data.

e	Theory:	$F_2(0)$	$= 0.001\,159\,652\,46(15)$	[11]
	Expt:	$F_2(0)$	$= 0.001\,159\,652\,193(10)$	[1]
	Expt:	$eF_3(0)$	$= (-2.7 \pm 8.3) \cdot 10^{-27}\,e\,\text{cm}$	[2]
μ	Theory:	$F_2(0)$	$= 0.001\,165\,920\,2(20)$	[12]
	Expt:	$F_2(0)$	$= 0.001\,165\,923\,0(84)$	[1]
	Expt:	$eF_3(0)$	$= (3.7 \pm 3.4) \cdot 10^{-19}\,e\,\text{cm}$	[3]
τ	Theory:	$F_2(0)$	$= 0.001\,177\,3(3)$	[4]
	Expt:	$F_2(q^2 \neq 0) < 0.02$		[5]
		$-0.004 < F_2(q^2 \neq 0) < 0.006$		[6]
		$F_2(0) < 0.11$		[9]
	Expt:	$eF_3(q^2 \neq 0) < 1.4 \cdot 10^{-16}\,e\,\text{cm}$		[10]
		$eF_3(q^2 \neq 0) < 0.11 \cdot 10^{-16}\,e\,\text{cm}$		[6]
		$eF_3(0) < 6.0 \cdot 10^{-16}\,e\,\text{cm}$		[9]

Table 1
Theoretical predictions and experimental measurements of the anomalous magnetic moments and the experimental measurements of the electric dipole moments of charged leptons.

2 Calculation of the Effects of Anomalous Couplings in $e^+e^- \rightarrow \tau^+\tau^-\gamma$ Events

Using the parametrisation of the electromagnetic current given in Eq. 2, we consider all the Standard Model and anomalous amplitudes for the diagrams shown in Fig. 1. The corresponding matrix element is then evaluated using the symbolic manipulation package FORM [13] without making any simplifying assumptions. In particular, no interference terms are neglected and no fermion masses are assumed to be zero. The inclusion of a non-zero tau mass is essential, as the (significant) interference terms between the Standard Model and the anomalous amplitudes vanish in the limit of vanishing tau mass. Standard Model radiative corrections are incorporated by using the improved Born approximation [14].
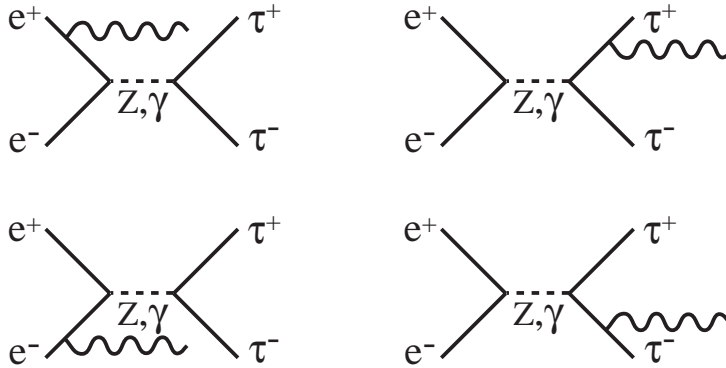

Fig. 1. Diagrams contributing to $e^+e^- \rightarrow \tau^+\tau^-\gamma$.

We use the FOWL program [15] to generate points in 3-body phase space, and for each point calculate the full squared matrix element. This procedure yields a sample of events with probabilistic weights which may used directly or applied in a Monte Carlo rejection method to produce a sample of events with weights of unity.

Infrared divergences for collinear or low energy photons may be naturally avoided by applying cuts on minimum photon energy and/or opening angle between the photon and tau. This method eliminates potential concerns about the consistency of cancelling divergences against vertex corrections which are calculated in QED assuming that the anomalous couplings vanish. Moreover, this procedure matches well the experimental reality, since electromagnetic calorimeters have a minimum energy cutoff and isolation cuts are required to distinguish tau decay products from radiated photons.

3   Anomalous Contribution to the Total Cross Section

It is illustrative to isolate the anomalous contribution to the cross section from the purely Standard Model contribution. The anomalous contribution is given by those terms in the matrix element which contain a factor of $F_2(0)$ or $F_3(0)$:

$$\sigma_{\mathrm{ano}} = \alpha F_2^2(0) + \beta F_2(0) + \gamma F_3^2(0) + \delta F_3(0). \qquad (4)$$

Terms linear in $F_2(0)$ and $F_3(0)$ arise from interference between Standard Model and anomalous amplitudes, whereas the quadratic terms are purely anomalous. Figure 2a shows the contribution to the total cross section [4] arising from these terms, with the linear and quadratic components shown separately. Interestingly, the interference terms do not vanish, and in fact are significant compared to the total anomalous cross section for small values of $F_2(0)$. Figure 2b shows the anomalous cross section as a function of $F_3(0)$, with the linear term again shown separately. Evidently the interference does vanish in the case of an electric dipole moment.

4   Anomalous Contribution to the Differential Cross Section

In addition to affecting the total cross section, anomalous couplings also affect the shapes of energy and angular distributions of the final state particles. Because of their experimental accessibility, we choose to study the photon energy, the three-dimensional opening angle between the photon and the closest tau, and the photon polar angle with respect to the electron direction. Figure 3a shows the anomalous contribution to the photon energy spectrum for various values [5] of $F_2(0)$. Because of interference, both the cross section and spectrum shape are asymmetric around $F_2(0) = 0$. Figure 3b shows the distribution of the opening angle between the photon and the closest tau for for various $F_2(0)$. The difference between spectrum shapes for $+ \mid F_2(0) \mid$ and $- \mid F_2(0) \mid$ is more striking for this distribution than for Figure 4, the energy and opening angle are plotted against one another for two different values of $F_2(0)$, showing that these two quantities provide independent information about the anomalous moment.

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[4] The calculation assumes $M_Z = 91.181 \, \mathrm{GeV}$, $M_\tau = 1.777 \, \mathrm{GeV}$, and $\sin^2(\theta_W^{eff}) = 0.2319$.

[5] Values of $F_2(0)$ used in plots in this paper were chosen to be in the neighborhood of the expected LEP sensitivity to $F_2(0)$. See section 6.
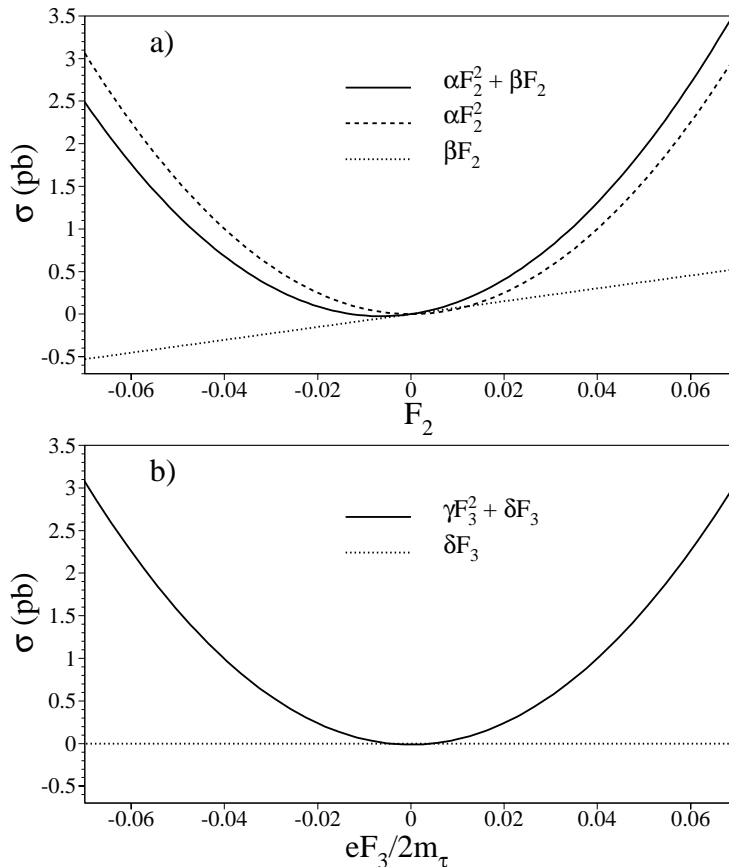
Fig. 2. Anomalous contribution to the $e^+e^- \rightarrow \tau^+\tau^-\gamma$ cross section as a function of a) $F_2(0)$ and b) $F_3(0)$. The contributions from the linear and quadratic terms are shown separately.

5 Comparisons with Other Results

As a technical crosscheck, we create a sample of events weighted using our matrix element calculation, henceforth referred to as "TTG," with $F_2(0) = F_3(0) = 0$ and compare the total cross section and energy and angular distributions with the predictions of the KORALZ [16] Monte Carlo. Since TTG provides an $\mathcal{O}(\alpha)$ calculation, KORALZ is also used in a mode in which only single photon radiation is considered. In order to prevent infrared divergences in TTG, cuts are placed on the photon energy and opening angles, as discussed in section 2. In particular, we require the photon energy to be greater than 1 GeV, the cosine of the opening angle between the photon and the nearest τ to be less than 0.995, and the cosine of the polar angle of the photon with respect to the electron direction to be less than 0.9. The total cross sections calculated by KORALZ and TTG for these cuts agree to better than 0.5%. Figure 5 shows a comparison of the photon energy spectra computed by the two programs.
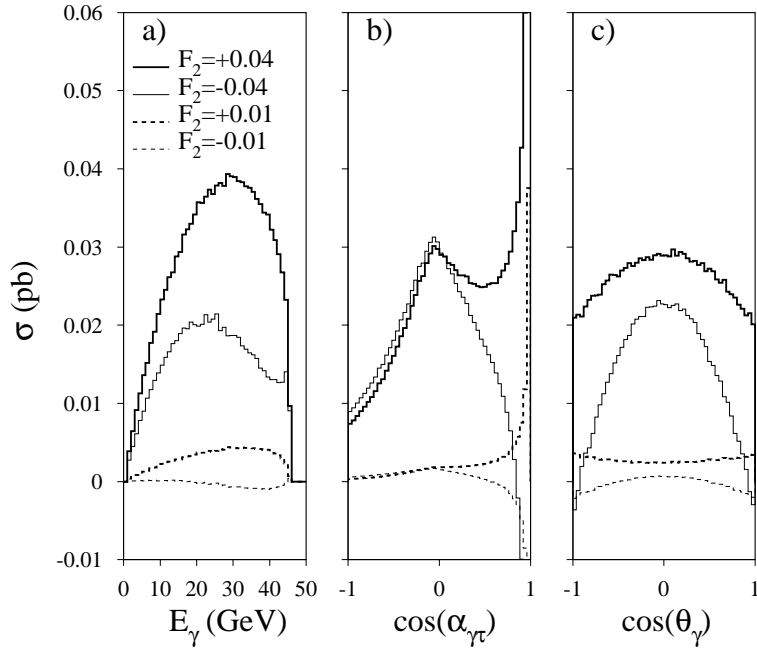
Fig. 3. Anomalous contribution to a) the photon energy spectrum for various values of $F_2(0)$, b) the distribution of the opening angle between the photon and the nearest tau, and c) the distribution of photon polar angle.
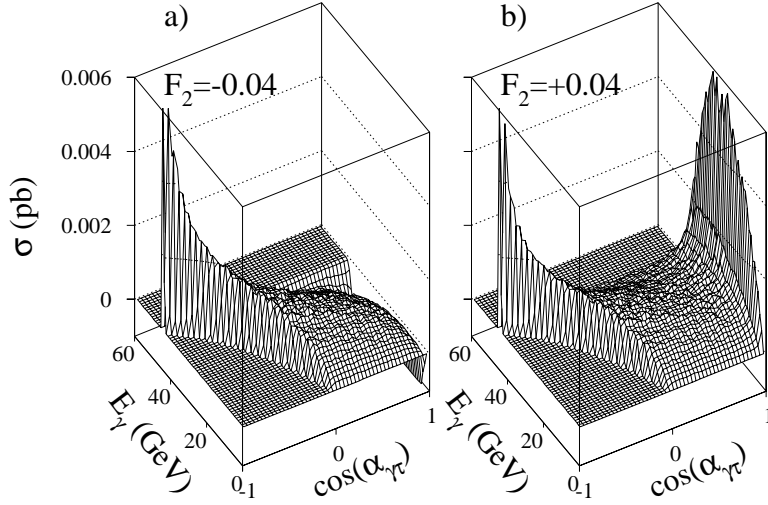

Fig. 4. Anomalous contribution to the cross section as a function of the photon energy versus the cosine of the opening angle between the photon and tau ($\alpha_{\gamma\tau}$) for a) $F_2(0) = -0.04$ and b) $F_2(0) = +0.04$.

Following an initial presentation of the results in this paper [17], an analytical calculation for the process $e^+e^- \to \tau^+\tau^-\gamma$ was carried out [18]. This calculation makes some approximations, but importantly does not assume zero tau mass and does not neglect interference between Standard Model and anomalous final states. As a mutual cross check, we compare our results with the
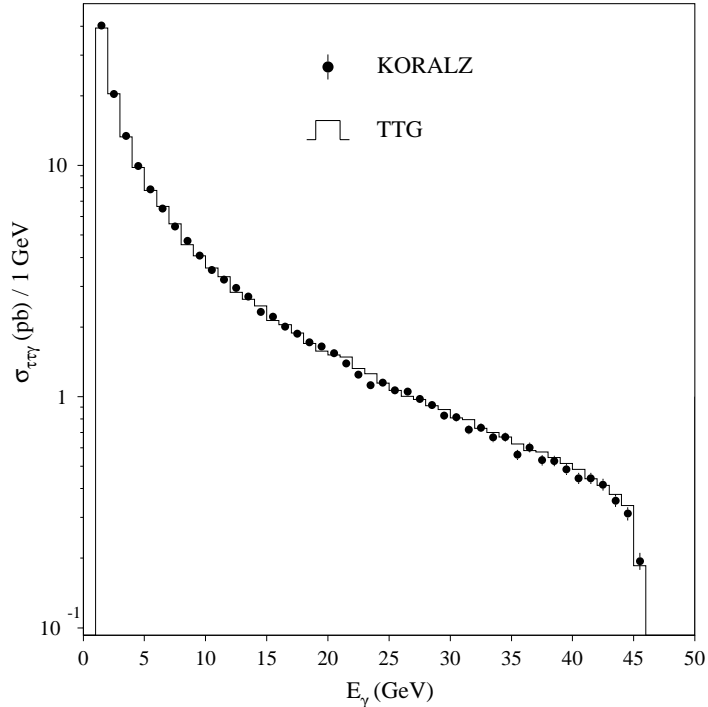
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Fig. 5. Photon energy spectrum for KORALZ (dots) and TTG (histogram) for the cuts described in the text.

total cross section and photon energy spectrum predicted by this calculation.

The analytical calculation neglects anomalous contributions from initial-final state interference, from γZ interference, and from γ exchange, so for purposes of comparison we remove these terms from our calculations as well. Figure 6 shows a comparison of anomalous cross sections as a function of $F_2(0)$ as determined by our simplified version of TTG and the analytical calculation, indicating agreement at the 1% level. Figure 7 gives a comparison of the photon energy spectra for the two calculations, showing good agreement between the spectrum shapes over a large range of $F_2(0)$ and the full range of photon energies accessible near the Z resonance.

At this point, one may ask whether all the possible anomalous contributions are important, or whether one need only consider the simplified case just discussed. To check this, we use our calculation to determine the full anomalous contribution to the cross section, including all the terms neglected in the analytical calculation, and compare with the simplified result. Figure 8 shows the difference between the cross sections as a function of $F_2(0)$ for the two cases. The discrepancy is roughly 1% of the total anomalous cross section,
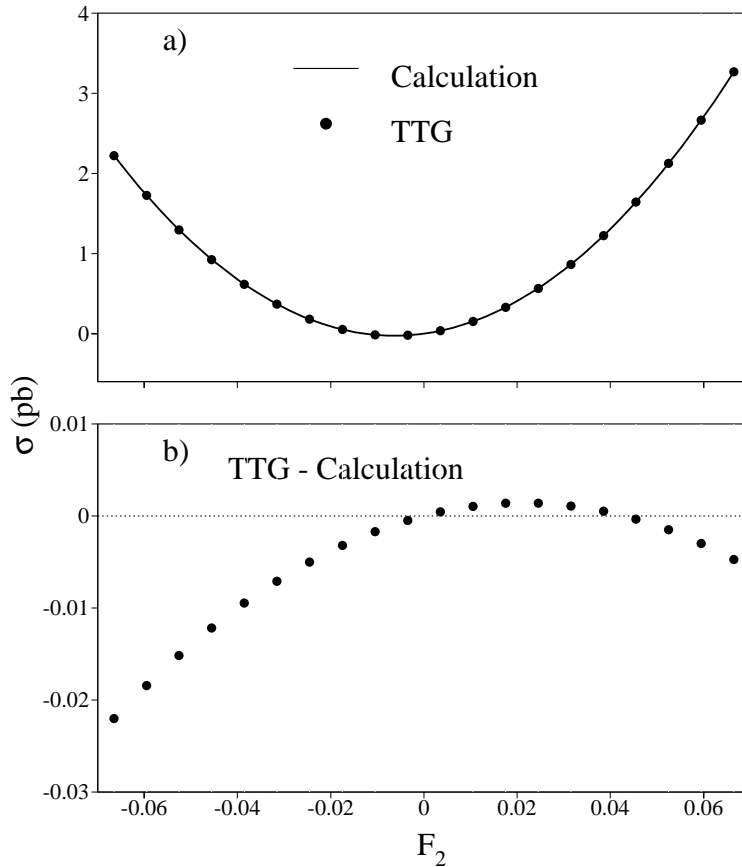
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Fig. 6. a) Comparison of anomalous cross section computed using a simplified analytical calculation (curve) and that computed using TTG with the same simplifications (dots). b) The difference between the two cross sections described in a).

showing that one can safely neglect anomalous contributions from initial-final state interference, γZ interference, and γ exchange.

6 Experimental Considerations

Previous experimental limits on $F_2(0)$ and $F_3(0)$ [9,19] have assumed that the interference between the Standard Model and anomalous contributions to the cross section can be neglected. As we have shown in Figure 2, this is not generally true for the anomalous magnetic dipole moment. If we take $\mid F_2(0) \mid = 0.04$, for example, then failure to account for interference leads to an underestimate of the anomalous contribution to the total cross section by about 25% if $F_2(0)$ is positive, and an overestimate of almost 50% if $F_2(0)$ is negative. These discrepancies become even more pronounced for smaller
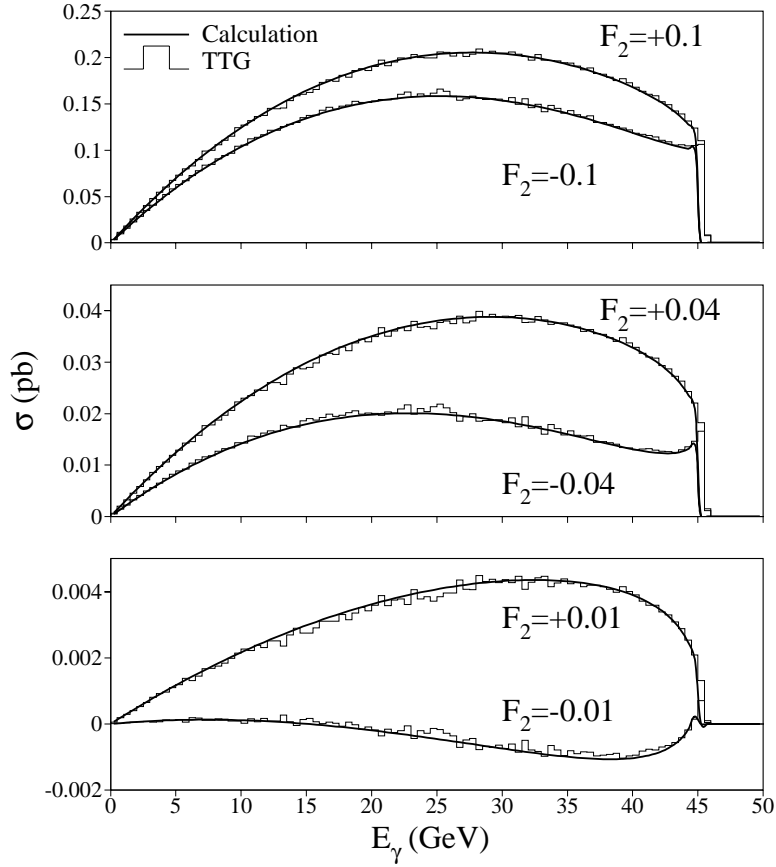
Fig. 7. Comparison for a range of $F_2(0)$ of the photon energy spectrum computed using a simplified analytical calculation (curve) and that computed using TTG with the same simplifications (histogram).

$| F_2(0) |$.

We have shown that the photon energy, the opening angle between the photon and the nearest tau, and the photon polar angle each contains independent information about $F_2(0)$ and $F_3(0)$, including the sign of $F_2(0)$ (Figures 3 and 4). One should note, however, that the distributions presented in these figures include no cuts on photon energies or angles. In practice, detector geometry imposes a cut on the photon polar angle with respect to the electron direction, and further cuts must be applied on the photon energy and minimum opening angle between the photon and tau in order to suppress background from tau decay products. In particular, the unavoidable cut on the opening angle must be placed in exactly the region where sensitivity to the sign of the anomalous coupling is highest.

Other theoretical treatments [9,18] have provided cross sections or differential distributions, which are generally not sufficient for meaningful analysis of the data. For example, we have just seen that the anomalous contribution to
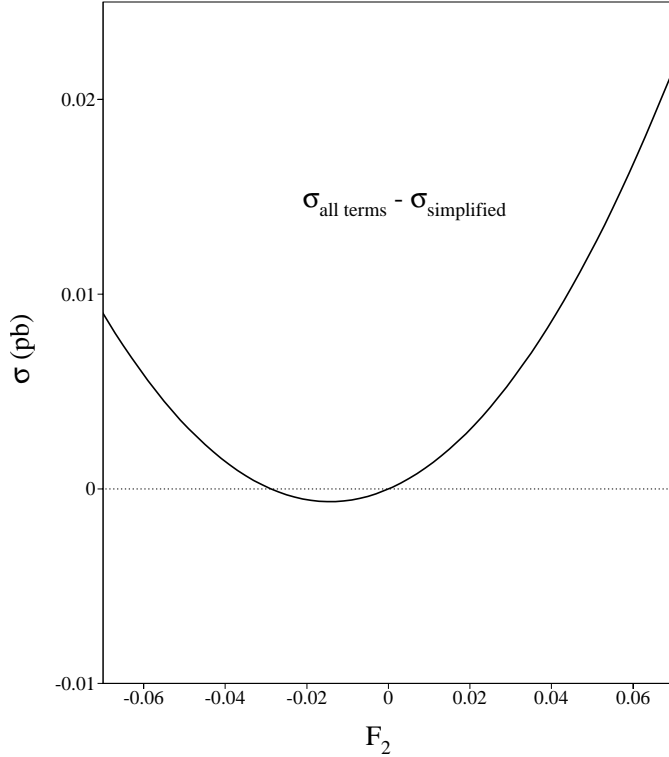
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Fig. 8. a) Difference as a function of $F_2(0)$ between the cross section including all anomalous contributions and cross section including all but anomalous contributions from initial-final state interference, γZ interference, and γ exchange.

the cross section depends strongly on the various experimental cuts applied, an effect which is difficult if not impossible to assess without a full Monte Carlo simulation. The calculation described here provides just such a tool. It may be employed to yield samples of 4-vectors for the final state particles in $e^+e^- \to \tau^+\tau^-\gamma$, which can be passed to the TAUOLA [20] program to perform the tau decays. The resulting final state particles can then be passed through the detector simulation program, which will account for such effects as energy loss and interactions in the detector, geometrical acceptance, and energy and spatial resolutions, together with obscure correlations which are not easily handled on a statistical basis. Alternatively, the matrix element calculation may be used to compute event-by-event weights for a given value of $F_2(0)$ or $F_3(0)$ using 4-vectors from previously simulated $e^+e^- \to \tau^+\tau^-\gamma$ events. This makes it possible to reweight existing samples for which detector effects have already been simulated, thus saving considerable computing time.

In order to estimate the sensitivity to anomalous moments expected for the LEP experiments running at $\sqrt{s} \approx M_Z$, we have performed a study of simulated $\tau^+\tau^-\gamma$ events as they would appear in the L3 detector [21]. The simulated sample corresponds to roughly the integrated luminosity collected by L3 at the Z peak (about $100 \ pb^{-1}$) and an anomalous magnetic moment $F_2(0) = 0$

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and electric dipole moment $F_3(0) = 0$. We estimate 40% efficiency for select-
ing $\tau^+\tau^-\gamma$ events, which is dominated by the requirement that at least one
jet be in the barrel region ($\cos\theta < 0.7$) of the detector. Events with good
quality photons are selected and background from tau decay products is re-
jected by asking that the transverse energy of the photon to the nearest jet be
larger than 2 GeV. We then perform a binned maximum likelihood fit of the
selected photon energy spectrum to an independent collection of simulated
samples which have been weighted, using the method outlined above, such
that they correspond to many different values of $F_2(0)$. The fit results show
that if the true value of $F_2(0) = 0$, one can expect to set a 95% confidence
level limit of roughly $|F_2(0)| < 0.05$. If $F_2(0) \neq 0$ or $F_3(0) \neq 0$, the sensitivity
improves. Of course, the exact value of the limit will depend on the details of
the experiment, selection applied, and the variables used in the fit.

7 Conclusions

We have carried out a calculation of the squared matrix element for the process
$e^+e^- \rightarrow \tau^+\tau^-\gamma$, allowing for possible anomalous magnetic and electric dipole
moments at the $\tau\tau\gamma$ vertex. No interferences are neglected and no fermion
masses are set to zero. The calculation shows that the total cross section as
well as the shapes of the photon energy and angular distributions are sensitive
to anomalous magnetic and electric dipole moments. We also find that inter-
ference between anomalous and Standard Model contributions is significant for
the case of the anomalous magnetic dipole moment, and increases in impor-
tance for smaller values $F_2(0)$. The bulk of the contribution from terms linear
in $F_2(0)$ arises from interference between anomalous and Standard Model fi-
nal states. Contributions from the electric dipole moment evidently do not
interfere. These results are in excellent agreement with a simplified analytical
calculation.

The calculation described here can be used to generate weights for different
values of $F_2(0)$ and $F_3(0)$ on an event-by-event basis. It is therefore an invalu-
able experimental tool, as it facilitates proper simulation of detector effects
and experimental cuts in analysis of the process $e^+e^- \rightarrow \tau^+\tau^-\gamma$. The matrix
element calculation is available from the authors in the form of a FORTRAN
subroutine.

Acknowledgement

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