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Massive Branes

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Abstract

We investigate the effective worldvolume theories of branes in a background given by (the bosonic sector of) 10-dimensional massive IIA supergravity ("massive branes") and their M-theoretic origin. In the case of the solitonic 5-brane of type IIA superstring theory the construction of the Wess-Zumino term in the worldvolume action requires a dualization of the *massive* Neveu-Schwarz/Neveu-Schwarz target space 2-form field. We find that, in general, the effective worldvolume theory of massive branes contains new worldvolume fields that are absent in the massless case, i.e. when the mass parameter m of massive IIA supergravity is set to zero. We show how these new worldvolume fields can be introduced in a systematic way.

In particular, we find new couplings between the massive solitonic 5-brane and the target space background, involving an additional worldvolume 1-form and 6-form. These new couplings have implications for the anomalous creation of branes. In particular, when a massive solitonic 5-brane passes through a D8-brane a stretched D6-brane is created. Similarly, in M-theory we find that when an M5-brane passes through an M9-brane a stretched Kaluza-Klein monopole is created.

We show that pairs of massive branes of type IIA string theory can be viewed as the direct and double dimensional reduction of a single "massive M-brane" whose worldvolume theory is described by a gauged sigma model. For D-branes, the worldvolume gauge vector field becomes the Born-Infeld field of the 10-dimensional brane. The construction of the gauged sigma model requires that the 11-dimensional background has a Killing isometry. This background can be viewed as an 11-dimensional rewriting of the 10-dimensional massive IIA supergravity theory. We present the explicit form and discuss the interpretation of (the bosonic sector of) this so-called "massive 11-dimensional supergravity theory".

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Introduction

In order to get a better understanding of the dynamics of branes in string theory, it is important to know precisely what the effective worldvolume theory is that describes the dynamics of these objects. This worldvolume theory contains a great deal of information. For instance, fields propagating on a brane's worldvolume describe the dynamics of the intersections with other branes ending on it [1, 2]. In particular, the Born-Infeld (BI) vector field present in the worldvolume of all D-p-branes describes the $U(1)$ field whose sources are open string endpoints. The anti-selfdual 2-form potential living on the M-5-brane worldvolume describes the dynamics of an M-5-brane intersecting with an M-2-brane over a 1-brane. The dynamics of these worldvolume fields has recently received some attention [3, 4, 5].

At present the structure of the worldvolume theory is quite well understood for the case in which the brane propagates in a supergravity background without a cosmological constant, the so-called "massless supergravity theories". As is well known, in the case of IIA supergravity, an extension is possible with a non-zero cosmological constant proportional to m^2 with m a mass parameter [6]. Such backgrounds are essential for the existence of D-8-branes whose charge is proportional to m [7, 8]. It is the purpose of this paper to investigate the worldvolume theory of branes that propagate in such a massive background. We will call such branes "massive branes"[4]. We will in turn use the worldvolume theories to investigate the 11-dimensional origin of these massive branes.

The basic branes of string theory are the fundamental string (p-1-brane), the solitonic 5-brane (p-5-brane), the Brinkmann wave, the Kaluza-Klein (KK) monopole and the D-p-branes ($0 \leq p \leq 9$). The charge of the first two objects is carried by a Neveu-Schwarz/Neveu-Schwarz (NS/NS) field whereas the D-p-branes carry a Ramond-Ramond (R-R) charge. For objects that carry a NS/NS charge there are three possibilities: the brane may move in a Heterotic, IIA or IIB supergravity background. Each of these three possibilities is described by a different worldvolume theory. We distinguish between these cases using an obvious notation, e.g. for the different p-5-branes

[4]In this paper we reserve the name "massive branes" for branes that propagate in a massive IIA supergravity background as opposed to branes that propagate in a background with zero mass parameter. Of course, all branes are massive in the sense that their physical mass is nonzero.

$$\text{p-5-brane} \rightarrow \begin{cases} \text{p-5H-brane} \\ \\ \text{p-5A-brane} \\ \\ \text{p-5B-brane} \end{cases}$$

On the other hand the D-p-branes carry a R-R charge and therefore always propagate in a IIA (p even) or IIB (p odd) background.

It is instructive to first remind the worldvolume theories that describe the dynamics of massless branes, i.e. branes that move in a background with zero cosmological constant.

The $(0+1)$-dimensional worldvolume theory corresponding to the Brinkmann wave is given by a kinetic term corresponding to a massless particle. The difference between the worldvolume theories of the Heterotic wave (WH), IIA-wave (WA) and IIB-wave (WB) is in the fermionic terms and in the coupling to the dilaton.

In the case of the p-1-brane (NS/NS string) there is no difference between the worldvolume theories of the p-1H-, p-1A- and p-1B-branes at the bosonic level. These actions are given by a Nambu-Goto kinetic term and a WZ term that describes the coupling of the brane to the NS/NS 2-form B. The dilaton does not appear in any of these two terms, since the tension of fundamental objects is independent of the string coupling constant g.

In the case of the p-5-brane the kinetic term is again of the Nambu-Goto form but now there is an extra dilaton factor $e^{-2\phi}$ in front of it showing that the p-5-brane is a solitonic object whose mass is proportional to $1/g^2$. Concerning the WZ term there is a distinction between the three different cases already at the bosonic level. The reason for this is that the p-5-brane couples to the 6-form dual of the NS/NS 2-form B. The definition of this dual 6-form depends on the background involved because either the field strengths or the couplings of B in the action are different. There are then three different dual 6-forms denoted by $\tilde{B}_{\rm H}, \tilde{B}_{\rm IIA}$ and $\tilde{B}_{\rm IIB}$.

The kinetic term of the (5+1)-dimensional worldvolume theory corresponding to the KK-monopole has only been recently constructed [9]. It is given by a gauged sigma model involving an auxiliary worldvolume 1-form. The kinetic term carries a factor $e^{-2\phi}k^2$ indicating that the effective tension of this object is proportional to $1/g^2$ (and so it is solitonic) and to R^2, R being the radius of the dimension to be compactified. This behavior is characteristic of KK monopoles. There is also a difference in the bosonic worldvolume content of the KK-H, KK-10A and KK-10B monopoles [10].

The WZ terms for these monopoles have not been constructed explicitly so far (see, however, [11]).

Finally, the worldvolume theory of the D-p-branes is given by a Dirac-Born-Infeld (DBI) kinetic term with a $e^{-\phi}$ dilaton coupling in front of it and a WZ term whose leading term is the R-R (p+1)-form field. The dilaton coupling shows that the mass of these objects is proportional to $1/g$.

In this work we will consider the massive extension of the worldvolume theory corresponding to the p-1A-brane, p-5A-brane and the D-p-branes (p even). In the latter case, the result has already been given in the literature [12, 13]. We find that the (bosonic part of the) worldvolume theory of the massive p-1A-brane is identical to that of the massless p-1A-brane. In the case of the p-5A-brane, however, there are striking differences. It turns out that there are extra couplings, involving an additional 1-form and 6-form worldvolume field, that are proportional to the mass parameter m of massive IIA supergravity. Both fields are auxiliary and do not contribute to the worldvolume degrees of freedom.

The reason for the presence of the worldvolume 6-form is that in order to build the WZ term one has to dualize the *massive* NS/NS target space 2-form field along the lines recently discussed in [14]. An interesting feature is that, whereas in the usual formulation of IIA supergravity the R-R 1-form $C^{(1)}$ is a Stueckelberg field that gets "eaten up" by the Neveu-Schwarz/Neveu-Schwarz (NS/NS) 2-form B which becomes massive:

$$\left\{ \begin{array}{rcl} C^{(1)} & \rightarrow & \text{Stueckelberg field}\,, \\ \\ B & \rightarrow & \text{massive field}\,, \end{array} \right.$$

in the dual formulation the situation is reversed: the dual NS/NS 6-form $\tilde{B}_{\rm IIA}$ becomes a Stueckelberg field giving mass to the dual R-R 7-form $C^{(7)}$:

$$\left\{ \begin{array}{rcl} C^{(7)} & \rightarrow & \text{massive field}\,, \\ \\ \tilde{B}_{\rm IIA} & \rightarrow & \text{Stueckelberg field}\,. \end{array} \right.$$

An immediate consequence of the above observation is that, since $\tilde{B}_{\rm IIA}$ occurs as the leading term of the WZ term of the p-5A-brane, we must introduce an independent auxiliary 6-form worldvolume field in order to cancel the Stueckelberg transformations of $\tilde{B}_{\rm IIA}$. This is on top of an auxiliary worldvolume 1-form which must be introduced in order to construct a WZ term that is invariant under the "massive gauge transformations" of massive IIA supergravity. The massive p-5A-brane therefore contains extra couplings to a worldvolume 1-form and 6-form that are absent in the massless case. The situation is similar to that of a D-0-brane in a massive background. In that case the Stueckelberg variation of the leading term $C^{(1)}$ in the WZ term is canceled by an auxiliary BI 1-form field that couples to the D-0-brane with a strength proportional to the mass parameter m.

As mentioned at the beginning, the second goal of this work is to investigate a possible "M-theoretic origin" of the massive branes considered above, thereby generalizing the massless case considered in [15]. This is a nontrivial problem in view of the fact that the massive IIA supergravity background fields have no known 11-dimensional interpretation.

We will deal with the above problem as follows: first, we will show that the massive brane worldvolume actions with 10-dimensional target space can be quite naturally rewritten as worldvolume actions of objects ("massive M-branes") moving in an 11-dimensional target space. These massive M-brane actions give the known massive brane actions of string theory upon direct or double dimensional reduction exactly as it happens in the massless case[5]. Therefore, a single massive M-brane unifies two different massive branes of string theory. The situation is depicted in Figure 1 (Section 3).

We find that the worldvolume theory of a general massive M-brane is given by a gauged sigma model[6]. To preserve the (p+1)-form potential gauge invariance in the gauging some modifications of the standard 11-dimensional supergravity background transformations have to be introduced. These modifications are the same for all massive M-branes and, thus, we are led to the construction of a "massive 11-dimensional supergravity theory", the natural background in which massive M-branes move.

The construction of this theory requires the existence of a Killing isometry (the one which is gauged in the sigma models), i.e. in adapted coordinates the fields of 11-dimensional supergravity do not depend on one of the coordinates, say y. In the limit in which the mass parameter is set to zero, the dependence on y can be restored and the action is given by ordinary (massless) 11-dimensional supergravity. Furthermore, the theory has the sought-after property that it gives type IIA supergravity upon dimensional reduction in the direction y.

Of course this so-called "massive 11-dimensional supergravity theory" is not a proper 11-dimensional theory in the sense that the fields do not depend on the special coordinate y[7]. Nevertheless, we believe that the massive 11-dimensional supergravity theory we propose should lead to a better understanding of a possible M-theoretic origin of the mass parameter m. An

[5] As we shall see, the situation for the M-5-brane is more subtle.

[6] The massive M-2-brane has already been considered in [16, 17] and the present work should be viewed as an extension of these works to the other M-branes.

[7] It is due to this feature that the no-go theorem of [18] can be circumvented.

interesting feature is that the massive 11-dimensional supergravity theory we consider in this work is on a rather similar footing with a recent proposal [19] for an 11-dimensional supergravity theory that, upon reduction, leads to a new 10-dimensional massive supergravity theory. A more detailed comparison between the two theories can be found in the Conclusion Section.

Finally, as a by-product of our work in which we were obliged to introduce additional worldvolume fields, we have found a systematic description for worldvolume p-forms, their gauge transformations and field strengths. These worldvolume p-forms can be defined as the dual of the BI field in the worldvolume action of the D-(p+2)-brane. The integrals of their (p+1)-form field strengths can be identified with the WZ terms of D-p-branes. Our description is valid both for type IIA and type IIB object worldvolumes and we believe that the structure of these fields has a deep significance.

The organization of the paper is as follows. The construction of the worldvolume theories describing the massive branes of string theory requires certain results both on the structure of the supergravity background fields, in particular their dual formulations, as well as on the structure of the different p-form worldvolume fields that will be needed. Therefore, as a preliminary, we first discuss in Section 1 the target space supergravity fields and in Section 2 the p-form worldvolume fields. Next, these results are applied in Section 3 to construct the sigma models describing the massive branes.

More specifically, in Section 1 we discuss massive supergravities in 10 and 11 dimensions together with their dual formulations. The massless supergravity theories are included as a special case by taking the mass parameter equal to zero. The different cases we consider are: (i) massive 11-dimensional supergravity (Section 1.1); (ii) the dual formulation of massless d=11 supergravity (Section 1.2); (iii) the dual formulation of massive 10-dimensional IIA supergravity (Section 1.3) and finally (iv) the dual formulation of massive d=11 supergravity (Section 1.4). Next, in Section 2 we introduce the different p-form worldvolume fields. Some of them will play the role of describing dynamical degrees of freedom, others will be auxiliary fields. All these different supergravity backgrounds and worldvolume fields are needed when we construct in Section 3 the worldvolume effective actions of the massive branes. The strategy we follow in this section will be to first construct the sigma models corresponding to the massive M-branes. In a second stage we perform the direct and double dimensional reduction of the different massive M-branes and show that they give rise to pairs of massive branes of string theory. This is represented in Figure 1. Our conclusions are given in Section 4. Finally, there are three appendices. Appendix A gives a general self-contained discussion on the duality of massive k-form fields; Appendix B (Appendix C) gives our notation for all the different target space (worldvolume) fields occurring in this paper and summarizes some useful formulae needed in the text.

1 Target Space Fields

1.1 Massive 11-Dimensional Supergravity

In this Section we present, using an 11-dimensional notation, an action that upon dimensional reduction gives massive 10-dimensional IIA supergravity. This theory provides the 11-dimensional background for the worldvolume actions of the *massive* M-branes (see Section 3). As discussed in the introduction the construction of the action requires that the 11-dimensional fields exhibit a Killing isometry. The action we will give below has the property that when the mass parameter m is set to zero the dependence of the fields on the Killing isometry direction (say y) in adapted coordinates, can be restored and the usual massless 11-dimensional supergravity theory is recovered. The purpose of this Section is to present the massive 11-dimensional supergravity theory, as a preliminary to the construction of the worldvolume sigma models in Section 3. In the Conclusion we will compare with the work of [19] and discuss a possible connection with M-theory.

The *massive* 11-dimensional theory has the same field content as the massless one[8]:

$$\left\{\hat{g}_{\hat{\mu}\hat{\nu}}, \hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}}\right\} . \tag{1.1}$$

The action for these fields is manifestly 11-dimensional Lorentz covariant but it does not correspond to a proper 11-dimensional theory because, in order to write down the action, we need to introduce an auxiliary non-dynamic vector field $\hat{k}^{\hat{\mu}}$ such that the Lie derivatives of the metric and 3-form potential with respect to it are zero[9]:

$$\pounds_{\hat{k}}\hat{g}_{\hat{\mu}\hat{\nu}} = \pounds_{\hat{k}}\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} = 0. \tag{1.3}$$

[8]Hats on spacetime fields and indices indicate they are 11-dimensional. Absence of hats indicates they are 10-dimensional.

[9]We will heavily rely on this property. In combination with the identity

$$i_{\hat{k}}\partial\hat{S}^{(r)} = \tfrac{(-1)^r}{r+1}\pounds_{\hat{k}}\hat{S}^{(r)} + \tfrac{r}{r+1}\partial\left(i_{\hat{k}}\hat{S}^{(r)}\right) \tag{1.2}$$

for r-forms, it will allow us to pull $\hat{k}$ through partial derivatives. $i_{\hat{k}}\hat{T}$ indicates the contraction of $\hat{k}$ with the last (by convention) index of the tensor $\hat{T}$.

To construct the action we start by defining the infinitesimal massive gauge transformations on any rank r 11-dimensional tensor with lower indices $\hat{L}_{\hat{\mu}_1\ldots\hat{\mu}_r}$ (not necessarily an r-form) with vanishing Lie derivative with respect to $\hat{k}$ (except for $\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}}$, whose transformation law we will define later) as follows[10]:

$$\delta_{\hat{\chi}}\hat{L}_{\hat{\mu}_1\ldots\hat{\mu}_r} = m\hat{\lambda}_{\hat{\mu}_1}\hat{k}^{\hat{\nu}}\hat{L}_{\hat{\nu}\hat{\mu}_2\ldots\hat{\mu}_r} + \ldots + m\hat{\lambda}_{\hat{\mu}_r}\hat{k}^{\hat{\nu}}\hat{L}_{\hat{\mu}_1\ldots\hat{\mu}_{r-1}\hat{\nu}}\,. \tag{1.4}$$

Here the 1-form parameter $\hat{\lambda}_{\hat{\mu}}$ is related to the infinitesimal 2-form parameter $\hat{\chi}_{\hat{\mu}\hat{\nu}}$, which generates the gauge transformations of the 3-form $\hat{C}$ and must also have vanishing Lie derivative $\pounds_{\hat{k}}\hat{\chi}_{\hat{\mu}\hat{\nu}} = 0$, by

$$\hat{\lambda}_{\hat{\mu}} \equiv -\tfrac{1}{2}\left(i_{\hat{k}}\hat{\chi}\right)_{\hat{\mu}} = -\tfrac{1}{2}\hat{k}^{\hat{\nu}}\hat{\chi}_{\hat{\mu}\hat{\nu}}\,. \tag{1.5}$$

In particular, for the metric $\hat{g}_{\hat{\mu}\hat{\nu}}$ and for any r-form $\hat{S}_{\hat{\mu}_1\ldots\hat{\mu}_r} = \hat{S}_{[\hat{\mu}_1\ldots\hat{\mu}_r]}$ (different from the 3-form $\hat{C}$) we have, according to the rule (1.4)

$$\left\{\begin{array}{rcl}\delta_{\hat{\chi}}\hat{g}_{\hat{\mu}\hat{\nu}} & = & 2m\hat{\lambda}_{(\hat{\mu}}\left(i_{\hat{k}}\hat{g}\right)_{\hat{\nu})}\,,\\[2ex] \delta_{\hat{\chi}}\hat{S}_{\hat{\mu}_1\ldots\hat{\mu}_r} & = & (-1)^{r-1}rm\hat{\lambda}_{[\hat{\mu}_1}\left(i_{\hat{k}}\hat{S}\right)_{\hat{\mu}_2\ldots\hat{\mu}_r]}\,.\end{array}\right. \tag{1.6}$$

Observe that the above transformation laws imply

$$\left\{\begin{array}{rcl}\delta_{\hat{\chi}}\sqrt{|\hat{g}|} & = & 0\,,\\[2ex] \delta_{\hat{\chi}}\hat{S}^2 & = & 0\,.\end{array}\right. \tag{1.7}$$

The 3-form is going to play the role of a connection-field with respect to the massive gauge transformations and therefore it does not transform covariantly:

$$\delta_{\hat{\chi}}\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} = 3\partial_{[\hat{\mu}}\hat{\chi}_{\hat{\nu}\hat{\rho}]} + 3m\hat{\lambda}_{[\hat{\mu}}\left(i_{\hat{k}}\hat{C}\right)_{\hat{\nu}\hat{\rho}]}\,. \tag{1.8}$$

For further use we also quote the transformation laws

$$\left\{\begin{array}{rcl}\delta_{\hat{\chi}}\left(i_{\hat{k}}\hat{C}\right)_{\hat{\mu}\hat{\nu}} & = & -4\partial_{[\hat{\mu}}\hat{\lambda}_{\hat{\nu}]}\,,\\[2ex] \delta_{\hat{\chi}}\left(i_{\hat{k}}\hat{g}\right)_{\hat{\mu}} & = & m\hat{\lambda}_{\hat{\mu}}\hat{k}^2\,.\end{array}\right. \tag{1.9}$$

[10] All tensors in this theory transform according to this rule, except for $\hat{C}$ and $\hat{\tilde{C}}$. However, tensors of higher rank also transform under *dual massive transformations*. These have as infinitesimal parameter the 6-form $\hat{\tilde{\lambda}}_{\hat{\mu}_1\ldots\hat{\mu}_6}$ (for more details, see Section 1.3).

The next step in our construction is to build a connection for the massive gauge transformations. The new total connection is

$$\hat{\Omega}_{\hat{a}}{}^{\hat{b}\hat{c}} = \hat{\omega}_{\hat{a}}{}^{\hat{b}\hat{c}}(\hat{e}) + \hat{K}_{\hat{a}}{}^{\hat{b}\hat{c}}\,, \tag{1.10}$$

where the piece that we have added is defined by

$$\hat{K}_{\hat{a}\hat{b}\hat{c}} = \tfrac{m}{4}\left[\hat{k}_{\hat{a}}\left(i_{\hat{k}}\hat{C}\right)_{\hat{b}\hat{c}} + \hat{k}_{\hat{b}}\left(i_{\hat{k}}\hat{C}\right)_{\hat{a}\hat{c}} - \hat{k}_{\hat{c}}\left(i_{\hat{k}}\hat{C}\right)_{\hat{a}\hat{b}}\right]. \tag{1.11}$$

The tensor $\hat{K}$ coincides with the contorsion tensor while the torsion tensor is

$$\hat{T}_{\hat{\mu}\hat{\nu}}{}^{\hat{\rho}} = -2\hat{K}_{[\hat{\mu}\hat{\nu}]}{}^{\hat{\rho}} = \tfrac{m}{2}\left(i_{\hat{k}}\hat{C}\right)_{\hat{\mu}\hat{\nu}}\hat{k}^{\hat{\rho}}\,. \tag{1.12}$$

Observe that in the limit $m \to 0$ one recovers the standard 11-dimensional supergravity connections and gauge transformation laws (assuming that the dependence of the fields on the isometry direction has also been restored).

The exterior covariant derivative on the 3-form $\hat{C}$ is defined by

$$D_{[\hat{\mu}}\hat{C}_{\hat{\nu}\hat{\rho}\hat{\sigma}]} = \partial_{[\hat{\mu}}\hat{C}_{\hat{\nu}\hat{\rho}\hat{\sigma}]} - \tfrac{3}{2}\hat{\Gamma}_{[\hat{\mu}\hat{\nu}}{}^{\hat{\alpha}}\hat{C}_{\hat{\rho}\hat{\sigma}]\hat{\alpha}} = \partial_{[\hat{\mu}}\hat{C}_{\hat{\nu}\hat{\rho}\hat{\sigma}]} + \tfrac{3}{8}m\left(i_{\hat{k}}\hat{C}\right)_{[\hat{\mu}\hat{\nu}}\left(i_{\hat{k}}\hat{C}\right)_{\hat{\rho}\hat{\sigma}]}\,. \tag{1.13}$$

Using the vanishing of the Lie derivative with respect to $\hat{k}$ of $\hat{C}$ it is easy to prove that the covariant derivative does indeed transform covariantly under massive gauge transformations. The 4-form field strength $\hat{G}$ is defined in terms of this covariant derivative

$$\hat{G} = 4D\hat{C}\,, \Rightarrow \delta_{\hat{\chi}}\hat{G} = -4m\hat{\lambda}\left(i_{\hat{k}}\hat{G}\right)\,, \tag{1.14}$$

and thus the obvious kinetic term for $\hat{C}$ in the action, $\hat{G}^2$, is invariant.

We are now ready to write the 11-dimensional action:

$$\begin{aligned}\hat{S} \;=\; & \tfrac{1}{16\pi G_N^{(11)}}\int d^{11}x\,\sqrt{|\hat{g}|}\,\Big\{\hat{R}(\hat{\Omega}) - \tfrac{1}{2\cdot 4!}\hat{G}^2 - \tfrac{1}{8}m^2|\hat{k}^2|^2 \\ & + \tfrac{1}{(144)^2}\tfrac{1}{\sqrt{|\hat{g}|}}\hat{\epsilon}\left[2^4\partial\hat{C}\partial\hat{C}\hat{C} + 3^2 m\partial\hat{C}\hat{C}\left(i_{\hat{k}}\hat{C}\right)^2 + \tfrac{3^3}{20}m^2\hat{C}\left(i_{\hat{k}}\hat{C}\right)^4\right]\Big\}\,.\end{aligned} \tag{1.15}$$

When $m = 0$ and upon restoring the dependence on y this action becomes that of the usual 11-dimensional supergravity theory.

Now we are going to perform the dimensional reduction of this action in the direction associated to the Killing vector $\hat{k}$, parametrized by the coordinate y. From now on we will use coordinates adapted to it, so $\hat{k}^{\hat{\mu}} = \delta^{\hat{\mu}y}$. First we define the 10-dimensional fields. We use the notation and conventions for the gauge transformations explained in Appendix B. The bosonic fields of the 10-dimensional type IIA theory are

$$\left\{g_{\mu\nu}, B_{\mu\nu}, \phi, C^{(1)}, C^{(3)}\right\} . \tag{1.16}$$

The R-R 1-form $C^{(1)}$ plays the role of a Stueckelberg field in the massive theory and could in principle be eliminated by an appropriate gauge choice, but it is important for us to keep it. Later we will also introduce the fields dual to $B, C^{(1)}, C^{(3)}$, namely $\tilde{B}_{\rm IIA}, C^{(7)}, C^{(5)}$. These fields occur in the effective worldvolume actions we will deal with in Section 3 and it will be important to introduce them properly.

The relation between the 11- and the above 10-dimensional fields is the same as in the massless case (see, e.g. [20]). In particular, we can use the same elfbein basis:

$$\begin{aligned}
\left(\hat{e}_{\hat{\mu}}{}^{\hat{a}}\right) &= \begin{pmatrix} e^{-\frac{1}{3}\phi} e_{\mu}{}^{a} & e^{\frac{2}{3}\phi} C^{(1)}{}_{\mu} \\ \\ 0 & e^{\frac{2}{3}\phi} \end{pmatrix} , \\
\left(\hat{e}_{\hat{a}}{}^{\hat{\mu}}\right) &= \begin{pmatrix} e^{\frac{1}{3}\phi} e_{a}{}^{\mu} & -e^{\frac{1}{3}\phi} C^{(1)}{}_{a} \\ \\ 0 & e^{-\frac{2}{3}\phi} \end{pmatrix} .
\end{aligned} \tag{1.17}$$

The 11-dimensional fields are expressed in terms of the 10-dimensional ones by

$$\left\{\begin{array}{rcl} \hat{g}_{yy} &=& -e^{\frac{4}{3}\phi} , \\ \\ \hat{g}_{\mu y} &=& \left(i_{\hat{k}} \hat{g}\right)_{\mu} = -e^{\frac{4}{3}\phi} C^{(1)}{}_{\mu} , \\ \\ \hat{g}_{\mu\nu} &=& e^{-\frac{2}{3}\phi} g_{\mu\nu} - e^{\frac{4}{3}\phi} C^{(1)}{}_{\mu} C^{(1)}{}_{\nu} , \end{array}\right. \qquad \left\{\begin{array}{rcl} \hat{C}_{\mu\nu\rho} &=& C^{(3)}{}_{\mu\nu\rho} , \\ \\ \hat{C}_{\mu\nu y} &=& \left(i_{\hat{k}} \hat{C}\right)_{\mu\nu} = B_{\mu\nu} . \end{array}\right. \tag{1.18}$$

Now we are going to perform the reduction of the action. We first consider the Ricci scalar term. To reduce this term we use a slight generalization of Palatini's identity which in d dimensions takes the form

$$\begin{aligned}
&\int d^{d}x \, \sqrt{|g|} \, e^{-2\varphi} \, [R] = \\
&\qquad \int d^{d}x \, \sqrt{|g|} \, \left\{ -e^{-2\varphi} \left[\omega_{b}{}^{ba} \omega_{c}{}^{c}{}_{a} + \omega_{a}{}^{bc} \omega_{bc}{}^{a} + 4 \omega_{b}{}^{ba} (\partial_{a} \varphi) \right] \right\} .
\end{aligned} \tag{1.19}$$

With the above ansatz for the elfbeins the non-vanishing components of the 11-dimensional connection are

$$
\begin{aligned}
\hat{\Omega}_{yay} &= -\tfrac{2}{3}e^{\frac{1}{3}\phi}\partial_a\phi\,, & \hat{\Omega}_{yab} &= -\tfrac{1}{2}e^{\frac{4}{3}\phi}G^{(2)}{}_{ab}\,, \\
\hat{\Omega}_{aby} &= \tfrac{1}{2}e^{\frac{4}{3}\phi}G^{(2)}{}_{ab}\,, & \hat{\Omega}_{abc} &= e^{\frac{1}{3}\phi}\left(\omega_{abc}(e) + \tfrac{2}{3}\delta_{a[b}\partial_{c]}\phi\right)\,,
\end{aligned}
\tag{1.20}
$$

where[11]

$$
G^{(2)} = 2\partial C^{(1)} + \tfrac{m}{2}B\,, \tag{1.22}
$$

is the field strength of the 10-dimensional vector field $C^{(1)}{}_\mu$ and the term linear in m is a Chern-Simons term[12]. Using

$$
\sqrt{|\hat{g}|} = \sqrt{|g|}\, e^{-\frac{8}{3}\phi}\,, \tag{1.23}
$$

plus Palatini's identity Eq. (1.19) for $d = 11$ and $\varphi = 0$, and the fact that the coordinate y conventionally lives in a circle of radius equal to the string length $\ell_s = \sqrt{\alpha'}$ we find

$$
\begin{aligned}
&\int d^{11}x\,\sqrt{|\hat{g}|}\,[\hat{R}(\hat{\Omega})] = \\
&\quad 2\pi\ell_s \int d^{10}x\,\sqrt{|g|}\left\{-e^{-2\phi}\left[\left(\omega_b{}^{ba} + 2\partial^a\phi\right)^2 + \omega_a{}^{bc}\omega_{bc}{}^a\right]\right. \\
&\quad \left. -\tfrac{1}{4}\left(G^{(2)}\right)^2\right\}\,.
\end{aligned}
\tag{1.24}
$$

Finally, using Palatini's identity Eq. (1.19) again, but now for $d = 10$ and $\varphi = \phi$, we get for the Ricci scalar term:

$$
\begin{aligned}
&\int d^{11}x\,\sqrt{|\hat{g}|}\,\left[\hat{R}(\hat{\Omega})\right] = \\
&\quad 2\pi\ell_s \int d^{10}x\,\sqrt{|g|}\,\left\{e^{-2\phi}\left[R - 4(\partial\phi)^2\right] - \tfrac{1}{4}\left(G^{(2)}\right)^2\right\}\,.
\end{aligned}
\tag{1.25}
$$

---

[11] When indices are not shown explicitly and partial derivatives are used, we assume that all indices are completely antisymmetrized in the obvious order. For instance the equation below means

$$
G^{(2)}{}_{\mu\nu} = 2\partial_{[\mu}C^{(1)}{}_{\nu]} + \tfrac{m}{2}B_{\mu\nu}\,. \tag{1.21}
$$

[12] All the definitions for the field strengths and gauge transformations can be found in Appendix B in differential form language.

Now we have to reduce the $\hat{G}^2$-term in the action. We identify field strengths in eleven and ten dimensions with flat indices (this automatically ensures gauge invariance) taking into account the scaling of the 10-dimensional metric

$$G^{(4)}{}_{abcd} = e^{-\frac{4}{3}\phi}\ \hat{G}_{abcd}\,, \tag{1.26}$$

which leads to

$$G^{(4)} = 4\left(\partial C^{(3)} - 3\partial BC^{(1)} + \tfrac{3}{8}mB^2\right)\,. \tag{1.27}$$

The remaining components of $\hat{G}$ are given by

$$\hat{G}_{abcy} = e^{\frac{1}{3}\phi}H_{abc}\,, \tag{1.28}$$

where H is the field strength of the two-form B

$$H = 3\partial B\,, \tag{1.29}$$

and the contribution of the $\hat{G}$-term to the 10-dimensional action becomes

$$\begin{aligned}\int d^{11}x\ \sqrt{|\hat{g}|}\ \left[-\tfrac{1}{2\cdot 4!}\left(\hat{G}\right)^2\right] &= \\ 2\pi\ell_s\int d^{10}x\ \sqrt{|g|}\ \left[\tfrac{1}{2\cdot 3!}e^{-2\phi}H^2 - \tfrac{1}{2\cdot 4!}\left(G^{(4)}\right)^2\right]\,.\end{aligned} \tag{1.30}$$

Applying the identity $\hat{k}^2 = -e^{\frac{4}{3}\phi}$ the cosmological constant term contribution is

$$\int d^{11}x\ \sqrt{|\hat{g}|}\ \left[-\tfrac{1}{8}m^2|\hat{k}^2|^2\right] = 2\pi\ell_s\int d^{10}x\ \sqrt{|g|}\ \left[-\tfrac{1}{8}m^2\right]\,. \tag{1.31}$$

Next we consider the first term in the WZ term. Taking into account that

$$\hat{\epsilon}^{\ \mu_0\cdots\mu_9 y} = \epsilon^{\ \mu_0\cdots\mu_9}\,, \tag{1.32}$$

and using curved indices we find that:

$$2^4\hat{\epsilon}\partial\hat{C}\partial\hat{C}\hat{C} = 2^4\cdot 3\epsilon\partial C^{(3)}\partial C^{(3)}B - 2^5\cdot 3\epsilon\partial C^{(3)}\partial BC^{(3)}\,. \tag{1.33}$$

Integrating by parts we get

$$\int d^{11}x \ \left[\tfrac{1}{(144)^2} \ \hat{\epsilon} \ 2^4 \partial\hat{C}\partial\hat{C}\hat{C}\right] = 2\pi\ell_s \int d^{10}x \ \left[\tfrac{1}{144}\epsilon\partial C^{(3)}\partial C^{(3)} B\right] . \tag{1.34}$$

Next we have

$$3^2 m\hat{\epsilon}\partial\hat{C}\hat{C}\left(i_{\hat{k}}\hat{C}\right)^2 = 3^3 m\epsilon\left[\partial C^{(3)}B^3 - \partial B C^{(3)} B^2\right] , \tag{1.35}$$

and integrating by parts the second term we find

$$\int d^{11}x \ \left[\tfrac{1}{(144)^2} \ \hat{\epsilon} \ 3^2 m\partial\hat{C}\hat{C}\left(i_{\hat{k}}\hat{C}\right)^2\right] = 2\pi\ell_s \int d^{10}x \ \left[\tfrac{1}{144} \ \epsilon \ \tfrac{1}{4} m\partial C^{(3)} B^3\right] . \tag{1.36}$$

Finally,

$$\tfrac{3^3}{20} m^2 \hat{\epsilon}\hat{C}\left(i_{\hat{k}}\hat{C}\right)^4 = \tfrac{3^4}{20} m^2 \epsilon B^5 , \tag{1.37}$$

and

$$\int d^{11}x \ \left[\tfrac{1}{(144)^2}\hat{\epsilon}\tfrac{3^3}{20} m^2 \hat{C}\left(i_{\hat{k}}\hat{C}\right)^4\right] = 2\pi\ell_s \int d^{10}x \ \left[\tfrac{1}{144}\epsilon\tfrac{3^2}{320} m^2 B^5\right] . \tag{1.38}$$

Putting all these results together we find the bosonic part of the massive $N = 2A, d = 10$ supergravity action [6] in the string frame, as given in [8]

$$\begin{aligned} S \ = \ & \tfrac{1}{16\pi G_N^{(10)}} \int d^{10}x \ \sqrt{|g|}\left\{e^{-2\phi}\left[R(\omega) - 4\left(\partial\phi\right)^2 + \tfrac{1}{2\cdot 3!}H^2\right]\right. \\ & -\left[\tfrac{1}{4}\left(G^{(2)}\right)^2 + \tfrac{1}{2\cdot 4!}\left(G^{(4)}\right)^2 + \tfrac{1}{8}m^2\right] \\ & \left. +\tfrac{1}{144}\tfrac{1}{\sqrt{|g|}} \ \epsilon\left[\partial C^{(3)}\partial C^{(3)} B + \tfrac{1}{4} m\partial C^{(3)} B^3 + \tfrac{9}{320} m^2 B^5\right]\right\} , \end{aligned} \tag{1.39}$$

where

$$G_N^{(10)} = (2\pi\ell_s)^{-1} G_N^{(11)} . \tag{1.40}$$

This action is invariant under the gauge transformations of the R-R fields

$$\left\{\begin{array}{rcl} \delta_{\Lambda^{(0)}} C^{(1)} & = & \partial\Lambda^{(0)} , \\ \delta_{\Lambda^{(0)}} C^{(3)} & = & 3\partial\Lambda^{(0)} B , \\ \delta_{\Lambda^{(2)}} C^{(3)} & = & 3\partial\Lambda^{(2)} . \end{array}\right. \tag{1.41}$$

The $\delta_{\Lambda^{(0)}}$ transformations are associated to reparametrizations of the compact coordinate of the form

$$\delta_{\Lambda^{(0)}} \hat{X}^{\hat{\mu}} = -\Lambda^{(0)}(x) \delta^{\hat{\mu} y} \,, \tag{1.42}$$

while the gauge transformation of the 3-form potential is inherited from the massless gauge transformation of the 11-dimensional 3-form potential.

The above action is also invariant under the massive gauge transformations

$$\left\{ \begin{array}{rcl} \delta_\lambda C^{(1)} & = & m\lambda \,, \\ & & \\ \delta_\lambda C^{(3)} & = & 3m\lambda B \,, \\ & & \\ \delta_\lambda B & = & -4\partial\lambda \,, \end{array} \right. \tag{1.43}$$

which are directly inherited from the 11-dimensional massive gauge transformations. Observe that $C^{(1)}$ transforms by shifts of the parameter of the gauge transformations of B. Thus, $C^{(1)}$ can be completely eliminated by a massive gauge transformation, leaving in the action a mass term for B: $m^2 B^2$. It is usually said that $C^{(1)}$ is the Stueckelberg field which is "eaten up" by B, which then gets a mass.

This concludes our description of the massive 11-dimensional supergravity theory.

1.2 Dual Massless 11-Dimensional Supergravity

It is known that in the worldvolume effective actions of supersymmetric solitons all the potentials of the theory may appear: not only those that occur in the usual formulation but also their (electro-magnetic) duals. For this reason we explore here the dualization of 11-dimensional supergravity. Since additional subtleties occur in the massive case we will first discuss in this Subsection the massless case. Next we will discuss in the next Subsection dual massive IIA supergravity and, finally, discuss in Section 1.4 dual massive 11-dimensional supergravity.

The usual procedure for formulating a theory in terms of the dual of some potential consists in finding an intermediate first-order action with an extra field. Integration of the extra field yields the Bianchi identity for the original potential field strength and integration of the field strength leads to the dual action with the auxiliary field playing now the role of dual potential.

There is a systematic way of constructing the first-order intermediate action: consider the action as a function of the field strength and add a

Lagrange multiplier term to enforce its Bianchi identity. This recipe fails when the action cannot be written in terms of the field strength only and the potential itself (i.e. not its derivative) appears explicitly in it. This is what happens in the action of massless d=11 supergravity and this is the reason why a formulation of 11-dimensional supergravity in terms of the dual 6-form $\hat{\tilde{C}}$ only has not been found.

Strictly speaking, for the sake of constructing worldvolume actions, it is enough to introduce dual potentials only on-shell [21] and one can also allow for the introduction of potentials and dual potentials at the same time. However, inspired by the work of [16], we would like to point out that there is a way of finding the dual theory, even if in the original action the potential does not only occur as a derivative. The idea consists in finding an intermediate action with an auxiliary field such that the 3-form potential $\hat{C}$ only appears through its derivatives. Elimination of the auxiliary field leads to the usual 11-dimensional supergravity action. To this intermediate action one can now apply the usual procedure for dualization, eliminating the auxiliary field in the last stage.

An intermediate action with the required properties is

$$\begin{aligned} S[\hat{g},\hat{C},\hat{L}] &= \tfrac{1}{16\pi G_N^{(11)}}\int d^{11}x\,\sqrt{|\hat{g}|}\,\Big\{R-\tfrac{1}{2\cdot 4!}\hat{G}^2 \\ &\quad -\tfrac{1}{(144)^2}\tfrac{\hat{\epsilon}}{\sqrt{|\hat{g}|}}\Big[\hat{G}\hat{G}\hat{L}+\tfrac{4}{3}\hat{G}\partial\hat{L}\hat{L}+\tfrac{16}{27}\partial\hat{L}\partial\hat{L}\hat{L}\Big]\Big\}\,. \end{aligned} \tag{1.44}$$

The equation of motion for the auxiliary field $\hat{L}_{\hat{\mu}_1\hat{\mu}_2\hat{\mu}_3}$ is

$$\hat{\epsilon}^{\hat{\alpha}_1\hat{\alpha}_2\hat{\alpha}_3\hat{\mu}_1\ldots\hat{\mu}_4\hat{\nu}_1\ldots\hat{\nu}_4}\left(\partial\hat{C}+\tfrac{1}{3}\partial\hat{L}\right)_{\hat{\mu}_1\ldots\hat{\mu}_4}\left(\partial\hat{C}+\tfrac{1}{3}\partial\hat{L}\right)_{\hat{\nu}_1\ldots\hat{\nu}_4}=0\,. \tag{1.45}$$

Substituting the solution

$$\hat{L}=-3\hat{C} \tag{1.46}$$

into the above auxiliary action one recovers the usual action of 11-dimensional supergravity.

We can now consider this action as an action for $\hat{G}$ and add to it the Lagrange multiplier term

$$\tfrac{1}{16\pi G_N^{(11)}}\int d^{11}x\tfrac{1}{4!\cdot 6!}\hat{\epsilon}\partial\hat{\tilde{C}}\hat{G}\,, \tag{1.47}$$

where we have already integrated it by parts, and try to eliminate $\hat{G}$ by using its equation of motion. However, in this equation of motion both $\hat{G}$ and ${}^{\star}\hat{G}$ appear:

$$\hat{G} = {}^{\star}\left\{7\left[\partial\hat{\hat{C}} - \tfrac{5}{3}\left(\hat{G} + \tfrac{2}{3}\partial\hat{L}\right)\hat{L}\right]\right\} . \tag{1.48}$$

After some tensor gymnastics we find the following equation for $\hat{G}$ as a function of $\hat{\hat{C}}$ and $\hat{L}$:

$$\hat{G}^{\hat{\alpha}_1\ldots\hat{\alpha}_4} = \left(f^{-1}\right)^{\hat{\alpha}_1\ldots\hat{\alpha}_4}{}_{\hat{\beta}_1\ldots\hat{\beta}_4}\left(\hat{B}^{\hat{\beta}_1\ldots\hat{\beta}_4} - {}^{\star}\hat{B}^{\hat{\beta}_1\ldots\hat{\beta}_7}\hat{A}_{\hat{\beta}_5\hat{\beta}_6\hat{\beta}_7}\right) , \tag{1.49}$$

where

$$\begin{aligned}
f_{\hat{\alpha}_1\ldots\hat{\alpha}_4}{}^{\hat{\beta}_1\ldots\hat{\beta}_4} &= \delta_{\hat{\alpha}_1\ldots\hat{\alpha}_4}{}^{\hat{\beta}_1\ldots\hat{\beta}_4} - \tfrac{7!}{4!}\delta_{\hat{\alpha}_1\ldots\hat{\alpha}_7}{}^{\hat{\beta}_1\ldots\hat{\beta}_7}\hat{A}_{\hat{\beta}_5\hat{\beta}_6\hat{\beta}_7}\hat{A}^{\hat{\alpha}_5\hat{\alpha}_6\hat{\alpha}_7} . \\
\hat{A}_{\hat{\alpha}_1\hat{\alpha}_2\hat{\alpha}_3} &= \tfrac{1}{3^3 \cdot 2^4}\hat{L}_{\hat{\alpha}_1\hat{\alpha}_2\hat{\alpha}_3} , \\
\hat{B}^{\hat{\alpha}_1\ldots\hat{\alpha}_4} &= \tfrac{1}{6!\sqrt{|g|}}\hat{\epsilon}^{\hat{\alpha}_1\ldots\hat{\alpha}_{10}}\left(\partial\hat{\hat{C}} - \tfrac{10}{9}\partial\hat{L}\hat{L}\right)_{\hat{\alpha}_5\ldots\hat{\alpha}_{10}} .
\end{aligned} \tag{1.50}$$

The next step in the dualization procedure, namely the elimination of the auxiliary field, can now be done. The result (too complicated to be meaningful) would be $\hat{L}$ as a complicated non-linear and perhaps non-local function of $\hat{\hat{C}}$ and $\partial\hat{\hat{C}}$. This relation, together with the relation between $\hat{L}$ and $\hat{C}$ would give us the relation between $\hat{C}$ and its dual field $\hat{\hat{C}}$. The lesson to be learned here is that an 11-dimensional supergravity theory formulated in terms of only the 6-form $\hat{\hat{C}}$ does exist although it is highly non-linear.

What we know about $\hat{\hat{C}}$ is, however, enough to determine its gauge transformation laws. We can use Eq. (1.48) plus the relation between $\hat{L}$ and $\hat{C}$ and the fact that it has to be gauge-invariant to find that [21]

$$\left\{\begin{aligned}
\delta_{\hat{\hat{\chi}}}\hat{\hat{C}} &= 6\partial\hat{\hat{\chi}} , \\
\delta_{\hat{\chi}}\hat{\hat{C}} &= 30\partial\hat{\chi}\hat{C} ,
\end{aligned}\right. \tag{1.51}$$

with the understanding that $\hat{C}$ is a complicated function of $\hat{\hat{C}}$. We can write the field strength of the latter as

$$\hat{\hat{G}} = 7\left(\partial\hat{\hat{C}} + 10\hat{C}\partial\hat{C}\right) , \tag{1.52}$$

so Eq. (1.48) is nothing but the usual duality relation

$$\hat{G} = {}^{\star}\hat{\tilde{G}}\,. \tag{1.53}$$

1.3 Dual Massive IIA Supergravity

In this Section we are going to discuss several aspects concerning the duals of the potentials appearing in massive IIA supergravity.

First we consider the massless theory as a special case. It is easy to see that in dualizing the NS/NS 2-form B or the R-R 3-form $C^{(3)}$ one runs into the non-linearity problem we found in eleven dimensions. The R-R 1-form $C^{(1)}$ cannot be immediately dualized off-shell since it appears explicitly in $G^{(4)}$. However, it can be reabsorbed into a redefinition of $C^{(3)}$ (after an integration by parts in the action). In the end, though, the same problem occurs. One can only define field strengths for the dual potentials if one uses also the original potentials (understood as functions of the dual ones) and the gauge transformation laws can then be determined. We will do so after discussing the massive case.

In the massive theory on top of the above-mentioned problems we have the problem of having to dualize the *massive* NS/NS 2-form B. In principle it is not clear what the dual of a massive k-form potential is, since a $(d-k-2)$-form potential (which is the dual in absence of mass) always has a different number of degrees of freedom. However, a massive $(d-k-1)$-form does describe the same number of degrees of freedom as a massive k-form. Dualization of the corresponding action can be achieved through the use of an intermediate action (see Ref. [14] and references therein). The construction of this intermediate action with an auxiliary field is not systematic and has to be made on a case by case basis, which obscures the meaning of the dualization.

In order to set up a more systematic procedure, it is convenient to rewrite the action for the massive k-form using an auxiliary $(k-1)$-form which plays the role of a Stueckelberg field. The resulting action is invariant under massive gauge transformations. That permits the elimination of the Stueckelberg field so that the usual action is recovered. One now dualizes first the Stueckelberg $(k-1)$-form field and then the k-form potential. The latter dualization is possible because after the first dualization the action only depends on the k-form potential via its field strength. This procedure is described in full detail in Appendix A. Here we only need to know the qualitative result: after the two dualizations we find that the $(d-k-2)$ dual (in the massless sense) of the massive k-form field becomes the Stueckelberg field of the $(d-k-1)$

dual (in the massless sense) of the original Stueckelberg $(k-1)$-form. The mass parameter is the same after dualization.

In our case, the R-R 1-form $C^{(1)}$ dualizes into the R-R 7-form $C^{(7)}$ which now is massive, while the NS/NS 2-form B, which is massive, dualizes into the NS/NS 6-form $\tilde{B}_{\rm IIA}$ which is nothing but the Stueckelberg field for $C^{(7)}$. There is a dual massive transformation but the mass parameter is the same. This result has to be taken into account when trying to find the field strength of the dual fields, specially of $\tilde{B}_{\rm IIA}$. From now on, since confusion with the Heterotic and IIB cases is not possible, we will denote $\tilde{B}_{\rm IIA}$ simply as $\tilde{B}$.

In all other respects the dualization of the massive type IIA theory proceeds as in the massless case. To find the field strengths we have to allow for the presence of the original potentials. Now we are going to construct those field strengths starting with the massless case. For $\tilde{B}$ and $C^{(5)}$ we can use the dimensional reduction of the field strength of $\hat{\tilde{C}}$. This 11-dimensional field splits as follows in terms of 10-dimensional fields:

$$\left\{\begin{array}{rcl} \hat{\tilde{C}}_{\mu_1\cdots\mu_5 y} & = & \left(i_{\hat{k}}\hat{\tilde{C}}\right)_{\mu_1\cdots\mu_5} \\ & = & C^{(5)}{}_{\mu_1\cdots\mu_5} - 5C^{(3)}{}_{[\mu_1\mu_2\mu_3}B_{\mu_4\mu_5]}\,, \\ \hat{\tilde{C}}_{\mu_1\cdots\mu_6} & = & -\tilde{B}_{\mu_1\cdots\mu_6}\,. \end{array}\right. \tag{1.54}$$

It is now immediate to find the massless field strengths[13]:

$$\left\{\begin{array}{rcl} G^{(6)}_{(0)} & = & 6\left[\partial C^{(5)} - 10\partial B C^{(3)}\right]\,, \\ \tilde{H}_{(0)} & = & 7\left[\partial\tilde{B} + G^{(6)}_{(0)}C^{(1)} - 10C^{(3)}\partial C^{(3)}\right]\,. \end{array}\right. \tag{1.55}$$

To find the massless field strength of the R-R 7-form $C^{(7)}$ (dual to $C^{(1)}$) we rewrite the equation of motion of $C^{(1)}$ as follows:

$$\partial\left\{{}^\star G^{(2)}_{(0)} - 28\left[{}^\star G^{(4)}_{(0)}B - 30\partial C^{(3)}B^2\right]\right\} = 0\,. \tag{1.56}$$

This equation should be equivalent to the Bianchi identity of the dual $C^{(7)}$. We can identify the expression in curly brackets with $8\partial C^{(7)}$ up to a total derivative. (The factor of 8 ensures canonical normalization of $C^{(7)}$.) This total derivative is arbitrary and it amounts to a different definition of $C^{(7)}$ with different gauge transformations. The choice that gives a $C^{(7)}$ in the form proposed in Refs. [13, 22] is

[13] The subscript (0) indicates that they correspond to the massless case.

$$
\begin{aligned}
&8\partial\left[C^{(7)}+3\cdot 5\cdot 7C^{(3)}B^2-3\cdot 7C^{(5)}B\right]= \\
&\qquad\qquad ={}^\star G^{(2)}_{(0)}-28\left[{}^\star G^{(4)}_{(0)}B-30\partial C^{(3)}B^2\right],
\end{aligned}
\tag{1.57}
$$

and so, using that ${}^\star G^{(2)}_{(0)}=G^{(8)}_{(0)}$ we find

$$
G^{(8)}_{(0)}=8\left[\partial C^{(7)}-3\cdot 7\partial BC^{(5)}\right]. \tag{1.58}
$$

The gauge transformation laws of $C^{(5)},\tilde{B},C^{(7)}$ can be found by using the gauge invariance (by construction) of the above field strengths. We find:

$$
\left\{
\begin{array}{rcl}
\delta C^{(5)} &=& 15\partial\Lambda^{(0)}B^2+30\partial\Lambda^{(2)}B+5\partial\Lambda^{(4)}\,, \\
\delta\tilde{B} &=& 6\partial\Lambda^{(0)}\left(C^{(5)}-5C^{(3)}B\right)-30\partial\Lambda^{(2)}C^{(3)}+6\partial\tilde{\Lambda}\,, \\
\delta C^{(7)} &=& 3\cdot 5\cdot 7\partial\Lambda^{(0)}B^3+3^2\cdot 5\cdot 7\partial\Lambda^{(2)}B^2 \\
&& +3\cdot 5\cdot 7\partial\Lambda^{(4)}B+7\partial\Lambda^{(6)}\,.
\end{array}
\right.
\tag{1.59}
$$

The massive field strengths are found by adding extra terms proportional to m, which are uniquely determined by requiring invariance under massive gauge transformations. The gauge transformations in the massive theory can be found from those of the massless theory (above) by the replacements

$$
\left\{
\begin{array}{rcl}
\Lambda &\to& -2\lambda\,, \\
\partial\Lambda^{(0)} &\to& \partial\Lambda^{(0)}+m\lambda\,,
\end{array}
\right.
\qquad
\left\{
\begin{array}{rcl}
\Lambda^{(6)} &\to& 2\tilde{\lambda}\,, \\
\partial\tilde{\Lambda} &\to& \partial\tilde{\Lambda}+\frac{m}{6}\tilde{\lambda}\,,
\end{array}
\right.
\tag{1.60}
$$

where we have taken into account that now $\tilde{B}$ is the Stueckelberg field for $C^{(7)}$ (for this reason we expect a term $mC^{(7)}$ in $\tilde{H}$). The result is[14]

$$
\left\{
\begin{array}{rcl}
G^{(6)} &=& 6\left[\partial C^{(5)}-10\partial BC^{(3)}+\frac{5}{4}mB^3\right], \\
\tilde{H} &=& 7\left[\partial\tilde{B}+G^{(6)}C^{(1)}-10C^{(3)}\partial C^{(3)}\right. \\
&& \left.-\frac{1}{14}m\left(C^{(7)}-3\cdot 7C^{(5)}B+3\cdot 5\cdot 7C^{(3)}B^2\right)\right], \\
G^{(8)} &=& 8\left[\partial C^{(7)}-3\cdot 7\partial BC^{(5)}+\frac{3\cdot 5\cdot 7}{16}mB^4\right].
\end{array}
\right.
\tag{1.61}
$$

[14] All these formulae are collected in Appendix B.

It is also useful to have the relation between the infinitesimal gauge transformation parameters of the 11- and 10-dimensional theories:

$$\left\{\begin{array}{rcl}\Lambda^{(2)}{}_{\mu\nu} & = & \hat{\chi}_{\mu\nu}\,,\\ \lambda_{\mu} & = & \hat{\lambda}_{\mu} = \\ & = & -\frac{1}{2}\left(i_{\hat{k}}\hat{\chi}\right)_{\mu} = -\frac{1}{2}\hat{\chi}_{\mu y}\,,\end{array}\right. \qquad \left\{\begin{array}{rcl}\Lambda^{(4)}{}_{\mu_1\ldots\mu_4} & = & \left(i_{\hat{k}}\hat{\tilde{\chi}}\right)_{\mu_1\ldots\mu_4}\\ & = & \hat{\tilde{\chi}}_{\mu_1\ldots\mu_4 y}\,,\\ \tilde{\Lambda}_{\mu_1\ldots\mu_5} & = & -\hat{\tilde{\chi}}_{\mu_1\ldots\mu_5}\,.\end{array}\right. \tag{1.62}$$

The gauge parameter $\Lambda^{(0)}$ is related to general coordinate transformations of the compact eleventh coordinate y. The parameters $\Lambda^{(6)}$ and $\tilde{\lambda}$ will be discussed in the next Subsection in which we will try to understand the higher-dimensional origin of $C^{(7)}$.

1.4 Dual Massive 11-Dimensional Supergravity

Dual massive 11-dimensional supergravity is by definition the theory that upon dimensional reduction leads to dual massive IIA supergravity. Using the relations between 10- and 11-dimensional fields and gauge-transformation parameters it is immediate to find the massive gauge-transformation rules for $\hat{\tilde{C}}$, which we write together with those of $\hat{C}$ for convenience:

$$\left\{\begin{array}{rcl}\delta\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} & = & 3\partial_{[\hat{\mu}}\hat{\chi}_{\hat{\nu}\hat{\rho}]} + 3m\hat{\lambda}_{[\hat{\mu}}\left(i_{\hat{k}}\hat{C}\right)_{\hat{\nu}\hat{\rho}]}\,,\\ \delta\hat{\tilde{C}}_{\hat{\mu}_1\ldots\hat{\mu}_6} & = & 6\partial_{[\hat{\mu}_1}\hat{\tilde{\chi}}_{\hat{\mu}_2\ldots\hat{\mu}_6]} + 30\partial_{[\hat{\mu}_1}\hat{\chi}_{\hat{\mu}_2\hat{\mu}_3}\hat{C}_{\mu_4\hat{\mu}_5\hat{\mu}_6]}\\ & & -6m\hat{\lambda}_{[\hat{\mu}_1}\left(i_{\hat{k}}\hat{\tilde{C}}\right)_{\hat{\mu}_2\ldots\hat{\mu}_6]} - m\hat{\tilde{\lambda}}_{\hat{\mu}_1\ldots\hat{\mu}_6}\,.\end{array}\right. \tag{1.63}$$

The first and second terms are also present in the massless case. The third term is just the massive gauge transformation of a 6-form according to the general rule Eq. (1.4)[15]. The fourth term corresponds to a *dual massive gauge transformation.* In order to understand its origin, we first observe that the above transformation law only reduces to those of $\tilde{B}, C^{(5)}$ that we found in the previous Subsection if

[15]Observe, though, that just as $\hat{C}$, $\hat{\tilde{C}}$ has additional terms and does not transform covariantly under massive gauge transformations. Extra terms are needed to construct a covariant gauge field strength.

$$\left(i_{\hat{k}}\hat{\hat{\lambda}}\right)_{\mu_1\ldots\mu_5} = \hat{\hat{\lambda}}_{\mu_1\ldots\mu_5 y} = 0\,, \tag{1.64}$$

because otherwise $C^{(5)}$ would have an extra symmetry under massive shifts by a 5-form. The same property is also satisfied by $\hat{\lambda}$ precisely because it is equal to $(i_{\hat{k}}\hat{\chi})$. This analogy then leads us to identify

$$\hat{\hat{\lambda}} \equiv a\left(i_{\hat{k}}\hat{\Sigma}\right)\,, \tag{1.65}$$

where $\hat{\Sigma}$ is a 7-form and a a constant to be determined. A 7-form gauge parameter is associated to an 8-form gauge potential. The only such potential available is the dual of the Killing vector, that we can consider as a 1-form potential $\hat{k}_{\hat{\mu}}$.

The same conclusion is reached if one writes the 11-dimensional massive field strength of $\hat{\tilde{C}}$:

$$\hat{\tilde{G}} = 7\left\{D\hat{\tilde{C}} + 10\hat{C}D\hat{C} - \tfrac{5}{4}m\left(i_{\hat{k}}\hat{C}\right)\left(i_{\hat{k}}\hat{C}\right)\hat{C} + \tfrac{1}{14}m\left(i_{\hat{k}}\hat{\tilde{N}}\right)\right\}\,, \tag{1.66}$$

where $\hat{\tilde{N}}$ is an 8-form potential such that

$$\left(i_{\hat{k}}\hat{\tilde{N}}\right)_{\mu_1\ldots\mu_7} = C^{(7)}{}_{\mu_1\ldots\mu_7} - 5\cdot 7C^{(3)}{}_{[\mu_1\mu_2\mu_3}B_{\mu_4\mu_5}B_{\mu_6\mu_7]}\,. \tag{1.67}$$

A 7-form potential was needed to absorb the dual massive gauge transformations of $\hat{\tilde{C}}$ but the contraction of the 7-form potential with the Killing vector would again be zero. Thus, the 7-form potential has to be the contraction of the Killing vector with an 8-form potential that we have denoted by $\hat{\tilde{N}}$.

The field strength of $\hat{\tilde{C}}$ does transform covariantly under massive gauge transformations:

$$\delta_{\hat{\chi}}\hat{\tilde{G}} = 7m\hat{\lambda}\left(i_{\hat{k}}\hat{\tilde{G}}\right)\,. \tag{1.68}$$

To complete the picture we have to determine the gauge transformation laws and field strength of $\hat{\tilde{N}}$ and then check that it leads to the field strength of $C^{(7)}$.

In a first step we find that

$$
\begin{aligned}
\delta\hat{\tilde{N}}_{\mu_1\ldots\mu_7 y} &= 16a\partial_{[\mu_1}\hat{\Sigma}_{\mu_2\ldots\mu_7 y]} + \tfrac{8!}{3\cdot 4!}\partial_{[\mu_1}\hat{\chi}_{\mu_2\mu_3}\hat{C}_{\mu_4\mu_5\mu_6}\left(i_{\hat{k}}\hat{C}\right)_{\mu_7 y]} \\
&\quad +3\tfrac{8!}{6!}\partial_{[\mu_1}\hat{\tilde{\chi}}_{\mu_2\ldots\mu_6}\left(i_{\hat{k}}\hat{C}\right)_{\mu_7 y]}\ ,
\end{aligned}
\tag{1.69}
$$

which looks somewhat strange because one would expect terms of the form $\left(i_{\hat{k}}\hat{C}\right)$ to appear only at order m. However, since $\hat{\tilde{N}}$ is the dual of the Killing vector it only exists when there is an isometry and, thus, those terms are allowed (since they do not appear in the massless case).

This suggests the following 11-dimensional transformation law

$$
\delta\hat{\tilde{N}} = 16a\partial\hat{\Sigma} + \tfrac{8!}{3\cdot 4!}\partial\hat{\chi}\hat{C}\left(i_{\hat{k}}\hat{C}\right) + 3\tfrac{8!}{6!}\partial\hat{\tilde{\chi}}\left(i_{\hat{k}}\hat{C}\right) - 8m\hat{\lambda}\left(i_{\hat{k}}\hat{\tilde{N}}\right)\ , \tag{1.70}
$$

where we have added a term corresponding to massive gauge transformations of an 8-form. The contraction of this term with the Killing vector vanishes and one recovers the previous formula.

If we accept the existence of an 8-form in 11 dimensions then this leads naturally to a 7-form and an 8-form in 10 dimensions. The 7-form is $C^{(7)}$, the dual of $C^{(1)}$. Similarly, it seems that the 8-form naturally corresponds to the dual of the dilaton. If true, this suggests the existence of a related NS/NS 7-brane to which this 8-form couples. We will not pursue these ideas further here.

2 Worldvolume Fields

To construct the worldvolume effective actions of massive branes in the next Section a knowledge of the background fields and worldvolume fields will be necessary. In the previous Section we have studied the former and in this Section we are going to study the gauge transformation laws and field strengths of the different worldvolume fields. The results of this Section are valid both for type IIA and type IIB worldvolume theories.

All D-p-brane worldvolume theories contain the BI vector field, that we call here b because it transforms by shifts of Λ, the gauge transformation parameter of B.

We are going to see that other worldvolume fields may occur in p- and D-p-brane actions. In the case of D-p-branes, these are the duals of the BI field b. They are forms of different rank depending on the dimension of the worldvolume at hand. These worldvolume fields transform by shifts of the

Target space Field	Field Strength	Gauge Parameter	World Volume Field	Field Strength
B	H	Λ (λ)	b	$\mathcal{F}$
$C^{(1)}$	$G^{(2)}$	$\Lambda^{(0)}$	$c^{(0)}$	$\mathcal{G}^{(1)}$
$C^{(3)}$	$G^{(4)}$	$\Lambda^{(2)}$	$c^{(2)}$	$\mathcal{G}^{(3)}$
$C^{(5)}$	$G^{(6)}$	$\Lambda^{(4)}$	$c^{(4)}$	$\mathcal{G}^{(5)}$
$C^{(7)}$	$G^{(8)}$	$\Lambda^{(6)}$ ($\tilde{\lambda}$)	$c^{(6)}$	$\mathcal{G}^{(7)}$

Table 1: This table shows the correspondence between type IIA target-space potentials and worldvolume potentials, together with their field strengths and the gauge parameters. The worldvolume fields transform by shifts of the gauge parameter of the associated target-space potential.

gauge parameter of a R-R form. We will denote them by $c^{(n)}$, since they are associated to $C^{(n+1)}$ (see Table 1 for some type IIA examples).

For instance, the D-4-brane can be described using the BI 1-form b (in the "1-form formalism") but it may also be described by a 2-form $c^{(2)}$ together with b ("1-2-form formalism") [23] or by a 2-form field $c^{(2)}$ alone ("2-form formalism") [24]. The worldvolume 2-form $c^{(2)}$ transforms by shifts of $\Lambda^{(2)}$, the gauge parameter associated to $C^{(3)}$.

Our goal in this Section is to find an homogeneous description (*independent of the worldvolume dimension*) of the gauge transformation laws and field strengths of all worldvolume p-form fields $c^{(p)}$ that will be needed in the next Section.

Our work will be greatly simplified by the following observation: the WZ terms of D-p-branes $WZ^{(p+1)}$ are $(p+1)$-forms that are invariant up to a total derivative[16]

$$\delta(WZ)^{(p+1)} \equiv dA^{(p)} \,. \tag{2.1}$$

In particular, $A^{(p)}$ contains $\Lambda^{(p)}$. Furthermore, $A^{(p)}$ contains only the pullbacks of the $\Lambda^{(p)}$'s and worldvolume fields but no background fields whatsoever. Thus, it is natural to identify $A^{(p)}$ with minus the variation of $c^{(p)}$ up to a total derivative which we denote by $d\kappa^{(p-1)}$. Then, by construction we have[17]

[16]In this Section and in Appendix B we will use, as opposed to the rest of the paper, a differential form notation. The relation between both notations is explained in Appendix B. Some of the formulae in this Section can be found in Appendix C in component notation.

[17]Note that, strictly speaking, we first introduce each $c^{(p)}$ for a fixed value (p+1) of the

$$\begin{cases} \mathcal{G}^{(p+1)} &=& dc^{(p)} + \frac{1}{2\pi\alpha'} WZ^{(p+1)}\,, \\ \\ \delta c^{(p)} &=& d\kappa^{(p-1)} - \frac{1}{2\pi\alpha'} A^{(p)}\,. \end{cases} \tag{2.2}$$

In order to give explicit expressions in the simplest possible way we use the formalism introduced in Refs. [25, 13] in which k-forms of different degrees are formally combined into a single entity:

$$\begin{cases} C &=& C^{(0)} + C^{(1)} + C^{(2)} + \ldots\,, \\ \\ G &=& G^{(0)} + G^{(1)} + G^{(2)} + \ldots\,, \\ \\ \Lambda^{(\cdot)} &=& \Lambda^{(0)} + \Lambda^{(1)} + \ldots\,, \end{cases} \tag{2.3}$$

and we extend it to the worldvolume fields[18]:

$$\begin{cases} c &=& c^{(0)} + c^{(1)} + c^{(2)} + \ldots\,, \\ \\ \mathcal{G} &=& \mathcal{G}^{(0)} + \mathcal{G}^{(1)} + \mathcal{G}^{(2)} + \ldots\,, \\ \\ \kappa &=& \kappa^{(0)} + \kappa^{(1)} + \ldots\,. \end{cases} \tag{2.4}$$

In this language the gauge transformations and field strengths of the R-R fields and the B field can be written in the more compact form[19]

$$\begin{cases} \delta C &=& \left(d\Lambda^{(\cdot)} + m\lambda\right) e^{B}\,, \\ \\ \delta B &=& -2d\lambda\,, \\ \\ G &=& dC - dBC + \frac{m}{2} e^{B}\,. \\ \\ H &=& dB\,. \end{cases} \tag{2.5}$$

The BI field gauge transformation rule and field strength are[20]

$$\begin{cases} \delta b &=& \frac{1}{2\pi\alpha'} 2\lambda + d\rho^{(0)}\,, \\ \\ \mathcal{F} &=& db + \frac{1}{2\pi\alpha'} B\,. \end{cases} \tag{2.6}$$

---

worldvolume. Next, we may consider the same $c^{(p)}$ for other worldvolume dimensions.

[18]This formal sum has also been introduced in [26].

[19]Observe that our $\Lambda^{(\cdot)}$ differs form that of Ref. [13] by a factor of e^{B}. All products of forms here are exterior products.

[20]It is worth remarking that $\kappa^{(0)} \neq \rho^{(0)}$.

We also define

$$\omega(b) = \sum_{r=0} \tfrac{(-1)^{r+1}}{(r+1)!}(2\pi\alpha')^{r+1} b(db)^r \,, \tag{2.7}$$

which has the property

$$d\omega = e^{-(2\pi\alpha')db} - 1 = e^{-(2\pi\alpha')\mathcal{F}} e^{B} - 1 \,. \tag{2.8}$$

With these elements we find that the gauge transformation law and field strength of c are given by

$$\begin{cases} \delta c &= d\kappa - \frac{1}{2\pi\alpha'}\Lambda^{(\cdot)} e^{-(2\pi\alpha')db} - \frac{m}{2}\rho^{(0)} \sum_{r=0} \frac{(-1)^{r+1}}{(r+1)!}(2\pi\alpha')^r (db)^r \\ & \quad -\frac{m}{2} 2\lambda \sum_{r=1} \frac{(-1)^{r+1} r}{(r+1)!}(2\pi\alpha')^{r-1} b(db)^{r-1} \,, \\ \mathcal{G} &= dc + \frac{1}{2\pi\alpha'}\left\{ C e^{-(2\pi\alpha')\mathcal{F}} + \frac{m}{2}\omega \right\} \,. \end{cases} \tag{2.9}$$

Observe that the above definition of $\mathcal{G}$ has the following property

$$G e^{-(2\pi\alpha')\mathcal{F}} = (2\pi\alpha') d\mathcal{G} \,, \tag{2.10}$$

where G is the pullback of the R-R field strengths.

Knowing the existence of the p-form worldvolume fields it is natural to redefine the WZ terms of the D-p-brane effective actions as

$$WZ^{(p+1)\,\prime} = (2\pi\alpha')\mathcal{G}^{(p+1)} \,. \tag{2.11}$$

The new WZ term is *exactly gauge-invariant*, which permits the extension of previous results to non-trivial worldvolume topologies. Observe that the new worldvolume field is non-dynamical and no new degrees of freedom are added. Furthermore, the equations of motion remain the old ones.

An alternative way to motivate the introduction of the worldvolume p-form fields $\{c^{(p)}\}$ is to consider Poincaré-duality transformations of D-p-brane actions.

To perform the Poincaré-duality transformation with respect to b one first has to eliminate any explicit dependence on b so that it only appears through its derivative db. It turns out that b occurs explicitly in the WZ term for non zero mass. Now, it is straightforward to obtain a form of the WZ term in which there is no explicit dependence on b by introducing an auxiliary vector field, that we call e, such that its integration gives back the original WZ term. The resulting WZ term is

$$\widetilde{WZ}^{(p+1)} = (2\pi\alpha')\left\{dc + \tfrac{1}{2\pi\alpha'}\left[C + \tfrac{m}{2}\omega(e)e^{2\pi\alpha'\mathcal{F}(e)}\right]e^{-2\pi\alpha'\mathcal{F}(b)}\right\}\,. \tag{2.12}$$

Here we have specified the definitions of ω and $\mathcal{F}$ in terms of the BI field (the auxiliary field e) as $\omega(b)$, $\mathcal{F}(b)$ ($\omega(e)$, $\mathcal{F}(e)$). Integration over e implies $e = b$ and the usual WZ term (2.11) is obtained.

Although the full Poincaré-dualization of a general D-brane action is complicated, given that this transformation is consistent with gauge-invariance it can in principle be performed in the quadratic approximation, and the field strength of the field dual to b (in a $(p+1)$-dimensional worldvolume, a $(p-2)$-form) can be immediately read off and turns out to be precisely $\mathcal{G}^{(p-1)}$. Thus, the $c^{(p-2)}$ worldvolume fields are nothing but the duals of the BI field in the quadratic approximation and one can also say that the field strength $\mathcal{G}^{(p+1)}$ is obtained via Poincaré duality from the BI field strength $\mathcal{F}$ in the $(p+3)$-dimensional worldvolume effective action of the D-$(\mathrm{p}+2)$-brane.

The set of worldvolume fields $\{c^{(p)}\}$ that we have just introduced is, of course, not unique. In order to build duals of D-brane actions keeping track of all total derivatives it is more appropriate to use a basis of worldvolume fields $\{a^{(p)}\}$ that do not transform under Hodge duality of the BI field[21]. The gauge transformation rules and field strengths ($\mathcal{H}^{(p+1)}$) of the $a^{(p)}$'s are given by

$$\left\{\begin{array}{rcl}
\delta a & = & d\mu - \frac{1}{2\pi\alpha'}\Lambda^{(\cdot)} - 2da\lambda \\
 & & -\frac{m}{2}e^{(2\pi\alpha')db}\left[\rho^{(0)}\sum_{r=0}\frac{(-1)^{r+1}}{(r+1)!}(2\pi\alpha')^r(db)^r\right. \\
 & & \left.+2\lambda\sum_{r=1}\frac{(-1)^{r+1}r}{(r+1)!}(2\pi\alpha')^{r-1}b(db)^{r-1}\right]\,, \\
\mathcal{H} & = & da\; e^B + \frac{1}{2\pi\alpha'}C + \frac{1}{2\pi\alpha'}\frac{m}{2}e^{2\pi\alpha'\mathcal{F}(b)}\omega(b)\,.
\end{array}\right. \tag{2.13}$$

$\mathcal{H}$ satisfies

$$G = (2\pi\alpha')d\mathcal{H} + (2\pi\alpha')\mathcal{H}H\,. \tag{2.14}$$

The relation between the two sets of worldvolume fields is:

[21] The basis that we have introduced so far does not satisfy this requirement because there is an explicit dependence on b in the gauge variations of the fields. Note that b appears explicitly in the massive contribution to the gauge transformation law of the new fields, however this dependence disappears when in the actual dualization one introduces the auxiliary field e as indicated above.

$$
\begin{cases}
a(c,b) & = & c\, e^{(2\pi\alpha')db}\,, \\
\mathcal{H}(a) & = & e^{(2\pi\alpha')\mathcal{F}}\mathcal{G}\left[c(a,b)\right]\,, \\
\mu & = & \kappa e^{(2\pi\alpha')db}\,.
\end{cases}
\tag{2.15}
$$

From the above relations the gauge-invariance of $\mathcal{H}$ is evident. Observe also that $a^{(0)} = c^{(0)}$.

The main difference between the $\{c^{(p)}\}$ and $\{a^{(p)}\}$ basis is that, while in the field strengths of the $c^{(n)}$'s many R-R potentials $C^{(r)}$ appear but only one $c^{(n)}$, in the field strength of each $a^{(n)}$ many other worldvolume fields $a^{(r)}$ appear but only one R-R potential $C^{(n+1)}$.

This second basis of worldvolume fields can be used as follows: if, in the WZ term of the *massless* D-p-brane action one substitutes everywhere the R-R field $C^{(n)}$ by the field strength $\mathcal{H}^{(n)}$

$$
C \to (2\pi\alpha')\mathcal{H}\,, \tag{2.16}
$$

one gets a new WZ term which is invariant under all gauge transformations, including the massive ones. Conversely, given a massive D-brane action we can absorb all the mass dependence into modified RR fields as above, and actually eliminate the explicit dependence on b by the introduction of the auxiliary field e as:

$$
\mathcal{H} = da\; e^{B} + \tfrac{1}{2\pi\alpha'}C + \tfrac{1}{2\pi\alpha'}\tfrac{m}{2}e^{2\pi\alpha'\mathcal{F}(e)}\omega(e). \tag{2.17}
$$

With this trick we can now apply the usual duality procedure to build the dual action, since the resulting action is a massless D-brane with modified RR fields. In the end we just substitute in the dual action these RR fields and read the final action in terms of the dual variable and the auxiliary fields. One can then check that the resulting action is gauge invariant, *including* total derivative terms.

We can extend our reasoning to the p-brane case. In the case of the fundamental string, the WZ term can be corrected by the inclusion of a total derivative db. The new WZ term is now

$$
WZ'_{string} = (2\pi\alpha')\mathcal{F}\,, \tag{2.18}
$$

and satisfies

$$
H = (2\pi\alpha')d\mathcal{F}\,. \tag{2.19}
$$

The case of the (solitonic) p-5-brane is different for each string theory for the reasons explained in the Introduction and here we will only consider the p-5A-brane. One can introduce again a worldvolume 5-form field $\tilde{b}$ whose role is in a sense dual to that of the BI vector field b. In the massless case, its field strength and gauge transformation law are:

$$\left\{\begin{array}{rcl} \tilde{\mathcal{F}}_{(0)} & = & d\tilde{b} + \frac{1}{2\pi\alpha'}\tilde{B} - \left(C^{(5)} - \frac{1}{2}C^{(3)}B\right) dc^{(0)} \\ & & -\frac{1}{2}da^{(2)}\left(C^{(3)} + (2\pi\alpha')Bdc^{(0)}\right) , \\ \delta\tilde{b} & = & -\frac{1}{2\pi\alpha'}\tilde{\Lambda} + \Lambda^{(4)}dc^{(0)} \\ & & +\frac{1}{2}(2\pi\alpha')da^{(2)}\left(\delta_{(0)}a^{(2)} - d\mu^{(1)}\right) , \end{array}\right. \tag{2.20}$$

and the gauge field strength satisfies

$$(2\pi\alpha')d\tilde{\mathcal{F}}_{(0)} = \tilde{H}_{(0)} - (2\pi\alpha')G^{(6)}_{(0)}\mathcal{G}^{(1)}_{(0)} + 3(2\pi\alpha')^2\mathcal{H}^{(3)}_{(0)}d\mathcal{H}^{(3)}_{(0)} . \tag{2.21}$$

In the massive case, $\tilde{B}$ transforms under shifts of the 6-form $\tilde{\lambda}$. There is only one way to make the field strength of $\tilde{b}$ invariant: to include $c^{(6)}$ in it. This, in turn forces $\tilde{b}$ to transform under shifts of the 5-form $\rho^{(5)}$, the gauge parameter for $c^{(6)}$. Thus, $c^{(6)}$ may become massive by "eating" $\tilde{b}$, which would be completely eliminated by a gauge transformation. In more detail, we find

$$\left\{\begin{array}{rcl} \tilde{\mathcal{F}} & = & d\tilde{b} + \frac{1}{2\pi\alpha'}\tilde{B} - \left(C^{(5)} - \frac{1}{2}C^{(3)}B\right)\left(dc^{(0)} - \frac{m}{2}b\right) \\ & & -\frac{1}{2}\left[da^{(2)} - \frac{m}{2}(2\pi\alpha')bdb\right] \times \\ & & \left[C^{(3)} + (2\pi\alpha')B\left(dc^{(0)} - \frac{m}{2}b\right) - \frac{m}{4}(2\pi\alpha')^2 bdb\right] \\ & & -\frac{1}{6}\frac{m}{2}(2\pi\alpha')^3 dc^{(0)}bdbdb + \frac{m}{2}c^{(6)} , \\ \delta\tilde{b} & = & -\frac{1}{2\pi\alpha'}\tilde{\Lambda} + \frac{m}{2}\rho^{(5)} + \Lambda^{(4)}dc^{(0)} \\ & & +\frac{1}{2}(2\pi\alpha')da^{(2)}\left(\delta a^{(2)} - d\mu^{(1)}\right) , \end{array}\right. \tag{2.22}$$

and the gauge field strength satisfies now

$$\tilde{H} = (2\pi\alpha')d\tilde{\mathcal{F}} - (2\pi\alpha')G^{(6)}\mathcal{G}^{(1)} + 3(2\pi\alpha')^2\mathcal{H}^{(3)}d\mathcal{H}^{(3)} + \tfrac{m}{2}\mathcal{G}^{(7)}\,. \qquad (2.23)$$

This ends our description of the worldvolume fields of 10-dimensional extended objects. Explicit formulae for the worldvolumes fields we will be considering in the next Section can be found in Appendix C.

We will see in the next sections that additional world-volume fields need also be introduced for eleven dimensional branes. Their corresponding gauge transformations and field strengths will be introduced case by case.

3 Effective Actions of Massive Branes

The purpose of this Section is to construct effective worldvolume actions describing the dynamics of branes whose target space is one of the massive supergravity theories described in Section 1. Our strategy will be to first construct the action for a massive M-brane. These actions turn out to have as a common characteristic that they are gauged sigma models. The gauged isometry coincides with the isometry needed to define the massive 11-dimensional supergravity theory. The original (ungauged) sigma model describes an object of the same dimension in the massless theory, i.e. the corresponding massless brane. The worldvolume actions are invariant under 11-dimensional massive gauge transformations.

In a second stage we consider different kinds of dimensional reductions of the massive M-brane actions. In particular, we will show that the so-called direct and double dimensional reductions in the isometry direction lead to a pair of massive branes of IIA superstring theory as represented in Figure 1. This shows that a single massive M-brane unifies a pair of massive branes of string theory.

Below, we will consider in successive order the massive M-0-brane, M-2-brane and M-5-brane. The case of the d=11 KK monopole (or M-6-brane) and a conjectured M-9-brane will be discussed in the Conclusions.

3.1 The Massive M-0-Brane

Consider the action of the massless 11-dimensional particle (M-0-brane):

$$\hat{S}\;[\hat{X}^{\hat{\mu}},\gamma] = -\tfrac{p}{2}\int d\xi\sqrt{|\gamma|}\;\gamma^{-1}\;\partial_\xi\hat{X}^{\hat{\mu}}\partial_\xi\hat{X}^{\hat{\nu}}\hat{g}_{\hat{\mu}\hat{\nu}}\,, \qquad (3.1)$$

where p is a constant with dimensions of mass. This action is known to give upon direct dimensional reduction the action of the D-0-brane of Type IIA

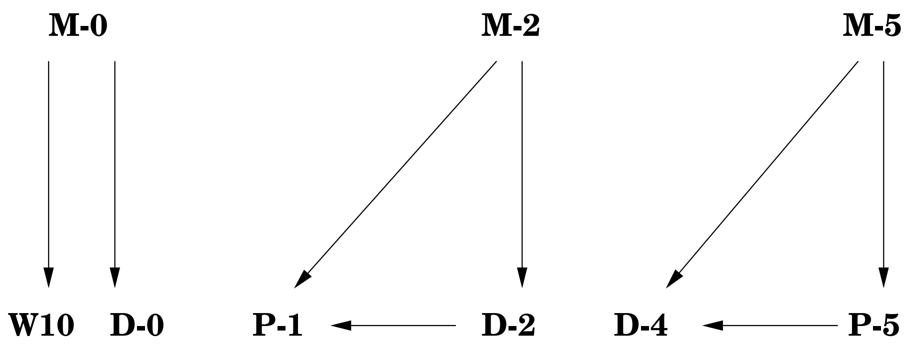

Figure 1: Each M-brane gives rise to a pair of type IIA branes both in the massive and massless cases. Vertical arrows indicate direct dimensional reduction (i.e. target-space dimensional reduction), slanted arrows indicate double dimensional reduction and horizontal arrows indicate worldvolume dimensional reduction together with gauge-fixing of some symmetries. As the figure indicates, double dimensional reduction is equivalent to direct dimensional reduction followed by worldvolume dimensional reduction and gauge-fixing. Upon direct dimensional reduction, the M-0-brane gives the D-0-brane or the ten-dimensional Brinkmann wave W10 depending on whether the M-0-brane has momentum along the 11th dimension or not, respectively.

superstring theory (see e.g. [27]). Our goal is to obtain an effective action with 11-dimensional target space from which one can derive the effective action of the massive D-0-brane. This object moves in 11-dimensional spacetime and will henceforth be referred to as the *massive* M-0-brane[22]. We proceed by analogy with the massive D-2-brane case, whose effective worldvolume action [12, 13] can be obtained from the M-2-brane effective action by gauging an isometry and introducing the massive gauge transformations in their 11-dimensional form [16, 17].

It is reasonable to expect that the same procedure will give us the action we are looking for. Thus, we assume that the metric has an isometry generated by a Killing vector $\hat{k}^{\hat{\mu}}$, so the above action is invariant under the infinitesimal transformations

$$\left\{ \begin{array}{rcl} \delta_\eta \hat{X}^{\hat{\mu}} & = & \eta\, \hat{k}^{\hat{\mu}}(\hat{X})\,, \\ \\ \delta_\eta \hat{g}_{\hat{\mu}\hat{\nu}} & = & \eta\, \hat{k}^{\hat{\lambda}} \partial_{\hat{\lambda}} \hat{g}_{\hat{\mu}\hat{\nu}}\,, \end{array} \right. \tag{3.2}$$

with constant η. To make the action invariant under the same transformations but now with $\eta(\xi)$ being an arbitrary worldline function, we simply substitute the derivative with respect to ξ by the covariant derivative

[22]We reserve the name massive M-branes for those branes that move in an 11-dimensional spacetime and have the property that they give upon dimensional reduction the massive branes of Type IIA superstring theory.

$$D_\xi \hat{X}^{\hat{\mu}} = \partial_\xi \hat{X}^{\hat{\mu}} + A_\xi(\xi)\hat{k}^{\hat{\mu}}\,, \tag{3.3}$$

where A_ξ is an auxiliary worldline variable transforming as follows

$$\delta_\eta A_\xi = -\partial_\xi \eta \;\Rightarrow\; \delta_\eta D_\xi \hat{X}^{\hat{\mu}} = \eta\; D_\xi \hat{X}^{\hat{\nu}} \partial_{\hat{\nu}} \hat{k}^{\hat{\mu}}\,. \tag{3.4}$$

We obtain the following action:

$$\hat{S}_{\text{gauged}}\;[\hat{X}^{\hat{\mu}}, A_\xi, \gamma] = -\tfrac{p}{2}\int d\xi \sqrt{|\gamma|}\;\gamma^{-1}\; D_\xi \hat{X}^{\hat{\mu}} D_\xi \hat{X}^{\hat{\nu}} \hat{g}_{\hat{\mu}\hat{\nu}}\,. \tag{3.5}$$

On top of being invariant under local transformations of the form (3.2), this action turns out to be invariant under the massive gauge transformations of the metric (1.6) and the auxiliary variable

$$\delta_{\hat{\chi}} A_\xi = -m\hat{\lambda}_\xi\,. \tag{3.6}$$

In order to make more uniform our notation and write all actions in such a way that the massless $m \to 0$ limit coincides with the ungauged limit, it is convenient to make the following redefinitions:

$$\begin{aligned} A_\xi &= -\tfrac{m}{2}(2\pi\alpha')\; b_\xi\,,\\ \eta &= \tfrac{m}{2}\;(2\pi\alpha')\;\rho^{(0)}\,. \end{aligned} \tag{3.7}$$

After these redefinitions the action is

$$\hat{S}_{\text{gauged}}\;[\hat{X}^{\hat{\mu}}, b_\xi, \gamma] = -\tfrac{p}{2}\int d\xi \sqrt{|\gamma|}\;\gamma^{-1}\; D_\xi \hat{X}^{\hat{\mu}} D_\xi \hat{X}^{\hat{\nu}} \hat{g}_{\hat{\mu}\hat{\nu}}\,, \tag{3.8}$$

where

$$D_\xi \hat{X}^{\hat{\mu}} = \partial_\xi \hat{X}^{\hat{\mu}} - \tfrac{m}{2}\;(2\pi\alpha')\; b_\xi\; \hat{k}^{\hat{\mu}}\,, \tag{3.9}$$

and the fields transform as follows:

$$\left\{ \begin{aligned} \delta \hat{X}^{\hat{\mu}} &= \tfrac{m}{2}\;(2\pi\alpha')\;\rho^{(0)}\,(\xi)\;\; \hat{k}^{\hat{\mu}}(\hat{X})\,,\\ \delta \hat{g}_{\hat{\mu}\hat{\nu}} &= \tfrac{m}{2}\;(2\pi\alpha')\;\rho^{(0)}\;\hat{k}^{\hat{\lambda}}\partial_{\hat{\lambda}}\hat{g}_{\hat{\mu}\hat{\nu}}\,,\\ \delta b_\xi &= \tfrac{1}{2\pi\alpha'}2\hat{\lambda}_\xi + \partial_\xi \rho^{(0)}\,. \end{aligned} \right. \tag{3.10}$$

Now we want to perform the dimensional reduction[23] of the gauged action in the direction associated to the isometry. As a first step, using a coordinate system adapted to the isometry ($\hat{k}^{\hat{\mu}} = \delta^{\hat{\mu}y}$) and using Eqs. (1.18) we rewrite the background fields in 10-dimensional form, obtaining

$$\hat{S}\ [X^\mu, c^{(0)}, b_\xi, \gamma] = -\tfrac{p}{2}\int d\xi\ \sqrt{|\gamma|}\ \gamma^{-1}\left[e^{-\frac{2}{3}\phi}g_{\xi\xi} - (2\pi\alpha')^2 e^{\frac{4}{3}\phi}\left(\mathcal{G}^{(1)}{}_\xi\right)^2\right], \tag{3.11}$$

with

$$\left\{\begin{array}{rcl} g_{\xi\xi} & = & \partial_\xi X^\mu \partial_\xi X^\nu g_{\mu\nu}\,, \\ C^{(1)}{}_\xi & = & \partial_\xi X^\mu\ C^{(1)}{}_\mu\,, \\ \mathcal{G}^{(1)}{}_\xi & = & \partial_\xi c^{(0)} + \frac{1}{2\pi\alpha'}C^{(1)}{}_\xi - \frac{m}{2}b_\xi\,. \end{array}\right. \tag{3.12}$$

$\mathcal{G}^{(1)}_\xi$ is the gauge-invariant "field" strength of $c^{(0)}$. The worldvolume 0-form $c^{(0)}$ is related to the original 11-dimensional coordinate Y by

$$Y = (2\pi\alpha')c^{(0)}\,, \tag{3.13}$$

and transforms as given by (2.9):

$$\delta c^{(0)} = -\tfrac{1}{2\pi\alpha'}\Lambda^{(0)} + \tfrac{m}{2}\rho^{(0)}\,. \tag{3.14}$$

Now we want to eliminate $c^{(0)}$ (or, equivalently, Y) by using its equation of motion, which essentially says that the momentum of the particle in the direction Y, P_y, is constant. The right way of doing this [11] is to first perform the Legendre transformation of the action with respect to Y (or $c^{(0)}$)

$$\tilde{S}'\ [X^\mu, P_y, b_\xi, \gamma] = \int d\xi\left[-P_y\partial_\xi Y + \mathcal{L}\right]\,, \tag{3.15}$$

with

$$P_y = \frac{\partial\mathcal{L}}{\partial(\partial_\xi Y)}\,. \tag{3.16}$$

We obtain

[23] Strictly speaking it is not "direct" dimensional reduction because we are going to eliminate one coordinate. Since this is not a field theory, but a quantum-mechanical theory, there is no counting of degrees of freedom and we cannot say how many of them we are eliminating.

$$
\begin{aligned}
\tilde{S}' \left[X^\mu, P_y, b_\xi, \gamma\right] &= -\tfrac{p}{2} \int d\xi\, \sqrt{|\gamma|}\, \gamma^{-1} \left[e^{-\frac{2}{3}\phi} g_{\xi\xi} + \gamma e^{-\frac{4}{3}\phi} \left(\tfrac{P_y}{p}\right)^2\right] \\
&\quad + \int d\xi P_y \left[C^{(1)}{}_\xi - \tfrac{m}{2}\,(2\pi\alpha')\, b_\xi\right] .
\end{aligned} \tag{3.17}
$$

We can now set P_y to a constant value and eliminate the auxiliary metric. For $P_y \neq 0$ (that is, the M-0-brane has some momentum in the compact dimension) we get[24]:

$$
\tilde{S}' \left[X^\mu, b_\xi\right] = -|P_y| \int d\xi\; e^{-\phi} \sqrt{|g_{\xi\xi}|} + P_y \int d\xi \left(C^{(1)}{}_\xi - \tfrac{m}{2}\,(2\pi\alpha')\, b_\xi\right) , \tag{3.18}
$$

which is the standard effective action of the massive D-0-brane [12], b_ξ being the BI vector "field". The mass of the D-0-brane is $|P_y|$ and its R-R charge is $-P_y$. Note that the constant p has disappeared from the action. As we said in Section 2 this action is gauge-invariant only up to total derivatives. The gauge-invariance of the Lagrangian has been lost in the Legendre transformation. It is convenient to introduce an auxiliary scalar field $c^{(0)}$ to make the above action exactly gauge invariant. Doing this, we have

$$
\tilde{S}' \left[X^\mu, b_\xi\right] = -|P_y| \int d\xi\; e^{-\phi} \sqrt{|g_{\xi\xi}|} + (2\pi\alpha') P_y \int d\xi\; \mathcal{G}^{(1)}{}_\xi\, . \tag{3.19}
$$

When $P_y = 0$ (the M-0-brane only moves in directions orthogonal to Y) we obtain the action for a null D-0-brane [28, 26]:

$$
\tilde{S}\left[X^\mu, \gamma\right] = -\tfrac{p}{2} \int d\xi\, \sqrt{|\gamma|}\, \gamma^{-1} e^{-\frac{2}{3}\phi} g_{\xi\xi}\, . \tag{3.20}
$$

Observe that the factor $e^{-\frac{2}{3}\phi}$ cannot be reabsorbed into γ at least when the above effective action is acting as a source for supergravity. The null D-0-brane does not qualify as fundamental, solitonic or D-type.

3.2 The Massive M-2-Brane

Starting from the massless M-2-brane action we perform a similar gauging as in the particle case. At the same time however we want the resulting gauged action to remain invariant under the gauge transformations of the 3-form $\hat{C}$.

[24] Note that the field equation for b_ξ seems to lead to $m = 0$. However, one should keep in mind that in the supersymmetric case b_ξ couples to further fermionic terms. We thank M.B. Green for a discussion on this point.

A straightforward Noether procedure leads to an action [17] that we rewrite as follows[25] [26]

$$
\begin{aligned}
\hat{S}_{\text{gauged}}\ [\hat{X}^{\hat{\mu}}, \hat{b}_{\hat{\imath}}] &= -T_{M2} \int d^3\hat{\xi}\ \sqrt{|D_{\hat{\imath}}\hat{X}^{\hat{\mu}} D_{\hat{\jmath}}\hat{X}^{\hat{\nu}} \hat{g}_{\hat{\mu}\hat{\nu}}|} \\
&\quad -(2\pi\alpha')\tfrac{T_{M2}}{3!} \int d^3\hat{\xi}\ \hat{\epsilon}^{\hat{\imath}\hat{\jmath}\hat{k}} \hat{\mathcal{K}}^{(3)}{}_{\hat{\imath}\hat{\jmath}\hat{k}}\,,
\end{aligned}
\tag{3.21}
$$

where

$$
D_{\hat{\imath}}\hat{X}^{\hat{\mu}} = \partial_{\hat{\imath}}\hat{X}^{\hat{\mu}} - \tfrac{m}{2}\ (2\pi\alpha')\ \hat{b}_{\hat{\imath}}\ \hat{k}^{\hat{\mu}}\,,
\tag{3.22}
$$

$\hat{k}^{\hat{\mu}}$ is a fixed (i.e. non-dynamical) spacetime vector and

$$
\hat{\mathcal{K}}^{(3)} = 3\left[\partial\hat{\omega}^{(2)} + \tfrac{1}{3(2\pi\alpha')} D\hat{X}^{\hat{\mu}} D\hat{X}^{\hat{\nu}} D\hat{X}^{\hat{\rho}} \hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} - \tfrac{m}{2}(2\pi\alpha')\hat{b}\partial\hat{b}\right]\,,
\tag{3.23}
$$

is the gauge-invariant field-strength of the auxiliary worldvolume field $\hat{\omega}^{(2)}$ that transforms as follows:

$$
\delta\hat{\omega}^{(2)}_{\hat{\imath}\hat{\jmath}} = -\tfrac{1}{2\pi\alpha'}\hat{\chi}_{\hat{\imath}\hat{\jmath}} - \tfrac{m}{2}(2\pi\alpha')\left(\tfrac{1}{2\pi\alpha'}2\hat{\lambda} + \partial\hat{\rho}^{(0)}\right)_{[\hat{\imath}}\hat{b}_{\hat{\jmath}]} + 2\partial_{[\hat{\imath}}\hat{\rho}^{(1)}{}_{\hat{\jmath}]}\,.
\tag{3.24}
$$

Finally, $|M_{\hat{\imath}\hat{\jmath}}|$ stands for the absolute value of the determinant of the matrix $M_{\hat{\imath}\hat{\jmath}}$. Like in the case of the particle we use conventions such that the massless and ungauged limits coincide.

Without the auxiliary worldvolume field $\hat{\omega}^{(2)}$, the above massive M-2-brane action would be invariant up to total derivatives under the infinitesimal massive gauge transformations with parameter $\hat{\chi}$ of $\hat{g}_{\hat{\mu}\hat{\nu}}$ and $\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}}$ given in Eqs. (1.6,1.8) together with the massive transformation of the auxiliary field $\hat{b}_{\hat{\imath}}$

$$
\delta_{\hat{\chi}}\hat{b}_{\hat{\imath}} = \tfrac{1}{2\pi\alpha'}2\hat{\lambda}_{\hat{\imath}}\,,
\tag{3.25}
$$

and the $\delta_{\hat{\rho}^{(0)}}$ transformations (for which $\hat{b}_{\hat{\imath}}$ plays the role of gauge field)

[25] In this Section we will consider a double dimensional reduction which involves not only a target space reduction (from d=11 to d=10) but also a worldvolume reduction (from 3 dimensions to 2 dimensions). To distinguish fields before and after the worldvolume reduction we use a notation where worldvolume hatted indices (Latin) and fields are 3-dimensional, while unhatted worldvolume indices and fields are 2-dimensional. The split is $\hat{\imath} = (i, 2)$, with $i = 0, 1$. In the next Section we will discuss the massive M-5-brane and use a similar notation.

[26] The three-dimensional gauged sigma model given below also occurs in [29]. The authors of [29] also note that the Killing isometry condition can be slightly weakened so that only the curvatures are Lie-invariant.

$$
\left\{
\begin{array}{rcl}
\delta_{\hat{\rho}^{(0)}} \hat{X}^{\hat{\mu}} & = & \frac{m}{2}\,(2\pi\alpha')\;\hat{\rho}^{(0)}(\hat{\xi})\;\hat{k}^{\hat{\mu}}(\hat{X})\,, \\
\delta_{\hat{\rho}^{(0)}} \hat{b}_{\hat{\imath}} & = & \partial\hat{\rho}^{(0)}\,, \\
\delta_{\hat{\rho}^{(0)}} \hat{g}_{\hat{\mu}\hat{\nu}} & = & \frac{m}{2}\,(2\pi\alpha')\;\hat{\rho}^{(0)}\hat{k}^{\hat{\lambda}}\partial_{\hat{\lambda}}\hat{g}_{\hat{\mu}\hat{\nu}}\,, \\
\delta_{\hat{\rho}^{(0)}} \hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} & = & \frac{m}{2}\,(2\pi\alpha')\;\hat{\rho}^{(0)}\hat{k}^{\hat{\lambda}}\partial_{\hat{\lambda}}\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}}\,,
\end{array}
\right.
\tag{3.26}
$$

assuming the conditions

$$
\pounds_{\hat{k}}\hat{g}_{\hat{\mu}\hat{\nu}} = \pounds_{\hat{k}}\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} = \pounds_{\hat{k}}\hat{\chi}_{\hat{\mu}\hat{\nu}} = 0 \tag{3.27}
$$

hold. We see that, in particular, $\hat{k}^{\hat{\mu}}$ must be a Killing vector. These conditions coincide with those necessary to build the massive M-0-brane and are exactly those satisfied by the massive 11-dimensional supergravity theory described in Section 1.1. This was our main motivation for constructing the massive 11-dimensional supergravity theory in Section 1.1.

Observe that the gauge transformation laws of the worldvolume field $\hat{b}_{\hat{\imath}}$ are identical to those of the auxiliary field b_{ξ} that we introduced in the M-0-brane case (taking into account the different dimensionality of the respective worldvolumes).

The auxiliary field $\hat{\omega}^{(2)}$ has been introduced to compensate for the total derivatives so the action is exactly gauge-invariant (the WZ term is precisely its field-strength). A field that transforms precisely in the same way and with the same field strength occurs in the M-5-brane effective action. There it is a dynamical field whose equation of motion is the anti-self-duality condition. Following our philosophy, we denote both worldvolume 2-forms by $\hat{\omega}^{(2)}$.

3.2.1 Direct Dimensional Reduction: the Massive D-2-Brane

By assumption the eleven dimensional background has an isometry and we can perform a direct dimensional reduction of the action (3.21) in the direction associated to the isometry that we have gauged. This amounts to a simple rewriting of the background fields from 11-dimensional to 10-dimensional form. Using a coordinate system adapted to the isometry ($\hat{k}^{\hat{\mu}} = \delta^{\hat{\mu}y}$) and the relations between 11- and 10-dimensional fields Eqs. (1.18) we obtain the dual action for the D-2-brane of massive type IIA theory found in [16] (plus the auxiliary field). Taking into account that $\hat{\omega}^{(2)}$ transforms as the $\partial\hat{a}^{(2)}$ defined in Section 2 and substituting accordingly, we can write that action as follows:

$$\tilde{S}\ [X^\mu, \hat{c}^{(0)}, \hat{b}_{\hat{\imath}}, \hat{a}^{(2)}_{\hat{\imath}\hat{\jmath}}] = -T_{M2} \int d^3\hat{\xi}\ e^{-\phi}\ \sqrt{|g_{\hat{\imath}\hat{\jmath}} - (2\pi\alpha')^2 e^{2\phi} \hat{\mathcal{G}}^{(1)}{}_{\hat{\imath}} \mathcal{G}^{(1)}{}_{\hat{\jmath}}|}$$
$$-(2\pi\alpha')\tfrac{T_{M2}}{3!} \int d^3\hat{\xi}\ \hat{\epsilon}^{\hat{\imath}\hat{\jmath}\hat{k}} \hat{\mathcal{H}}^{(3)}{}_{\hat{\imath}\hat{\jmath}\hat{k}}\,, \tag{3.28}$$

where

$$g_{\hat{\imath}\hat{\jmath}} = \partial_{\hat{\imath}} X^\mu \partial_{\hat{\jmath}} X^\nu g_{\mu\nu}\,, \tag{3.29}$$

and $\hat{\mathcal{G}}^{(1)}{}_{\hat{\imath}}$ and $\hat{\mathcal{H}}^{(3)}{}_{\hat{\imath}\hat{\jmath}\hat{k}}$ are defined in Section 2 and given explicitly in Appendix C.

As for the D-particle the worldvolume 0-form $\hat{c}^{(0)}$, whose transformation properties are also defined in Section 2, is related to the original 11-dimensional coordinate Y by

$$Y = (2\pi\alpha')\hat{c}^{(0)}\,. \tag{3.30}$$

The equivalence between this action and the usual effective action for the massive D-2-brane which contains the BI vector field can be proven via the intermediate action of Ref. [16] or by eliminating $\hat{c}^{(0)}$ in the above action. This can be done in two alternative but equivalent ways. First, one can try to use the equation of motion of $\hat{b}_{\hat{\imath}}$. To do this consistently one has to Legendre-transform the above action with respect to $\hat{c}^{(0)}$[27]:

$$\tilde{S}'\ [X^\mu, \hat{P}_{(0)}{}^{\hat{\imath}}, \hat{b}_{\hat{\imath}}, \hat{\gamma}_{\hat{\imath}\hat{\jmath}}] = \int d^3\hat{\xi} \left[-\hat{P}_{(0)}{}^{\hat{\imath}} \partial_{\hat{\imath}} \hat{c}^{(0)} + \hat{\mathcal{L}}\right]\,, \tag{3.31}$$

with

$$\hat{P}_{(0)}{}^{\hat{\imath}} = \frac{\partial \hat{\mathcal{L}}}{\partial(\partial_{\hat{\imath}} \hat{c}^{(0)})}\,. \tag{3.32}$$

The result is

[27] This is more easily done using an auxiliary worldvolume metric $\hat{\gamma}_{\hat{\imath}\hat{\jmath}}$. The corresponding worldvolume action can be found in Ref. [17].

$$
\begin{aligned}
&\tilde{S}' \; [X^\mu, \hat{\mathcal{P}}_{(0)}{}^{\hat{\imath}}, \hat{b}_{\hat{\imath}}, \hat{\gamma}_{\hat{\imath}\hat{\jmath}}] = \\
&\quad -\tfrac{T_{M2}}{2} \int d^3\hat{\xi}\ \sqrt{|\hat{\gamma}|}\ \hat{\gamma}^{\hat{\imath}\hat{\jmath}} \left\{ e^{-\frac{2}{3}\phi} g_{\hat{\imath}\hat{\jmath}} + e^{-\frac{4}{3}\phi} \left(\hat{\mathcal{P}}_{(0)} + {}^\star B\right)_{\hat{\imath}} \left(\hat{\mathcal{P}}_{(0)} + {}^\star B\right)_{\hat{\jmath}} \right. \\
&\qquad\qquad \left. -2\left(\hat{\mathcal{P}}_{(0)} + {}^\star B\right)_{\hat{\imath}} \left(C^{(1)} - \tfrac{m}{2}\ (2\pi\alpha')\ \hat{b}\right)_{\hat{\jmath}} - 1 \right\} \\
&\quad -\tfrac{T_{M2}}{3!} \int d^3\hat{\xi}\ \hat{\epsilon}^{\hat{\imath}\hat{\jmath}\hat{k}} \left\{ C^{(3)}{}_{\hat{\imath}\hat{\jmath}\hat{k}} - 3\ \tfrac{m}{2}\ (2\pi\alpha')^2 \hat{b}_{\hat{\imath}} \hat{\mathcal{F}}_{\hat{\jmath}\hat{k}} + \tfrac{3}{2}\ \tfrac{m}{2}\ (2\pi\alpha')\ \hat{b}_{\hat{\imath}} B_{\hat{\jmath}\hat{k}} \right. \\
&\quad \left. +3(2\pi\alpha')\partial_{\hat{\imath}} \hat{c}^{(2)}{}_{\hat{\jmath}\hat{k}} \right\} ,
\end{aligned} \tag{3.33}
$$

where $\hat{\mathcal{F}}$ is defined in Section 2, and we are using for simplicity

$$
\hat{\mathcal{P}}_{(0)}{}^{\hat{\imath}} = \tfrac{1}{2\pi\alpha' T_{M2}\sqrt{|\hat{\gamma}|}} \hat{P}_{(0)}{}^{\hat{\imath}} . \tag{3.34}
$$

We have also substituted the auxiliary field $\hat{a}^{(2)}$ by the auxiliary field $\hat{c}^{(2)}$ to compensate for the lack of exact gauge-invariance introduced by the Legendre transformation.

Now, the equation of motion for $\hat{b}_{\hat{\imath}}$ is a purely algebraic constraint for $\hat{\mathcal{P}}_{(0)}{}^{\hat{\imath}}$:

$$
\left(\hat{\mathcal{P}}_{(0)} + {}^\star B\right)^{\hat{\imath}} = (2\pi\alpha')\ {}^\star\hat{\mathcal{F}}^{\hat{\imath}} . \tag{3.35}
$$

Substituting this constraint into the action Eq. (3.33) and eliminating again the auxiliary metric we readily obtain the action of the massive D-2-brane [12, 13]:

$$
\begin{aligned}
S\ [X^\mu, \hat{b}_{\hat{\imath}}] &= -T_{M2} \int d^3\hat{\xi}\ e^{-\phi} \sqrt{|g_{\hat{\imath}\hat{\jmath}} + (2\pi\alpha')\hat{\mathcal{F}}_{\hat{\imath}\hat{\jmath}}|} \\
&\quad -(2\pi\alpha')\tfrac{T_{M2}}{3!} \int d^3\hat{\xi}\ \epsilon^{\hat{\imath}\hat{\jmath}\hat{k}} \hat{\mathcal{G}}^{(3)}{}_{\hat{\imath}\hat{\jmath}\hat{k}} ,
\end{aligned} \tag{3.36}
$$

where $\hat{\mathcal{G}}^{(3)}{}_{\hat{\imath}\hat{\jmath}\hat{k}}$ is defined in Section 2.

In this action the worldvolume 1-form $\hat{b}_{\hat{\imath}}$ plays the role of the Born-Infeld vector field and the auxiliary field $\hat{c}^{(2)}$ makes the action exactly gauge-invariant.

A more elegant way to proceed, keeping track as well of the total derivative induced in the dualization, is to perform the worldvolume canonical transformation [30]:

$$\left\{\begin{array}{lcl}\hat{P}_{(0)} & = & -(2\pi\alpha')^2 T_{M2}\hat{\epsilon}^{0\hat{\alpha}\hat{\beta}}\partial_{\hat{\alpha}}\hat{V}_{\hat{\beta}}\,, \\ \\ \hat{\Pi}^{\hat{\alpha}} & = & (2\pi\alpha')^2 T_{M2}\hat{\epsilon}^{0\hat{\alpha}\hat{\beta}}\partial_{\hat{\beta}}\hat{c}^{(0)}\,,\end{array}\right. \tag{3.37}$$

from $\{\hat{c}^{(0)}, \hat{P}_{(0)}\}$ to $\{\hat{V}_{\hat{\alpha}}, \hat{\Pi}^{\hat{\alpha}}\}$, where we have split the three dimensional index $\hat{\imath} = (0, \hat{\alpha})$ and $\hat{P}_{(0)}$ $(\equiv \hat{P}^0_{(0)})$, $\hat{\Pi}^{\hat{\alpha}}$ are the canonically conjugate momenta of $\hat{c}_{(0)}$ and $\hat{V}_{\hat{\alpha}}$ respectively. This transformation is generated by the functional

$$\Psi = (2\pi\alpha')^2 T_{M2}\int_{\text{t fixed}} \hat{\epsilon}^{0\hat{\alpha}\hat{\beta}}\hat{V}_{\hat{\alpha}}\partial_{\hat{\beta}}\hat{c}^{(0)}\,. \tag{3.38}$$

The canonically transformed action is given by:

$$\begin{array}{rcl} S[X^\mu, \hat{b}_{\hat{\imath}}, \hat{V}_{\hat{\imath}}] & = & -T_{M_2}\int d^3\hat{\xi} e^{-\phi}\sqrt{|g_{\hat{\imath}\hat{\jmath}} + (2\pi\alpha')\hat{\mathcal{F}}_{\hat{\imath}\hat{\jmath}}|} \\ \\ & & -\frac{T_{M_2}}{3!}\int d^3\hat{\xi}\epsilon^{\hat{\imath}\hat{\jmath}\hat{k}}\Big[C^{(3)}_{\hat{\imath}\hat{\jmath}\hat{k}} - 3(2\pi\alpha')C^{(1)}_{\hat{\imath}}\hat{\mathcal{F}}_{\hat{\jmath}\hat{k}} \\ \\ & & +6m\pi\alpha'(2\pi\alpha')\hat{b}_{\hat{\imath}}\partial_{\hat{\jmath}}\hat{V}_{\hat{k}} - 3\frac{m}{2}(2\pi\alpha')^2\hat{b}_{\hat{\imath}}\partial_{\hat{\jmath}}\hat{b}_{\hat{k}} \\ \\ & & +3(2\pi\alpha')\partial_{\hat{\imath}}\hat{a}^{(2)}{}_{\hat{\jmath}\hat{k}} - 6(2\pi\alpha')^2\partial_{\hat{\imath}}\hat{c}^{(0)}\partial_{\hat{\jmath}}V_{\hat{k}}\Big]\,, \end{array} \tag{3.39}$$

where $\hat{\mathcal{F}}_{\hat{\imath}\hat{\jmath}} = 2\partial_{[\hat{\imath}}\hat{V}_{\hat{\jmath}]} + \frac{1}{2\pi\alpha'}B_{\hat{\imath}\hat{\jmath}}$ and we have included the total derivative generated in the dualization procedure, namely

$$-(2\pi\alpha')^2 T_{M_2}\int d^3\hat{\xi}\epsilon\partial\hat{c}^{(0)}\partial\hat{V}\,. \tag{3.40}$$

Elimination of $\hat{b}_{\hat{\imath}}$ through its equation of motion gives $\hat{b}_{\hat{\imath}} = \hat{V}_{\hat{\imath}}$ and the usual action for the D-2-brane is recovered where now $\hat{V}_{\hat{\imath}}$ is the BI field and there are additional world-volume fields compensating for all the total derivatives coming from the gauge transformations. Note that $\hat{c}^{(0)}$ is now playing the role of an auxiliary field needed to compensate for the total derivatives, and not of the eleventh coordinate as in (3.28). In (3.39) $\hat{b}$ is an auxiliary field whose role is to eliminate the explicit dependence on the BI field (other than through its derivatives), such that the standard duality transformation can be applied even in the massive case. Once it is integrated out the usual action is recovered.

3.2.2 Double Dimensional Reduction: the Type IIA String

In this Section we consider the double dimensional reduction of the massive M-2-brane worldvolume effective action. We will see that this reduction leads to the usual ("massless") type IIA string action.

It is convenient to perform the double dimensional reduction of Eq. (3.21) in three steps. First, we perform a direct dimensional reduction, obtaining Eq. (3.28). The second step consists in a partial gauge-fixing of the action Eq. (3.28). We set

$$\hat{c}^{(0)} = \tfrac{1}{2\pi\alpha'}\hat{\xi}^{2}\,, \tag{3.41}$$

while all other worldvolume fields (and gauge parameters) are independent of $\hat{\xi}^2$:

$$\partial_2 X^{\mu} = \partial_2 \hat{b}_{\hat{\imath}} = \partial_2 \hat{a}^{(2)}{}_{\hat{\imath}\hat{\jmath}} = 0\,, \tag{3.42}$$

etc. We want to keep track of all gauge transformations. Thus, we introduce a compensating gauge transformation to keep our gauge choice Eq. (3.41). This transformation is the g.c.t.

$$\delta\hat{\xi}^{\hat{\imath}} = \delta^{\hat{\imath}\ 2}\left[-\Lambda^{(0)} + \tfrac{m}{2}(2\pi\alpha')\hat{\rho}^{(0)}\right]\,. \tag{3.43}$$

Including this compensating gauge transformation, we find the modified gauge transformations

$$\left\{\begin{array}{rcl}
\delta_M \hat{b}_2 & = & 0\,,\\
\delta_M \hat{b}_i & = & \tfrac{1}{2\pi\alpha'}2\lambda_i + \partial_i\hat{\rho}^{(0)} + \left[\partial_i\Lambda^{(0)} - \tfrac{m}{2}(2\pi\alpha')\partial_i\hat{\rho}^{(0)}\right]\hat{b}_2\,,\\
\delta_M \hat{a}^{(2)}{}_{i2} & = & \partial_i\hat{\mu}^{(1)}{}_2 + \tfrac{1}{2\pi\alpha'}2\lambda_i + \tfrac{m}{4}(2\pi\alpha')\hat{\rho}^{(0)}\partial_i\hat{b}_2 - \tfrac{m}{2}\lambda_i\hat{b}_2\,,\\
\delta_M \hat{a}^{(2)}{}_{ij} & = & 2\partial_{[i}\hat{\mu}^{(1)}{}_{j]} - \tfrac{1}{2\pi\alpha'}\Lambda^{(2)}{}_{ij} + \tfrac{m}{2}(2\pi\alpha')\hat{\rho}^{(0)}\partial_{[i}\hat{b}_{j]} - m\lambda_{[i}\hat{b}_{j]}\\
 & & -2\left[\partial\Lambda^{(0)} - \tfrac{m}{2}(2\pi\alpha')\partial\hat{\rho}^{(0)}\right]_{[i}\hat{a}^{(2)}{}_{j]2}\,.
\end{array}\right. \tag{3.44}$$

Now, we perform a field redefinition consisting of a $\Lambda^{(0)}$ gauge transformation where the gauge parameter $\Lambda^{(0)}$ is a new scalar field $c^{(0)}$. Since we are dealing with a gauge-invariant action, the final result is guaranteed not to depend on $c^{(0)}$, no matter how it transforms. It is convenient to asign to $c^{(0)}$ the gauge transformation rules introduced in Section 2 since then the redefined fields also happen to transform as combinations of the standard worldvolume fields introduced in Section 2. More explicitly, we find the following decompositions:

$$
\left\{
\begin{array}{rcl}
\hat{b}_2 & = & v^{(0)}\,, \\
\hat{b}_i & = & b_i - (2\pi\alpha')\partial_i c^{(0)} v^{(0)}\,, \\
\hat{a}^{(2)}{}_{i2} & = & \left[1 - \frac{m}{4}(2\pi\alpha')v^{(0)}\right] b_i + v^{(1)}{}_i\,, \\
\hat{a}^{(2)}{}_{ij} & = & a^{(2)}{}_{ij} + 2(2\pi\alpha')\partial_{[i}c^{(0)}\left\{\left[1 - \frac{m}{4}(2\pi\alpha')v^{(0)}\right] b + v^{(1)}\right\}_{j]}\,,
\end{array}
\right.
\tag{3.45}
$$

where $v^{(1)}{}_i$ is a vector field transforming according to

$$
\left\{
\begin{array}{rcl}
\delta v^{(1)} & = & \partial\Delta^{(0)}\,, \\
\Delta^{(0)} & = & \hat{\mu}^{(1)}{}_2 - \left[1 - \frac{m}{4}(2\pi\alpha')v^{(0)}\right]\hat{\rho}^{(0)}\,,
\end{array}
\right.
\tag{3.46}
$$

and we have identified $\hat{\rho}^{(0)} = \rho^{(0)}$ and $\hat{\mu}^{(1)}{}_i = \mu^{(1)}{}_i$.

Given the above identifications of the potentials, we find, for the field strength

$$
\hat{\mathcal{H}}^{(3)}{}_{ij2} = \left[1 - \frac{m}{2}(2\pi\alpha')v^{(0)}\right]\mathcal{F}_{ij} + \mathcal{L}^{(2)}{}_{ij}\,, \tag{3.47}
$$

where $\mathcal{F}$ is the BI field strength defined in Section 2 and

$$
\mathcal{L}^{(2)} = 2\partial v^{(1)}\,. \tag{3.48}
$$

Substituting these results and integrating over $\hat{\xi}^2$ the massive M-2-brane action one arrives at the action

$$
\begin{array}{rcl}
\tilde{S}\;[X^\mu, b, v^{(1)}] & = & -T_{M2}l \int d^2\xi\; \left[1 - \frac{m}{2}(2\pi\alpha')v^{(0)}\right]\sqrt{|g_{ij}|} \\
 & & -\frac{T_{M2}l}{2}\int d^2\xi\;\epsilon\left\{\left[1 - \frac{m}{2}(2\pi\alpha')v^{(0)}\right]\mathcal{F}_{ij} + \mathcal{L}^{(2)}{}_{ij}\right\}\,.
\end{array}
\tag{3.49}
$$

Now, the equation of motion for b implies that the scalar $v^{(0)}$ is constant. Substituting this result into the action and redefining

$$
b + \left[1 - \frac{m}{2}(2\pi\alpha')v^{(0)}\right]^{-1} v^{(1)} \to b\,, \tag{3.50}
$$

(plus an analogous redefinition of the gauge parameters) we get the action of the type IIA string in the Nambu-Goto form with an extra term which does not change the equations of motion but makes the WZ term exactly gauge-invariant[28]:

$$S\ [X^{\mu}, b_i] = -T\int d^2\xi\ \sqrt{|g_{ij}|} - (2\pi\alpha')\tfrac{T}{2}\int d^2\xi\ \epsilon\mathcal{F}\,, \tag{3.51}$$

where the tension T is now given by:

$$T = \left[1 - \tfrac{m}{2}(2\pi\alpha')v^{(0)}\right]T_{M_2}l\,. \tag{3.52}$$

Observe that the new tension depends on the constant value of the gauge vector field $\hat{b}$ in the compact direction $v^{(0)} = \hat{b}_2$ (a Wilson line). The Wilson line has a critical value for which the tension vanishes. It would be interesting to investigate the physical mechanism behind this phenomenon.

3.2.3 Reduction in a Non-Isotropic Direction

Let us now assume that there exists an additional isometry realized by translations of a coordinate other than the isotropic coordinate Y, and perform the double dimensional reduction along this direction. In this subsection we will ignore total derivatives and only analyze in detail the λ, $\rho^{(0)}$ gauge transformations for the sake of simplicity. We split the eleven coordinates into (Z, X^{μ}) and use the ansatz:

$$Z = \hat{\xi}^2\,, \qquad \partial_2 X^{\mu} = \partial_2\hat{b}_{\hat{\imath}} = 0\,. \tag{3.53}$$

We also split the vector field $\hat{b}$

$$\hat{b}_i = b_i\,, \qquad \hat{b}_2 = v^{(0)}\,. \tag{3.54}$$

Notice that now the isotropic coordinate Y is among the X^{μ} coordinates. As before, the ansatz above is compatible with the equations of motion of the action (3.28) and the resulting action. The latter is given by:

[28]Substituting the equation of motion of $v^{(0)}$ into the action one gets an action from which one cannot derive any equation of motion. As we have stressed before, the elimination of a field through its equation of motion can not always be done directly in the action and the equations of motion always have to be checked.

$$\tilde{S}[X^\mu, b_i, v^{(0)}] = -T_{M2}l \int d^2\xi \sqrt{|D_iX^\mu D_jX^\nu g_{\mu\nu}|}$$
$$-\tfrac{T_{M2}l}{2} \int d^2\xi\epsilon^{ij} \Big[D_iX^\mu D_jX^\nu B_{\mu\nu} - m\pi\alpha' v^{(0)} \left(i_k C^{(3)}\right)_{\mu\nu} \partial_i X^\mu \partial_j X^\nu$$
$$-m(2\pi\alpha')^2 v^{(0)} \partial_i b_j\Big] \, . \tag{3.55}$$

Integrating out $v^{(0)}$ we obtain the constraint:

$$\epsilon^{ij}\partial_i b_j = -\tfrac{1}{2(2\pi\alpha')}\epsilon^{ij}(i_k C^{(3)})_{ij} \, . \tag{3.56}$$

Substituting this constraint back into (3.55) the action of a gauged superstring in ten dimensions is obtained:

$$\begin{aligned} \tilde{S}[X^\mu, b_i] &= -T_{M2}l \int d^2\xi \sqrt{|D_iX^\mu D_jX^\nu g_{\mu\nu}|} \\ & \quad -\tfrac{T_{M2}l}{2} \int d^2\xi\epsilon^{ij} D_iX^\mu D_jX^\nu B_{\mu\nu} \, . \end{aligned} \tag{3.57}$$

This action is invariant under the transformations:

$$\left\{ \begin{aligned} \delta g_{\mu\nu} &= 2m\lambda_{(\mu}\left(i_k g\right)_{\nu)} + \tfrac{m}{2}(2\pi\alpha')\rho^{(0)}k^\gamma\partial_\gamma g_{\mu\nu} \, , \\ \delta B_{\mu\nu} &= -2m\lambda_{[\mu}\left(i_k B\right)_{\nu]} + 2\partial_{[\mu}\lambda_{\nu]} + \tfrac{m}{2}(2\pi\alpha')\rho^{(0)}k^\gamma\partial_\gamma B_{\mu\nu} \, , \\ \delta b &= \tfrac{1}{2\pi\alpha'}2\lambda + \partial\rho^{(0)} \, , \end{aligned} \right. \tag{3.58}$$

assuming the conditions:

$$\pounds_k g_{\mu\nu} = \pounds_k B_{\mu\nu} = \pounds_k \lambda_\mu = 0 \tag{3.59}$$

hold.

The above transformations are to be interpreted within the nine dimensional theory obtained by dimensional reduction along the isotropic coordinate. The ansatz for the decomposition of the ten dimensional NS-NS fields is[29] [20]:

[29] We split the ten dimensional coordinates as $X^\mu = (\tilde{X}^{\tilde{\mu}}, Y)$. In this section tilded fields are nine dimensional.

$$
\left\{\begin{array}{rcl} g_{\tilde{\mu}\tilde{\nu}} & = & \tilde{g}_{\tilde{\mu}\tilde{\nu}} - \tilde{k}^2 \tilde{A}^{(1)}{}_{\tilde{\mu}} \tilde{A}^{(1)}{}_{\tilde{\nu}}\,, \\ \\ g_{y\tilde{\mu}} & = & -\tilde{k}^2 \tilde{A}^{(1)}{}_{\tilde{\mu}}\,, \\ \\ g_{yy} & = & -\tilde{k}^2\,, \end{array}\right. \qquad \left\{\begin{array}{rcl} B_{\tilde{\mu}\tilde{\nu}} & = & \tilde{B}^{(2)}{}_{\tilde{\mu}\tilde{\nu}} + 2\tilde{A}^{(1)}{}_{[\tilde{\mu}} \tilde{B}^{(1)}{}_{\tilde{\nu}]}\,, \\ \\ B_{y\tilde{\mu}} & = & \tilde{B}^{(1)}{}_{\tilde{\mu}}\,, \\ \\ \phi & = & \tilde{\phi} + \frac{1}{2}\log \tilde{k}\,, \end{array}\right. \tag{3.60}
$$

and the dimensionally reduced action reads:

$$
\begin{aligned} S \;=\; & -T_{M_2} l \int d^2\xi \sqrt{|\tilde{g}_{ij} - \tilde{k}^2 (2\pi\alpha')^2 \tilde{\mathcal{G}}^{(1)}{}_i \tilde{\mathcal{G}}^{(1)}{}_j|} \\ & -\tfrac{T_{M_2} l}{2} \int d^2\xi \epsilon^{ij} \left[\tilde{B}^{(2)}{}_{ij} + 2(2\pi\alpha')\tilde{\mathcal{G}}^{(1)}{}_i \tilde{B}^{(1)}{}_j\right]\,, \end{aligned} \tag{3.61}
$$

where now the field strength $\tilde{\mathcal{G}}^{(1)}$ is defined by (recall $Y = (2\pi\alpha')c^{(0)}$):

$$
\tilde{\mathcal{G}}^{(1)} = \partial c^{(0)} + \tfrac{1}{2\pi\alpha'}\tilde{A}^{(1)} - \tfrac{m}{2} b\,. \tag{3.62}
$$

It is easy to see that this action is invariant under the nine dimensional massive transformations:

$$
\left\{\begin{array}{rcl} \delta_\lambda \tilde{A}^{(1)} & = & m\lambda\,, \\ \\ \delta_\lambda \tilde{B}^{(1)} & = & 0\,, \end{array}\right. \qquad \left\{\begin{array}{rcl} \delta_\lambda \tilde{B}^{(2)} & = & 2\partial\lambda\,, \\ \\ \delta_\lambda b & = & \frac{1}{2\pi\alpha'} 2\lambda\,. \end{array}\right. \tag{3.63}
$$

The existence of a string solution invariant under nine dimensional massive transformations seems to imply that a similar type of solution will exist in the type IIB theory after performing a T-duality transformation. However we will now show that after the T-duality transformation the auxiliary vector field b decouples from the other fields and its integration fixes the mass parameter to zero, in such a way that the resulting action corresponds to the dimensional reduction of the massless type IIB superstring.

In order to construct the T-dual of the action (3.61), which is nothing but a Poincaré duality transformation of the redundant coordinate, we add to it the following Lagrange multiplier term enforcing the Bianchi identity for $c^{(0)}$ (after an integration by parts):

$$
T_{M_2} l \int d^2\xi (2\pi\alpha') \epsilon^{ij} \partial_i \varrho \, \left(\tilde{\mathcal{G}}^{(1)}_j - \tfrac{1}{2\pi\alpha'}\tilde{A}^{(1)}{}_j + \tfrac{m}{2} b_j\right)\,. \tag{3.64}
$$

Integration over $\tilde{\mathcal{G}}^{(1)}$ gives the following dual action:

$$
\begin{aligned}
S_{\rm IIB} &= -T_{M_2} l \int d^2\xi \sqrt{|\tilde{g}_{ij} - \tfrac{1}{k^2}(\partial_i\varrho + \tilde{B}_i^{(1)})(\partial_j\varrho + \tilde{B}_j^{(1)})|} \\
& \quad -\tfrac{T_{M_2} l}{2} \int d^2\xi \epsilon^{ij} \left[\tilde{B}^{(2)}{}_{ij} + 2\partial_i\varrho \ \left(\tilde{A}^{(1)}{}_i + m\pi\alpha' b_j\right)\right] .
\end{aligned} \tag{3.65}
$$

Here the auxiliary field b can be integrated out, giving the constraint that m must be equal to zero[30]. The resulting action is the dimensional reduction of the ten dimensional type IIB superstring along the ϱ coordinate (see [20] for more details).

3.3 The Massive M-5-Brane

We start by reviewing the massless case. The worldvolume fields of the massless M-5-brane are the eleven embedding scalars $\hat{X}^{\hat{\mu}}$ ($\hat{\mu} = 0, 1, \ldots, 10$) and an (anti-self-dual[31]) worldvolume two-form $\hat{\omega}^{(2)}{}_{\hat{\imath}\hat{\jmath}}$ ($i = 0, \ldots, 5$). The curvature $\hat{\mathcal{K}}^{(3)}$ of $\hat{\omega}^{(2)}$ is given by[32]

$$
\hat{\mathcal{K}}^{(3)} = 3\left[\partial\hat{\omega}^{(2)} + \tfrac{1}{3(2\pi\alpha')}\hat{C}\right] , \tag{3.66}
$$

where $\hat{C}$ is the 3-form potential of 11-dimensional supergravity, with gauge transformation

$$
\delta\hat{C} = 3\partial\hat{\chi} \tag{3.67}
$$

under a 2-form gauge parameter $\hat{\chi}$. The gauge invariance of $\hat{\mathcal{K}}^{(3)}$ implies the following gauge transformation law of $\hat{\omega}^{(2)}$:

$$
\delta\hat{\omega}^{(2)} = -\tfrac{1}{2\pi\alpha'}\hat{\chi} + 2\partial\hat{\rho}^{(1)} . \tag{3.68}
$$

The anti-self-duality condition of the field strength can be written in the linear approximation (which we will use throughout this section) as follows:

$$
\hat{\mathcal{K}}^{(3)} = -{}^*\hat{\mathcal{K}}^{(3)} , \tag{3.69}
$$

[30] Or $\varrho =$ constant, which is a particular case in the final action.

[31] With our conventions the 3-form field strength has to be anti-self-dual, otherwise the kinetic term would have the wrong sign. In the absence of the WZ term, imposing self- or anti-self-duality is a matter of convention and either choice is possible. In the presence of the WZ term, with our conventions, only anti-self-duality can be consistently imposed.

[32] In this Section, worldvolume hatted indices (Latin) and fields are 6-dimensional, while unhatted worldvolume indices and fields are 5-dimensional. The split is $\hat{\imath} = (i, 5)$, with $i = 0, 1, \ldots, 4$. Note also that we have adapted our conventions from those in [23].

and therefore it is not possible to write a covariant action for $\hat{\omega}^{(2)}$[33]. We can write however an action in which this constraint is only imposed at the level of the equations of motion derived from it. This can be done consistently if the action is Poincaré anti-self-dual. To second order in $\hat{\mathcal{K}}^{(3)}$ the unique action with the required properties is [23]

$$\begin{aligned}
\hat{S}[\hat{X}^{\hat{\mu}}, \hat{\omega}^{(2)}{}_{\hat{\imath}\hat{\jmath}}] &= -T_{M5} \int d^6\hat{\xi}\ \sqrt{|\hat{g}_{\hat{\imath}\hat{\jmath}}|}\ \left\{1 - \tfrac{1}{4\cdot 3!}(2\pi\alpha')^2\ \left(\hat{\mathcal{K}}^{(3)}\right)^2 + \cdots\right\} \\
&\quad +(2\pi\alpha')\tfrac{T_{M5}}{6!} \int d^6\hat{\xi}\ \hat{\epsilon}^{\hat{\imath}_1\cdots\hat{\imath}_6}\hat{\mathcal{K}}^{(6)}{}_{\hat{\imath}_1\cdots\hat{\imath}_6}\ ,
\end{aligned} \tag{3.70}$$

where $\hat{\mathcal{K}}^{(6)}$ is the gauge-invariant 6-form field strength of the worldvolume 5-form field $\hat{\omega}^{(5)}$:

$$\hat{\mathcal{K}}^{(6)} = 6\left[\partial\hat{\omega}^{(5)} + \tfrac{1}{6(2\pi\alpha')}\hat{\tilde{C}} + \tfrac{5}{3}\hat{\mathcal{K}}^{(3)}\hat{C}\right]\ , \tag{3.71}$$

whose mission is to make the above action exactly gauge invariant. Thus

$$\delta\hat{\omega}^{(5)} = -\tfrac{1}{2\pi\alpha'}\hat{\tilde{\chi}} + 15\partial\hat{\omega}^{(2)}\hat{\chi} + 5\partial\hat{\rho}^{(4)}\ . \tag{3.72}$$

This worldvolume 5-form has also been recently considered in Ref. [34]. Observe that the M-5-brane couples naturally to the dual 6-form potential of d=11 supergravity [21]. The action (3.70) is invariant (including total derivatives) under the massless gauge transformations (3.67), (3.68) together with the gauge transformations of the 6-form (1.51), and it gives the massless D-4-brane action upon double dimensional reduction and the massless p-5A-brane action upon direct dimensional reduction [23].

Our goal is to construct a massive M-5-brane action which is invariant under the 11-dimensional massive gauge transformations. The double dimensional reduction of this action should give the massive D-4-brane action and the direct dimensional reduction the massive p-5A-brane action. These will be derived as by-products of our construction in a second step.

To construct the massive M-5-brane action we proceed as we did with the M-0- and M-2-brane actions: we assume that the background admits an isometry and fulfills all the requirements to be considered a "massive 11-dimensional background". Then we gauge the isometry in the usual M-5-brane action, substituting all partial derivatives of the embedding scalars

[33] A covariant action can be constructed by introducing an auxiliary scalar field [31]. The nonlinear form of the equations of motion of the massless M-5-brane has been given in [31, 32, 33]. We expect that the results given in this Section on the massive M-5-brane automatically carry over to the nonlinear case simply by replacing the massless curvatures in [31, 32, 33] by their massive extensions.

$\partial_{\hat\imath}\hat X^{\hat\mu}$ by covariant derivatives $D_{\hat\imath}\hat X^{\hat\mu}$ with gauge field $\hat b_{\hat\imath}$, trying to keep gauge invariance as well as Poincaré anti-self-duality (which are both broken by naive gauging). As we saw in the M-2-brane case this implies the introduction of new terms (in the present case not only the auxiliary gauge fields $\hat b_{\hat\imath}, \hat\omega^{(5)}$ but also an additional worldvolume 6-form $\hat\omega^{(6)}$), as well as a modification of the gauge transformations of both the background fields and the worldvolume fields. The new transformations of the background fields turn out to be precisely those of massive (dual) 11-dimensional supergravity.

The action that we get to second order is:

$$
\begin{aligned}
\hat S[\hat X^{\hat\mu}, \hat\omega^{(2)}{}_{\hat\imath\hat\jmath}, \hat b_{\hat\imath}] \;&= \\
&-T_{M5}\int d^6\hat\xi\, \sqrt{|D_{\hat\imath}\hat X^{\hat\mu} D_{\hat\jmath}\hat X^{\hat\nu}\hat g_{\hat\mu\hat\nu}|}\, \left\{1-\tfrac{1}{4\cdot 3!}(2\pi\alpha')^2\, \left(\hat{\mathcal K}^{(3)}\right)^2+\cdots\right\} \\
&+(2\pi\alpha')\tfrac{T_{M5}}{6!}\int d^6\hat\xi\;\hat\epsilon^{\hat\imath_1\ldots\hat\imath_6}\hat{\mathcal K}^{(6)}{}_{\hat\imath_1\ldots\hat\imath_6}\,,
\end{aligned}
\tag{3.73}
$$

where $\hat{\mathcal K}^{(3)}$ and $\hat{\mathcal K}^{(6)}$ are now the massive field strengths of $\hat\omega^{(2)}$ and $\hat\omega^{(5)}$:

$$
\left\{
\begin{aligned}
\hat{\mathcal K}^{(3)} &= 3\left[\partial\hat\omega^{(2)}+\tfrac{1}{3(2\pi\alpha')}D\hat X^{\hat\mu}D\hat X^{\hat\nu}D\hat X^{\hat\rho}\hat C_{\hat\mu\hat\nu\hat\rho}-\tfrac{m}{2}(2\pi\alpha')\hat b\partial\hat b\right], \\
\hat{\mathcal K}^{(6)} &= 6\partial\hat\omega^{(5)}+\tfrac{m}{2}\hat\omega^{(6)}+\tfrac{1}{2\pi\alpha'}D\hat X^{\hat\mu_1}\cdots D\hat X^{\hat\mu_6}\hat{\tilde C}_{\hat\mu_1\cdots\hat\mu_6} \\
&\quad +10\hat{\mathcal K}^{(3)}\left[D\hat X^{\hat\mu}D\hat X^{\hat\nu}D\hat X^{\hat\rho}\hat C_{\hat\mu\hat\nu\hat\rho}-3\tfrac{m}{2}(2\pi\alpha')^2\hat b\partial\hat b\right] \\
&\quad +30\tfrac{m}{2}(2\pi\alpha')D\hat X^{\hat\mu}D\hat X^{\hat\nu}D\hat X^{\hat\rho}\hat C_{\hat\mu\hat\nu\hat\rho}\hat b\partial\hat b\,,
\end{aligned}
\right.
\tag{3.74}
$$

and the indices of $\hat{\mathcal K}^{(3)}$ in the kinetic term are contracted using the metric

$$
D_{\hat\imath}\hat X^{\hat\mu} D_{\hat\jmath}\hat X^{\hat\nu}\hat g_{\hat\mu\hat\nu}\,. \tag{3.75}
$$

The covariant derivative is defined by

$$
D_{\hat\imath}\hat X^{\hat\mu} = \partial_{\hat\imath}\hat X^{\hat\mu}-\tfrac{m}{2}(2\pi\alpha')\hat b_{\hat\imath}\hat k^{\hat\mu}\,. \tag{3.76}
$$

Note that the metric above is also used to raise the indices of $\hat{\mathcal K}^{(3)}$ in the anti-self-duality constraint, which now takes the form[34]

[34]In our conventions the antisymmetric tensor $\hat\epsilon$ is defined to be independent of the metric with upper indices $\hat\epsilon^{012345}=+1$.

$$\hat{\mathcal{K}}^{(3)\ \hat{\imath}_1\hat{\imath}_3\hat{\imath}_3} = -\frac{\hat{\epsilon}^{\hat{\imath}_1\ldots\hat{\imath}_6}}{3!\sqrt{|D_{\hat{\imath}}\hat{X}^{\hat{\mu}}D_{\hat{\jmath}}\hat{X}^{\hat{\nu}}\hat{g}_{\hat{\mu}\hat{\nu}}|}}\hat{\mathcal{K}}^{(3)}{}_{\hat{\imath}_4\hat{\imath}_5\hat{\imath}_6}\,. \tag{3.77}$$

The massive field strengths $\hat{\mathcal{K}}^{(3)}$ and $\hat{\mathcal{K}}^{(6)}$, and therefore the full action, are invariant under the following transformations of spacetime fields

$$\left\{\begin{array}{rcl}
\delta\hat{g}_{\hat{\mu}\hat{\nu}} &=& -\left[\Lambda^{(0)} - \frac{m}{2}(2\pi\alpha')\hat{\rho}^{(0)}\right]\hat{k}^{\hat{\rho}}\partial_{\hat{\rho}}\hat{g}_{\hat{\mu}\hat{\nu}} \\
&& +2\left(\partial\Lambda^{(0)} + m\hat{\lambda}\right)_{(\hat{\mu}}\left(i_{\hat{k}}\hat{g}\right)_{\hat{\nu})}\ , \\
\delta\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} &=& 3\partial_{[\hat{\mu}}\hat{\chi}_{\hat{\nu}\hat{\rho}]} - \left[\Lambda^{(0)} - \frac{m}{2}(2\pi\alpha')\hat{\rho}^{(0)}\right]\hat{k}^{\hat{\lambda}}\partial_{\hat{\lambda}}\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} \\
&& +3\left(\partial\Lambda^{(0)} + m\hat{\lambda}\right)_{[\hat{\mu}}\left(i_{\hat{k}}\hat{C}\right)_{\hat{\nu}\hat{\rho}]}\ , \\
\delta\hat{\tilde{C}}_{\hat{\mu}_1\ldots\hat{\mu}_6} &=& 6\partial_{[\hat{\mu}_1}\hat{\tilde{\chi}}_{\hat{\mu}_2\ldots\hat{\mu}_6]} + 30\partial_{[\hat{\mu}_1}\hat{\chi}_{\hat{\mu}_2\hat{\mu}_3}\hat{C}_{\hat{\mu}_4\hat{\mu}_5\hat{\mu}_6]} \\
&& -\left[\Lambda^{(0)} - \frac{m}{2}(2\pi\alpha')\hat{\rho}^{(0)}\right]\hat{k}^{\hat{\lambda}}\partial_{\hat{\lambda}}\hat{\tilde{C}}_{\hat{\mu}_1\ldots\hat{\mu}_6} \\
&& -6\left(\partial\Lambda^{(0)} + m\hat{\lambda}\right)_{[\hat{\mu}_1}\left(i_{\hat{k}}\hat{\tilde{C}}\right)_{\hat{\mu}_2\ldots\hat{\mu}_6]} - m\hat{\tilde{\lambda}}_{\hat{\mu}_1\ldots\hat{\mu}_6}\ ,
\end{array}\right. \tag{3.78}$$

and worldvolume fields

$$
\left\{
\begin{array}{rcl}
\delta\hat{X}^{\hat{\mu}} & = & -\left[\Lambda^{(0)} - \frac{m}{2}(2\pi\alpha')\hat{\rho}^{(0)}\right]\hat{k}^{\hat{\mu}}\,, \\
\delta\hat{b}_{\hat{\imath}} & = & \frac{1}{2\pi\alpha'}2\hat{\lambda}_{\hat{\imath}} + \partial_{\hat{\imath}}\hat{\rho}^{(0)}\,, \\
\delta\hat{\omega}^{(2)}{}_{\hat{\imath}\hat{\jmath}} & = & -\frac{1}{2\pi\alpha'}\hat{\chi}_{\hat{\imath}\hat{\jmath}} - \frac{m}{2}(2\pi\alpha')\left(\frac{1}{2\pi\alpha'}2\hat{\lambda} + \partial\hat{\rho}^{(0)}\right)_{[\hat{\imath}}\hat{b}_{\hat{\jmath}]} + 2\partial_{[\hat{\imath}}\hat{\rho}^{(1)}{}_{\hat{\jmath}]}\,, \\
\delta\hat{\omega}^{(5)}{}_{\hat{\imath}_1\ldots\hat{\imath}_5} & = & -\frac{1}{2\pi\alpha'}\hat{\tilde{\chi}}_{\hat{\imath}_1\ldots\hat{\imath}_5} - \frac{m}{2}\hat{\rho}^{(5)} + 5\partial_{[\hat{\imath}_1}\hat{\rho}^{(4)}{}_{\hat{\imath}_2\ldots\hat{\imath}_5]} \\
 & & +15\partial\hat{\omega}^{(2)}{}_{[\hat{\imath}_1\hat{\imath}_2}\left[\hat{\chi} + \frac{m}{2}(2\pi\alpha')\left(\frac{1}{2\pi\alpha'}2\hat{\lambda} + \partial\hat{\rho}^{(0)}\right)\hat{b}\right]_{\hat{\imath}_3\hat{\imath}_4\hat{\imath}_5]}\,, \\
\delta\hat{\omega}^{(6)}{}_{\hat{\imath}_1\ldots\hat{\imath}_6} & = & \frac{1}{2\pi\alpha'}2\hat{\tilde{\lambda}}_{\hat{\imath}_1\ldots\hat{\imath}_6} \\
 & & +90\frac{m}{2}(2\pi\alpha')^3\left(\frac{1}{2\pi\alpha'}2\hat{\lambda} + \partial\hat{\rho}^{(0)}\right)_{[\hat{\imath}_1}\hat{b}_{\hat{\imath}_2}\partial_{\hat{\imath}_3}\hat{b}_{\hat{\imath}_4}\partial_{\hat{\imath}_5}\hat{b}_{\hat{\imath}_6]} \\
 & & -30\hat{b}_{[\hat{\imath}_1}\partial_{\hat{\imath}_2}\left(i_{\hat{k}}\hat{\tilde{\chi}}\right)_{\hat{\imath}_3\ldots\hat{\imath}_6]} - 180(2\pi\alpha')\partial_{[\hat{\imath}_1}\hat{\chi}_{\hat{\imath}_2\hat{\imath}_3}\hat{b}_{\hat{\imath}_4}\partial_{\hat{\imath}_5}\hat{b}_{\hat{\imath}_6]} \\
 & & +6\partial_{[\hat{\imath}_1}\hat{\rho}^{(5)}{}_{\hat{\imath}_2\ldots\hat{\imath}_6]}\,.
\end{array}
\right.
\tag{3.79}
$$

We have included in these transformations infinitesimal reparametrizations with parameter $\Lambda^{(0)}$ in the direction of the Killing vector $\hat{k}^{\hat{\mu}}$ since they will become gauge transformations after dimensional reduction. Observe that $\hat{\omega}^{(5)}$ transforms as a Stueckelberg field for $\hat{\omega}^{(6)}$.

We want to comment on a subtlety in the construction of the above action and transformation rules which will be relevant when discussing the double dimensional reduction of the massive M-5-brane. The point is the following. In the present construction we have used that the target-space 6-form $\hat{\tilde{C}}$ transforms with a shift under a dual massive gauge transformation with parameter $\hat{\tilde{\lambda}}_{\hat{\mu}_1\cdots\hat{\mu}_6}$. This shift was canceled, in the variation of the WZ term, by the introduction of the 6-form worldvolume field $\hat{\omega}^{(6)}$. However, we know that the parameter $\hat{\tilde{\lambda}}_{\hat{\mu}_1\cdots\hat{\mu}_6}$ is a constrained parameter satisfying

$$
\hat{k}^{\hat{\mu}_6}\hat{\tilde{\lambda}}_{\hat{\mu}_1\cdots\hat{\mu}_5\hat{\mu}_6} = 0\,. \tag{3.80}
$$

This means that, after a worldvolume reduction in the isometry direction, there is no shift variation to cancel in the WZ term and, consequently, there is no need to introduce a new worldvolume 5-form to cancel such a variation. Nevertheless, as we will see, our reduction procedure leads to the appearance of another 5-form, essentially decoupled from the rest of the action. This is consistent with the fact that the known standard massive D-4-brane action, which is the double dimensional reduction of the massive M-5-brane action, does not contain a worldvolume 5-form in its WZ term. Therefore, in order to obtain the standard D-4-brane action one must perform by hand a truncation of the theory. It is only with this understanding that the massive M-5-brane action given above leads to the massive D-4-brane action. At present it is not clear to us how to impose this truncation *before* the double dimensional reduction, at the level of the massive M-5-brane action itself.

We now take the massive M-5-brane action as our starting point and construct in the following two subsections two massive brane actions of type IIA superstring theory. First, we perform a direct dimensional reduction to obtain the massive p-5A-brane effective action and next we perform a double dimensional reduction to get the massive D-4-brane effective action. This last action is obtained in the so-called "1-2-form formalism", which uses a 1- and a 2-form worldvolume fields that are related to each other by a duality constraint (inherited from the anti-self-duality of $\hat{\mathcal{K}}^{(3)}$). Using this constraint we then obtain the usual formulation of the massive D-4-brane action which only uses a 1-form BI field.

3.3.1 Direct Dimensional Reduction: The Massive p-5A-Brane

We assume that the background fields do not depend on the coordinate Y and rewrite them in 10-dimensional form. At the same time the worldvolume scalar Y can be treated differently to the other embedding coordinate scalars. We rename it

$$Y \equiv (2\pi\alpha')\hat{c}^{(0)} \,. \tag{3.81}$$

This $\hat{c}^{(0)}$ transforms as the worldvolume scalar defined in Section 2.

Furthermore, it is convenient to redefine $\hat{\omega}^{(2)}, \hat{\omega}^{(5)}$ and $\hat{\omega}^{(6)}$ so that their relation with the worldvolume fields defined in Section 2 is manifest. Using 10-dimensional notation, the transformation rules of these fields read[35]

[35] We use that $\hat{\lambda}_\mu = \lambda_\mu$ and $\hat{\lambda}_y = 0$. Similarly, $\hat{\tilde{\lambda}}_{\mu_1\cdots\mu_6} = \tilde{\lambda}_{\mu_1\cdots\mu_6}$ and $\hat{\tilde{\lambda}}_{\mu_1\cdots\mu_5 y} = 0$. The relations between the other gauge parameters of the 11- and 10-dimensional theories are given in (1.62).

$$
\begin{cases}
\delta\hat{\omega}^{(2)} & = & -\frac{1}{2\pi\alpha'}\Lambda^{(2)} + 4\lambda\partial\hat{c}^{(0)} \\
& & -\frac{m}{2}(2\pi\alpha')\left[\frac{1}{2\pi\alpha'}2\lambda + \partial\hat{\rho}^{(0)}\right]\hat{b} + 2\partial\hat{\rho}^{(1)}\,, \\
\delta\hat{\omega}^{(5)} & = & \frac{1}{2\pi\alpha'}\tilde{\Lambda} - \frac{m}{2}\hat{\rho}^{(5)} - 5\Lambda^{(4)}\partial\hat{c}^{(0)} \\
& & -15(2\pi\alpha')\partial\hat{a}^{(2)}\left(\delta\hat{a}^{(2)} - 2\partial\hat{\mu}^{(1)}\right) + 5\partial\hat{\rho}^{(4)}\,, \\
\delta\hat{\omega}^{(6)} & = & \frac{1}{2\pi\alpha'}2\tilde{\lambda} + 90\frac{m}{2}(2\pi\alpha')^3\left[\frac{1}{2\pi\alpha'}2\lambda + \partial\hat{\rho}^{(0)}\right]\hat{b}\partial\hat{b}\partial\hat{b} \\
& & -180(2\pi\alpha')\partial\Lambda^{(2)}\hat{b}\partial\hat{b} + 720(2\pi\alpha')^2\partial\lambda\partial\hat{c}^{(0)}\hat{b}\partial\hat{b} \\
& & -30\hat{b}\partial\Lambda^{(4)} + 6\partial\hat{\rho}^{(5)}\,.
\end{cases}
\tag{3.82}
$$

It is easy to see that $\hat{\omega}^{(2)}$ transforms as $\hat{a}^{(2)}$ (just as it happened in the M-2-brane case), and $\hat{\omega}^{(5)}$ transforms as $-\hat{\hat{b}}$. Similarly, one may verify that the transformation rule of $\hat{\omega}^{(6)}$ is related to that of $\hat{c}^{(6)}$ via the relation

$$
\hat{\omega}^{(6)} \equiv -\hat{c}^{(6)} + 120(2\pi\alpha')^3\partial\hat{c}^{(0)}\hat{b}\partial\hat{b}\partial\hat{b}\,, \tag{3.83}
$$

if we make the identification

$$
\begin{aligned}
\hat{\rho}^{(5)} \quad = \quad & -\hat{\kappa}^{(5)} - 5\Lambda^{(4)}\hat{b} + 30(2\pi\alpha')\Lambda^{(2)}\hat{b}\partial\hat{b} - 20(2\pi\alpha')^2\Lambda^{(0)}\hat{b}\partial\hat{b}\partial\hat{b} \\
& -20(2\pi\alpha')^3\hat{\rho}^{(0)}\partial\hat{c}^{(0)}\partial\hat{b}\partial\hat{b} + 40(2\pi\alpha')^2\partial\hat{c}^{(0)}\lambda\hat{b}\partial\hat{b} \\
& +5\frac{m}{2}(2\pi\alpha')^3\hat{\rho}^{(0)}\hat{b}\partial\hat{b}\partial\hat{b}\,.
\end{aligned}
\tag{3.84}
$$

In doing the reduction, we have to take into account that there are background fields present in the 3-form field strength. The resulting action in the quadratic approximation (linear in the equations of motion) is

$$
\begin{aligned}
\hat{S}[X^{\mu}, \hat{c}^{(0)}, \hat{b}, \hat{a}^{(2)}] \;\; = & \\
& -T_{M5} \int d^6\hat{\xi}\; e^{-2\phi}\, \sqrt{|g_{\hat{\imath}\hat{\jmath}} - (2\pi\alpha')^2\; e^{2\phi}\; \hat{\mathcal{G}}^{(1)}{}_{\hat{\imath}} \hat{\mathcal{G}}^{(1)}{}_{\hat{\jmath}}|}\; \times \\
& \qquad\qquad \times \left\{1 - \tfrac{1}{4\cdot 3!}(2\pi\alpha')^2\; e^{2\phi} \left(\hat{\mathcal{H}}^{(3)}\right)^2 + \cdots \right\} \\
& -(2\pi\alpha')\tfrac{T_{M5}}{6!} \int d^6\hat{\xi}\; \hat{\epsilon}^{\hat{\imath}_1\cdots\hat{\imath}_6} \hat{\tilde{\mathcal{F}}}_{\hat{\imath}_1\cdots\hat{\imath}_6}\,,
\end{aligned}
\tag{3.85}
$$

where the field strengths $\hat{\mathcal{G}}^{(1)}, \hat{\mathcal{H}}^{(3)}$ and $\hat{\tilde{\mathcal{F}}}$ are defined in Section 2.

After getting the equations of motion from the action above one still has to impose the anti-self-duality constraint, which in the linear approximation used here reads:

$$
\hat{\mathcal{H}}^{(3)} = -e^{\phi}\; {}^{\star}\hat{\mathcal{H}}^{(3)}\,. \tag{3.86}
$$

The factor $e^{-2\phi}$ in front of the kinetic term indicates that the physical mass is proportional to g_A^{-2}, $g_A = e^{<\phi>}$ being the string coupling constant. This is the right behavior for a solitonic object.

3.3.2 Double Dimensional Reduction: the Massive D-4-brane

We take as our starting point the massive p-5A-brane action and gauge-fix the g.c.t. transformation in the $\hat{\xi}^5$-direction by imposing the condition

$$
\hat{c}^{(0)} = \tfrac{1}{2\pi\alpha'}\hat{\xi}^5\,. \tag{3.87}
$$

To preserve this gauge condition we have to perform a compensating world-volume g.c.t. transformation in the $\hat{\xi}^5$ direction

$$
\delta\hat{\xi}^{\hat{\imath}} = \delta^{\hat{\imath}5}\left[-\Lambda^{(0)} + \tfrac{m}{2}(2\pi\alpha')\hat{\rho}^{(0)}\right]\,, \tag{3.88}
$$

leading to modifications in the $\Lambda^{(0)}$ and $\hat{\rho}^{(0)}$ transformations of the reduced worldvolume fields. Now, however, we cannot perform the field redefinitions that we used in the M-2–brane case. In this case we have target space fields transforming under $\Lambda^{(0)}$ and we do not want to modify their gauge transformations with the introduction of $c^{(0)}$. This complicates somewhat our work.

We do the following identification:

$$\begin{aligned}\hat{c}^{(6)}{}_{ijklm5} &= -5(2\pi\alpha')\partial_{[i}v^{(0)}c^{(4)}{}_{jklm]} - 5\tfrac{m}{2}(2\pi\alpha')^3 v^{(0)} b_{[i}\partial_j b_k \partial_l b_{m]} \\ &\quad + v^{(5)}{}_{ijklm}\,,\end{aligned} \tag{3.89}$$

where $v^{(5)}$ is a vector field transforming according to

$$\left\{\begin{aligned}\delta v^{(5)} &= 5\partial\Delta^{(4)}\,, \\ \Delta^{(4)}{}_{ijkl} &= \hat{\kappa}^{(5)}{}_{ijkl5} + 4(2\pi\alpha')v^{(0)}\partial_{[i}\kappa^{(3)}{}_{jkl]} \\ &\quad + \tfrac{1}{2}m(2\pi\alpha')^3 v^{(0)}\rho^{(0)}\partial_{[i}b_j\partial_k b_{l]} + 2m(2\pi\alpha')^2 v^{(0)}\lambda_{[i}b_j\partial_k b_{l]}\,.\end{aligned}\right. \tag{3.90}$$

The equation of motion for $c^{(4)}$ would imply that $v^{(0)}$ is a constant and with this simplification we would proceed as in the M-2-brane case. This might be correct if we took into account fermionic terms, but without fermions it is not clear to us whether we can use the equation of motion of $c^{(4)}$ to set $v^{(0)}$ to a constant. For the time being we will simply truncate the theory and we will set $v^{(0)} = 0$ by hand and then we will extend the calculation to the non vanishing but constant value of $v^{(0)}$. We will also ignore total derivatives. Thus, at this level we find the following result

$$5\partial_{[i_1}\hat{\hat{b}}_{i_2\cdots i_5]\ 5} + \tfrac{m}{2}\hat{c}^{(6)}{}_{i_1\cdots i_5\ 5} = -5\partial_{[i_1}v^{(4)}{}_{i_2\cdots i_5]} + \tfrac{m}{2}v^{(5)}{}_{i_1\cdots i_5}\,, \tag{3.91}$$

where $v^{(4)}$ is an auxiliary 4-form transforming by shifts

$$\delta v^{(4)} = \tfrac{m}{2}\Delta^{(4)}\,. \tag{3.92}$$

This implies that the pair $v^{(4)}, v^{(5)}$ constitutes a Stueckelberg pair, in which $v^{(4)}$ gets eaten by $v^{(5)}$ which becomes massive. This is why we keep this total derivative. This part of the WZ term is completely decoupled from the rest.

The rest of the worldvolume fields can be indentified as follows:

$$\begin{aligned}\hat{b}_5 &= v^{(0)} = 0\,, & \hat{b}_i &= b_i\,, \\ \hat{a}^{(2)}{}_{i5} &= b_i + v^{(1)}{}_i\,, & \hat{a}^{(2)}{}_{ij} &= a^{(2)\prime}{}_{ij}\,,\end{aligned} \tag{3.93}$$

where $a^{(2)\prime} \neq a^{(2)}$ from the point of view of the gauge transformations.

Taking into account that the worldvolume metric is given by:

$$g_{\hat{\imath}\hat{\jmath}} = e^{-2/3\phi} \begin{pmatrix} g_{ij} - (2\pi\alpha')^2 e^{2\phi} \mathcal{G}^{(1)}_i \mathcal{G}^{(1)}_j & -(2\pi\alpha') e^{2\phi} \mathcal{G}^{(1)}{}_i \left[1 - \frac{m}{2}(2\pi\alpha') v^{(0)}\right] \\ -(2\pi\alpha') e^{2\phi} \mathcal{G}^{(1)}{}_i \left[1 - \frac{m}{2}(2\pi\alpha') v^{(0)}\right] & -e^{2\phi} \left[1 - \frac{m}{2}(2\pi\alpha') v^{(0)}\right]^2 \end{pmatrix} , \tag{3.94}$$

we find the action of the massive D-4-brane in the "1-2-form" formalism[36]:

$$\begin{aligned} & S\left[X^\mu, a^{(2)\prime}, b, v^{(1)}, v^{(4)}, v^{(5)}\right] \quad = \\ & -T_{M5} l \int d^5\xi \; e^{-\phi} \sqrt{|g_{ij}|} \left\{1 - \tfrac{1}{4\cdot 3!}(2\pi\alpha')^2 \left(\mathcal{R}^{(3)}\right)^2 + \tfrac{1}{4\cdot 2!}(2\pi\alpha')^2 \left(\mathcal{F} + \mathcal{L}^{(2)}\right)^2\right\} \\ & + \tfrac{T_{M5} l}{5!}(2\pi\alpha') \int d^5\xi \epsilon \left\{\tfrac{1}{2\pi\alpha'} C^{(5)} - 5 C^{(3)} \left(\mathcal{F} + \mathcal{L}^{(2)}\right) + 15 \partial a^{(2)\prime} B \right. \\ & -15 \tfrac{m}{2}(2\pi\alpha') b\partial b B + 30 \tfrac{m}{2}(2\pi\alpha') b\partial(b + v^{(1)}) B + 30 \tfrac{m}{2}(2\pi\alpha') b\partial b\partial(b + v^{(1)}) \\ & \left. -20 \tfrac{m}{2}(2\pi\alpha')^2 b\partial b\partial b - 5\partial v^{(4)} + \tfrac{m}{2} v^5 \right\} , \end{aligned} \tag{3.95}$$

where the double-dimensionally reduced anti-self-duality condition

$$\mathcal{R}^{(3)} = -e^{-\phi \star} \left(\mathcal{F} + \mathcal{L}^{(2)}\right) , \tag{3.96}$$

with

$$\begin{aligned} \mathcal{R}^{(3)} \quad = \quad & 3\partial a^{(2)\prime} + \tfrac{1}{2\pi\alpha'} C^{(3)} - 3\tfrac{m}{2}(2\pi\alpha') b\partial b - 3\tfrac{m}{2}(2\pi\alpha') B b \\ & - \left(C^{(1)} - \tfrac{m}{2} b\right) \left(\mathcal{F} + \mathcal{L}^{(2)}\right) , \end{aligned} \tag{3.97}$$

still needs to be imposed in the reduced action.

Either b or $a^{(2)\prime}$ can be considered an auxiliary field and eliminated in the equations of motion after using the constraint above, that relates their field strengths. This was the procedure followed in Ref. [23] to find the Born-Infeld effective action from the massless M-5-brane. Here we will follow an alternative, simpler, procedure which consists in introducing the constraint into the action by means of a Lagrange multiplier term. Integrating out $a^{(2)\prime}$ yields the usual D-4-brane action and integrating out b we obtain the dual action in terms of the 2-form. This action is the worldvolume dual of the usual massive D-4-brane. We will not study this one here but we will concentrate on recovering the standard D-4-brane action.

[36]Since it depends on the 1-form b plus the 2-form $a^{(2)\prime}$.

The Lagrange multiplier term, containing a Lagrange multiplier 1-form ϱ, that we add to the action is:

$$+\tfrac{T_{M_5}l}{4!}(2\pi\alpha')^2\int d^5\xi\epsilon(2\partial\varrho)(3\partial a^{(2)\prime}) \tag{3.98}$$

Integration over ϱ enforces the Bianchi identity for the two-form $a^{(2)\prime}$. On the other hand, integrating out the 3-form $\mathcal{R}^{(3)}$ we can impose the anti-self-duality constraint on-shell and obtain the action in the 1-form formalism. In particular, the equation of motion for $\mathcal{R}^{(3)}$ gives:

$$\mathcal{R}^{(3)} = -e^{-\phi}\mathcal{F}'\,, \qquad \mathcal{F}' = 2\partial\varrho + \tfrac{1}{2\pi\alpha'}B\,. \tag{3.99}$$

We can now substitute the above equation of motion to eliminate $a^{(2)\prime}$. Now one can use the anti-self-duality constraint which takes the form

$$\mathcal{F}' = \mathcal{F} + \mathcal{L}^{(2)}\,, \tag{3.100}$$

in the action. The result is

$$\begin{aligned} S\left[X^\mu, b, v^{(1)}, v^{(4)}, v^{(5)}\right] \;=\; & \\ & -T_{M5}l\int d^5\xi\; e^{-\phi}\sqrt{|g_{ij}|}\left\{1 + \tfrac{1}{2\cdot 2!}(2\pi\alpha')^2\left(\mathcal{F}+\mathcal{L}^{(2)}\right)^2\right\} \\ & +\tfrac{T_{M5}l}{5!}(2\pi\alpha')\int d^5\xi\epsilon\left\{\tfrac{1}{2\pi\alpha'}C^{(5)} - 10C^{(3)}\left(\mathcal{F}+\mathcal{L}^{(2)}\right)\right. \\ & +15(2\pi\alpha')C^{(1)}\left(\mathcal{F}+\mathcal{L}^{(2)}\right)^2 - 60\tfrac{m}{2}(2\pi\alpha')b\partial v^{(1)}\partial(b+v^{(1)}) \\ & \left.-20\tfrac{m}{2}(2\pi\alpha')^2 b\partial b\partial b - 5\partial v^{(4)} + \tfrac{m}{2}v^{(5)}\right\}\,. \end{aligned} \tag{3.101}$$

Now we make the field redefinition

$$b + v^{(1)} \to b\,, \tag{3.102}$$

and adding the total derivative $5\partial c^{(4)}$ to get a more compact expression we find the usual action of the D-4–brane ("1-form formalism") to quadratic order in $\mathcal{F}$ containing three extra terms which are completely decoupled

$$S\left[X^{\mu},b,v^{(1)},v^{(4)},v^{(5)}\right] \ =$$

$$-T_{M5}l\int d^5\xi\ e^{-\phi}\sqrt{|g_{ij}|}\left\{1+\tfrac{1}{2\cdot 2!}(2\pi\alpha')^2\left(\mathcal{F}\right)^2\right\}$$

$$+\tfrac{T_{M5}l}{5!}(2\pi\alpha')\int d^5\xi\epsilon\left\{\mathcal{G}^{(5)}-20\tfrac{m}{2}(2\pi\alpha')^2v^{(1)}\partial v^{(1)}\partial v^{(1)}-5\partial v^{(4)}+\tfrac{m}{2}v^5\right\}. \tag{3.103}$$

Observe that $v^{(1)}$ appears only through a topological term. The consequences of the presence of a 5-form field in the WZ term of the massive D-4-brane will be discussed in the next Section.

One can easily see that if we consider a constant but non-vanishing $v^{(0)}$ the only change in the above result is that the tension has to be replaced by

$$T=\left[1-\tfrac{m}{2}(2\pi\alpha')v^{(0)}\right]T_{M_5}l\,, \tag{3.104}$$

and the terms with v-forms get an extra factor $\left[1-\frac{m}{2}(2\pi\alpha')v^{(0)}\right]^{-1}$ for each power of v.

4 Conclusions

In this paper we have investigated the worldvolume effective theory of massive branes, i.e. branes moving in a background with a nonzero cosmological constant, both in string theory and M-theory. The construction of the worldvolume actions requires new results on the structure of both the target-space background fields as well as the worldvolume fields.

As far as the target-space fields are concerned we encountered two issues. First of all the coupling of massive IIA supergravity to the type IIA five-brane requires a dualization of the *massive* NS-NS target-space 2-form B. Inspired by the work of [14] we succeeded in doing this and encountered an interesting mechanism where, after dualization, the roles of the NS-NS and R-R fields get interchanged. Whereas in the usual formulation the R-R 1-form $C^{(1)}$ is a Stueckelberg field and B is a massive 2-form, we find that, after dualization, the dual R-R 7-form $C^{(7)}$ becomes massive while the dual NS-NS 6-form is a Stueckelberg field.

The second issue concerns the question of whether or not the massive IIA supergravity theory has an eleven-dimensional origin. Inspired by the worldvolume sigma model approach we made a proposal for a massive eleven-dimensional supergravity theory. A word of caution is needed here. We do not claim that a cosmological constant (containing a mass parameter m) can be added to the usual (massless) eleven-dimensional supergravity theory. Indeed, a no-go theorem for such a theory exists [18]. There is a simple argument for this obstruction. One can easily verify that any cosmological constant in eleven dimensions of the form

$$\mathcal{L}_{\mathrm{d}=11} \sim \int d^{11}x \; m^2 \sqrt{|g|} \tag{4.1}$$

leads, upon dimensional reduction to ten dimensions, to a cosmological constant with a dilaton coupling that differs from the one in massive IIA supergravity. What we find from the sigma model approach is that it can be done only under the assumption that the d=11 target space background fields have an isometry. This assumption follows naturally from the fact that the underlying sigma model turns out to be a *gauged* sigma model whose construction requires an isometry direction. Only in the massless case, when $m = 0$, this assumption can be deleted and we obtain the usual massless supergravity theory with no isometry whatsoever. The obstruction mentioned above is now avoided by considering a cosmological constant of the form

$$\mathcal{L}_{massive\ d=11\ supergravity} \sim \int d^{11}x \; m^2 |\hat{k}^2|^2 \sqrt{|g|}\,, \tag{4.2}$$

involving the Killing vector $\hat{k}^\mu$. This term gives the correct dilaton coupling.

It is of interest to compare our proposal of massive eleven-dimensional supergravity with the work of [19] where a new d=11 supergravity theory is introduced that, upon reduction to ten dimensions, leads to an alternative massive IIA supergravity theory. In both cases the massive corrections to the massless d=11 supergravity theory are triggered by modifying the definition of the spin connection. The specific form of the modification, however, differs in the two cases. In our case (see Eq. (1.10)) the extra term in the connection involves the 3-form $\hat{C}$ whereas the extra term in the connection of [19] involves only the Killing vector, leading to a different massive supergravity theory. In the case of [19] the modified spin connection has a geometric interpretation in terms of a Weyl superspace (see e.g. [35])[37]. It would be interesting to see whether the modified connection we have introduced has a similar geometric interpretation.

Another question is whether the massive eleven-dimensional supergravity theory considered in this work has something to do with M-theory. We remind that the relation between the usual massless d=11 supergravity theory

[37]This construction is based on the fact that the equations of motion of d=11 supergravity are invariant under scale transformations (see, e.g., [36]). It has recently been shown that the new massive d=10 supergravity of [19] can alternatively be understood as a generalized Scherk-Schwarz reduction (making use of the scaling symmetry) of the ordinary massless d=11 supergravity theory [37].

and M-theory goes via the observation that massless d=11 supergravity can be viewed as the decompactification limit ($R_{11} \to \infty$) of massless IIA supergravity and that this decompactification limit is equivalent to the strong coupling limit ($g_s \to \infty$) of type IIA superstring theory since $R_{11} = g_s^{2/3}$ [38]. It is natural to consider the isometry direction as an eleventh direction, since the massive M-brane configurations we introduce in this work can be positioned in this direction. Indeed, this feature enables us to construct two different massive branes of string theory out of a single massive M-brane. The two possibilities arise due to the fact that the isometry direction can be transverse to the brane (direct dimensional reduction) or along one of the worldvolume directions of the brane (double dimensional reduction). It would be interesting to consider the massive eleven-dimensional supergravity theory of this work in the context of the Hořava-Witten approach [39] where there are two M-9-branes positioned at the end of spacetime.

As far as the worldvolume fields are concerned the construction of the worldvolume actions for the massive branes of string theory requires that we do not only introduce the usual embedding coordinates X^μ and BI 1-form b but also a whole set of worldvolume p-form fields $c^{(p)}$, which are in one to one correspondence to the target space R-R fields $C^{(p+1)}$. Such p-form fields have been encountered before in a scale-invariant formulation of branes [40] and in a study of branes ending on branes [1] (see also [41]). They have also been recently used in the construction of a manifestly $SL(2,\mathbb{R})$ covariant formulation of IIB brane worldvolume actions [42, 43, 44]. In the latter case all $c^{(p)}$ (p odd) occur in definite $SL(2,\mathbb{R})$ representations: for instance, there are doublets $(b^{(1)}, c^{(1)}), (b^{(5)}, c^{(5)})$ and singlets $c^{(3)}$, etc., where $b^{(1)}(b^{(5)})$ describes the tension of a type IIB string (type IIB five-brane). Finally, p-form worldvolume fields also play an important role in the construction of null super D-branes [26]. We have shown in this paper that the worldvolume p-form fields of IIA string theory can be obtained from corresponding p-form worldvolume fields in M-theory via reduction. We plan to discuss these M-theory worldvolume fields in a future publication [11].

With the required new knowledge on the structure of the target-space and worldvolume fields described above, we proceeded to construct the worldvolume actions for massive branes. Our strategy was to first construct the action of a massive M-brane, i.e. an M-brane moving in a massive d=11 supergravity background, and next obtain, via direct and double dimensional reduction, a pair of massive branes of string theory. We first discussed the massive M-0-brane which is a bit special and only allows a direct reduction to the massive D-0-brane. Next, we treated the massive M-2-brane [16, 17] and obtained the massive fundamental string and the massive D-2-brane. The last case we considered was the massive M-5-brane whose reduction leads to the massive type IIA five-brane and the massive D-4-brane, although we must impose a truncation in order to make contact with the massive D-4-brane action given in the literature.

One interesting outcome of our work is that we find that the massive type IIA 5-brane has an additional coupling to a worldvolume 6-form $c^{(6)}$, describing the tension of a D-6-brane, with the strength of the coupling proportional to m:

$$S_{\text{massive NS5-brane}} \sim \int d^6\xi \; m \; \epsilon^{i_1\cdots i_6} c^{(6)}{}_{i_1\cdots i_6} \, . \tag{4.3}$$

A similar coupling occurs in the massive D-0-brane action:

$$S_{\text{massive D0-brane}} \sim \int d\xi \; m \; b_\xi \, , \tag{4.4}$$

where b_ξ describes the tension of a fundamental string. In [45, 46] it was pointed out, for the massive D-0-brane, that this new coupling has implications for the anomalous creation of branes [47, 48] (for a more recent discussion see [49]). To be precise, if a D-0-particle crosses a D-8-brane a stretched fundamental string is created in the single direction transverse to the D-8-brane. This process is, via duality, related to the creation of a stretched D3-brane if a D-5-brane crosses a NS5-brane [47]. In the latter case the intersecting configuration is given by

$$\begin{array}{ll|lllllllll} \text{D5}: & \times & \times & \times & - & - & - & \times & \times & \times & - \\ \text{NS5}: & \times & \times & \times & \times & \times & \times & - & - & - & - \\ \text{D3}: & \times & \times & \times & - & - & - & - & - & - & \times \end{array} \tag{4.5}$$

where we have used the notation of [50] [38]. The intersecting configuration of [45, 46] is obtained by first applying T-duality in the directions 1 and 2, next applying an S-duality and, finally, applying a T-duality in the directions 6,7 and 8 [45, 46]:

$$\begin{array}{ll|lllllllll} \text{D0}: & \times & - & - & - & - & - & - & - & - & - \\ \text{D8}: & \times & \times & \times & \times & \times & \times & \times & \times & \times & - \\ \text{F1}: & \times & - & - & - & - & - & - & - & - & \times \end{array} \tag{4.6}$$

One would expect that the new coupling we find for the massive 5-brane has similar implications[39]. To be precise, one would expect that if a NS5-brane crosses a D-8-brane a D-6-brane stretched between them is created. This process would be the dual of the one considered in [45, 46]. Indeed, this

[38] Each horizontal line indicates the 10 directions $0, 1, \cdots 9$ in spacetime. A $\times(-)$ means that the corresponding direction is in the worldvolume of (transverse to) the brane.

[39] We thank C. Bachas for an illuminating discussion on this point.

is exactly the process which has been considered in [51] where it was used to construct $N = 1$ supersymmetric gauge theories in four dimensions with chiral matter[40].

The intersecting configuration corresponding to this process is obtained by applying T-duality in the directions 3,4 and 5 on the configuration (4.5) [51]:

$$\begin{array}{rc|cccccccccc} \mathrm{D8}: & \times & \times & \times & \times & \times & \times & \times & \times & \times & - \\ \mathrm{NS5}: & \times & \times & \times & \times & \times & \times & - & - & - & - \\ \mathrm{D6}: & \times & \times & \times & \times & \times & \times & - & - & - & \times \end{array} \tag{4.7}$$

A special feature of this configuration is that when the NS5-brane passes through the D-8-brane it is completely embedded within the D-8-brane. After passing through the D-8-brane a D-6-brane is created that has 5 of its directions on the worldvolume of the D-8-brane and in the remaining directions is stretched in the single direction transverse to the D-8-brane.

We may consider the above process from an M-theory perspective. The intersecting M-brane configuration that, via reduction (over the 10 direction), yields the intersecting configuration given in (4.7) is given by[41]:

$$\begin{array}{rc|ccccccccccc} \mathrm{M9}: & \times & \times & \times & \times & \times & \times & \times & \times & \times & - & \times \\ \mathrm{M5}: & \times & \times & \times & \times & \times & \times & - & - & - & - & - \\ \mathrm{KK11}: & \times & \times & \times & \times & \times & \times & - & - & - & \times & z \end{array} \tag{4.8}$$

where z refers to the $U(1)$ isometry direction in the Taub-NUT space of the monopole. This intersecting configuration suggests that when an M-5-brane passes through a (hypothetical) M-9-brane a stretched 11-dimensional KK monopole is created. The anomalous creation of this KK monopole is related to the following term we found in the massive M5-brane action:

$$S_{\text{massive M5}-\text{brane}} \sim \int d^6\hat{\xi}\ m\ \epsilon^{\hat{\imath}_1\cdots\hat{\imath}_6}\hat{\omega}^{(6)}{}_{\hat{\imath}_1\cdots\hat{\imath}_6}\,. \tag{4.9}$$

This suggests that, in the same way as $c^{(6)}$ describes the tension of a D-6-brane, the worldvolume 6-form $\hat{\omega}^{(6)}$ is related to the tension of an 11-dimensional KK monopole. A gauged version of this 6-form can be used to construct the WZ term in the KK11-monopole worldvolume action [11].

We have found in the action of the massive D-4-brane a term of the form

$$S_{\text{massive D4}-\text{brane}} \sim \int d^5\xi\ m\ v^{(5)}\,, \tag{4.10}$$

[40] We thank P. Townsend for pointing reference [51] out to us.

[41] This configuration, alternatively denoted by $(5|M9, KK)$, is known to preserve 1/4 of the supersymmetry [52].

where $v^{(5)}$ is related to the tension of a KK10-monopole. This coupling suggests that whenever a D4-brane passes through a D8-brane a stretched KK10-monopole is created with 4 of its worldvolume directions inside the D8-brane. Indeed, performing first a T-duality in the 9 direction and next a T-duality in the 3 and 4 directions on the configuration (4.5) we obtain the intersecting brane configuration corresponding to this process:

$$
\begin{array}{rc|l}
\mathrm{D8}: & \times & \times\times\times\times - \times\times\times\times \\
\mathrm{KK10}: & \times & \times\times\times\times\times - - - z \\
\mathrm{D4}: & \times & \times\times\times\times - - - - -
\end{array}
\tag{4.11}
$$

The same intersecting configuration is obtained if one reduces the intersecting M-brane configuration (4.8) over the 5-direction.

There is one more argument in favor of the above process. It turns out that, of all branes in string theory, only $D0$, $NS5$, $D4$ (and a wave) can be embedded in a D8-brane and the intersections are given by

$$
\begin{array}{l}
(0|D0, D8)\,, \\
(5|NS5, D8)\,, \\
(4|D4, D8)\,,
\end{array}
\tag{4.12}
$$

respectively. The first 2 cases are required for the creation of a fundamental string (D-6-brane) whenever a D-0-brane (NS5-brane) passes through a D-8-brane. The third and only other possible embedding of a brane into a D-8-brane is that of a D-4-brane and this would correspond to the process (4.11) where an anomalous KK10-monopole is created.

There is one more potential role to be played by the worldvolume 6-form field $\hat{\omega}^{(6)}$ we found in the massive M-5-brane action. It has been shown that the selfdual 2-form $\hat{\omega}^{(2)}$ occurring in the same action can be used to construct a 1-brane soliton on the M-5-brane worldvolume [4]. This 1-brane soliton corresponds to the occurrence of a 1-form central charge in the 6-dimensional (2,0) worldvolume supersymmetry algebra. The complete (2,0) algebra also suggests a 3-brane and a 5-brane soliton. The 3-brane soliton has been recently constructed by dualizing one of the embedding scalars into a worldvolume 4-form [4]. The 5-brane soliton should describe the embedding of the M-5-brane into something else and a natural possibility is an M-9-brane. It would be interesting to see whether such a 5-brane soliton can indeed be constructed and whether the 6-form $\hat{\omega}^{(6)}$ (which does not describe a dynamical degree of freedom) could describe the charge of such a 5-brane soliton.

In this work we have shown that the massive branes of M-theory, i.e. the M-0-brane, M-2-brane and M-5-brane, are described by a *gauged* sigma model. It would be interesting to see whether the gauged sigma model approach can also be applied to describe a massive KK11 monopole and/or the conjectured M9-brane. Concerning the KK11-monopole, a new feature in this case is that the gauged sigma model is already needed to describe the dynamics of the KK11-monopole in a *massless* background [9][42].

It would be interesting to see whether our results shed new light on the evasive 11-dimensional M-9-brane[43]. A standard argument against a *freely moving* M-9-brane is that the corresponding 10-dimensional worldvolume field theory does not allow multiplets containing a single scalar to indicate the position of the M9-brane[44]. A way out of this is to assume that the M-9-brane is really an 8-brane with an extra isometry in one of the 2 transverse directions, leading to a gauged sigma-model, like it happened in the case of the KK11-monopole. Now, we are dealing with a nine-dimensional field theory which naturally contains a vector multiplet with a single scalar. We will not pursue this line of thought further here.

We would like to end with the following remark. Although this paper is dealing with *massive* branes, our results also have implications, via *massive* T-duality, for the worldvolume theories of *massless* branes. To be specific, it is known that the term (4.4), via the T-duality rule [12]

$$C^{(0)} = mx \tag{4.13}$$

in the direction x, leads to the following gauge-invariant term in the D-1-brane action:

$$S_{\mathrm{D1-brane}} \sim \int d^2\xi\ C^{(0)}\epsilon^{ij}\mathcal{F}_{ij}\,. \tag{4.14}$$

This term is needed for an $SL(2,\mathbb{R})$-covariant formulation of (p,q)-strings [42]. Similarly, the new term (4.3) we have found in the worldvolume action of the p-5A-brane leads, via massive T-duality, to a gauge-invariant term in the worldvolume action of the type IIB KK-monopole and the NS5B-brane containing the RR-scalar $C^{(0)}$. The latter case suggests the following gauge-invariant term in the NS5B-brane action:

[42]Note that the gauge transformation rule of the 6-form $\hat{\omega}^{(6)}$ contains m-dependent terms. This shows that the WZ term of a massive KK11-monopole differs from the WZ term of a massless KK11-monopole.

[43]For other discussions of the M-9-brane, see e.g. [39, 8, 53, 54, 55, 56, 10, 33].

[44]Such a scalar would not be needed if the M9-brane was positioned at the end of spacetime, like in [39].

$$S_{\mathrm{NS5B-brane}} \sim \int d^6\xi \ C^{(0)}\mathcal{G}^{(6)} \,, \tag{4.15}$$

with $\mathcal{G}^{(6)}$ the curvature of the D-5-brane tension. Via S-duality this implies that the D-5-brane action contains the term

$$S_{\mathrm{D5-brane}} \sim \int d^6\xi \ C^{(0)}\mathcal{F}_{IIB} \,, \tag{4.16}$$

with $\mathcal{F}_{IIB}$ the curvature of the NS5B-brane tension. It would be interesting to see whether the above modifications are indeed present, in which case they are likely to play a role in the construction of an $SL(2,\mathbb{R})$-covariant formulation of (p,q)-5-branes. In order to verify this one needs to explore the T-duality properties of the different worldvolume p-forms introduced in this work.

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A Duality of Massive k-Form Fields

In this Appendix we focus on the study of duality for massive d-dimensional k-form fields in general to then try to apply the ideas to massive type IIA supergravity. Following Ref. [14] we start by briefly discussing duality of massless k-form fields in the presence of magnetic sources.

Let us consider a $(p+1)$-form potential $A_{(p+1)}$ with field strength

$$F_{(p+2)} = (p+2)\partial A_{(p+1)} \,, \tag{A.1}$$

and action

$$S[A_{(p+1)}] = \int d^dx \, \sqrt{|g|} \, \left[\tfrac{(-1)^{(p+1)}}{2\cdot(p+2)!} \tfrac{1}{e^2_{(p)}} F^2_{(p+2)} \right] . \tag{A.2}$$

If $A_{(p+1)}$ is defined everywhere, then the field strength satisfies the Bianchi identity[45]

$$\nabla_\nu \left({}^\star F_{(p+2)} \right)^{\nu\mu_1\dots\mu_{\tilde{p}+1}} = 0 \, . \tag{A.3}$$

If we consider the presence of (non-dynamical) magnetic sources then the Bianchi identity becomes

$$\nabla_\nu \left({}^\star F_{(p+2)} \right)^{\nu\mu_1\dots\mu_{\tilde{p}+1}} = \tilde{J}^{\mu_1\dots\mu_{\tilde{p}+1}} \, , \tag{A.4}$$

which means that $F_{(p+2)}$ cannot be $(p+2)\partial A_{(p+1)}$ everywhere (i.e. it is not an exact form anymore) but is singular at the Dirac singularity. In general we can write

$$F_{(p+2)} = (p+2)\partial A_{(p+1)} + B_{(p+2)} \, , \tag{A.5}$$

where $A_{(p+1)}$ is defined everywhere and

$$\nabla_\nu \left({}^\star B_{(p+2)} \right)^{\nu\mu_1\dots\mu_{\tilde{p}+1}} = \tilde{J}^{\mu_1\dots\mu_{\tilde{p}+1}} \, . \tag{A.6}$$

With this modification the action is still the same (A.2). This modification makes sense as long as we deal with a number of (non-dynamical) magnetic objects. If we allow the magnetic objects to condense then we have to promote the field that describes them ($B_{(p+2)}$) to a dynamical one by the simple addition of a kinetic term for it:

$$S[A_{(p+1)}, B_{(p+2)}] = \int d^dx \, \sqrt{|g|} \, \left[\tfrac{(-1)^{(p+1)}}{2\cdot(p+2)!} \tfrac{1}{e^2_{(p)}} F^2_{(p+2)} + \tfrac{(-1)^{(p+2)}}{2\cdot(p+3)!} \tfrac{1}{e^2_{(p+1)}} H^2_{(p+3)} \right] , \tag{A.7}$$

where

$$H_{(p+3)} = (p+3)\partial B_{(p+2)} \, . \tag{A.8}$$

The action above is invariant under the usual gauge transformations of the $A_{(p+1)}$ potential

$$\delta A_{(p+1)} = (p+1)\partial \chi_{(p)} \, , \tag{A.9}$$

[45] Here $\tilde{p} = d - p - 4$ so the Hodge dual of a $(p+2)$-form is a $(\tilde{p}+2)$-form.

and under the *massive gauge transformations*

$$
\left\{
\begin{array}{rcl}
\delta B_{(p+2)} & = & -(p+2)\partial\lambda_{(p+1)}\,, \\
\\
\delta A_{(p+1)} & = & \lambda_{(p+1)}\,,
\end{array}
\right.
\tag{A.10}
$$

which allow us to eliminate $A_{(p+1)}$ by setting it to zero. Doing this, we get the following action for a massive $(p+2)$-form potential[46]:

$$
S[B_{(p+2)}] = \int d^d x\ \sqrt{|g|}\ \left[\tfrac{(-1)^{(p+2)}}{2\cdot(p+3)!}\tfrac{1}{e^2_{(p+1)}}H^2_{(p+3)} + \tfrac{(-1)^{(p+1)}}{2\cdot(p+2)!}\tfrac{1}{e^2_{(p)}}B^2_{(p+2)}\right]\,,
\tag{A.11}
$$

which is the one studied in Ref. [14]. This action is also a truncation of the massive type IIA supergravity action for $p = 0$[47] with $e_{(1)} = 1$ and $e_{(0)} = 1/m$.

In Ref. [14] it is mentioned that the above action enjoys no evident global symmetry, but that in spite of this it can be dualized. What we have just seen is that the action for the massive $(p+2)$-form potential is equivalent to the action with $A_{(p+1)}$ (which plays the role of Stueckelberg field here) with a massive gauge symmetry, and that in this formulation the dualization can be done systematically, which may mean that a global symmetry (which we have not identified) is present.

In any case, in Ref. [14] the dual action is found via an intermediate first-order action with an auxiliary $(\tilde{p}+1)$-form field. Integration of the auxiliary field gives (A.11) and integration of the field strength $H_{(p+3)}$ gives the dual action. Instead of using this intermediate action, which in principle one has to construct in a case by case basis, we are going to follow a different, more systematic, procedure using the usual recipe for Poincaré duality in the action with the Stueckelberg field[48]. This recipe cannot be applied to the dualization of $B_{(p+2)}$ because it occurs explicitly in the action instead of appearing only through $H_{(p+3)}$ (hence the need of an intermediate action) but it can be applied to the dualization of the Stueckelberg field $A_{(p+1)}$ as a first step. Thus, we consider now $F_{(p+2)}$ as a fundamental field and we add to

[46] It is often said that the $(p+2)$-form has "eaten" the $(p+1)$-form, thus getting a mass.

[47] This does not necessarily mean that the mechanism through which B gets mass in the type IIA theory must be the condensation of 6-branes. The origin of the mass in the type IIA theory is the presence of 8-branes [7, 8]. Still, the reason why it is precisely B the field that becomes massive is not clear to us, and other mechanisms for giving mass to different fields of the type IIA theory should not be excluded. We thank J.L.F. Barbón for discussions on this point.

[48] The usual recipe also involves the construction of an intermediate action, but our construction is systematic.

the action (A.7) a Lagrange multiplier term to enforce its Bianchi identity. The resulting action is, after integration by parts

$$
\begin{aligned}
S[F_{(p+2)}, B_{(p+2)}, \tilde{A}_{(\tilde{p}+1)}] = \\
&\int d^d x \,\sqrt{|g|}\, \left\{ \tfrac{(-1)^{(p+1)}}{2\cdot(p+2)!} \tfrac{1}{e^2_{(p)}} F^2_{(p+2)} + \tfrac{(-1)^{(p+2)}}{2\cdot(p+3)!} \tfrac{1}{e^2_{(p+1)}} H^2_{(p+3)} \right. \\
&\left. + \tfrac{(-1)^p}{(p+2)!(\tilde{p}+1)!} \tfrac{\epsilon}{\sqrt{|g|}} \left[F_{(p+2)} - B_{(p+2)} \right] \partial \tilde{A}_{(\tilde{p}+1)} \right\} .
\end{aligned}
\tag{A.12}
$$

Substituting into this action the equation of motion for $F_{(p+2)}$

$$
F_{(p+2)} = e^2_{(p)} \, {}^\star \tilde{F}_{(\tilde{p}+2)} \,, \qquad \tilde{F}_{(\tilde{p}+2)} = (\tilde{p}+2)\partial \tilde{A}_{(\tilde{p}+1)} \,, \tag{A.13}
$$

we find (after integration by parts) the dual action

$$
\begin{aligned}
S[B_{(p+2)}, \tilde{A}_{(\tilde{p}+1)}] = \\
&\int d^d x \,\sqrt{|g|}\, \left\{ \tfrac{(-1)^{(p+2)}}{2\cdot(p+3)!} \tfrac{1}{e^2_{(p+1)}} H^2_{(p+3)} + \tfrac{(-1)^{(\tilde{p}+1)}}{2\cdot(\tilde{p}+2)!} e^2_{(p)} \tilde{F}^2_{(\tilde{p}+2)} \right. \\
&\left. + \tfrac{1}{(p+3)!(\tilde{p}+1)!} \tfrac{\epsilon}{\sqrt{|g|}} H_{(p+3)} \tilde{A}_{(\tilde{p}+1)} \right\} .
\end{aligned}
\tag{A.14}
$$

We have substituted a massive $(p+2)$-form potential by a massless $(p+2)$-form potential and a massless $(\tilde{p}+1)$-form potential. None of these fields is auxiliary nor it can be eliminated by a gauge transformation of some kind. Using that the number of degrees of freedom of a massive d-dimensional k-form potential is

$$
N(d,k)_{\text{massive}} = \frac{(d-1)!}{k!(d-1-k)!} \,, \tag{A.15}
$$

and that the number of degrees of freedom of a massless d-dimensional k-form potential is

$$
N(d,k)_{\text{massless}} = \frac{(d-2)!}{k!(d-2-k)!} \,, \tag{A.16}
$$

one can check that the number of degrees of freedom of the theory (A.14) is the same as that of the original (A.7)[49]:

[49] The Higgs mechanism by which the massless $(p+2)$-form "eats" the $(p+1)$-form becoming a massive $(p+2)$-form is based in it.

$$N(d, p+2)_{\rm massive} = N(d, p+2)_{\rm massless} + N(d, \tilde{p}+1)_{\rm massless}\,. \tag{A.17}$$

The usual electric-magnetic duality for massless d-dimensional $(p+1)$- and $(\tilde{p}+1)$-form potentials is based on the identity

$$N(d, p+1)_{\rm massless} = N(d, \tilde{p}+1)_{\rm massless}\,, \tag{A.18}$$

while the electric-magnetic duality between massive d-dimensional $(p+2)$- and $(\tilde{p}+1)$-form potentials (which we are going to see next) is based on

$$N(d, p+2)_{\rm massive} = N(d, \tilde{p}+1)_{\rm massive}\,. \tag{A.19}$$

Now, the action (A.14) does not depend explicitly on $B_{(p+2)}$ anymore and we can follow again the usual prescription for dualization of this field. We find the intermediate action (after integration by parts)

$$\begin{aligned} S[H_{(p+3)}, \tilde{A}_{(\tilde{p}+1)}, \tilde{B}_{(\tilde{p})}] = & \\ & \int d^dx\,\sqrt{|g|}\,\left\{\tfrac{(-1)^{(p+2)}}{2\cdot(p+3)!}\tfrac{1}{e^2_{(p+1)}}H^2_{(p+3)} + \tfrac{(-1)^{(\tilde{p}+1)}}{2\cdot(\tilde{p}+2)!}e^2_{(p)}\tilde{F}^2_{(\tilde{p}+2)}\right. \\ & \left. + \tfrac{1}{(p+3)!}H_{(p+3)}{}^{\star}\tilde{H}_{(\tilde{p}+1)}\right\}\,, \end{aligned} \tag{A.20}$$

where

$$\tilde{H}_{(\tilde{p}+1)} = (\tilde{p}+1)\partial\tilde{B}_{(\tilde{p})} + \tilde{A}_{(\tilde{p}+1)}\,. \tag{A.21}$$

Substituting the equation of motion of $H_{(p+3)}$

$$H_{(p+3)} = (-1)^{(p+1)}e^2_{(p+1)}\,{}^{\star}\tilde{H}_{(\tilde{p}+1)}\,, \tag{A.22}$$

we arrive at the action

$$S[\tilde{B}_{(\tilde{p})}, \tilde{A}_{(\tilde{p}+1)}] = \int d^dx\,\sqrt{|g|}\,\left\{\tfrac{(-1)^{(\tilde{p}+1)}}{2\cdot(\tilde{p}+2)!}e^2_{(p)}\tilde{F}^2_{(\tilde{p}+2)} + \tfrac{(-1)^{\tilde{p}}}{2\cdot(\tilde{p}+1)!}e^2_{(p+1)}\tilde{H}^2_{(\tilde{p}+1)}\right\} \tag{A.23}$$

This action is now invariant under the usual gauge transformations of $\tilde{B}_{(\tilde{p})}$

$$\delta\tilde{B}_{(\tilde{p})} = \tilde{p}\partial\tilde{\chi}_{(\tilde{p}-1)}\,, \tag{A.24}$$

and the following massive gauge transformations ($\tilde{B}_{(\tilde{p})}$ playing now the role of Stueckelberg field):

$$\left\{ \begin{array}{rcl} \delta \tilde{A}_{(\tilde{p}+1)} & = & -(\tilde{p}+1)\partial\tilde{\lambda}_{(\tilde{p})}\,, \\ \\ \delta \tilde{B}_{(\tilde{p})} & = & \tilde{\lambda}_{(\tilde{p})}\,. \end{array} \right. \tag{A.25}$$

These transformations allow us to eliminate $\tilde{B}_{(\tilde{p})}$ by setting it to zero. We then get the following action

$$S[\tilde{A}_{(\tilde{p}+1)}] = \int d^dx\,\sqrt{|g|}\,\left[\tfrac{(-1)^{(\tilde{p}+1)}}{2\cdot(\tilde{p}+2)!}e^2_{(p+1)}\tilde{F}^2_{(\tilde{p}+2)} + \tfrac{(-1)^{\tilde{p}}}{2\cdot(\tilde{p}+1)!}e^2_{(p)}\tilde{A}^2_{(\tilde{p}+1)}\right]\,, \tag{A.26}$$

which is the one given in Ref. [14]. Had we set $p=0$, $e_{(1)}=1$ and $e_{(0)}=1/m$ at the beginning, rescaling now $\tilde{A}_{(\tilde{p}+1)}$ we would have found that the dual mass parameter is identical to the original one $\tilde{m}=m$. We would also have found that the dual NS/NS 6-form $\tilde{B}$ is the Stueckelberg field for the R-R 7-form $C^{(7)}$.

It seems strange that one can replace a massive field by a pair of massless fields. Observe that one of them (the $(\tilde{p}+1)$-form potential) corresponds to the $\tilde{p}$-extended objects whose condensation gives mass to the $(p+2)$-form potential which was previously massless. Then, the action makes sense as a slightly more general action which can describe the situation in which there is no condensation of $\tilde{p}$-branes and the $(p+1)$-branes are massless, and also the opposite situation. The WZ interaction term is crucial for this.

B Target Space Fields

In this Appendix we collect the field strengths and the gauge transformations of the different fields that appear throughout the paper. We also give the relation with 11-dimensional fields.

Let us start with some conventions: when indices are not shown explicitly and partial derivatives are used, we assume that all indices are completely antisymmetrized in the obvious order. For instance:

$$G^{(4)} = 4\left(\partial C^{(3)} - 3\partial B C^{(1)} + \tfrac{3}{8}mB^2\right)\,, \tag{B.1}$$

means

$$G^{(4)}{}_{\mu\nu\rho\sigma} = 4\left[\partial_{[\mu}C^{(3)}{}_{\nu\rho\sigma]} - 3\left(\partial_{[\mu}B_{\nu\rho}\right)C^{(1)}{}_{\sigma]} + \tfrac{3}{8}mB_{[\mu\nu}B_{\rho\sigma]}\right]\,. \tag{B.2}$$

Differential form components are defined as follows:

$$A_{(k)} = \tfrac{1}{k!} A_{(k)\mu_1 \ldots \mu_k} dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_k} \,, \tag{B.3}$$

but we will often omit the $\wedge$ symbol.

The 11-dimensional 3-form and dual 6-form fields are denoted by $\hat{C}$ and $\hat{\tilde{C}}$ respectively. In the massless theory their field strengths $\hat{G}$ and $\hat{\tilde{G}}$ are:

$$\left\{ \begin{array}{rcl} \hat{G} & = & d\hat{C} \,, \\ & & \\ \hat{\tilde{G}} & = & d\hat{\tilde{C}} + \tfrac{1}{2}\hat{C} d\hat{C} = {}^\star\hat{G} \,, \end{array} \right. \tag{B.4}$$

and their gauge transformation laws are

$$\left\{ \begin{array}{rcl} \delta\hat{C} & = & d\hat{\chi} \,, \\ & & \\ \delta\hat{\tilde{C}} & = & d\hat{\tilde{\chi}} + \tfrac{1}{2} d\hat{\chi}\hat{C} \,. \end{array} \right. \tag{B.5}$$

The action of 11-dimensional supergravity in differential forms language is

$$S = \tfrac{1}{16\pi G_N^{(11)}} \int \left\{ \Omega_{(11)} \hat{R} - \tfrac{1}{2} \hat{G} \wedge {}^\star\hat{G} + \tfrac{1}{6} d\hat{C} \wedge d\hat{C} \wedge \hat{C} \right\} \,. \tag{B.6}$$

Ten-dimensional R-R k-form fields are denoted by $C^{(k)}$ and their field strengths by $G^{(k+1)}$. B is the NS/NS 2-form and $\tilde{B}$ its dual NS/NS 6-form, and their field strengths are H and $\tilde{H}$. The $(k-1)$-form parameters of the R-R gauge transformations are denoted by $\Lambda^{(k-1)}$, and those of the NS/NS fields $B, \tilde{B}$ are denoted by $\Lambda, \tilde{\Lambda}$. The parameters of the massive gauge transformations are $\lambda, \tilde{\lambda}$.

The definitions of the field strengths of all these fields are[50] [13, 22]

[50] The gauge parameters used in Ref. [13] are related to ours by $\Lambda = \Lambda_{GHT} e^B$. Of course, other differences in conventions need also be taken into account.

$$
\begin{aligned}
G^{(2)} &= dC^{(1)} + \tfrac{m}{2}B\,, \\
H &= dB\,, \\
G^{(4)} &= dC^{(3)} - dBC^{(1)} + \tfrac{1}{2!}\tfrac{m}{2}B^2\,, \\
G^{(6)} &= dC^{(5)} - dBC^{(3)} + \tfrac{1}{3!}\tfrac{m}{2}B^3\,, \\
\tilde{H} &= d\tilde{B} + G^{(6)}C^{(1)} - \tfrac{1}{2}C^{(3)}dC^{(3)} \\
&\quad -\tfrac{m}{2}\left[C^{(7)} - C^{(5)}B + \tfrac{1}{2}C^{(3)}B^2\right]\,, \\
G^{(8)} &= dC^{(7)} - dBC^{(5)} + \tfrac{1}{4!}\tfrac{m}{2}B^4\,.
\end{aligned}
\tag{B.7}
$$

The field strengths of the massless theory are obtained by setting $m = 0$. In this case the gauge transformations of the fields are:

$$
\begin{aligned}
\delta C^{(1)} &= d\Lambda^{(0)}\,, \\
\delta B &= d\Lambda\,, \\
\delta C^{(3)} &= d\Lambda^{(2)} + d\Lambda^{(0)}B\,, \\
\delta C^{(5)} &= d\Lambda^{(4)} + d\Lambda^{(2)}B + \tfrac{1}{2!}d\Lambda^{(0)}B^2\,, \\
\delta\tilde{B} &= d\tilde{\Lambda} - \tfrac{1}{2}d\Lambda^{(2)}C^{(3)} + d\Lambda^{(0)}\left(C^{(5)} - \tfrac{1}{2}C^{(3)}B\right)\,, \\
\delta C^{(7)} &= d\Lambda^{(6)} + d\Lambda^{(4)}B + \tfrac{1}{2}d\Lambda^{(2)}B^2 + \tfrac{1}{3!}d\Lambda^{(0)}B^3\,,
\end{aligned}
\tag{B.8}
$$

and those of the massive theory are obtained by the replacements

$$
\left\{
\begin{aligned}
d\Lambda^{(0)} &\to d\Lambda^{(0)} + m\lambda\,, \\
\Lambda &\to -2\lambda\,, \\
d\tilde{\Lambda} &\to d\tilde{\Lambda} + m\tilde{\lambda}\,, \\
\Lambda^{(6)} &\to 2\tilde{\lambda}\,,
\end{aligned}
\right.
\tag{B.9}
$$

that is

$$
\begin{aligned}
\delta C^{(1)} &= d\Lambda^{(0)} + m\lambda\,, \\
\delta B &= -2d\lambda\,, \\
\delta C^{(3)} &= d\Lambda^{(2)} + \left(d\Lambda^{(0)} + m\lambda\right) B\,, \\
\delta C^{(5)} &= d\Lambda^{(4)} + d\Lambda^{(2)} B + \tfrac{1}{2!}\left(d\Lambda^{(0)} + m\lambda\right) B^2\,, \\
\delta \tilde{B} &= \left(d\tilde{\Lambda} + m\tilde{\lambda}\right) - \tfrac{1}{2} d\Lambda^{(2)} C^{(3)} + \left(d\Lambda^{(0)} + m\lambda\right)\left(C^{(5)} - \tfrac{1}{2} C^{(3)} B\right)\,, \\
\delta C^{(7)} &= 2d\tilde{\lambda} + d\Lambda^{(4)} B + \tfrac{1}{2} d\Lambda^{(2)} B^2 + \tfrac{1}{3!}\left(d\Lambda^{(0)} + m\lambda\right) B^3\,.
\end{aligned} \tag{B.10}
$$

Defining, as in Ref. [13] the Grassmann algebra objects

$$
\begin{aligned}
C &= C^{(0)} + C^{(1)} + C^{(2)} + \ldots \\
G &= G^{(0)} + G^{(1)} + G^{(2)} + \ldots \\
\Lambda^{(\cdot)} &= \Lambda^{(0)} + \Lambda^{(1)} + \ldots\,,
\end{aligned} \tag{B.11}
$$

the gauge transformations and field strengths of the R-R fields can be written in the more compact form

$$
\begin{aligned}
\delta C &= \left(\Lambda^{(\cdot)} + m\lambda\right) e^B\,, \\
G &= dC - dBC + \tfrac{m}{2} e^B\,.
\end{aligned} \tag{B.12}
$$

The action of massive ten dimensional IIA supergravity can be written in differential forms language as follows:

$$
\begin{aligned}
S = \; & \tfrac{1}{16\pi G_N^{(10)}} \int \Big\{ e^{-2\phi}\left[\Omega_{(10)} R - 4d\phi \wedge {}^\star d\phi + \tfrac{1}{2} H \wedge {}^\star H\right] \\
& + \tfrac{1}{2} G^{(2)} \wedge {}^\star G^{(2)} + \tfrac{1}{2} G^{(4)} \wedge {}^\star G^{(4)} + \tfrac{1}{2}\left(\tfrac{m}{2}\right) \wedge {}^\star \left(\tfrac{m}{2}\right) \\
& + \tfrac{1}{2} dC^{(3)} \wedge dC^{(3)} \wedge B + \tfrac{1}{3!}\tfrac{m}{2} dC^{(3)} \wedge B^3 + \tfrac{3}{5!}\left(\tfrac{m}{2}\right)^2 B^5 \Big\}\,.
\end{aligned} \tag{B.13}
$$

C Worldvolume Fields

Below we collect some useful gauge transformation rules of worldvolume fields needed in the text. The compact form notation of these rules was given in Section 2. The gauge transformation rules of the $c^{(p)}$ (p=0,2,4,6) are given by

$$\left\{\begin{array}{lcl} \delta c^{(0)} & = & -\frac{1}{2\pi\alpha'}\Lambda^{(0)} + \frac{m}{2}\rho^{(0)}\,, \\ \delta c^{(2)} & = & 2\partial\kappa^{(1)} - \frac{1}{2\pi\alpha'}\Lambda^{(2)} + 2\Lambda^{(0)}\partial b - \frac{m}{2}(2\pi\alpha')\rho^{(0)}\partial b - m\lambda b\,, \\ \delta c^{(4)} & = & 4\partial\kappa^{(3)} - \frac{1}{2\pi\alpha'}\Lambda^{(4)} + 12\Lambda^{(2)}\partial b - 12(2\pi\alpha')\Lambda^{(0)}\partial b\partial b \\ & & +2m(2\pi\alpha')^2\rho^{(0)}\partial b\partial b + 8m(2\pi\alpha')\lambda b\partial b\,, \\ \delta c^{(6)} & = & 6\partial\kappa^{(5)} - \frac{1}{2\pi\alpha'}2\tilde{\lambda} + 30\Lambda^{(4)}\partial b - 180(2\pi\alpha')\Lambda^{(2)}\partial b\partial b \\ & & +120(2\pi\alpha')^2\Lambda^{(0)}\partial b\partial b\partial b - 15m(2\pi\alpha')^3\rho^{(0)}\partial b\partial b\partial b \\ & & -90m(2\pi\alpha')^2\lambda b\partial b\partial b\,. \end{array}\right. \tag{C.1}$$

The corresponding field strengths are

$$\left\{\begin{array}{lcl} \mathcal{G}^{(1)} & = & \partial c^{(0)} + \frac{1}{2\pi\alpha'}C^{(1)} - \frac{m}{2}b,\,, \\ \mathcal{G}^{(3)} & = & 3\partial c^{(2)} + \frac{1}{2\pi\alpha'}C^{(3)} - 3C^{(1)}\mathcal{F} + 3\frac{m}{2}\,(2\pi\alpha')\,b\partial b\,, \\ \mathcal{G}^{(5)} & = & 5\partial c^{(4)} + \frac{1}{2\pi\alpha'}C^{(5)} - 10C^{(3)}\mathcal{F} + 15(2\pi\alpha')C^{(1)}\mathcal{F}\mathcal{F} \\ & & -20\frac{m}{2}(2\pi\alpha')^2 b\partial b\partial b\,. \end{array}\right. \tag{C.2}$$

The following gauge transformations of the $a^{(p)}$ fields are also used in the paper:

$$
\begin{cases}
\delta a^{(0)} &= \delta c^{(0)} \\
&= -\frac{1}{2\pi\alpha'}\Lambda^{(0)} + \frac{m}{2}\rho^{(0)}\,, \\
\delta a^{(2)} &= 2\partial\mu^{(1)} - \frac{1}{2\pi\alpha'}\Lambda^{(2)} - 4\partial a^{(0)}\lambda + \frac{m}{2}(2\pi\alpha')\rho^{(0)}\partial b - m\lambda b\,, \\
\delta a^{(4)} &= 4\partial\mu^{(3)} - \frac{1}{2\pi\alpha'}\Lambda^{(4)} - 24\partial a^{(2)}\lambda + 2(2\pi\alpha')^2 m\rho^{(0)}(\partial b)^2 \\
& \quad -4(2\pi\alpha')m\lambda b\partial b\,.
\end{cases}
\tag{C.3}
$$

The corresponding field strengths are

$$
\begin{cases}
\mathcal{H}^{(1)} &= \mathcal{G}^{(1)} \\
&= \partial a^{(0)} + \frac{1}{2\pi\alpha'}C^{(1)} - \frac{m}{2}b,\,, \\
\mathcal{H}^{(3)} &= 3\partial a^{(2)} + 3\partial a^{(0)}B + \frac{1}{2\pi\alpha'}C^{(3)} - 3\ \frac{m}{2}\ (2\pi\alpha')b\partial b - 3\ \frac{m}{2}\ Bb\,, \\
\mathcal{H}^{(5)} &= 5\partial a^{(4)} + 30\partial^{(2)}B + 15\partial a^{(0)}B^2 + \frac{1}{2\pi\alpha'}C^{(5)} - 20\frac{m}{2}(2\pi\alpha')^2 b(\partial b)^2 \\
& \quad -30m(2\pi\alpha')Bb\partial b - 15\frac{m}{2}bB^2\,.
\end{cases}
\tag{C.4}
$$

For the fields that compensate the total derivatives in p-brane WZ terms we have

$$
\begin{cases}
\delta b &= \frac{1}{2\pi\alpha'}2\lambda + \partial\rho^{(0)}\,, \\
\delta\tilde{b} &= -5\partial\rho^{(4)} - \frac{1}{2\pi\alpha'}\tilde{\Lambda} + \frac{m}{2}\rho^{(5)} + 5\Lambda^{(4)}\partial c^{(0)} \\
& \quad +15(2\pi\alpha')\partial a^{(2)}\left(\delta a^{(2)} - 2\partial\mu^{(1)}\right)\,,
\end{cases}
\tag{C.5}
$$

and their respective field strengths are

$$
\begin{cases}
\mathcal{F} &= 2\partial b + \frac{1}{2\pi\alpha'}B\,, \\
\tilde{\mathcal{F}} &= 6\partial\tilde{b} + \frac{m}{2}c^{(6)} + \frac{1}{2\pi\alpha'}\tilde{B} - 6\left(C^{(5)} - 5C^{(3)}B\right)\left(\partial c^{(0)} - \frac{m}{2}b\right) \\
& \quad -30(2\pi\alpha')\left[\partial a^{(2)} - \frac{m}{2}(2\pi\alpha')b\partial b\right]\left(\mathcal{H}^{(3)} - 3\partial a^{(2)}\right) \\
& \quad -120\frac{m}{2}(2\pi\alpha')^3\partial c^{(0)}b\partial b\partial b\,.
\end{cases}
\tag{C.6}
$$

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