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# The Consequence of Self-Field and Non-Uniform Current Distribution on Short Sample Tests of Superconducting Cables

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**Abstract**

Electrical measurements on samples of superconducting cables are usually performed in order to determine the critical current $I_c$ and the $n$-value, assuming that the voltage $U$ at the transition from the superconducting to the normal state follows the power law, $U \sim (I/I_c)^n$. An accurate measurement of $I_c$ and $n$ demands, first of all, good control of temperature and field, and precise measurement of current and voltage.

The critical current and $n$-value of a cable are influenced by the self-field of the cable, an effect that has to be known in order to compare the electrical characteristics of the cable with those of the strands from which it is made. The effect of the self-field is dealt with taking into account the orientation and magnitude of the applied field and the $n$-value of the strands.

An important source of inaccuracy is related to the distribution of the currents among the strands. Non-uniform distributions, mainly caused by non-equal resistances of the connections between the strands of the cable and the current leads, can easily result in a misinterpretation of the measured critical current and $n$-value by 5% and 50% respectively. In this paper this effect is explained in detail, taking also into account the influence of the current ramp-rate (during a voltage-current measurement), the sample length, the contact resistance between the strands and the placement of the voltage taps.

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## 1. INTRODUCTION

The critical current $I_C$ of a superconducting cable at a given temperature $T$ and field $B$ is usually defined by the current at which the cable has an effective resistivity equal to $\rho_C$. Typically, $\rho_C$ is taken as $10^{-14}$ Ωm using the total cross-section $A$ of the conductor. The transition between the superconducting and the normal state can often be expressed by:

$$U = \rho_C \frac{l}{A} I_C \left( \frac{I}{I_C} \right)^n \quad , \tag{1}$$

with $l$ the length over which the voltage is taken. A large $n$-value represents a sharp transition and a small $n$-value a smooth one. The $n$-value can therefore often be regarded as a measure of the homogeneity of the $I_C$-values of the filaments and strands along the length and over the cross-section.

The $I_C(B, T)$ relation of a cable can be obtained by multiplying Lubell's formula [1] for the critical surface of a NbTi superconductor by $\beta_{man}\beta_{SF}N_s$:

$$I_C = \beta_{man}\beta_{SF} N_s I_{C,str}(B,T) = \beta_{man}\beta_{SF} N_s \left( C_1 + C_2|B| \right)\left( 1 - (T/9.2)(1-|B|/14.5)^{-0.59} \right), \tag{2}$$

with $N_s$ the number of strands in the cable, $C_1$ and $C_2$ constants related to the critical current of a strand, $\beta_{man}$ a factor denoting the degradation of the critical current due to the cable manufacturing, and $\beta_{SF}$ a factor related to the self-field caused by the cable current.

An accurate measurement of $I_C$ and $n$ demands, first of all, good control of temperature and field, and precise measurement of current and voltage. The total errors in $I_C$ due to errors $\Delta I$, $\Delta U$, $\Delta T$ and $\Delta B$ in current, voltage, temperature and field can be estimated using eqs. 1-2 and calculating:

$$\Delta I_{C,tot} = \Delta I_I + \Delta I_U + \Delta I_T + \Delta I_B = \Delta I_I + (dI_C/dU)\Delta U + (dI_C/dT)\Delta T + (dI_C/dB)\Delta B. \tag{3}$$

Typical currents for SC cables are of the order of 10 kA, a value which can easily be measured with $\Delta I_I$ less than 0.1% error. Typical voltages at $I$=$I_C$ are of the order of 10 μV/m. A low-noise current supply and some filtering are therefore required to reduce the noise in the voltage over the sample to a few μV/m. The noise can be reduced to very low levels of less than 100 nV/m if a superconducting transformer is used in stead of an external current supply.

A stable temperature of the *helium bath* during the measurement can be achieved by good and fast temperature regulation and/or by having a sufficient amount of helium acting as a buffer. One also has to take care that the cable itself is properly cooled by the helium. In the resistive transition the heat generated in the cable is more difficult to transfer to the helium than in the case of a single strand. This results in a more pronounced increase in temperature at the transition, so that the *UI*-curve becomes steeper, and $I_C$ slightly smaller. The effect of self heating is usually small up to resistivities of about $10^{-14}$ Ωm. Determining $I_C$ at higher resistivities can lead to errors of a few percent.

Important work on the influence of the self-field on critical current measurements on cables has been done at BNL [2], concluding that the critical current is only determined by the properties of the superconductor in the peak field region. It was then possible to have results which were independent of the short sample test geometry. A slightly different approach is made in section 2, where the factor $\beta_{SF}$, related to the self-field of the cable, is not only a function of the peak field but on the complete field pattern in the conductor.

Besides the above given sources of error the shape of the *UI*-curve (and hence the deduced critical current and *n*-value) is also influenced by the current distribution among the strands. This effect is treated in section 3.

The influences of both the self-field and the current distribution are quantitatively illustrated by means of simulations performed in the case of a 15.1 mm wide Rutherford-type cable with rectangular cross-section, having 28 strands of 1.065 mm diameter, $C_1$=2970.5 A, $C_2$=-213.6 A/T, $T$=1.9 K, $I_C$(10 T, 1.9 K)=13.75 kA, $n_{str}$=40 (*n*-value of the strands).

## 2. SELF-FIELD OF THE CABLE

In a cable each strand is subject to the field caused by the transport current (i.e. the self-field) superposed to the external field.

In this paper, the self-field at each strand is taken as the average field over the cross-section of the strand. Due to the cable twist, each strand is therefore subject to a longitudinally changing self-field with a period equal to the cable pitch $L_P$. Fig. 1 shows the typical total field (i.e. the vector summation of the applied field and the self-field) along one strand over a length of one cable pitch. Two geometries are shown:

1. "single cable" geometry: one cable carrying 500 A per strand, with an applied field parallel ($B_{par}$) and perpendicular ($B_{perp}$) to the wide cable face,
2. "double cable" geometry: two cables spaced 0.5 mm apart and carrying opposite currents of ±500 A per strand, with the applied field parallel ($\pm B_{par}$) and perpendicular ($B_{perp}$) to the wide cable face.

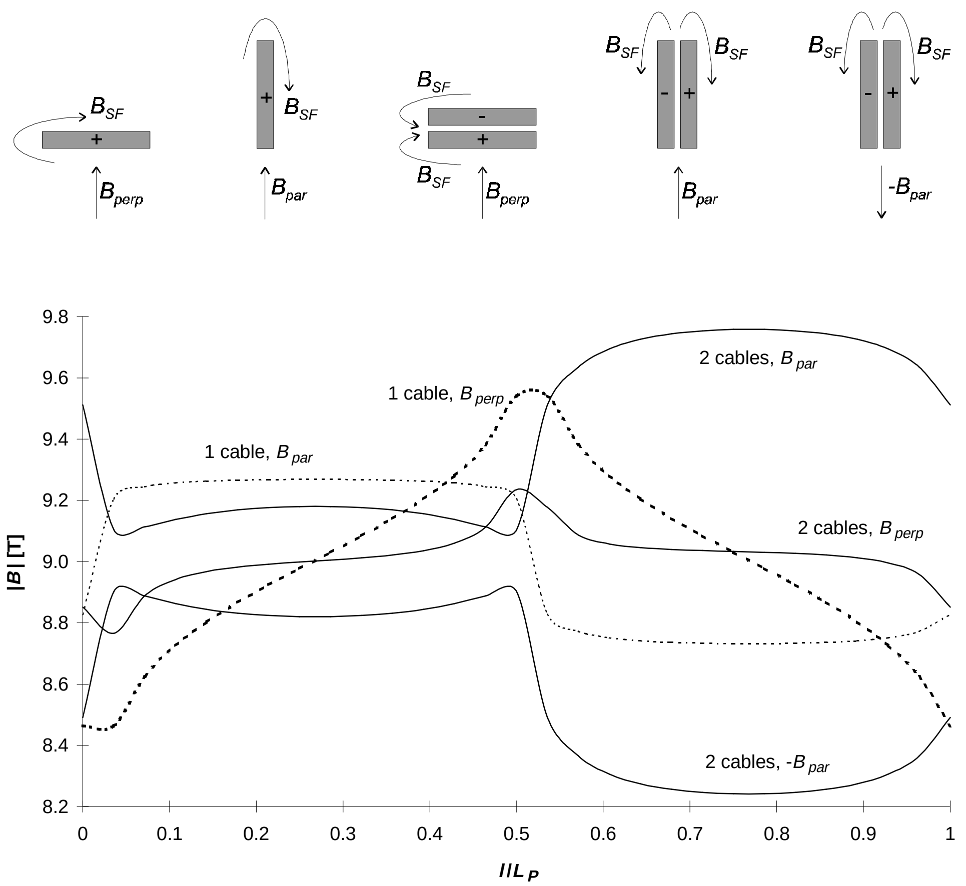


**Figure 1.** The total field seen by a strand along one cable pitch of a Rutherford-type cable in "single cable" (dotted curves) and "double cable" (solid curves) geometries. The applied field is 9 T parallel ($\pm B_{par}$) and perpendicular ($B_{perp}$) to the wide face of the cable. The strand current is 500 A. The edges of the flat cable are at $l/L_P$=0 and 0.5.

It is clear that, due to the self-field, the ratio $I/I_C$ varies along the length of the strand. If the first part of the strand reaches $I_C$ only ca. 10% of the strand is near the critical current (for perpendicular fields) and ca. 50% for parallel fields. Note that for *keystoned* Rutherford-type cables the curves become slightly asymmetric (around $|B|$=9 T). In this case there are two "double cable" configurations for a perpendicular applied field where the peak field is either at the thin or at the thick edges of the cables.

The self-field causes a change in the $I_C$-value of the cable as compared to the strands from which the cable is wound. This change, given by the factor $\beta_{SF}$ in eq. 2, is not a fixed percentage but depends on $I_{C,str}(B, T)$, $n_{str}$, the applied field and the cable geometry. This is illustrated in Fig. 2 in the case of the cable (as mentioned in section 1) in the "double cable geometry" subject to a perpendicular applied field (assuming $\beta_{man}$=1). The factor $\beta_{SF}$ becomes smaller for larger $n_{str}$ and lower applied fields (and hence higher critical currents) were the self-fields are larger compared to the background field. It is important that this reduction is inherent to the self-field variation along the strands and should not be attributed to a possible degradation of the strand performances due to the cable manufacturing, given by the factor $\beta_{man}$ in eq. 2. To know the factor $\beta_{man}$ one therefore has to know the factor $\beta_{SF}$ and hence an approximate value for $n_{str}$.

In a similar way, the differences between the cable and the strands can be calculated for a field parallel to the wide face of the cable. The $I_C$-values are typically 20% smaller (for positive applied parallel field) or 5-10% larger (for negative applied parallel field).

Due to the self-field also the sharpness of the transition (i.e. the $n$-value) changes. The $n$-values are typically 5% larger (for perpendicular applied field), 5-10% larger (for positive applied parallel field) or 20% smaller (for negative applied parallel field).

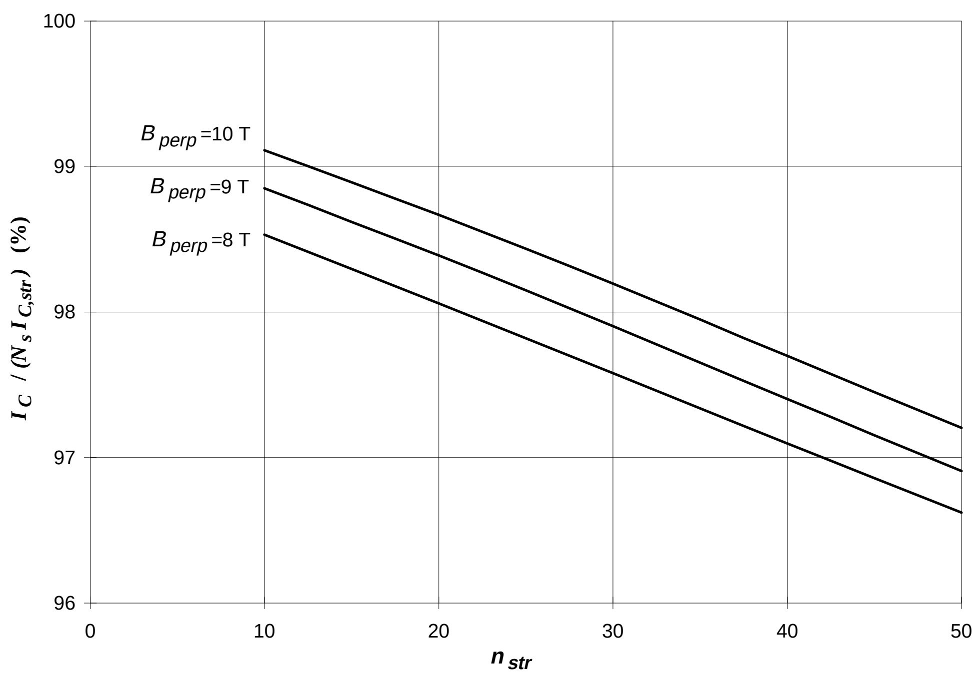


**Figure 2.** The reduction in $I_C$ of a cable as compared to the strands as a function of the $n$-value of the strands (in a two-cable stack geometry). The applied field is perpendicular to the wide face of the cables.

## 3. CURRENT DISTRIBUTION AMONG THE STRANDS

The current in a multistrand cable is distributed among the $N_s$ strands in such a way that the total voltage over each strand is the same. The total voltage consists of the resistive voltage $U_{sc}$ in the high field region, the inductive voltage $U_{ind}$ and the resistive voltage $U_{conn}$ in the connections (with the current leads or with another cable).

The highest critical current of a cable is obtained when the currents distribute according to $U_{sc}$ only (which will be called an *optimal distribution*). Note that this distribution is uniform if each strand has the same critical current and non-uniform if the strands have different $I_C$ values.

In the following the influence of the strand connection resistances $R_{conn}$ and the strand inductances on $I_C$ and $n_{cab}$ will be illustrated by means of several simulations on the above given 15.1 mm wide cable. The self inductance of a strand ($L_{str}$) and the total mutual inductance ($\Sigma M_{str}$) between a strand and all the other strands (per unit length), calculated using the network model CUDI.FOR[3], are:

$$L_{str}=1.54\ \mu\text{H/m} \qquad \Sigma M_{str}=15\ \mu\text{H/m} \qquad \Rightarrow\ L_{cab}=(L_{str}+\Sigma M_{str})/N_s=0.59\ \mu\text{H/m} \tag{6}$$

It will be assumed that all strands have the same critical current and $n$-value. The length of the cable $L_{cab}$ is taken as 2 m. Only part of the cable (with length $L_B$=0.6 m) is located in the high field area. The contact resistance between the strands is assumed to be infinitely large. $I_C$ is taken at the criterium $\rho_C=10^{-14}\ \Omega$m.

$R_{cab}$ denotes the average connection resistance on both sides of the cable. The average strand connection resistance $R_{conn,av}$ is therefore $2R_{cab}N_s$. The resistances $R_{conn}$ are taken as a normal distribution within 20% of the average. During a ramp the current will start to distribute according to the inductances and connection resistances. If the voltage would be measured on such a cable, different *UI*-relations would be found on different strands. The two extreme curves (for those strands having the smallest and largest $R_{conn}$) are shown in Fig. 3, along with the average voltage over all the strands. Depending on which strands the voltages are taken to determine $I_C$ one can find values between ca. 12000 A (called $I_{C,min}$) and 13950 A (called $I_{C,max}$), whereas the average $I_{C,av}$=13500 A (defined if the average measured voltage over the strands is equal to $U_0$). These values have to be compared to $I_C$=13750 A in case of a uniform distribution of the currents.

It is important to note that it is well possible to have *UI*-curves with an initial negative slope $dU/dI$ if the voltage taps would be soldered on two different strands.

Due to the continuous distribution of currents among the strands it is impossible to speak about ONE $n$-value. In stead, the local $n$-value of the *UI*-curves changes with current. An average $n$-value $n_{av}$ can then be defined as the local $n$-value of the average *UI*-curve at the current $I_{C,av}$.

At high current, where the voltages developed in each strand are much larger than $R_{conn}$, each strand shows an $n$-value close to $n_{str}$. At intermediate currents, strands with a small $R_{conn}$ have an $n$-value smaller than $n_{str}$ whereas strands with a large $R_{conn}$ show high $n$-values. This is caused by the fact that at intermediate currents an increase in cable current is mainly distributed over the not-yet resistive strands.

A representative $n$-value of the cable can only be determined at large currents. At large currents, however, the cable can be slightly warmer than the helium bath, especially if the cable is not properly cooled (due to the sample holder or the cable insulation). Warming-up of the cable in the resistive transition leads to a sharper transition and hence results in a larger $n$-value.

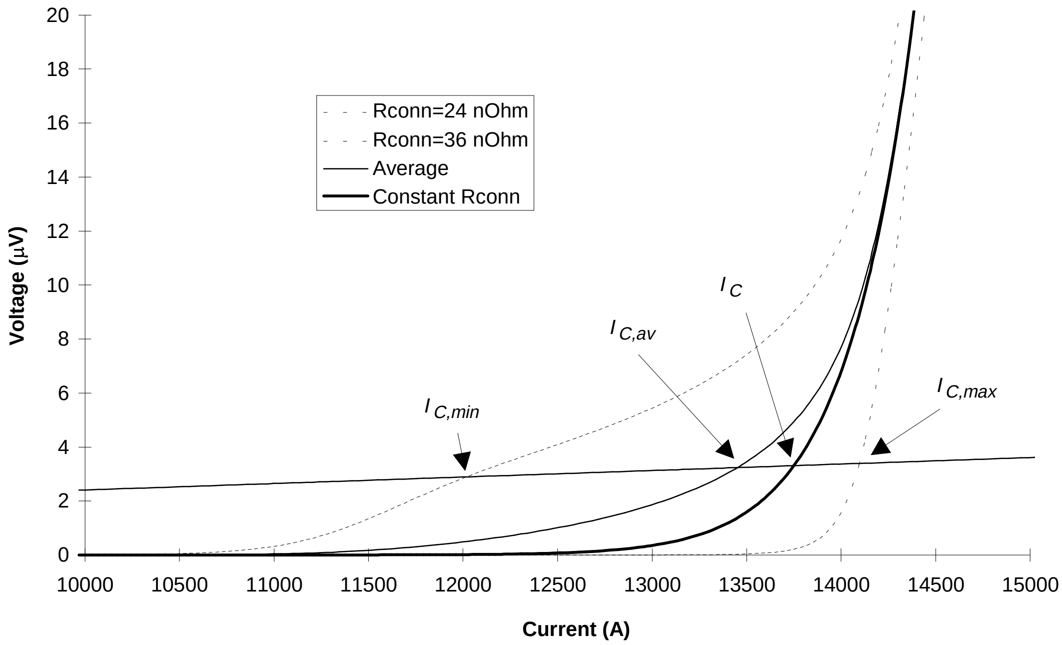


**Figure 3.** The *UI*-curves of several strands of a 28-strand cable for a 20% variation in the strands connection resistances ($n_{str}$ =40). The simulation is performed for a ramp from 0 A with 30 A/s.

The shape of the *UI*-curves and hence the $I_C$-values of each strand vary as a function of the ramp rate and depend on $n_{str}$, the variation in $R_{conn}$, the average connection resistance $R_{cab}$, the length $L_{cab}$ of the cable, and the length $L_B$ subject to a high field. Typical results of $I_{C,min}$, $I_{C,max}$ and $I_{C,av}$ as a function of the ramp rate are shown in Fig. 4.

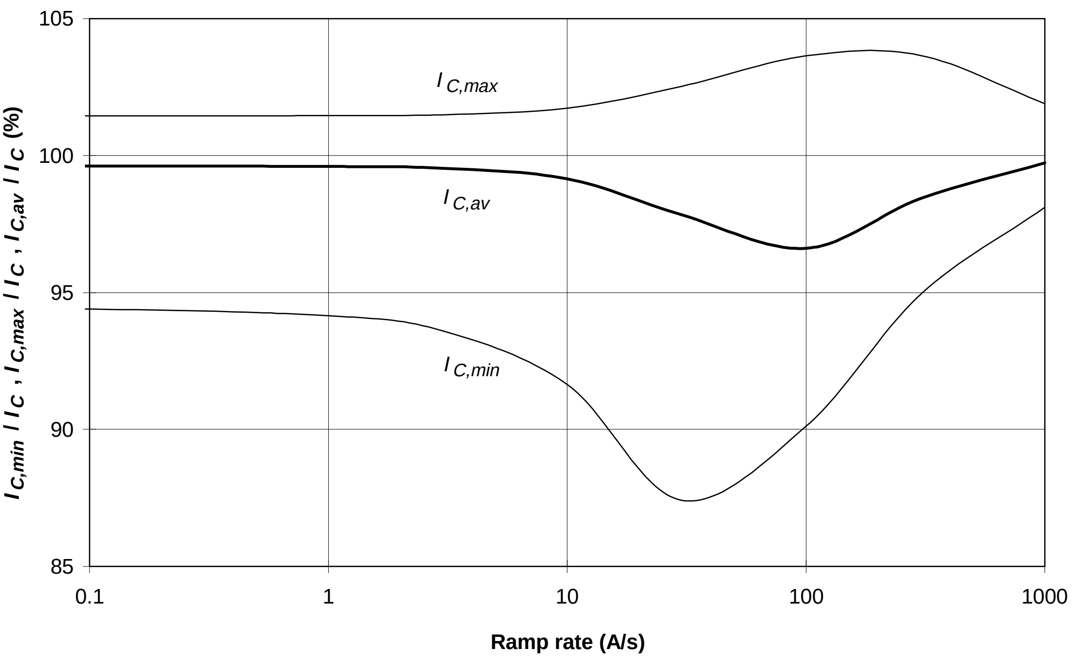


**Figure 4.** Typical calculated curves of $I_{C,min}$, $I_{C,max}$ and $I_{C,av}$ (as a percentage of $I_C$) of a 28-strand cable as a function of the ramp rate during the *UI*-curve ($n_{str}$ =40).

The curves show clearly that the *UI*-curves (and hence the critical currents) can be most accurately measured at low ramp rates, especially because at high ramp rates the inductive voltage is too large compared to the resistive signal. In a practical measurement on a cable one should always measure several *UI*-curves (on different strands) in order to obtain a more accurate value of $I_C$.

Some results of simulations (for the cable as mentioned in section 1) are shown in Table 1, where the values of the critical currents and $n_{av}$ are given both at zero ramp rate and at that ramp rate where the values are minimum (for $I_{C,min}$, $I_{C,max}$ and $n_{av}$) or maximum (for $I_{C,max}$).

**Table 1.** Simulated values for the reduction in $I_C$ and $n$ due to non-uniform connection resistances. ($I_{C,min}$, $I_{C,max}$ and $I_{C,av}$ are given as a percentage of $I_C$, $n_{av}$ is given as a percentage of $n_{str}$)

| | | | | | At zero ramp rate | | | | At most extreme ramp rate | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $L_{cab}$ (m) | $L_B$ (m) | $n_{str}$ | $R_{cab}$ (nΩ) | $R_{conn}$ (nΩ) | $I_{C,min}$ (%) | $I_{C,max}$ (%) | $I_{C,av}$ (%) | $n_{av}$ (%) | $I_{C,min}$ (%) | $I_{C,max}$ (%) | $I_{C,av}$ (%) | $n_{av}$ (%) |
| 2.0 | 0.6 | 40 | 0.54 | 24-36 | 94 | 101 | 100 | 77 | 87 | 104 | 97 | 44 |
| 2.0 | 0.6 | 20 | 0.54 | 24-36 | 93 | 103 | 99 | 84 | 88 | 105 | 98 | 69 |
| 2.0 | 0.6 | 40 | 0.54 | 27-33 | 99 | 101 | 100 | 94 | 95 | 103 | 99 | 70 |
| 2.0 | 0.6 | 40 | 0.27 | 9-21 | 90 | 102 | 99 | 71 | 78 | 104 | 95 | 28 |
| 2.0 | 0.6 | 40 | 0.27 | 12-18 | 98 | 101 | 100 | 93 | 91 | 103 | 98 | 53 |
| 1.0 | 0.6 | 40 | 0.54 | 24-36 | 94 | 101 | 100 | 77 | 88 | 104 | 97 | 42 |
| 2.0 | 0.3 | 40 | 0.54 | 24-36 | 89 | 102 | 98 | 47 | 85 | 105 | 95 | 37 |
| 2.0 | 0.1 | 40 | 0.54 | 24-36 | 84 | 104 | 94 | 39 | 83 | 106 | 91 | 39 |
| 2.0 | 0.04 | 40 | 0.54 | 24-36 | 82 | 105 | 91 | 44 | 82 | 108 | 89 | 44 |

The results show that it is very important:

- to have connections with a low average connection resistance and with a small variation among the strands. The length of the connection should therefore be an integer times the cable pitch.
- that the length which is subject to a high field is as large as possible. So called "hairpin" samples with only a few cm sample length (i.e. $L_B$<0.1 m) can easily result in erratic results.
- that considerable variations in $I_C$ and $n$ can be measured on different strands, even for long sample lengths. This implies that for a proper measurement the average should be determined of several strands, each being equipped with a pair of voltage taps. Note that the average voltage on a cable can not simply be measured using voltage taps placed on e.g. two soldered parts of the cable.
- to perform *UI* measurements at a few ramp rates. If there is a variation of several percent among the $I_C$ values at different ramp rates, it is likely that a non-uniform connection resistance influences the measurement. In this case the *n*-values are probably not significant and should not be correlated to the *n*-value of the strands.

In the above given results the contact resistance in the cable (besides the connections) is assumed to be infinite. A finite contact resistance will improve the current distribution and hence reduce the errors $|I_{C,av}-I_C|$ and $|n_{av}-n_{str}|$. The reduction can be significant if

$R_c < R_{conn}(l_{cab}\text{-}l_{conn})N_s$, with $l_{conn}$ the length of the connections and $R_c$ the average contact resistance between the strands in the cable.

## 4. CONCLUSIONS

Simulations are performed in order to investigate how the critical current $I_C$ and $n$-values of a multistrand cable can be correctly measured. An accurate measurement demands, first of all, good control of temperature and field, and precise measurement of current and voltage. The variation of $I_C$ caused by errors in these parameters can be easily determined using the estimated $I_C(B,T)$ relation of the conductor. Besides the measurement errors of current, voltage, field and temperature, $I_C$ and $n$ are influenced by the self field of the cable and the current distribution among the strands.

***Self-field of the cable:*** At large currents the self-field of a cable is not negligible compared to the applied field. The self-field causes that each strand is subject to a longitudinal varying total field, which implies that the critical current in the strand varies as well longitudinally. The result is that $I_C$ measured on a cable is a few percent smaller than the sum of $I_C$'s on the strands. The exact percentage depends on the field, the temperature, the $n$-value of the strands and the sample geometry (e.g. one cable, two cables spaced apart etc.).

Only if the effect of the self field and the current distribution on the $I_C$-value are quantified, it is possible to determine the degradation of cable performances due to the cable manufacturing.

***Current distribution among the strands:*** In a multistrand cable the current distribution among the strands can be non-uniform due to variations in the contact resistances between the strands and the current lead. This results in different *UI*-curves between different strands, where strands with a small connection resistance become resistive at lower cable currents than strands with large connection resistances. In practice it is even possible that the *UI*-curve has an initial negative slope, especially if the voltage taps are soldered on different strands. It is not possible to ensure a proper $I_C$ measurement on a cable if the sample length is too small, or if the connections of the cable sample with the leads are very non-uniform.

Whether in an experimental set-up the *UI*-measurement (and hence $I_C$) is affected by a non-uniform current distribution can be deduced by monitoring the strand currents (using for example Hall probes) or by measuring the *UI*-curves of several strands in the cable and at various ramp rates. One has to take care however that a possible non-uniform distribution can also be caused by a variation in the critical currents among the strands.

Both the self-field and the current distribution not only influence the accuracy with which $I_C$ can be measured but also affect the maximum voltage that can be measured on a cable. In a cable measurement each strand is subject to a longitudinally varying field and only part of the strand enters the resistive transition. In this part the resistivity can be easily a factor 10 larger than the average resistivity in the cable. This factor can become even larger if, due to a non-uniform connection resistance, some strands carry more current than others. A high local resistivity results in a large local dissipation which can cause a quench, already at a relatively low average resistivity, and possibly even before all strands have entered the resistive transition.

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