On velocities beyond the speed of light c

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Abstract

From a mathematical point of view velocities can be larger than c. Lorentz transformations are easily extended in Minkowski space to discuss velocities beyond the speed of light. Energy and momentum conservation fixes the relation between masses and velocities larger than c, and some interesting consequences are drawn. Current data make neutrinos as possible candidates for having speed larger than c. Assuming this is true, a well known enigma on the arrival times of the neutrino bursts from the SN1987A supernova can be explained quite naturally. Finally, experimental research is proposed to verify the theory.

1 Theory

In Minkowski's space, Lorentz transformations are expressed as rotations in the plane $x_1 x_4$, where x_1 is the space coordinate associated to the direction of the relative velocity v between two reference systems, and x_4 is equal to ict [1]. A relative velocity of c corresponds to a rotation dividing the first quadrant of the complex plane $x_1 x_4$ into two equivalent parts. However, both velocities greater and lower than c correspond to valid rotations (of course ending on different sides of the diagonal defined by $v = c$). The Lorentz transformations

$$x_1' = \frac{x_1 + ix_4 v/c}{\sqrt[2]{1 - v^2/c^2}} \ , \quad x_4' = \frac{x_4 - ix_1 v/c}{\sqrt[2]{1 - v^2/c^2}} \tag{1}$$

show that the effect of setting $v > c$ is to swap the roles (real vs imaginary) of x_1' and x_4'. In this sense, the speed of light is a threshold velocity, but not the maximum allowed velocity. The speed of light remains the only velocity which is constant in any reference system, making the invariant $s^2 = \Delta x_1^2 + \Delta x_4^2$ equal to 0. Particles with velocities lower/greater than c have a negative/positive s^2 (and different velocity for different reference systems). From this discussion, it does not follow that it is possible to accelerate a particle in vacuum from a velocity lower than c to velocities greater than c (similarly, it is not possible to

decelerate a photon in vaccum to a speed lower than c). However it is argued that special Relativity does not exclude [2] the possibility for particles to exist in three different regimes of velocities V: $0 \leq V < c$; $V = c$; $c \leq V < \infty$.

All known particles and interactions are observed to conserve energy and momentum. If any known particle has speed larger than c, this should not affect the (measured) energy and momentum balance in the interactions that are undertaken by such a particle. The particle energy is given by [3]

$$E = \frac{m_0 c^2}{\sqrt[2]{1 - v^2/c^2}}.$$ (2)

If the velocity v is greater than c, the denominator becomes imaginary, but if, at the same time, the mass (intended as rest mass here and in the following) is imaginary, the value of the energy is real [4]: it would be the same energy value obtainable for a particle of real mass and with velocity lower than c such that $|c - v|$ is the same as above. The same argument is valid for the relativistic momentum expression and conservation. A physics consequence is obvious: particles with velocity larger than c cannot exist at rest (as it is also true for the photons) and cannot be decelerated in vacuum to speed equal c .

The following principle is proposed [4]:
Particles with velocity larger than c have imaginary rest mass and, viceversa, particles with imaginary rest mass have velocity larger than c.

Equation (2) becomes:

$$E = Re(\frac{m_0 c^2}{\sqrt[2]{1 - v^2/c^2}}) \; ; \quad 0 = Im(\frac{m_0 c^2}{\sqrt[2]{1 - v^2/c^2}}).$$ (3)

Since the denominator of the ratio in equation (3) can only be purely real or purely imaginary, the second condition of equation (3) ensures that the numerator and the denominator are either both real or both imaginary (from the expression of the division between complex numbers), while the first condition of equation (3) allows to compute the energy in both cases.

Consequences [4]:

- m_0^2 would be negative. It is still the relativistic invariant of the four-momentum, and it would be positive for velocities lower than c, it would be null for velocities equal to c, and it would be negative for velocities greater than c (in analogy to the space-time invariant s^2).

- The greater (assuming it is always larger than c) is the velocity, the lower would be the particle'e energy. It would reamin true that the energy tends to infinity when the velocity tends to c. In cases where the interaction probability of such particles in a material is correlated to the time they spend flying in it, we can expect the interaction probability to increase with the particle's energy.

- For a given energy, heavier imaginary masses imply higher velocities. In the limit of immaginary mass tending to zero, the velocity tends to c.

2   Experimental data

Existing experimental data on neutrinos seem to be consistent with the hypothesis that their mass square is negative [5] (see also [5], [6], [7], [8] for several theoretical treatments of superluminal neutrinos). Seven independent measurements of the electron neutrino mass square are reported in [9]. Six of them yeld a negative value for the result, though each experiment reports of large statistical and systematic errors. Their range is from $-147 \pm 68 \pm 41 \, (eV/c^2)^2$ to $1.5 \pm 5.9 \pm 3.6 (eV/c^2)^2$. In addition, it has been computed [10] that the weighted average of the electron neutrino mass square measured in tritium β decay experiments is $m_{\nu_e}^2 = -96 \pm 21 (eV/c^2)^2$. Finally, the best fit of the measurements considered by the Particle Data Group [11] gives $m_{\nu_e}^2 = -27 \pm 20 (eV/c^2)^2$ (see [11] also for a preliminary discussion on the relation between the electron neutrino mass and the values associated to the neutrino mass eigenstates). Concerning the muon neutrino mass square, two independent measurements find a negative value [11], though given the reported errors it is safer to give only an upper limit for the mass [11].

3   SN1987A data: an experimental signature of superluminal neutrinos?

At the level of astrophysical data, another hint on the possible superluminal speed of neutrinos can be found. An alternative analysis of the neutrino bursts detected in 1987, before the optical observation of the SN1987A supernova, is proposed here. A summary of the relevant data follows below:

- The distance between the Earth and SN1987A (in the Large Magellanic Cloud) is computed to be 170000 light-years [12], [13], [14], [15] (though the value 160000 light-years is found as well [16]).

- Sophisticated and different models of stars implosion and supernovae explosion agree on the fact that neutrinos are emitted within an interval of a "few" seconds (peaked, especially for electron neutrinos, at a few hundreds of milli-seconds, and consistently also with the eventual neutrino heating mechanism) [17], [18], [19].

- Indeed, several detectors revealed neutrino bursts on 23rd February 1987:

    - IMB found 8 events within 5.58 sec, with energy between 19 MeV and 39 MeV [12], [13], [17], [20] (errors between 5 MeV and 9 MeV).

– K2 found 12 events (1 discarded later) within 12.439 sec, with energy between 6.3 MeV and 35.4 MeV [12], [13], [17], [20] (errors between 1.7 Mev and 8 MeV).

– Baksan found 5-6 events [12], [13] with energy between 12 MeV and 23.3 MeV.

The syncronization between the three experiments was not better than about one minute [12], [13], though all neutrinos are normally considered to have arrived within about 15 sec. It has to be noted also that the energy values reported above refer to the electrons which produced the Cherenkov light signal in the detectors: whenever the detected Cherenkov light was instead produced by a positron, the neutrino energy should be increased by about 2 MeV [21].

• Supernova models predict that the emission of visible light follows the neutrino burst after a few hours (for blue giants, like SN1987A) [15], [21], or after ten(s) hours (for red giants) [20], [21]: in fact the explosion of the supernova occurs when the shock-wave generated by the material rebounded by the neutronized core, reaches the outer star layers.

• Indeed, it has been possible to reconstruct that the first photografic observation of SN1987A was recorded between 09h:36min and 10h:38min GMT, while the neutrino burst was observed at 07h:36min GMT [12], [15], [19]. Hence the delay was within 2 and 3 hours.

The picture seems consistent, however there is an experimental observation which is currently un-explained: the LSD experiment observed 5 events within 7 sec, with energy between 5.8 and 7.8 MeV [13], [20] (errors of about 1-2 MeV according to [20], energy resolution of about 15% or 25% according to [22]), earlier than K2 and IMB by 4h:43min:4.58sec [13], [20]. A thourough analysis showing that those events could not be due to statistical noise was performed in [20], also combined with the fact that K2 recorded at least one event 4h43min13±2sec before the published burst [20], [23]. It should also be noted that IMB could not have recorded events at those energies because of its higher thereshold. Moreover, it should be considered that LSD detected 2 candidate events at IMB+K2 time, of energies between 7 and 9 MeV within an interval of 13 sec [22], [23]. Finally, it is recalled that the so-called electron-neutrino, muon-neutrino and tau-neutrino are eigenstates of the weak interaction, each of them being a linear combination of mass eigenstates [24], [25], [26], assuming neutrino masses different from zero:

$$\nu_f = \sum_m U_{fm}\nu_m, \qquad (4)$$

where f refers to the flavour and m to the mass eigenstates. A proper wave-packet treatment (including spreading) of the neutrino [24], [25], [27] emphasises that each mass eigenstate has its own energy and momentum [27] (the

momentum difference between eigenstates cannot be neglected for extremely long distances or times-of-flight, i.e. for supernova neutrinos). This leads to different speeds for the mass eigenstates and hence to their eventual separation after long enough distances [24], which could be observed for neutrinos coming from supernovae [28]. Then the following approach is proposed to explain the enigma of the LSD events:

- *All the SN1987A neutrinos have been emitted in a "few" seconds.*

- *The faster superluminal mass eigenstate of all neutrinos reached the Earth about 4h43min before the corresponding slower superluminal mass eigenstate.*

- *LSD (and probably K2) detected the faster superluminal mass eigenstate (at the corresponding energies) of the neutrinos, while IMB, K2 and Baksan detected their slower superluminal mass eigenstate (at the corresponding energies).*

The mass eigenstate $|m1>$ is assumed to be closely associated to the electron-neutrino flavour (so the measured m_{ν_e} will be used for it), while the mass eigenstate $|m2>$ is assumed to be closely associated to the muon-neutrino flavour (so m_{ν_μ} will be used for it) [11]. The tau-neutrino is for the moment neglected. However, it cannot be excluded that LSD detected $|m3>$, associated to the tau-neutrino, rather than $|m2>$. Therefore, in the following, the notations μ and $|m2>$ should be considered as indicating either μ or τ and $|m2>$ or $|m3>$. In any case, even in regime of non-oscillations, any flavour can be selected at detection time with a given probability [24], [25], [27], as it can be easily guessed inverting equation (4). For a quantitative treatment of the approach outlined above, the first step is to show that the IMB+K2 burst can correspond to the detection of the $|m1>$ mass eigenstate of the SN1987A neutrinos. Equation (3) is applied, at first, to the mass eigenstate $|m1>$, setting the mass to be $m_{\nu_e} = i\sqrt{(27)}eV/c^2$, and varying the energy between the limits of the measured spectrum, from E_{min} = 6.3 MeV (K2) to E_{max} = 39 MeV (IMB). Then, the corresponding velocities can be computed. These are used to compute the time delays introduced by the different energies within the burst of neutrinos of mass eigenstate $|m1>$ (using the distance from SN1987A to the Earth to compute the time differences ΔT). The results (expressed in terms of the time gained vs hypothetical neutrinos travelling at the speed of light) are:

$$\Delta T(E_{min}^{\nu_e}; Light) = 1.8sec \; ; \; \Delta T(E_{max}^{\nu_e}; Light) = 0.05sec. \tag{5}$$

This gives two interesting consequences:

- For the $|m1>$ neutrinos eigenstate, the computed spread of the arrival times due to different velocities (energies), found to be lower than 2 sec, is well contained in the observed time intervals at IMB and K2. This

means that basically it does not affect the arrival time spread due to the emission spread of the SN1987A neutrinos, and hence is consistent with the observations (this can indeed allow to improve the fits of the K2 data). Anyway note that, for the moment, similar results could be obtained by applying equation (2) with a real mass and velocities lower than c.

- However, if it is true that $|m1>$ is the eigenstate seen at IMB+K2 time, the $|m2>$ eigenstate, in order to have arrived 4h:43 min earlier, must definitively have traveled at a speed greater than c. This is also consistent with an heavier (imaginary) mass for $|m2>$ (as from equation (3)).

Therefore, the second step in this quantitative treatment is to show that the LSD burst is consistent with the detection of the $|m2>$ eigenstate of the SN1987A neutrinos. If equation (3) is applied using an average energy measured at LSD (= 6.8 MeV) and using the velocity needed to justify an advance of 4h43min over the $|m1>$ eigenstate, it is possible to compute a value for the mass associated to the $|m2>$ eigenstate. The result is of the order of

$$m_{\nu_\mu} \simeq i541eV/c^2, \qquad (6)$$

which is not currently favoured but also not exluded (for suitable mixing angles) by most of the neutrino oscillation experiments [27], [29]. The reason why the average energy measured at LSD can be used needs to be discussed:

- The mass and velocities needed to satisfy equation (3) and, at the same time, the requirement of ΔT = 4h:43min (before $|m1>$ arrival) are such that the energy of $|m2>$ is required to be constant within about 0.1%, if the time spread due to different velocities (energies) has to be contained in the observed time spread between LSD events.

- Indeed, differently from the IMB and K2 events, the LSD events can be addressed by a fit at constant energy, as it can be evidently deduced from the LSD data and relative errors reported above. In addition, if the K2 event measured at LSD time is considered, its energy of 7 MeV [23] is consistent with the fit at constant energy.

However, a proper error analysis goes beyond the scope of this paper, since the result is sensitive to the uncertainties on several quantities, such as on the errors on the energy of the LSD (K2) events and on the exact distance from SN1987A, hence an average of the LSD energies is used for a proof-of-concept, rather than trying to perform a proper fit. In addition, no attempt is made here to investigate the relations between the observed energy of the mass eigenstates and the energies at which neutrinos were produced in the supernova (which imply model-dependent considerations and red-shifting plus eventual gravitational effects), since only time-intervals and energies meaured on Earth are consistently used in the computations.

So far, it has been demonstrated that experimental data are consistent with the hypothesis that LSD (and probably K2) detected the faster and heavier $|m2>$ mass eigenstate, while IMB, K2 and Baksan detected the slower and lighter $|m1>$ mass eigenstate of a single superluminal neutrino burst emitted by SN1987A in a few seconds.

However, it has not been excluded yet the possibility that LSD detected a faster and lighter $|m1>$, while IMB, K2 and Baksan detected a slower and heavier $|m2>$, both associated to real masses, of a single burst of neutrinos traveling at velocities lower than c, according to equation (2). Anyway, this alternative is discarded because of the following arguments:

- Applying equation (2) to the supposed slower and heavier (real mass) $|m2>$, the spread of energies measured at IMB+K2 (and the consequent spread of velocities) provokes a spread of arrival times much bigger (> 20 hours) than the observed one (≈ 15 sec, certainly < 1 minute).

- Since the speed difference between the $|m1>$ eigenstate (at LSD energy) and c is negligible, and since the optical recording of the 1987A explosion occurred 2-3 hours after IMB+K2 time, it follows that the first photons emission of SN1987A would have started about 7-8 hours after the neutrinos emission. This is not favoured (compared with a 2-3 hours delay) by the current modelling of a blue giant such as the progenitor of 1987A.

- Though the statistics is low, all the neutrino events detected from 1987A are consistent with electron-neutrino flavour [13], [21]. Hence it is more likely that the $|m1>$ eigenstate was revealed at IMB+K2 time because, overall, more neutrinos were detected there than at LSD time. However, it would be necessary to take into account the mixing angles, the detectors' thresholds and sensitivities, and to have sufficiently high statistics, in order to be able to use this argument.

4   Future Research

From a theoretical perspective, it would be interesting to re-discuss the concept of simultaneous events in the framework of Special Relativity when considering velocities greater than c [30]. In particular, the possibility to have events in multiple space points for a given time could be compared with the quantum-mechanical view. In addition, some gravitational effects, in particular for black-holes, could be revisited in view of the existence of superluminal neutrinos of known imaginary mass.

From an experimental perspective, it would be interesting to perform direct measurements of the neutrino velocities in vacuum, to be compared with c. According to equations (3) and (6), a $|m2>$ neutrino eigenstate of 6.8 MeV energy should have a velocity approximatively equal to 3.0000000095047575e+05

km/sec. For distances of the order of the Earth-Sun distance, this gives time differences (between neutrinos and light) of the order of micro-seconds. An Earth-based accelerator and a spacecraft-based detector (or viceversa) seem the only reasonable fit for such an experiment. However, managing to generate and detect neutrinos of energy equal to 68 KeV, would allow the observation of time differences (between neutrinos and light) of the order of a micro-second for distances of the order of the Earth radius. This would make it possible to use existing accelerators and Earthly detectors (or detectors on satellites) to measure the neutrinos velocities. Another interesting experience could be related to the deflection angles of neutrinos by strong gravitational fields.

In conclusion, if some estimation of m_{ν_τ} would be available (and in case it is assumed that LSD detected the mass eigenstate closer to ν_μ), it would be interestng to scan backwards the IMB, K2 and LSD data, looking for signals at times prior to the LSD time. Viceversa, if the LSD burst corresponds to the detection of the mass eigenstate closer to m_{ν_τ}, then it would be interesting to scan the time interval between LSD and IMB+K2 times, searching for a burst that might correspond to the mass eigenstate closer to ν_μ. Alternatively, if two of the mass eigenstates would have very close masses, they could have appeared in the same burst (at LSD or IMB+K2 time depending on which eigenstates are considered). Finally, individual events during the LSD − IMB+K2 interval, could account for neutrinos of the same mass eigenstate as detected at LSD, but with higher energies.

5   Conclusions

Particles with velocities greater than c and imaginary mass are allowed by the conservation of the relativistic energy-momentum. Current data for neutrinos are consistent with this hypothesis. The expression of neutrinos as combination of superluminal mass eigenstates is consistent with all data (including LSD) recorded from the SN1987A supernova. Direct measurement of the neutrino velocities in vacuum at different energies could confirm the theory or definitively reject the neutrino as a candidate for superluminal velocities.

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