

CERN-TH/97-320
BUTP-97/32
November 1997

Aspects of Chiral Dynamics*

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Plenary talk given at the Workshop on Chiral Dynamics 1997
Mainz, Germany, Sept. 1 – 5, 1997
To appear in the Proceedings

* Work supported in part by Schweizerischer Nationalfonds.

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1 Introduction

In my talk I first discussed the symmetry properties of the QCD hamiltonian and its ground state. In particular, I considered flavour (isospin and $SU(3)$) and chiral symmetries in some detail. Here, I followed closely the article by [?], to which I refer the reader. Then, I outlined the effective low-energy theory of QCD in the meson and baryon sector and illustrated it with a few examples. There are many review articles on chiral perturbation theory available on the market, see e.g. Bijnens et al. (1995), Ecker (1995a,b), Gasser (1995), Leutwyler (1991,1994b) and Meißner (1993). Here, I shall therefore concentrate on some aspects of baryon chiral perturbation theory and illustrate why the low-energy expansion is rather involved in this case.

2 Effective theory

The QCD lagrangian can be replaced at low energies with an effective lagrangian that is formulated in terms of the asymptotically observable fields, see Weinberg (1979), Gasser and Leutwyler (1984,1985). This effective lagrangian reads for processes with pions alone

$$\mathcal{L}_M = \frac{F^2}{4} \langle \partial_\mu U \partial^\mu U^\dagger + M^2(U + U^\dagger) \rangle .$$

Here, the matrix field U is an element of $SU(2)$, and the symbol $\langle A \rangle$ denotes the trace of the matrix A . In the following, I use the parametrization

$$U = \sigma + \frac{i\phi}{F} ; \phi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix} , \sigma = [\mathbf{1} - \phi^2/F^2]^{\frac{1}{2}} ,$$

and the notation

$$\phi = \sum_{i=1}^3 \tau^i \phi_i , \pi = (\phi_1, \phi_2, \phi_3) .$$

The coupling constant $F \simeq 93$ MeV measures the strength of the $\pi\pi$ interaction, and the quantity M^2 denotes the square of the physical pion mass (that I denote with M_π) at lowest order in an expansion in powers of $1/F$, see below. It is proportional to the light quark masses m_u, m_d ,

$$M^2 = 2\hat{m}B \quad , \quad \hat{m} = \frac{1}{2}(m_u + m_d) \quad ,$$

where B itself is related to the quark condensate, see [?]. Note that the quantity M^2 occurs not only in the kinetic term of the pion lagrangian, but also in the interaction: it acts both as a mass parameter and as a coupling constant. The lagrangian \mathcal{L}_M is called the "non-linear sigma-model lagrangian". This name has led to some confusion in the literature about the meaning of the effective lagrangian: one is not replacing QCD with a "chiral model", as this procedure is often called. To the contrary, \mathcal{L}_M can be used to calculate processes at low energies, with a result that is – as shown by [?] – identical to the one in QCD.

In case we wish to consider also nucleons, one has to enlarge the above lagrangian. In the following, I will consider processes where a single baryon (proton or neutron) travels in space, emitting and absorbing pions in all possible ways allowed by chiral symmetry, see Fig. 1. I do not consider pro-

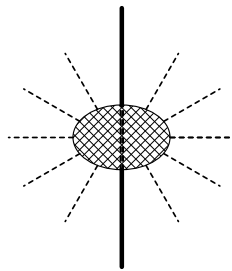


Fig. 1. The nucleon traveling through space, emitting and absorbing pions.

cesses with closed nucleon lines. These contributions may be absorbed in a renormalization of the coupling constants in the effective lagrangian

$$\mathcal{L}_{eff} = \mathcal{L}_M + \mathcal{L}_{MB} \quad , \quad (1)$$

where the meson–nucleon interaction is given by

$$\mathcal{L}_{MB} = \bar{\Psi} \left\{ i\gamma^\mu \partial_\mu - m - \frac{g_A}{2F} \gamma^\mu \gamma_5 \partial_\mu \pi + O(\pi^2) \right\} \Psi \quad . \quad (2)$$

Here, m is the nucleon mass in the chiral limit, and g_A is the neutron decay constant $g_A \simeq 1.25$. The effective lagrangian (1) contains the three couplings $1/F, M^2$ and g_A .

2.1 Tree level

According to the rules set up in the sixties and seventies, one has simply to evaluate tree graphs with \mathcal{L}_{eff} to generate S -matrix elements that are in agreement with current algebra predictions. As is known today, this procedure generates the leading order term in a systematic low-energy expansion of the Green functions, see [?] and [?]. I illustrate it with two examples.

The pion mass It suffices to consider the terms in \mathcal{L}_M that are quadratic in the pion fields,

$$\mathcal{L}_M = \frac{1}{2} \{ \partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi} - M^2 \boldsymbol{\pi}^2 \} + O(\boldsymbol{\pi}^4) .$$

Therefore, the effective theory contains at tree level three mass degenerate bosons π^+ , π^- , π^0 , with

$$M_{\pi^\pm}^2 = M_{\pi^0}^2 = M^2 . \quad (3)$$

At the leading order considered here, there is no isospin splitting: the masses of the charged and of the neutral pions are identical, see [?]. A small mass difference due to $m_u \neq m_d$ does show up only at next order in the chiral expansion.

$\pi\pi$ scattering The full power of the effective lagrangian method comes into play when one starts to evaluate scattering matrix elements. Consider for this purpose elastic $\pi\pi$ scattering. The interaction part of the effective lagrangian is

$$\mathcal{L}_{int} = \frac{1}{8F^2} \{ \partial_\mu \boldsymbol{\pi}^2 \partial^\mu \boldsymbol{\pi}^2 - M^2 (\boldsymbol{\pi} \cdot \boldsymbol{\pi})^2 \} + O(\boldsymbol{\pi}^6) .$$

Since we calculate tree matrix elements, the terms at order $O(\boldsymbol{\pi}^6)$ do not contribute. The contributions with four fields in the lagrangian contain two types of vertices: the first one has two derivatives, while the second contains the parameter M^2 as a coupling constant. In the following I consider the isospin symmetry limit $m_u = m_d$ and use the standard notation

$$T^{ab;cd} = \delta^{ab;cd} A(s, t, u) + \delta^{ac;bd} A(t, u, s) + \delta^{ad;bc} A(u, s, t)$$

for the matrix element of the process

$$\pi^a(p_1) \pi^b(p_2) \rightarrow \pi^c(p_3) \pi^d(p_4) ,$$

with the Mandelstam variables

$$s = (p_1 + p_2)^2 , t = (p_1 - p_3)^2 , u = (p_1 - p_4)^2 ; s + t + u = 4M_\pi^2 .$$

The result of the calculation is

$$A \stackrel{\text{tree}}{=} \frac{s - M^2}{F^2} \stackrel{\text{tree}}{=} \frac{s - M_\pi^2}{F_\pi^2}. \quad (4)$$

The second equal sign in Eq. (4) is based on the fact that the coupling M^2 can be replaced at tree level with the square of the physical pion mass, see Eq. (3), and that the physical pion decay constant F_π is equal to F in the same approximation. Of course, the result Eq. (4) agrees with the expression found by [?] using current algebra techniques.

In order to compare the above expression for the scattering matrix element with the data, it is useful to consider the partial wave expansion of the amplitude. For an illustration of this procedure, I refer the reader to the contribution by [?].

2.2 Loops

As is well-known, unitarity requires that one considers loops with the above effective lagrangian, see [?] – tree level results do not obey the unitarity constraints for S -matrix elements. I illustrate in the following chapter some features of loop contributions in the baryon sector.

3 Mass shifts – relativistic framework

To start with, we note that the interactions between the nucleon and the pions, mediated through the effective lagrangian, will shift the value of the nucleon mass m . In particular, as the coupling constant M^2 is proportional to the quark mass, the physical nucleon mass will depend on \hat{m} as well. So, I start with a simple question: How does the nucleon mass depend on the quark masses according to the effective lagrangian (1)? At lowest order in the coupling g_A , we have to consider the graph displayed in Fig. 2a, where

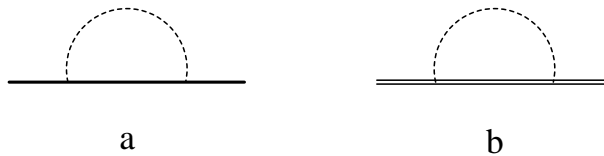


Fig. 2. Selfenergy graphs for a heavy particle. Fig. a: The solid (dashed) line denotes the propagator of the heavy (light) particle. Fig. b: The double line indicates a modified propagator for the heavy field. See text for details.

the dashed line denotes a pion with mass M , and the solid line stands for the nucleon propagator. Note that this graph is of the type considered in Fig. 1.

The nucleon mass The integral over the meson momentum in the graph Fig. 2a is ultraviolet divergent. Regularizing this divergence by performing the calculation in d space–time dimensions, the shift becomes

$$\Delta m = -mg(1+z) \frac{\Gamma(1-d/2)}{(4\pi)^{d/2-2}} + O(1) ,$$

with

$$g = \frac{3g_A^2 m^2}{32\pi^2 F^2} , \quad z = \frac{M^2}{m^2}$$

near the physical space–time dimension $d = 4$. In order to eliminate this divergence, I introduce the counterterms

$$\delta\mathcal{L} = g(c_0 + c_1 z)m\bar{\Psi}\Psi .$$

Note that the structure of $\delta\mathcal{L}$ is different from the original lagrangian (2), which thus corresponds to a non–renormalizable interaction. The result for the nucleon mass will be finite, provided that we tune the couplings c_0, c_1 appropriately as $d \rightarrow 4$. One obtains, see [?],

$$m_N = m [1 + gh(z)] ,$$

$$h(z) = \bar{c}_0 + z(\bar{c}_1 - 1) - z \int_0^1 \frac{x(2-x) dx}{x^2 + z(1-x)} - z \ln z . \quad (5)$$

The quantities $\bar{c}_{0,1}$ denote renormalized, scale independent coupling constants, independent of M^2 . The exact relation to the $c_{1,2}$ introduced above is of no relevance in the following, and I do therefore not display it here.

Comparison with the pion mass In order to discuss the special feature of the result Eq. (5), I also display the corresponding formula for the shift of the pion mass, due to the graph Fig. 3. Including the contribution from the counterterm l_3 in the effective lagrangian at order p^4 , one obtains at $m_u = m_d$

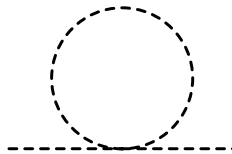


Fig. 3. Tadpole contribution to the pion propagator. This graph generates the leading correction to the pion mass in the chiral expansion.

$$M_\pi^2 = M^2 \left\{ 1 - \frac{M^2}{32\pi^2 F^2} \bar{l}_3 \right\} ,$$

where the renormalized coupling \bar{l}_3 depends logarithmically on the quark mass,

$$M^2 \frac{d\bar{l}_3}{dM^2} = -1 .$$

The following comments are in order.

1. The counterterms needed in the case of the nucleon mass are of $O(1)$ and $O(p^2)$, whereas the tadpole contribution to the pion mass requires a counterterm of $O(p^4)$. The fact that, in the pion case, only this counterterm is required, is a feature of the particular regularization scheme used. Had we introduced a momentum cutoff, one would have to add a counterterm of order p^2 for the pion as well, see [?].
2. The physical pion mass at one-loop order contains terms linear and quadratic in the quark mass (up to the logarithm in \bar{l}_3). On the other hand, the expression for the nucleon mass as evaluated above is a complicated function of \hat{m} . Indeed, expanding the quantity $h(z)$ in Eq. (5) around $z = 0$ gives

$$m_N = m + mg \left\{ \bar{c}_0 + \bar{c}_1 z - \pi z^{3/2} - \frac{1}{2} z^2 \ln z + \sum_{\nu=4}^{\infty} c_\nu z^{\nu/2} \right\} . \quad (6)$$

The origin of the different character of the one-loop expressions for the nucleon (pion) mass is easy to identify: as the nucleon mass does not vanish in the chiral limit, it provides an additional scale m in the calculation – besides \hat{m}/F , one may also form m/F . It is obvious that this generates a problem with the chiral counting: in the meson case, loops contribute at a definite order in dimensional regularization. On the other hand, any power of the quark mass can be generated by chiral loops in the nucleon case. Below, I will illustrate how one can avoid this problem in heavy baryon chiral perturbation theory (HBCHPT).

Non-analytic terms I now discuss the result Eq. (6) in some detail. First, consider the chiral limit. It is convenient for the following to normalize the counterterm at order p^0 such that the nucleon mass stays at m when $\hat{m} \rightarrow 0$, i.e., $\bar{c}_0 = 0$. Next, consider the term linear in the quark mass. It contains the counterterm \bar{c}_1 , which is related to the pion-nucleon sigma-term, defined by

$$\sigma = \hat{m} \frac{\partial m_N}{\partial \hat{m}} .$$

From the above expression for the nucleon mass we obtain

$$\sigma = mg \left\{ \bar{c}_1 z - \frac{3\pi}{2} z^{3/2} - z^2 \ln z + O(z^2) \right\} .$$

Therefore, the nucleon mass as well as the sigma-term contain non-analytic contributions of order $\hat{m}^{3/2}$ and $\hat{m}^2 \ln \hat{m}$, see [?, ?]. One may wonder what happens to these terms once higher loop contributions are considered. Of course, these will start at order p^0 as well and give again rise to an infinite tower of terms. However, it can be shown that the leading non-analytic term in the expansion of the nucleon mass,

$$\delta m_N = -3 \frac{g_A M^3}{32\pi F^2} , \quad (7)$$

is not touched by these contributions – the coefficient of the term proportional to $\hat{m}^{3/2}$ is fixed by chiral symmetry [[?]], in contrast to the coefficient of the logarithmic singularity $\sim \hat{m}^2 \ln \hat{m}$, see [?].

For an evaluation of all the terms at order p^4 in the chiral expansion of the baryon octet, see [?], and the contribution of [?] to these Proceedings.

4 Non-relativistic formulation

Heavy baryon chiral perturbation theory is a quantum field theory in which pure power counting for the baryons is restored, see [?] and [?]: Each loop generates exactly one term in the low-energy expansion of the quantity in question. For example, in case of the nucleon mass, the graph Fig. 2b generates the term (7) and nothing else. In that graph, the double line denotes a properly modified nucleon propagator. I wish to illustrate in this section how this is achieved. In order to simplify the presentation, I consider the case of a scalar theory.

Scalar theory Let

$$\mathcal{L} = \partial_\mu H^\dagger \partial^\mu H - m_H^2 H^\dagger H + \frac{1}{2} (\partial_\mu l \partial^\mu l - m_l^2 l^2) + \kappa H^\dagger H l ,$$

where H (l) denotes a heavy (light) field of mass m_H (m_l). The shift in the heavy mass at lowest order in the expansion in the coupling κ is due to the graph Fig. 2a, where the solid line now denotes the propagator of the heavy scalar field H ,

$$\delta m_H^2 = i\kappa^2 \int \frac{d^d l}{(2\pi)^d} \frac{1}{m_l^2 - l^2} \frac{1}{m_H^2 - (p-l)^2} , \quad p^2 = m_H^2 . \quad (8)$$

Here, I have again regularized the expression by performing the integral in d dimensions. Expanding the result in powers of the light mass gives

$$\delta m_H^2 = \frac{\kappa^2}{16\pi^2} \left\{ a_d + \pi \frac{m_l}{m_H} + O(m_l^2 \ln m_l) \right\} ,$$

where a_d is independent of m_l and contains a pole at $d = 4$, which is removed by standard mass renormalization. The next term illustrates that the shift in the mass contains a non-analytic term of the square root type. This term can be picked out directly from the original integral (8) in the following manner.

First, I consider the rest frame $p^\mu = (m_H, \mathbf{0})$, where

$$\delta m_H^2 = i \frac{\kappa^2}{2m_H} \int \frac{d^d l}{(2\pi)^d} \frac{1}{m_l^2 - l^2} \frac{1}{l^0 - l^2/2m_H} .$$

Now, in the large m_H limit, I neglect the mass in the denominator of the integrand and consider the integral

$$J_m \doteq i \frac{\kappa^2}{2m_H} \int \frac{d^d l}{(2\pi)^d} \frac{1}{m_l - l^2} \frac{1}{l^0} ,$$

which is linearly divergent. By performing the integral in d dimensions, one finds that J_m is finite at $d = 4$,

$$J_m = \frac{\kappa^2}{16\pi} \frac{m_l}{m_H} , \quad (9)$$

which is exactly the non-analytic term in the mass shift found above! [One could as well introduce e.g. a momentum cutoff. The integral J_m then contains, aside from the non-analytic piece (9), a linear divergent part which is independent of the light mass, and terms that vanish as the cutoff is removed.] Note that neglecting $l^2/2m_H$ in the denominator does not represent a legal mathematical procedure: the result of the operation does not correspond to the large m_H expansion of the original integral – on the other hand, it does correctly reproduce the leading mass correction, as we have just seen.¹

HBCHPT is the science how to achieve these manipulations systematically and legally in a lagrangian framework. Again, I illustrate it with the scalar theory.

Non-relativistic formulation First, I replace the heavy field H with a non-relativistic complex scalar field Φ ,

$$\mathcal{L} \rightarrow \mathcal{L}_{NR} = \Phi^\dagger \left(i\partial_t - \sqrt{m_H^2 - \Delta} \right) \Phi + \frac{1}{2} (\partial_\mu l \partial^\mu l - m_l^2 l^2) + \frac{\kappa}{2m_H} \Phi^\dagger \Phi l .$$

The coupling constant has been adjusted in order to generate the correct low-energy behavior of the tree amplitudes. Next, I expand the non-local differential operator,

$$\sqrt{m_H^2 - \Delta} = m_H - \frac{\Delta}{2m_H} + \dots ,$$

¹ Tang (1996) has proposed similar manipulations in ordinary relativistic baryon chiral perturbation theory, recovering the results of HBCHPT at one-loop order. See also Ellis and Tang (1997).

and put the derivative terms in the interaction,

$$\mathcal{L}_{NR} = \Phi^\dagger (i\partial_t - m_H)\Phi + \frac{1}{2} (\partial_\mu l \partial^\mu l - m_l^2 l^2) + \frac{\kappa}{2m_H} \Phi^\dagger \Phi l + \Phi^\dagger \frac{\Delta}{2m_H} \Phi + \dots$$

The propagator of the non-relativistic field is $(p^0 - m_H)^{-1}$ in Fourier space. Dropping the terms with derivatives, the graph Fig. 2b gives

$$\delta m_H^2 = \frac{\kappa^2}{16\pi} \frac{m_l}{m_H},$$

and nothing else, which is exactly the needed result.

HBCHPT allows one to perform the low-energy expansion in the baryon sector (one external nucleon) in a systematic manner, by proceeding similarly to the scalar field just discussed. The chiral expansion of the quantities evaluated earlier in the relativistic framework can then be obtained much easier – I refer the interested reader to the review by [?]. In fact, an impressive amount of calculations has been done in recent years e.g. by Bernard, Kaiser and Meißner and others in this framework, see [?], where also an outline of HBCHPT is presented.

Comment There is one point that I wish to mention concerning this way of performing the chiral expansion. As I have just illustrated, HBCHPT is a clever method to organize the calculations and to keep track of power counting. On the other hand, the physics does, of course, not change. To illustrate, consider e.g. the elastic pion–nucleon scattering amplitude. It has been evaluated to one loop in the relativistic formulation some time ago by [?]. Expanding that result in powers of momenta and of quark masses, one would obtain

$$A^{1loop} = A_1 + A_2 + A_3 + \dots \quad (\text{relativistic framework}),$$

where A_n is of order p^n . I see no reason to doubt that the one-loop calculation in HBCHPT, performed recently by [?] and by [?], is identical to

$$A^{1loop} = A_1 + A_2 + A_3 \quad (\text{HBCHPT}).$$

In this sense, the physics of HBCHPT is the same as the one of the original relativistic formulation. On the other hand, HBCHPT has the advantage that one is certain to have collected all the terms at a given order in the chiral expansion even for nonzero quark mass – something that would be very difficult to prove in the relativistic framework.

5 Rate of convergence

Convergence of the chiral series is sometimes very slow in the nucleon sector. To illustrate this, I consider the scalar form factor of the nucleon,

$$\langle N(p') | \hat{m}(\bar{u}u + \bar{d}d) | N(p) \rangle = \bar{u}(p')u(p)\sigma(t) \quad ; \quad t = (p' - p)^2 .$$

At zero momentum transfer, the scalar form factor coincides with the sigma-term considered above, $\sigma(0) = \sigma$. The difference

$$\Delta_\sigma = \sigma(2M_\pi^2) - \sigma(0)$$

plays a central role in the extraction of the sigma-term from the elastic pion-nucleon scattering amplitude. The chiral expansion for this difference gives at leading order

$$\Delta_\sigma = \frac{3g_A^2 M_\pi^3}{64\pi F_\pi^2} \simeq 7.5 \text{ MeV} \quad (\text{leading order}) ,$$

see [?]. On the other hand, a dispersive analysis – that includes all orders in the quark mass expansion – leads to

$$\Delta_\sigma = 15 \text{ MeV} \quad (\text{dispersive analysis}) , \quad (10)$$

see [?]. This example shows quite drastically that higher orders in the quark mass expansion may be large – even as large as the leading term, as the present example shows. Indeed, by including the Delta resonance as an explicit degree of freedom in the effective lagrangian, [?] also find $\Delta_\sigma \simeq 15 \text{ MeV}$. The difference between this value and the leading order result $\Delta_\sigma = 7.5 \text{ MeV}$ is due to terms at order p^4 and higher. Of course, if one would not know the result (10) of the dispersive analysis, one could only conclude from their calculation that there are potentially large corrections to the leading order result – and nothing more. Whether the remaining terms at order p^4 or even higher order contributions are large cannot be decided from this one-loop calculation. To pin them down in a purely chiral expansion framework is very difficult – in this case, the dispersive analysis is more efficient.

As this example illustrates, the Delta degree of freedom may generate large perturbations. Hemmert, Holstein and Kambor (1997) have therefore developed a framework where the Delta resonance is taken into account in a systematic manner. One counts the pion mass, as well as the difference between the Delta and the nucleon mass, as quantities of order ϵ . For example, the ratio

$$\frac{M_\pi^3}{M_\pi + m_{Delta} - m_N}$$

is then considered as order ϵ^2 , whereas it is order p^3 in conventional power counting. For details concerning this framework, I refer the reader to the contribution of [?].

There are several reasons for the slow convergence of the chiral expansion in the nucleon sector. First, as we have just seen, the proximity of the Delta resonance may cause large corrections. Although there is a mass gap between the Delta and the nucleon also in the chiral limit, the Delta does stay nearby in the real world and cause large effects through small energy denominators. Second, the ratio $M_\pi/M_N \simeq 1/7$, in which the amplitudes are expanded, is not that small. Third and most importantly in my opinion, the chiral expansion of e.g. the full elastic pion–nucleon scattering amplitude is of the form

$$A = A_1 + A_2 + A_3 + A_4 + \dots ,$$

i.e., there is a chain of even and odd powers in the momenta. In each chain, one needs at least the leading and the next-to-leading order term to have a reliable prediction. I see no reason to trust any calculation that does not include all these terms – only in this case can one check to some extent whether one has obtained a satisfactory approximation. This means that we need the terms of order p^4 in the pion–nucleon amplitude. For a discussion of the results at order p^3 , see Mojžiš (1997a,b) and [?].

In fact, the calculation of the terms at order p^4 in the baryon mass, recently carried out by [?], allows for such a check. I consider the mass of the Ξ and write again

$$m_\Xi = m_0 + m_2 + m_3 + m_4 + \dots .$$

According to these authors, the first chain reads

$$(m_0, m_3) = (770, -893) \text{ MeV} ,$$

whereas

$$(m_2, m_4) = (847, 600) \text{ MeV} .$$

Since $m_3(m_4)$ should be a correction to $m_0(m_2)$, I consider this a disaster for the chiral expansion. For a different opinion, see [?], and the contribution of [?] to this workshop.

Note that, in the meson sector, the situation is very much different: The effective action contains only even powers of the momenta – a one-loop calculation therefore often suffices in the case where the leading order term starts at tree level.

6 Mass effects in the low–energy constants

There is one more feature of chiral expansions that one can nicely illustrate with the nucleon mass and the sigma–term, that I write as

$$\begin{aligned} m_N &= m + \sigma + \frac{\pi}{2} m g z^{3/2} + O(z^2 \ln z) , \\ \sigma &= m g \bar{c}_1 z - \frac{3\pi}{2} m g z^{3/2} + O(z^2 \ln z) . \end{aligned}$$

These expressions contain the two low-energy constants m and \bar{c}_1 , which are not determined by chiral symmetry. As is usual, one may rely on experimental information to pin them down. I illustrate the procedure in the following, using

$$(m_N, \sigma, F, M) = (940, 45, 93, 135) \text{ MeV} ; g_A = 1.25.$$

At leading order in the chiral expansion, one has

$$m_N = m , \sigma = 0 \Rightarrow m = 940 \text{ MeV} .$$

At next order,

$$m_N = m + \sigma , \sigma = mg\bar{c}_1 z \Rightarrow m = 895 \text{ MeV} , \bar{c}_1 = 1.6 .$$

Finally, from the expressions at order p^3 , I find

$$m = 888 \text{ MeV} , \bar{c}_1 = 2.3 .$$

The fact that the values of the low-energy constants depend on the order we are considering seems to be in contradiction with calling them “constants”. Of course, these quantities indeed are quark mass independent. However, once we determine them from data, one is using a specific order in the chiral expansion, whereas the data do include the quark mass effects to all orders. Some of these are therefore effectively absorbed in the low-energy constants, as a result of which one is faced with a systematic uncertainty in the determination of their values, even with infinitely precise data, as the above chain

$$m = 940 \text{ MeV} \rightarrow 895 \text{ MeV} \rightarrow 888 \text{ MeV} , \\ \bar{c}_1 = 1.6 \rightarrow 2.3 ,$$

nicely illustrates.

There is, on the other hand, at least in principle a possibility to generate data without quark mass effects: lattice calculations. Indeed, once it will be possible to e.g. determine the value of the nucleon mass in the chiral limit from lattice simulations, we may simply take that value for m . The other parameter at hand, \bar{c}_1 , can be obtained by evaluating the derivative of the nucleon mass with respect to the quark mass in the chiral limit. Needless to say that these are very difficult quantities to measure on the lattice.

Acknowledgements

I thank Alex Gall for enjoyable discussions concerning the material in section four, and Gerhard Ecker, Joachim Kambor, Ulf Meißner and Martin Mojžiš for discussions on atrocities in baryon chiral perturbation theory. Furthermore, I thank Aron Bernstein, Dieter Drechsel and Thomas Walcher for the efficient organization of this Workshop.

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